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DESIGN OPTIMIZATION OF THE JPL PHASE B TESTBED

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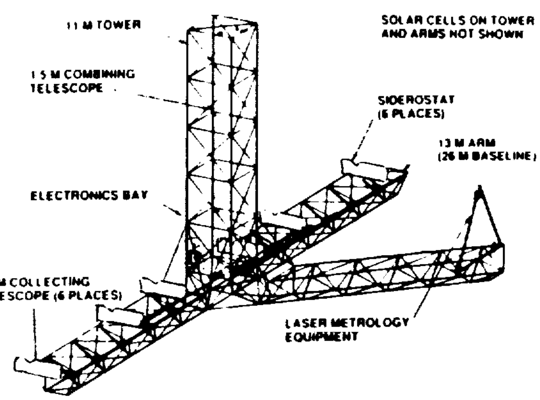
INTRODUCTION

Increasingly complex spacecraft will benefit from integrated design and optimization of structural, optical, and control subsystems. Integrated design optimization will allow designers to make tradeoffs in objectives and constraints across these subsystems. The location, number, and types of passive and active devices distributed along the structure can have a dramatic impact on overall system performance. In addition, the manner in which structural mass is distributed can also serve as an effective mechanism for attenuating disturbance transmission between source and sensitive system components. This paper presents recent experience using optimization tools that have been developed for addressing some of these issues on a challenging testbed design problem. This particular testbed is one of a series of testbeds at the Jet Propulsion Laboratory under the sponsorship of the NASA Control Structure Interaction (CSI) Program to demonstrate nanometer level optical pathlength control on a flexible truss structure that emulates a spaceborne interferometer.

- o GOALS
  - Minimize wavefront/LOS error
  - Minimize total system mass
  - Minimize power consumption
  - Others
- o DESIGN VARIABLES
  - Structure parameters
  - Control gains
  - Optical design variables
  - Placement of active/passive devices

Proposed Space Interferometer



METHODOLOGY VALIDATION TESTBED

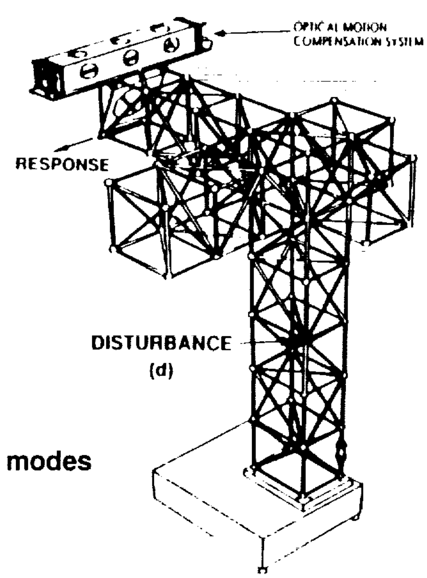
To demonstrate nanometer level optical pathlength control for the proposed interferometer, the Phase B CSI testbed structure was designed and built to emulate the dynamic characteristics of the interferometer. It consists of a vertical tower of length 2.5m with two horizontal arms at right angles to each other attached at the top of the tower. The tower and arms are trusses built from a total of 14 rectangular bays each with dimension 40.6 x 40.6 x 28.7 cm. Attached to one of the horizontal arms is the optical motion compensation system which is encased in a flexure mounted frame. The system was modeled using NASTRAN with beam elements for the individual struts. The resulting 666 degrees--of--freedom (dof) system was reduced by Guyan reduction to 252 dof.

The objective of the current study is to modify the structure using two methods: by resizing truss element diameters and by strategically placing and selecting parameters for passive dampers in the structure to attenuate motion at the optical compensation subsystem attachment points to the trolley due to a colored noise input disturbance at the location shown in the figure. An optimization approach is adopted to achieve this objective. The resizing optimization problem employs a multiobjective cost functional consisting of total structural mass and the H_2 norm of the transfer function between the input disturbance and the trolley attachment points. The multiobjective formulation generates a family of optimal designs that allows trades to be made between the competing components of the cost functional. In the damper placement and tuning optimization problems several performance metrics were considered. These include the H_2 metric described above and various metrics for damping specific structural modes.

This paper addresses two main questions regarding the optimization approach. First, can these methods be applied to the large scale applications that they are ultimately targeted for? And secondly, is there substantial payoff to implementing them? A combination of experimental and numerical simulation studies indicate affirmative answers to each of these questions.

- **Objective: Demonstrate nanometer level optical pathlength control on a flexible structure that emulates the proposed interferometer.**

  - **660 D.O.F. (252 mass D.O.F.)**
  - **22 modes < 100. Hz**
    7 modes < 30. Hz
  - **186 possible locations for dampers**
  - **Disturbance: shaker force at 412 along diagonal (xy)**
  - **Outputs: disp. at 8-attach pts. (trolley/truss)**
  - **Perf. measure: H_2 norm of TF_{io}/damping of selected modes**



- **Approach:**

  - **Opt. placement of passive dampers, followed by tuning and resizing of truss members**

PLACEMENT/TUNING PHILOSOPHY

Passive damping is a highly effective means for improving performance of both passively and actively controlled systems, and is especially critical for the problematic class of lightly damped systems. Reducing peak responses in the vicinity of resonant frequencies not only enhances the stability of the open loop system, but it also allows for the implementation of more aggressive control strategies. Furthermore, since many current approaches to robust control design require a bound on the uncertainty in the model on which the design is based, when models are obtained from an identification process the resulting uncertainty (as measured by the H_∞ norm) can be reduced by first damping the resonant peaks.

The effectiveness of viscous elements in introducing damping is a function of several variables including their number, their location in the structure and their physical parameters. The optimization studies to follow will treat the location and tuning problems in an independent manner. Future work will address hybrid approaches for developing strategies that combine these problems into a single framework.

- Introduce passive damping into structure to
  - Passively and optimally attenuate disturbances
  - Enhance plant identification process/robustness
  - Improve closed loop system performance

- Determine
  - Element locations
  - Element stiffness/damping
  - Number/combination of elements

APPROACH
OPTIMAL PLACEMENT/TUNING

The viscous dampers are modeled as collocated position and velocity feedback elements in the nominal structure. In the case of a single damper, the feedback gains are scalar quantities corresponding to the damper spring and dashpot coefficients. In the multiple damper case the same system model results, with the exception that the feedback gains are now diagonal matrices with nonzero terms corresponding to the spring and dashpot terms of the individual dampers.

The placement problem is to optimize the control influence matrix B over the set of admissible locations for the dampers. The tuning problem is to optimize the matrices K_p and K_v over the parameter range of feasible designs for the dampers.

- System Model: $M\ddot{x} + Kx = Bu + f$

  Single Element:

  $$u = -b^T[k_p x + k_v \dot{x}]$$

  $$M\ddot{x} + k_v bb^T\dot{x} + (K + k_p bb^T)x = f$$

  Multiple Elements:

  $$x = \Phi q; \quad \Phi^T M \Phi = I; \quad \Phi^T K \Phi = D = \mathrm{diag}(\omega_1, \ldots, \omega_n)$$

  $$\ddot{q} + BK_v B^T \dot{q} + (D + BK_p B^T)q = \Phi^T f,$$

M = Mass; K = Stiffness; B = Damping selection matrix;

u = Damper force; f = Disturbance force

k_p = Damper spring coef.; k_v = Dashpot coef.; K_v, K_p = Diagonal damping/stiffness matrices.

TECHNICAL APPROACH
OPTIMIZATION PROBLEM FORMULATION

The choice of performance metric in these optimization problems plays a central role in the character of the solution. Criteria should simultaneously reflect the physical objectives and be numerically tractable. For large scale problems this latter property is especially important. Thus cost functional evaluation must involve both numerically stable and efficient computations. In addition, the availability of analytical gradients is another very desirable feature. Two general sets of criteria meeting these requirements were implemented---an H_2 metric and a damping metric. The H_2 functional evaluation requires solving large scale Lyapunov equations, while computing the damping requires solving large nonsymmetric eigenvalue problems. Methods for accelerating these computations were developed, and will be discussed later.

Two distinct optimization strategies were used to solve the placement and tuning problems. The placement problem involves discrete optimization over a finite (but large) set, while the tuning problem can be treated in a continuous framework. A simulated annealing program was developed for optimizing the control influence matrix B for the placement problem, and a sequential quadratic programming (SQP) using the Stanford Computer Science Laboratory's NPSOL software was used to tune the damper gain matrices K_p and K_v.

- **Determine placements and damper parameters by optimizing physically meaningful performance criteria**

  Performance Functionals:
  - **Damping of selected modes**
  - **Minimizing total system energy**
  - **Optimizing H_2 norm of transfer function from disturbance inputs to outputs**

  Design Variables:
  - **Damper stiffness (K_p), damping coefficient (K_v), location (B)**

  Optimization Strategies:
  - **Sequential Quadratic Programming (SQP) for K_p, K_v**
  - **Simulated annealing for B**

- **Challenges**

  - **Solving problems on big models**

OPTIMAL PLACEMENT WITH RESPECT TO DAMPING OF SELECTED MODES

The ability to damp selected modes is very useful in control design applications where to ensure robustness and closed loop stability it is necessary to damp modes in the loop gain crossover region. Once these modes are targeted, a number of functionals can be introduced for this purpose. Two typical functionals are shown below. The first functional is simply the sum of the damping of the targeted modes, while the second is a weighted sum of the exponentials of the individual damping values. Although these functionals involve the same set of modes, they can lead to quite different results. For example if a particular mode can be heavily damped, the first of these metrics would have the propensity to find a solution that introduces large damping into this mode but ignores the others. The second metric is an approximation to a minimax functional and is better suited to more evenly distributing the damping in the individual modes. A possible problem that can arise here is that there may be a mode that is not easily damped, and additional damping in other modes may be sacrificed to introduce a modest amount of damping in the problem mode.

Let ξ_i = damping in i^{th} mode of system

$$\ddot{q} + BK_vB^T\dot{q} + (D + BK_pB^T)q = \Phi^T f$$

Optimization Problem

$$\min_{K_p,K_v,B} g(\xi_1,\ldots,\xi_m)$$

where g is a smooth function

Examples:

$$\text{(i)} \quad g(\xi_1,\ldots,\xi_m) = \sum_{i\in I} \xi_i,$$

$$\text{(ii)} \quad g(\xi_1,\ldots,\xi_m) = \frac{1}{\gamma} \sum_{i\in I} \mu_i \exp\{\gamma\xi_i\}$$

where I = set of targeted modes

***Solving large eigenvalue problem is difficult in optimization loop**

OPTIMAL PLACEMENT WITH RESPECT TO H_2 METRIC

The H_2 norm of a transfer function is a general performance metric that can capture a variety of physical objectives. Penalties on the velocities and displacements of various nodal points can be accommodated by this formulation, as well as the total system energy. More generally any quadratic functional of the system state can be expressed in these terms. The stochastic physical interpretation of the H_2 norm is as the steady state rms error of the output due to a white noise input.

Computing the H_2 norm of a transfer function requires putting the system into first order form, and then solving a Lyapunov equation involving the state matrix, the control input matrix and the intensity matrix of the input disturbance. The solution of this equation is the covariance of the state. Obtaining the steady state value of a linear combination of state variables then requires only matrix multiply and trace operations.

- **System:**

$$\dot{x} = Ax + Bv$$
$$z = Cx,$$

  where

$$A = \begin{bmatrix} O & I \\ -D - BK_pB^T & -BK_vB^T \end{bmatrix}, \quad B = \begin{bmatrix} O \\ \bar{B} \end{bmatrix}$$

  v = zero mean white noise, $E(vv^T) = V$

- **H_2 Cost:**

$$\min_{K_p, K_v, B} J = \lim_{t \to \infty} E\{|z(t)|^2\};$$

$$J = tr(CC^TQ),$$

$$AQ + QA^T + BVB^T = 0.$$

- **Examples:**

  (i) C = selects displacement/velocity components at certain nodes
  (ii) C = determines system energy

$$C = \begin{bmatrix} D^{1/2} & 0 \\ 0 & I \end{bmatrix}$$

*Direct evaluation of J not tractable in optimization loop

COMPARISON OF METHODS FOR COST FUNCTIONAL EVACUATION

Solving large unsymmetric eigenvalue problems or Lyapunov equations within an optimization loop is extremely cumbersome, and can render the optimization routine ineffective. Two methods were developed to circumvent these difficulties.

A Newton method for updating specific eigenvalues as a function of the damper stiffness and dashpot coefficient matrices, K_p and K_v was developed. The method is based on finding the roots of an associated rational function whose zeros coincide with the system eigenvalues. Within the optimization loop the scheme is initialized by the current eigenvalues corresponding to the values of K_p and K_v. As K_p and K_v are updated in the optimization, the eigenvalues are also updated. The method is highly accurate and very efficient. To facilitate the solution of the large scale Lyapunov equation associated with computing the transfer function H_2 norm, a model reduction method based on augmenting a modally reduced model with Ritz vectors to statically correct the reduced model transfer function was implemented. This method also produces high fidelity approximations to the damped system eigenvalues.

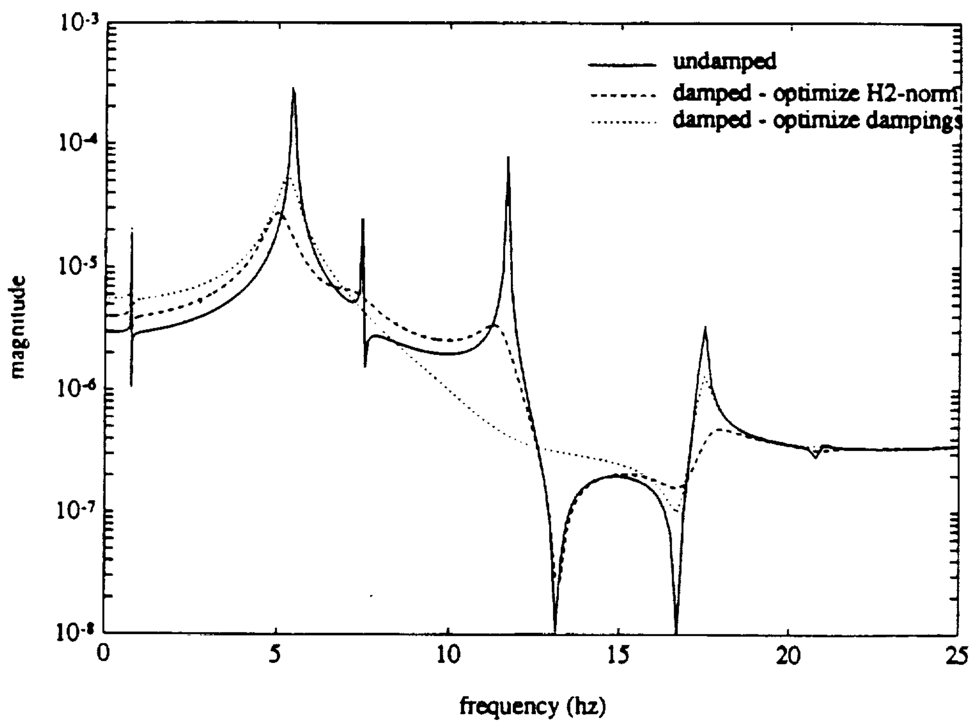
The tables below demonstrate the effectiveness of these two methods in approximating the eigenvalues of the structure with three dampers placed. The full testbed model in this study contains 249 modes and three dampers were inserted. The first column in the table contains the frequencies of the undamped system. The other values correspond respectively to the full order model, the Ritz reduction model containing the first twelve undamped modes plus three Ritz vectors corresponding to the damper inputs, the Newton method, and finally a modally reduced model obtained by retaining the first 15 undamped modes. The conclusion here is that the Ritz reduction technique and the Newton method yield high precision estimates with enormous reduction in operation count, while the modally reduced model produces inaccurate results.

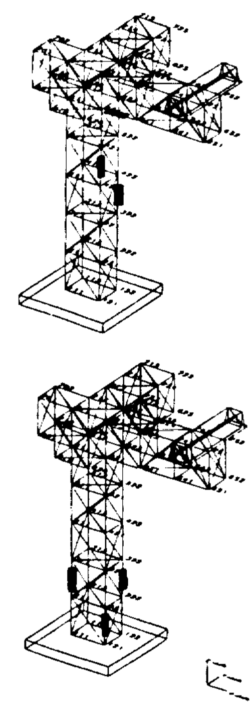
Mode No.	Undamped System Frequency (Hz)	Damped System Frequency (Hz)			
		249 Modes (full order)	12 Modes Plus 3 Ritz Vectors	Newton Method	15 Modes (truncation)
1	0.7427	0.7420	0.7420	0.7420	0.7425
2	5.4263	5.2940	5.2940	5.2940	5.3262
3	7.4564	7.0376	7.0376	7.0376	6.9540
4	11.6777	10.4862	10.4862	10.4862	10.4493
5	17.4248	17.4386	17.4386	17.4386	17.3444
6	20.8423	20.8236	20.8236	20.8236	20.7055
7	31.1387	31.2231	31.2231	31.2231	31.0481

Mode No.	Damped System Damping (%)			
	249 Modes (full order)	12 Modes Plus 3 Ritz Vectors	Newton Method	15 Modes (truncation)
1	0.0179	0.0179	0.0179	0.0012
2	4.5744	4.5744	4.5744	0.6125
3	25.5358	25.5357	25.5358	2.3228
4	32.6380	32.6379	32.6380	5.5664
5	0.9033	0.9034	0.9033	0.4066
6	1.3197	1.3197	1.3197	0.5709
7	0.5013	0.5016	0.5013	0.5031

SIMULATED ANNEALING SOLUTION COMPARISON OF H_2 AND DAMPING OPTIMIZATION

A simulated annealing algorithm was used to optimally place three dampers in the Phase B Testbed structure. The algorithm was implemented twice. First, it was implemented with an H_2 metric with disturbance input generated by a sixth order low--pass filter with a bandwidth of 25Hz. The second time it was implemented with a metric to maximally damp the sum of the first seven modes. A comparison of the respective Bode plots of the resulting transfer functions is given in the figure below. As observed, large damping is introduced into the third and fourth modes as a result of optimizing the damping. However, this is at a sacrifice to the damping attained in the other modes. The H_2 norm optimization metric distributes the damping across the modes in a more even fashion. Values for the sum of the damping and H_2 norms for both performance metrics are given at the bottom of the page. This example indicates the disparity in performance that can result from choosing various cost functionals.





H_2 Optimization: H_2 error = 1.514 x 10^{-4}; $\sum_i \xi_i$ = .261

Damping Optimization: H_2 error = 2.143x10^{-4}; $\sum_i \xi_i$ = .655

TUNING 3 DAMPERS

The optimal damping solution shown on the previous chart added very little damping to the fifth, sixth, and seventh modes. To remedy this situation the simulated annealing algorithm was implemented again, but with the performance metric $g = \exp(10\xi_5) + \exp(10\xi_6) + \exp(10\xi_7)$, where ξ_i= damping in the i^{th} mode. A new damper configuration emerges, and the resulting damping in these modes improves considerably. Furthermore, because g represents a "minimax" approximation, the damping in these modes are all within a factor of two of one another.

Once these damper locations were selected, we next optimized the damping coefficient matrices K_p and K_v using the SQP algorithm. The figure below contains the Bode plots for the resulting two closed loop systems, and the table at the bottom of the page contains the damping values for modes 5, 6, and 7 for the two solutions. As can be seen from these values, significant improvement in damping is obtained by optimizing the coefficients. In this case the solution called for *reducing* the spring constant of the damper to the minimum allowed and simultaneously *reducing* the dashpot coefficient by a factor of 2/3.

The damping obtained in these modes by a combination of placement and tuning optimization techniques demonstrates that very specific tailoring of the system response is possible by these methods.

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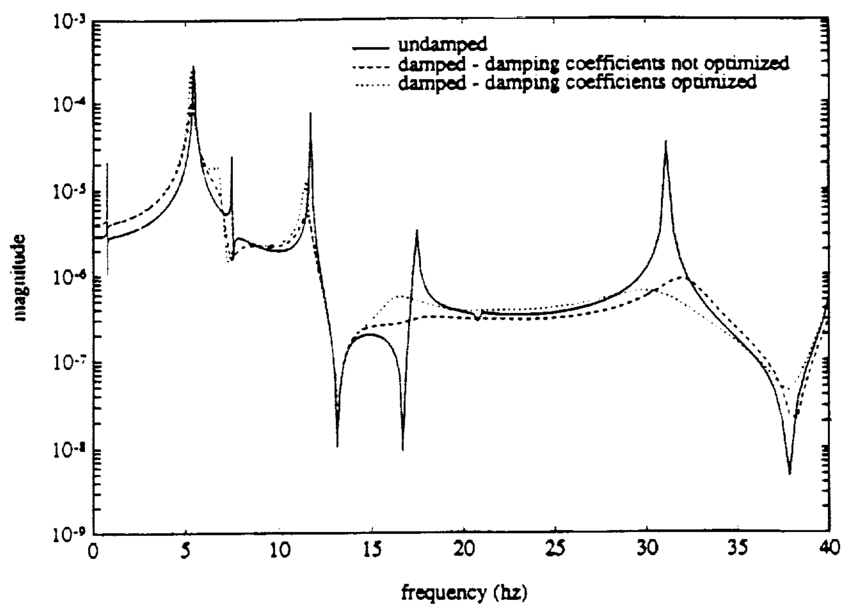
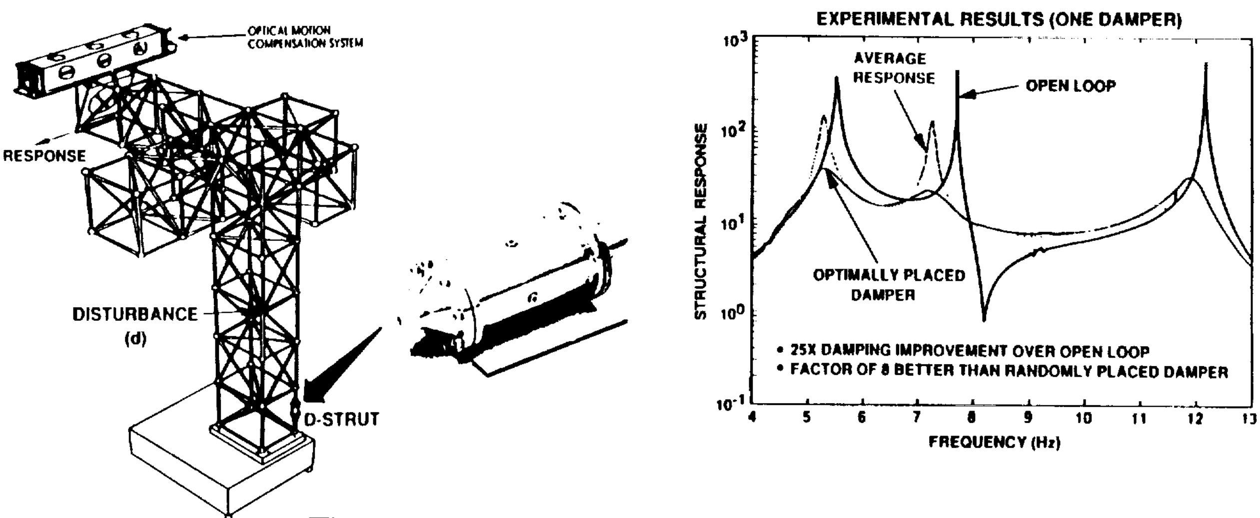
o Optimizing K_p, K_v with respect to $\sum_{i=5}^{7} \exp\{10\xi_i\}$



	Undamped	Damped (Kv - 320 lb/in)		Damped (Kv - 105 lb/in)	
Mode	freq	freq	damping	freq	damping
5	17.4248	17.2668	8.1733	16.1907	6.5787
6	20.8423	21.3353	3.9098	20.1644	8.0746
7	31.1387	32.1426	3.7256	30.5854	7.4840

OPTIMAL PLACEMENT AND TUNING OF PASSIVE ELEMENTS

The foregoing discussions centered around simulated data. The figure below presents experimental data taken from the Phase B Testbed. The optimal placement of the damper here was determined via the H_2 performance metric with disturbance input and response locations as shown. An exhaustive search over the 186 locations was performed using the Ritz reduction technique to compute the solution to the associated Lyapunov equation. The figure contains the open loop response, the response obtained by the optimally placed damper, and finally an "average" response computed by averaging the transfer functions over all possible locations of the damper. The experimental results indicate a 25 times improvement in damping in the first three modes over open loop, and 8 times improvement in damping over the "averaged" locations.



- **OBJECTIVE: OPTIMALLY PLACE/TUNE DAMPERS FOR STRUCTURE QUIETING**
- **DEMONSTRATED ANALYTICAL TECHNIQUES ON PRELIMINARY DESIGN PROBLEM**
- **WILL EXTEND TO MULTIPLE PASSIVE AND ACTIVE ELEMENTS**

OPTIMAL RESIZING APPROACH

Given the optimum placement and characteristics of a number of passive dampers, the objective here is to find the optimal sizes (α_i^*) of the truss members. In this regard, one may choose any one or combination of the listed criteria $J_1, \ldots, J_6$ as the objective function for the resizing problem. The optimization can be achieved by varying design parameters such as member sizes, control gains, and mirror geometry and relative position. The challenge one faces in this type of multi-criteria problem is the potential conflict among criteria, so that there is no single design that optimizes all criteria simultaneously. Here, one must look for Pareto optimal solutions. A solution is said to be "Pareto optimal" if there is no other solution that improves any criteria without making at least one other criterion worse.

Statement: For given damper placement, optimize the structure.

Possible Criteria:

- J_1: LOS error of chief ray at some D.O.F.
- J_2: RMS wavefront error of multiple rays at focal plane
- J_3: Strains/stresses in selected members
- J_4: Displacements at selected D.O.F.
- J_5: Total mass
- J_6: Control cost

Design Variables:

Structural - Member sizes
Control - Gains
Optical - Mirror sizes, rel. position

Criteria:

Non-Commensurate and conflicting
Must look for Pareto optimal solutions

OPTIMAL RESIZING APPROACH (CONT'D)

In the results that follow, two criteria were selected for optimization; the total mass, J_1, and the steady state mean square displacement, J_2, (at the truss-to-trolley attachment points) in response to a white noise disturbance. Designating the matrix that maps the state variables x into the attachment points by H, we have $J2 = tr(H\,Q\,H^T)$, where Q is the solution of the matrix Lyapunov equation shown, and $A = A(\alpha)$, $B_2 = B_2(\alpha)$, respectively, are the plant matrix and the disturbance coefficient matrix. Pareto optimal solutions can be found by determining the stationary values of the functional $J(J_1, J_2, \lambda)$ for various values of the weighting parameter $\lambda \in [0,1]$. By allowing λ to take on discrete values at small intervals from zero to one, one can propagate the solution along the λ path to provide a subset of Pareto optimal designs.

- **Problem Statement:** For a given damper placement, optimize cross sectional areas of truss members to attenuate disturbance transmission from source to output D.O.F.

- **For testbed, let** J_1 = total mass

  J_2 = steady state mean square displacements at trolley attach.

  $= tr(H_1 Q\, H_1^T)$

  H_1: Maps x into observed displacements at trolley attachment D.O.F.

  Q: Solution of $(AQ + QA^T + B_2 B_2^T = 0)$

- **Construct weighted criterion:** $J = (1-\lambda)J_1 + \lambda\, J_2;\ \lambda \in [0,1]$

  λ: TBD from system study of mission (not known in advance)
  Starting with $\lambda = 0$; → Find $J^*(\lambda)$ → $\lambda \leftarrow \lambda + \Delta\lambda$ (repeat), till $\lambda = 1$

- **Will provide a set of feasible Pareto optimal designs to optimally trade mass vs. performance**

CSI PHASE B TRUSS - MODEL REDUCTION

Optimization of large order models makes model reduction a required step due to the need to solve one or more Lyapunov or Riccati equations. Model reduction consists of applying a model reduction transformation, T_r, to a full-order model. The transformation T_r is typically not recomputed for each parameterization of the model and hence the transformation T_r computed for one set of parameter values may be a poor transformation to use for another parameterization. The addition of a linear correction term to T_r, constructed from eigenvector gradients (EGs), may extend the range of parameter variations over which T_r is valid.

Our optimization program uses a "Parameter Variance Tolerance" (PVTol) to allow the user to specify how much the parameters may vary before T_r is recomputed. The program also allows the user to specify whether EGs are used in the construction of T_r. The computation of T_r is a CPU time intensive task. Therefore, there exists a tradeoff between added computation time and accuracy of the models used in optimization. Questions which naturally arise are *"What impact do these options have on run time?"* and *"What impact do these options have on the results of the optimization?"*

The left figure shows a plot of CPU time versus PVTol for a certain problem. The "x"-marks show the timings for runs without EGs and the "+"-marks show timings for runs with EGs. As one can see, the change in PVTol 0.10 to 0.50 causes roughly a factor of two in CPU time. The use of EGs increased the CPU time by roughly a factor of four over not using EGs.

The right figure illustrates how the optimal cost from several runs changed with varying PVTol and whether or not EGs were used. For each PVTol (0.10, 0.25, and 0.50) three optimization points were obtained both with and without use of EGs. All costs are shown relative to the best (i.e. smallest) optimal cost for each of the optimization runs. As one can see, the runs which used the EGs always produced the smallest optimal cost and the results were consistent for all values of PVTol. The results for no EGs show that the resulting optimal costs were not really optimal and varied somewhat with PVTol.

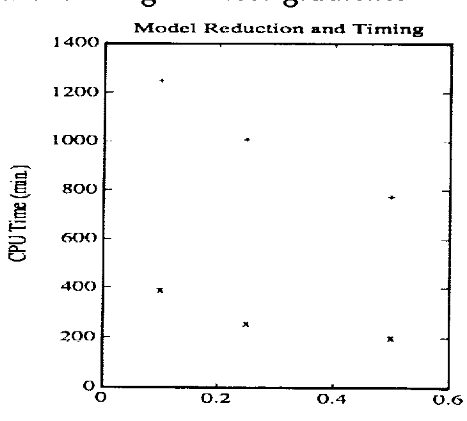
The conclusion drawn from this analysis was that using EGs is preferred. The timing was brought down to a manageable size by setting the PVTol to a value of 0.50. The computation time is about twice what would be obtained by using no EGs with a PVTol of 0.10.

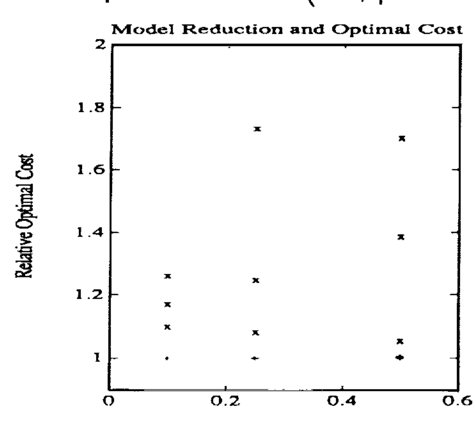
What effect does

- parameter variation tolerance
- use/non-use of eigenvector gradients

have on ...

- CPU Time
- Optimal control (i.e., performance) cost





OPTIMAL RESIZING APPROACH

An important step in the optimization process is the parameterization of the structure. The parameters chosen for optimization of the truss were the rod member cross-sectional areas. Independent parameterization of each member would have led to 186 parameters, probably making the optimization run time quite long: we needed to reduce the number of parameters. Choosing too few parameters, and thus a low order design space, would have lead to poorer results. A natural way to compromise is to take advantage of symmetry in the structure. We did this in the following way:

1. Each bay in the tower was budgeted three parameters: one for the longerons, one for the battens, and one for the diagonal elements. However, the bottom bay required only two parameters because its longerons were attached to the base. Thus the tower gave us 14 parameters.
2. The first, second and third pairs of bays of the two three-bay arms were budgeted three parameters each as above. This gave us another 9 parameters.
3. The two single-bay arms were parameterized with one parameter.
4. The bay at the intersection of the tower and the arms was parameterized with one parameter, giving a total of 25 parameters.

- Issues
  - Number of parameters
  - Distribution of parameters - symmetry
  - Solving large scale eigenvalue and Lyapunov equations
- Solution
  - Three parameters per bay for tower and three-bay arms
  - One parameter per bay for one-bay arms and intersection
  - Total of 25 design variables
  - Ritz model reduction using eigenvectors/eigenvector derivatives

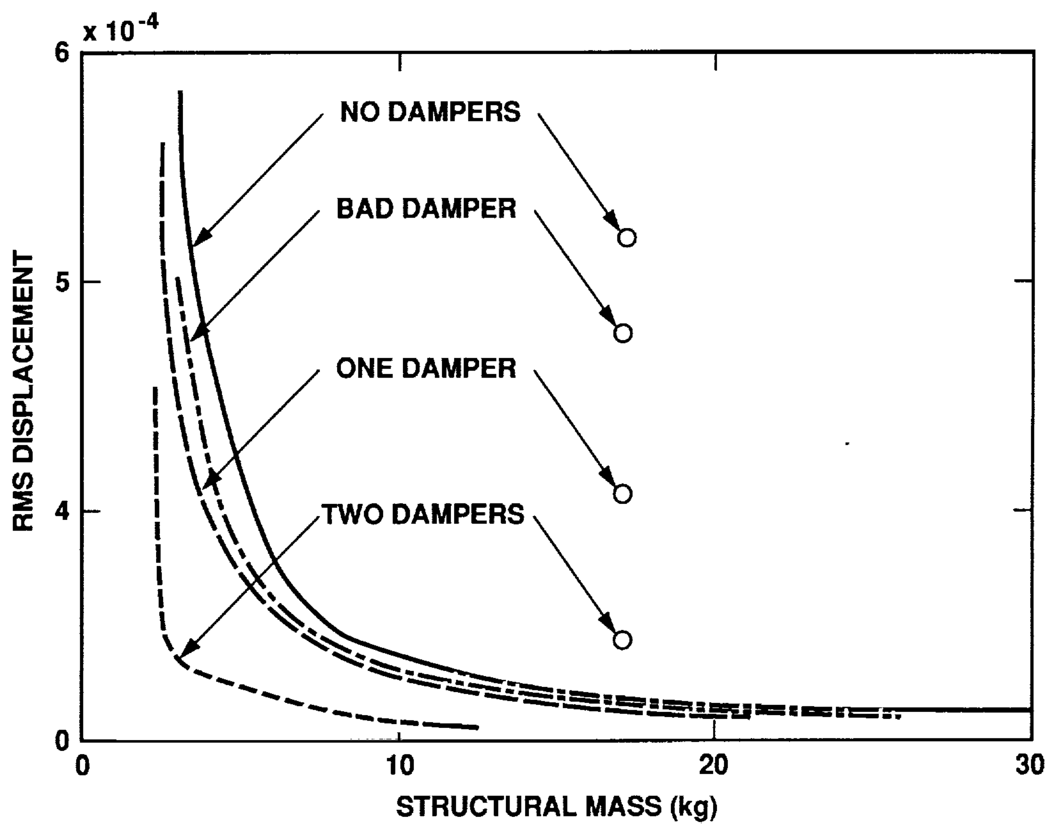
CSI PHASE B TRUSS OPTIMIZATION RESULTS

In this figure the results for four cases are depicted in the criteria space (J_1, J_2). The four cases represent (1) no dampers are present, (2) one arbitrarily placed damper is used, (3) one damper is optimally placed, and (4) two optimally placed dampers. For each case, the performance of the nominal (initial) design is represented by a single point in the (J_1, J_2) space, and the performance of the Pareto optimal design is represented by a curve joining multiple points, each point corresponds to a λ-value $\lambda \in [0,1]$. A number of observations can be made

(1) Considering the damper optimization alone, for a fixed mass an arbitrarily placed damper shows only 25% improvement in transfer function over the no damper case, while an optimally placed damper shows a factor of two improvement. When two dampers are optimally placed, the improvement is increased from a factor of two to five.

(2) For the same mass (~17.00lb), optimal resizing of the truss members results in impressive improvement in the transfer function, where a reduction by a factor of 7 to 12 is achieved, depending on the number and location of the dampers.

(3) A definite corner point near the origin of the Pareto optimal curve is typical of non-competitive criteria that admit a single optimal solution. Optimally placed dampers tend to sharpen the (J_1, J_2) curve, and thereby make minimization of the mass and rms response more achievable.

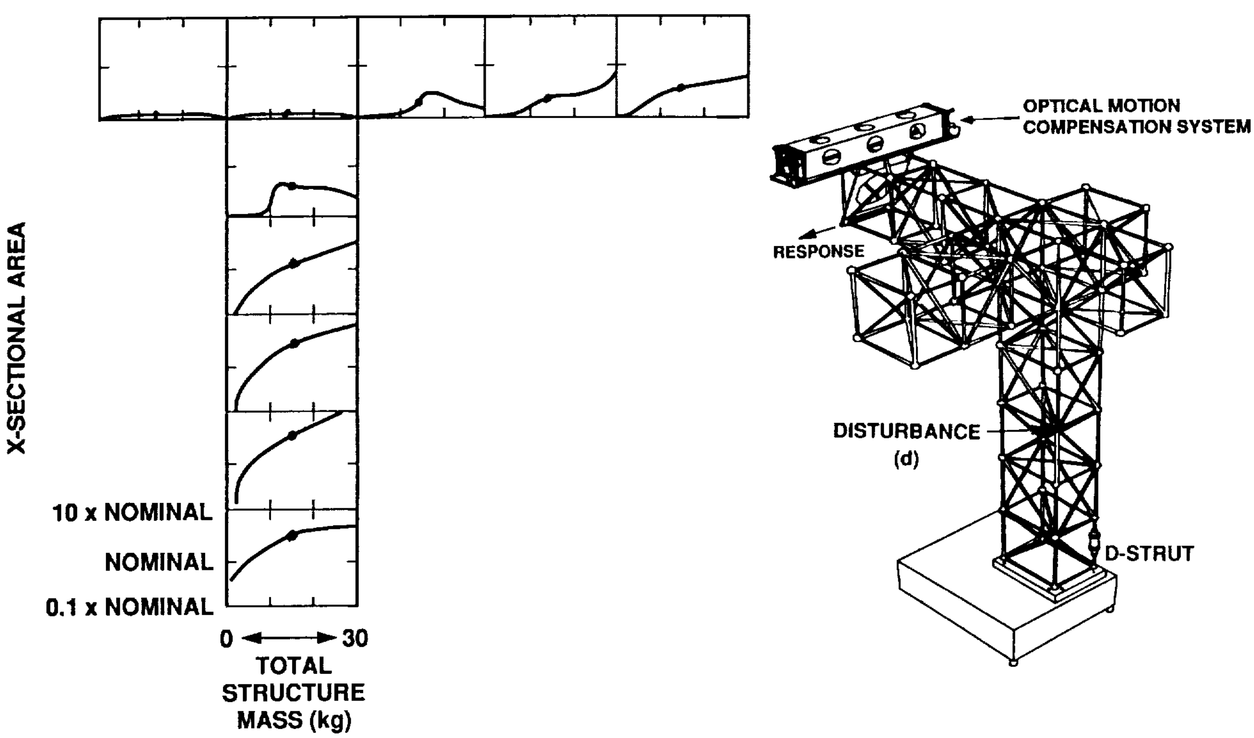


CSI PHASE B TRUSS OPTIMIZATION RESULTS
DISTRIBUTION OF MASS IN PARETO OPTIMAL SET

The chart illustrates the distribution of mass in the set of optimized structures for the single optimally placed damper on the previous page. In the figure on the left each plot shows variation of the structural mass of individual bays versus the total structural mass of the optimized structure. The ordinate is a logarithmic scale covering the range of the allowed bay mass (corresponding to the allowed variation of parameters). For this case the parameters were allowed to vary in a range of plus-or-minus an order of magnitude from nominal. For the two single-bay and two three-bay arms, the plots show the combined structural mass of the corresponding bays on each arm.

As one can see, the majority of the mass for the optimized structures lies in the tower near the base. A suitable explanation is that the mass is being added to stiffen the structure between the disturbance input and the grounded base. There is also some mass added to the three-bay arms, to make the structure near the trolley heavier, and hence more resistant to vibration.

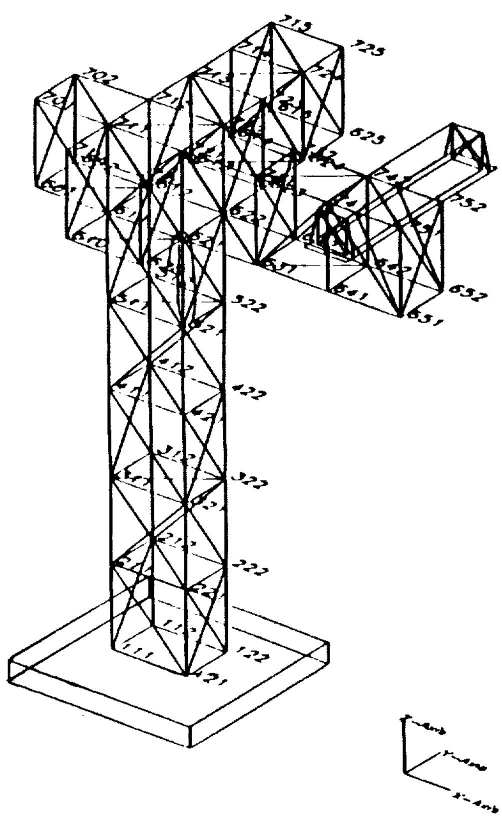
The general trend in the way mass was added within each buy was that the majority of mass was added to the longerons and diagonals, implying that the battens play a small role in the system performance.



BUCKLING OF OPTIMAL DESIGN UNDER ONE - "G"

During optimization, lower and upper bounds on the design vector α, with components α_i, were imposed so that $(5. \times 10^{-6}) \leq \alpha_i \leq (5. \times 10^{-4})$. As a result of the optimization, some members were driven toward the lower bound where buckling instability may be of concern. To check whether or not the resulting design is viable, a worst case buckling analysis under one-g loading was conducted for the case of one optimally placed damper. To simplify this analysis, instead of using 25 different cross sectional areas, the member sizes were grouped in three intervals: $(5.\text{ to }9.9)\times10^{-6}$, $(1.0\text{ to }9.9)\times10^{-5}$, and $(1.0\text{ to }5.0)\times10^{-4}$. After computing the forces in all members under one-g, the maximum compressive load (labeled P_{max}) on any of the members belonging to a given group was checked against the fundamental critical buckling load (labeled P_{cr}) for the member with the smallest cross section and greatest length in that group. The smaller the ratio $(P_{max}/P_{cr}) < 1.0$, the greater the degree of conservatism of the design against buckling.

One damper optimal design at mass = 15.54 Kg
Considered 3-intervals for x-sec. areas: $(5.-9.)\times10^{-6}$; $(1.-9.)\times10^{-5}$; $(1.-5.)\times10^{-4}$



P_{1max} = 312.N; $P_{1cr.}$ = 544.N;
Ratio = 0.57
A=5.3×10^{-6}; =0.41; R=$5.\times10^{-3}$; t=0.17×10^{-3}

P_{2max} = 509.N; P_{2cr} = 1,954.N
Ratio = 0.26
A=1.79×10^{-5}; =0.41; R=$5.\times10^{-3}$; t=0.6×10^{-3}

P_{3max} = 563.N; P_{3cr} = 150,000.N
Ratio = 0.004
A=4.67×10^{-4}; =.575; R=12.5×10^{-3}; t=6.0×10^{-3}

SUMMARY/CONCLUSIONS

This paper has addressed several design optimization problems for the JPL testbed structure. The two classes of problems considered were the optimal resizing of truss element areas, and the placement and tuning of viscous dampers. Various measures of performance were defined for these problems including minimizing RMS error due to input disturbances and maximizing damping in selected structural modes.

The testbed structure is of sufficient complexity to expose the numerical challenges and issues related to practically solving these optimization problems. A number of different economizing techniques were introduced and validated to meet these challenges. Each of these methods proved to be very well suited for their particular target optimization application.

The resulting optimized design in each instance led to significant improvement in performance. Although each optimization problem was attacked individually, future work will focus on integrating the approaches.

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o Developed and applied analysis/design tools for optimal placement/tuning of passive dampers and optimal resizing of structural mass

o Developed efficient solution techniques for optimization problems

o Demonstrated significant improvement in performance both analytically and experimentally on CSI testbed structure using optimization approach

o Established real benefits to the methods, and their applicability to large systems