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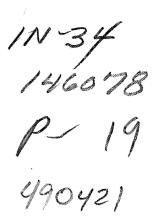
A Kinetic Theory Treatment of Heat Transfer in Plane Poiseuille Flow with Uniform Pressure

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(NASA-TM-103993)   A KINETIC THEORY          N93-18884
TREATMENT OF HEAT TRANSFER IN PLANE
POISEUILLE FLOW WITH UNIFORM
PRESSURE  (NASA)  19 p                       Unclas

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December 1992

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490690
20p.

A Kinetic Theory Treatment of Heat Transfer in Plane Poiseuille Flow with Uniform Pressure

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December 1992

NOMENCLATURE

$\underset{\sim}{c}$ molecular velocity vector

d   distance between the plates

f   velocity distribution function

f_1, f_2 components of two-stream Maxwellian

$\underset{\sim}{F}$ external body force

g, G   functions defined by equation (45)

$I_n(\beta)$ integral defined by equation (3)

$\hat{i}, \hat{j}, \hat{k}$ unit vectors in the x, y and z directions, respectively

k   thermal conductivity

L   plate temperature ratio parameter, equation (43)

m   mass of molecule

n   molecular number density and parameter in equation (3)

n_1, n_2 number density functions

$\hat{n}$ unit normal vector to an area element

p   pressure

$\underset{\sim}{q}$ heat flux vector

q_2 heat flux in y direction

Q, ψ molecular property

R   gas constant

T_1, T_2 temperature functions

T_U, T_L upper and lower plate temperatures

u, v, w components of $\underset{\sim}{C}$ in the x, y and z directions, respectively

$\underset{\sim}{V}$ macroscopic velocity vector

α_i, a_i constants, equations(51--53)

β constant in equation (3), and parameter defined by equation (54)

γ the ratio of specific heats

ε azimuthal angle in binary collision

μ dynamic viscosity

ρ mass density

SUMMARY

Plane compressible Poiseuille flow with uniform pressure (Couette flow with stationary boundaries) is revisited where the Lees two-stream method with the Enskog equation of change is applied. Single particle velocity distribution functions are chosen, which preserve the essential physical features of this flow with arbitrary but uniform plate temperatures and gas pressure. Lower moments are shown to lead to expressions for the parameter functions, molecular number densities, and temperatures which are entirely in agreement with those obtained in the analysis of Lees for compressible plane Couette flow in the limit of low Mach number and vanishing mean gas velocity. Important simplifications result, which are helpful in gaining insight into the power of kinetic theory in fluid mechanics. The temperature distribution, heat flux, as well as density, are completely determined for the whole range of Knudson numbers from free molecular flow to the continuum regime, when the pressure level is specified.

INTRODUCTION

The Kinetic theory of gases is one of the most intriguing inventions in the theory of matter, with its foundations in statistical physics and applications in fundamental fluid physics. The main success of the kinetic theory is explaining the transport properties of gases set forth by its founders, Clasius, Maxwell (ref.1) and Boltzmann (refs. 2 and 3), and later by the more exact works of Chapman and Enskog (ref.4). There are as yet unanswered intriguing questions, --for example, why the Navier--Stokes equations describe fluid motions so satisfactorily. They can be obtained from the Boltzmann equations by the Chapman--Enskog theory by successive approximation with $\Delta\lambda$ as the expansion parameter and Δ acts on the five macroscopic variables, density ρ, velocity V, and temperature T, where λ is the mean free path. Second order hydrodynamical equations derived by Burnett, however, lead to unrealistic physical phenomenon, --for example, negative absorption coefficient in very high frequency sound waves. Turbulence, of course, remains an important unsolved problem in fluid mechanics. Attempts by several researchers have been made to develop a kinetic theory for turbulence. Despite their modest success, description of turbulent flow must clearly be statistical and this makes kinetic theory an attractive guiding light.

From the foregoing it may be clear that much insight into the physics of fluids, not only turbulence but, for example, the structure of shock waves can be obtained by carefully studying the works of masters and understanding their logic in deriving fundamental concepts and equations and expanding upon them. Even seemingly simple flow geometries may serve this purpose enormously. Obtaining exact solutions of Boltzmann equations is undoubtedly a formidable task. Lees (ref.5) developed a method utilizing a distribution function that is two-sided in character and contains parameter functions of space and time variables. The method is formulated in terms of Maxwellian functions which is a natural extension and generalization of Mott-Smith's (ref.6) approach for normal shock waves. Sufficient numbers of moments are taken to ensure a complete set of first order partial differential equations to solve for the functions. As pointed out by Lees (ref.5), this method amounts to satisfying the Maxwell--Boltzmann equation in a certain average sense rather than point by point. The distribution function is then a suitable weighting function and not an exact solution; therefore, the rigidity inherent in the polynomial of Chapman--Enskog or Grad's moment method (ref.7) is avoided. Kinetic theory is often obscured by mathematical formalism. In this paper we start with a brief discussion of the fundamental concepts and the two-stream method and proceed with the solution of the present problem in a most straightforward way.

FUNDAMENTAL CONCEPTS

Maxwell and Enskog Equations of Change

The Boltzmann equation is fundamental to the kinetic theory. It represents conservation of molecules in the (c, r) phase space and accounts for all the mechanisms which can change the number of particles in (dc, dr). Here, the molecular velocity is denoted by c and r is the position vector. The RHS of the Boltzmann equation pertains to gas phase binary collisions, where no correlation between particle velocities prior to collisions exists. This is generally referred to as independent a priori probabilities or molecular chaos. A spherically symmetric intermolecular force field which is much stronger than all external forces is assumed. The duration of collisions is also much smaller than the time between collisions. In an integral form the Boltzmann equation leads to the Maxwell equation of change

$$\frac{\partial}{\partial t}\int_c Qf dc + \frac{\partial}{\partial r} \cdot \int_c Qf c\, dc - \int_c f F \cdot \frac{\partial Q}{\partial c} dc = \int_{c'}\int_c\int_\varepsilon\int_b Q\,(f'f'_1 - ff_1)\, gb\, db\, d\varepsilon\, dc\, dc' \qquad (1)$$

where, $\frac{\partial}{\partial t}$ represents the partial derivative with respect to time. Here F denotes the external force field and ε is the azimuthal angle in the collision. The distribution functions f and f_1 represent the binary (probe and colliding) particles before the collision and the primed functions after the collision. The speed of approach of binary particles is denoted by g, and b is an impact parameter (a hypothetical distance of closest approach if there was no interacting force.) The molecular property $Q = Q(c)$ only. In its more general form, when $Q = Q(c, r, t)$, where t is the time, the Enskog equation of change is obtained.

$$\frac{\partial}{\partial t}\int_c Qf dc + \frac{\partial}{\partial r} \cdot \int_c Qf c\, dc - \int_c f\left[\frac{\partial Q}{\partial t} + c \cdot \frac{\partial Q}{\partial r} + F \cdot \frac{\partial Q}{\partial c}\right] dc = \int Q\,(f'f'_1 - ff_1)\, gb\, db\, d\varepsilon\, dc\, dc' \quad (2)$$

Averaging Over the Velocity Space

Since the transport coefficients ρ, μ, k (density, viscosity, and thermal conductivity, respectively) are given in terms of the averages over the velocity space involving the distribution function, the conservation laws can be obtained when Qs are the collisional invariants, mass m, linear momentum mc, and translational kinetic energy of particles $1/2\, mc^2$. The RHS of equation (2) is then identically zero.

In evaluating the three-fold integrals over the velocity space, since the three components of the particle velocity are independent, the integrals can be separated into single-fold integrals. The single-fold integrals can then be evaluated individually. We will present summaries of the integral results, employing the following definition

5

$$I_n(\beta) = \int_0^\infty \xi^n e^{-\beta \xi^2} d\xi \qquad , \quad \beta > 0 \qquad (3)$$

For n = 0, 1:

$$I_0(\beta) = \frac{1}{2}\sqrt{\frac{\pi}{\beta}} \qquad (4)$$

$$I_1(\beta) = \frac{1}{2\beta} \qquad (5)$$

For n>1, a recursion formula may be written

$$I_n(\beta) = \left(\frac{n-1}{2\beta}\right) I_{n-2}(\beta) \qquad (6)$$

Lees Two-Stream Method

The integral equations of transfer, equations (1) and (2) possess an important property. They permit a large degree of flexibility in the choice of the distribution function. The distribution function f can be expressed in terms of arbitrary functions of space and time that preserve the essential physical features of a problem and are, therefore, tailored to the particular case. There is no sequence of logic which enables one to find such functions; the tools seem to be accumulated experience and intuition. As mentioned earlier, Lees (ref. 5) developed a method utilizing a distribution function which is two-sided in character and contains parameter functions of space and time variables. The method is formulated in terms of Maxwellian functions, which is a natural extension and generalization of Mott-Smith's (ref. 7) approach for normal shock waves. Sufficient numbers of moments are taken to obtain a complete set of first order differential equations to solve for the functions. As pointed out earlier, this procedure amounts to satisfying the Maxwell--Boltzmann integral equation and, therefore, the Enskog equation of change in an average sense rather than point by point. The search for higher order macroscopic equations in terms of the mean quantities is not persued, but surface boundary conditions can be incorporated. Lees (ref. 5) applied this method to steady plane Couette flow for the limiting case of low Mach number and low temperature difference. Reasonable results were obtained for macroscopic quantities over the whole range of densities from free molecular flow to the Navier--Stokes. Later, Lees and Liu (ref. 8) expanded this work to compressible plane Couette flow with arbitrary Mach number and uniform plate temperature.

We will start with a two-sided velocity distribution function, which appears sensible for the present boundary conditions and carry out the integrations over the corresponding velocity space. The results are then shown to be entirely consistent with those obtained in the more general analysis of Lees and Liu (ref. 8) for compressible plane Couette flow in the limit of low Mach number. Important simplifications result which are helpful in gaining insight into the power of kinetic theory in fluid mechanics. The temperature distribution, heat flux, as well as density, are therefore completely determined for the whole range of Knudson numbers from free molecular flow to the continuum regime when the pressure level is specified.

PLANE POISEUILLE AND COUETTE FLOWS

Plain Poiseuille flow and plain Couette flow must be represented by different velocity distribution functions in the kinetic theory. They are, however, physically identical when, in each flow respectively, uniform gas pressure and stationary boundaries are approached. We consider two stationary infinite parallel plates designated by the subscripts U (upper) and L (lower) which are held a distance d apart and are at temperatures T_U and T_L, respectively. We will assume that the gas between the plates is monatomic and is at a uniform pressure p . Despite the non-zero molecular velocity $\underset{\sim}{c}$, no shear stresses exist, and determination of the heat flux and temperature distribution is of primary interest.

We choose a two-sided Maxwellian particle velocity distribution function f and assume that the plates scatter molecules diffusely with complete thermal accommodation.

$$f_1 = \frac{n_1}{(2\pi RT_1)^{3/2}} e^{-c^2/2RT_1} \qquad , v < 0, \text{ all u and w} \tag{7}$$

$$f_2 = \frac{n_2}{(2\pi RT_1)^{3/2}} e^{-c^2/2RT_2} \qquad , v > 0, \text{ all u and w} \tag{8}$$

where n_1, n_2, T_1, and T_2 are parameter functions of y, and R is the gas constant. The boundary conditions for completely diffuse re-emission are

$$T_1 = T_U \qquad \text{at y= +1/2 d} \tag{9}$$

$$T_2 = T_L \qquad \text{at y= -1/2 d} \tag{10}$$

The coordinate system denotes the components of velocity by u, v, and w in the x, y, and z ($\hat{i}, \hat{j}, \hat{k}$ unit vectors) directions, respectively. The plates are in the x, z plane.

The Heat Flux Vector

The net flux of any molecular property $\psi(\underset{\sim}{c})$ across an area element with unit normal vector $\hat{n}$ may be represented by

$$\int_{\underset{\sim}{c}} (\hat{n} \cdot \underset{\sim}{c}) \, \psi(\underset{\sim}{c}) \, f(\underset{\sim}{c},\underset{\sim}{r},t) \, d\underset{\sim}{c} \tag{11}$$

The heat flux vector between the plates is directed perpendicular to the plates, i.e.,

7

$$\underset{\sim}{q} = q_2 \hat{j} \qquad (12)$$

Here $\psi(\underset{\sim}{c})$ is the molecular kinetic energy $1/2\ mc^2$. Therefore,

$$q_2 = \int_{\underset{\sim}{c}} \frac{mvc^2}{2} f d\underset{\sim}{c} \qquad (13)$$

Introducing the two-stream distribution function f and integrating over the velocity space,

$$q_2 = \frac{mn_1}{2\,(2\pi RT_1)^{3/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{0} \int_{-\infty}^{\infty} v\,(u^2 + v^2 + w^2)\,e^{-(u^2 + v^2 + w^2)/2RT_1} dudvdw$$

$$+ \frac{mn_2}{2\,(2\pi RT_2)^{3/2}} \int_{-\infty}^{\infty} \int_{0}^{\infty} \int_{-\infty}^{\infty} v\,(u^2 + v^2 + w^2)\,e^{-(u^2 + v^2 + w^2)/2RT_2} dudvdw \qquad (14)$$

$$q_2 = \frac{-mn_1}{2\,(2\pi RT_1)^{3/2}} [8I_2 I_1 I_0 + 4I_0^2 I_3]_1 + \frac{mn_2}{2\,(2\pi RT_2)^{3/2}} [8I_2 I_1 I_0 + 4I_0^2 I_3]_2 \qquad (15)$$

Here, subscripts 1 and 2 to the square brackets refer to the integrals evaluated by equations (3)--(5) containing the respective parameter functions T_1 and T_2. Therefore,

$$\therefore q_2 = m\sqrt{\frac{2}{\pi}} R^{3/2} \Big[-n_1 T_1^{3/2} + n_2 T_2^{3/2} \Big] \qquad (16)$$

We note that q_2 is independent of y and therefore

$$\therefore -n_1 T_1^{3/2} + n_2 T_2^{3/2} = const \qquad (17)$$

The Temperature and Density Distributions

We now proceed with the definition of temperature as the mean molecular kinetic energy,

8

$$\frac{3}{2}mRT = \ <\frac{mc^2}{2}> \tag{18}$$

where

$$<\frac{mc^2}{2}> \ = \frac{1}{n}\int_{\underset{\sim}{c}}\frac{mc^2}{2}fd\underline{c} \tag{19}$$

$$T = \frac{1}{3nR}\int_{(\underline{c},\,v<0)}c^2f_1d\underline{c} + \int_{(\underline{c},\,v>0)}c^2f_2d\underline{c} \tag{20}$$

$$T = \frac{3}{(2\pi RT_1)^{3/2}}\left[4I_2I_0^2\right]_1 + \frac{3}{(2\pi RT_2)^{3/2}}\left[4I_2I_0^2\right]_2 \tag{21}$$

$$T = \frac{T_1n_1 + T_2n_2}{2n} \tag{22}$$

The gas density may be represented as the integral of the distribution function itself.

$$n = \int_{\underset{\sim}{c}}fd\underline{c} \tag{23}$$

$$n = \frac{n_1}{(2\pi RT_1)^{3/2}}\int_{(\underline{c},\,v<0)}e^{-c^2/2RT_1}d\underline{c} + \frac{n_2}{(2\pi RT_2)^{3/2}}\int_{(\underline{c},\,v>0)}e^{-c^2/2RT_2}d\underline{c} \tag{24}$$

$$n = \frac{4n_1}{(2\pi RT_1)^{3/2}}\left[I_0^3\right]_1 + \frac{4n_2}{(2\pi RT_2)^{3/2}}\left[I_0^3\right]_2 \tag{25}$$

$$\therefore n = \frac{1}{2}(n_1 + n_2) \tag{26}$$

The temperature T is, therefore,

$$\therefore T = \frac{T_1n_1 + T_2n_2}{n_1 + n_2} \tag{27}$$

The Moments

Moments may now be taken to establish additional equations for determination of the functions n_1, n_2, T_1, and T_2. Since there is no net flux of mass in the y direction at any 'y',

$$\int\limits_{(\underset{\sim}{c}, v < 0)} v f_1 d\underset{\sim}{c} + \int\limits_{(\underset{\sim}{c}, v > 0)} v f_2 d\underset{\sim}{c} = 0 \tag{28}$$

$$\frac{-4n_1}{(2\pi R T_1)^{3/2}} [I_0^2 I_1]_1 + \frac{4n_2}{(2\pi R T_2)^{3/2}} [I_0^2 I_1]_2 = 0 \tag{29}$$

$$\therefore n_1 \sqrt{T_1} = n_2 \sqrt{T_2} \tag{30}$$

This is the first moment, or a statement of the conservation of mass. The second and forth moments related to the x and z components of velocity are not needed.

The Enskog equation of change (eq.(2)) for this problem, when no external forces are present is reduced to

$$\frac{d}{dy} \int\limits_{\underset{\sim}{c}} Q v f d\underset{\sim}{c} - \int\limits_{\underset{\sim}{c}} f v \frac{dQ}{dy} = J_Q \tag{31}$$

where only the derivatives with respect to y are retained, and J_Q denotes the collision integral.

For the third moment ($Q = mv$) , the conservation of momentum in the y direction, the collision integral vanishes. Furthermore, $\frac{dQ}{dy} = 0$, since v is independent of y. Therefore,

$$\int\limits_{(\underset{\sim}{c}, v < 0)} v^2 f_1 d\underset{\sim}{c} + \int\limits_{(\underset{\sim}{c}, v > 0)} v^2 f_2 d\underset{\sim}{c} = \text{const} \tag{32}$$

$$\frac{4n_1}{(2\pi R T_1)^{3/2}} [I_0^2 I_2]_1 + \frac{4n_2}{(2\pi R T_2)^{3/2}} [I_0^2 I_2]_2 = \text{const} \tag{33}$$

$$\therefore n_1 T_1 + n_2 T_2 = \text{const} \tag{34}$$

For the fifth moment, $Q = mc^2/2$, the RHS of the equation of change is zero, since it is

10

also a collisional invariant, and $\dfrac{dQ}{dy}$ is also zero.

$$\frac{d}{dy}\int_{c} \frac{mvc^2}{2} f\,dc = 0 \tag{35}$$

We have already evaluated this integral in determining the heat flux and shown that equation (17) is satisfied.

The sixth moment, $Q = mvc^2$, is not a collisional invariant and evaluation of the collision integral is required, although $\dfrac{dQ}{dy}$ again vanishes. We have

$$\frac{d}{dy}\int_{c} mv^2c^2 f\,dc = J_{mvc^2} \tag{36}$$

where J_{mvc^2} denotes the collision integral.

For molecules obeying an inverse fifth power intermolecular force law, the collision integrals in a full-range moment method are completely independent of the form of the distribution function. This advantage is preserved in the half-range moment method, at least for certain moments. Following Maxwell (ref. 1) and Jeans (ref. 9), Lees (ref. 5) considered collision dynamics between particles by means of vector geometry and showed that the first nine moments beyond the collisional invariants are, in fact, completely independent of the form of the distribution function. Lees then evaluated and corrected once and for all the numerical errors in the expressions given by Jeans . The result for $Q = mvc^2$ is simply

$$J_{mvc^2} = \frac{pq_2}{6\mu} \tag{37}$$

where p is the gas pressure and μ the classical coefficient of viscosity.

As pointed out earlier, q_2 is a constant, therefore J_{mvc^2} is a constant. Again, the integral in equation (36) is evaluated using the two-stream distribution function f_1 and f_2.

$$\frac{d}{dy}\int_{(c,\,v<0)} (2v^2u^2+v^4)\,f_1\,dc + \frac{d}{dy}\int_{(c,\,v>0)} (2v^2u^2+v^4)\,f_2\,dc = \text{const} \tag{38}$$

11

$$\frac{d}{dy}\left[\frac{n_1}{(2\pi RT_1)^{3/2}} [8I_2^2 I_0 + 4I_0^2 I_4]_1 + \frac{n_2}{(2\pi RT_2)^{3/2}} [8I_2^2 I_0 + 4I_0^2 I_4]_2 \right] = \text{const} \tag{39}$$

$$\therefore \frac{d}{dy}[n_1 T_1 + n_2 T_2] = \text{const} \tag{40}$$

Agreement With The Results of Lees and Liu

Lees and Liu (ref. 8) expanded the analysis of Lees (ref. 5) and applied the two-stream method to plane compressible Couette flow with arbitrary plate temperatures. Mach and Reynolds numbers were defined in terms of the characteristic quantities n_L and T_L (density and temperature at the lower plate surface respectively), U the relative plate velocity, and the separation distance d.

$$M = \frac{U}{\sqrt{\gamma RT_L}} \tag{41}$$

$$Re = \frac{mn_L Ud}{\mu_L} \tag{42}$$

Classical fluid viscosity coefficient μ_L evaluated at the lower plate temperature, the ratio of specific heats γ, and a plate temperature ratio parameter L were also employed.

$$L = \sqrt{T_U / T_L} \tag{43}$$

It was shown that when the Mach number is small there is a split in the system of equations containing the thermodynamic variables n_1, n_2, T_1, and T_2 from the dynamical variables u_1 and u_2 which were included in their analysis for moving plates. In the present analysis the Mach and Reynolds numbers are vanishingly small. Their ratio Re/M, however, is finite and is proportional to the ratio of the mean free path λ to the characteristic length d. It characterizes the density level of the gas. We note that there are close similarities between the results of Lees and Liu (ref. 8) and the present analysis when $M \ll 1$.

As mentioned earlier, Plain Poiseuille flow and plain Couette flow must be represented by different velocity distribution functions in kinetic theory. They are, however, physically identical when in each flow respectively, uniform gas pressure and stationary boundaries are approached. The present analysis must, therefore, be in agreement with the results of Lees and Liu (ref. 8) for $M \ll 1$. This, in fact, is the case. We note that by substituting equation (30) into equation (17), one obtains the following equation.

$$n_1\sqrt{T_1}\,(T_2 - T_1) = \text{const} \tag{44}$$

Inspection of reference 8 reveals, that equations (22a), (22c), and (22d) of reference (8) are identical to equations (30), (34), and (44) of the present work, respectively, except for the dimensionality difference. A new function g(y) may now be conveniently defined as

$$g(y) = 1/(n_1\sqrt{T_1}) = 1/(n_2\sqrt{T_2}) \tag{45}$$

As shown in reference 8, the present equations (30), (34), (44), and (45) can be easily manipulated to result in relations for T_1, T_2, n_1, and n_2 in terms of g(y) and constants a_i. For the present problem $a_1 = 0$.

$$T_1(y) = \frac{1}{4}(a_2 g - a_3)^2 \tag{46}$$

$$T_2(y) = \frac{1}{4}(a_2 g + a_3)^2 \tag{47}$$

$$n_1(y) = \frac{2}{g(a_2 g - a_3)} \tag{48}$$

$$n_2(y) = \frac{2}{g(a_2 g + a_3)} \tag{49}$$

Once g(y) is determined, T_1, T_2, n_1, and n_2 are also completely determined. In order to obtain g(y) explicitly, the expressions for T_1, T_2, n_1, and n_2 are substituted in equation(40) and the integration is carried out, which results in an additional constant a_4. The boundary conditions (eqs. (9) and (10)), as well as equations (46)--(49), are fully applicable for determination of the constants. The details of the algebra are rather lengthy and will not be presented here. We will, for completeness, however, include the expressions for the a_i and g(x) as presented in reference 8. For consistency with reference 8, a_i can be replaced by α_i, and g(x) by G(x). This implies that the quantities n_1, n_2, T_1, and T_2 will be accordingly dimensionless (normalized by n_L and T_L, respectively).

$$G(y) = \left[(\alpha_3/\alpha_2)^2 + (2/\alpha_2)\,(\alpha_4 - \frac{32}{15}\sqrt{\frac{2}{\pi\gamma}}\frac{\alpha_3}{\alpha_2} \times \frac{Re}{M}y)^{1/2} \right]^{1/2} \tag{50}$$

$$\alpha_2 = \frac{[\,(1+\beta)\,(L^4 + 2L^3 + \beta)\,L^2]^{1/2} - (L^3 + 2L^2 - \beta - 1)}{1 + \beta - L^2} \tag{51}$$

$$\alpha_3 = 2 - \alpha_2 \tag{52}$$

$$\alpha_4 = 1 - 2(1 + \beta) \times (\alpha_3/\alpha_2) + (\alpha_3/\alpha_2)^2 \tag{53}$$

where

$$\beta = (8/15)\sqrt{2/\pi\gamma}\,Re/M \tag{54}$$

The results of both analysis are entirely consistent with each other. As shown by Lees and Liu (ref. 8), the heat flux and temperature distribution are, therefore, fully determined for the whole range of Knudson numbers from free molecular flow to the continuum regime, when the pressure level is specified. Furthermore, when $u_1 = u_2 = 0$ in Lees and Liu (ref. 8), the distribution functions lead to identical expressions. The present analysis, therefore, may be viewed as an alternative to the more general analysis of Lees and Liu (ref. 8) in which identical results are arrived at independently .

CONCLUDING REMARKS

Plane compressible Poiseuille flow with uniform pressure (which is, in fact, Couette flow with stationary boundaries) was chosen to discuss the mathematical and physical features of an approximate technique for the solution of the Boltzmann equation. Lees two-stream method, a clever invention in kinetic theory, shows that in a rigorous field such as statistical physics, fundamental and physically realistic treatments can be made with relative ease. The Enskog equation of change, which is more general than the Maxwell integral equation, was shown to lead to results that completely describe the heat transfer process. A simple form of the two-stream single particle velocity distribution function which preserves the essential physical features of Poiseuille flow with arbitrary but uniform plate temperatures and gas pressure was chosen. Important simplifications result which are helpful in gaining insight into the power of kinetic theory in fluid mechanics. Lower moments are shown to lead to equations for four parameter functions, molecular number densities, and temperatures employed in the distribution function, which are entirely consistent with those obtained by Lees and Liu (ref. 8) in the limit of low Mach number and vanishing mean gas velocity. The temperature distribution, heat flux, as well as density, are therefore completely determined for the whole range of Knudson numbers from free molecular flow to the continuum regime, when the pressure level is specified.

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REPORT DOCUMENTATION PAGE

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	December 1992	Technical Memorandum

4. TITLE AND SUBTITLE

A Kinetic Theory Treatment of Heat Transfer in Plane Poiseuille Flow with Uniform Pressure

6. AUTHOR(S)

Parviz A. Bahrami

5. FUNDING NUMBERS

505-59-40

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Ames Research Center
Moffett Field, CA 94035-1000

8. PERFORMING ORGANIZATION REPORT NUMBER

A-93021

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Washington, DC 20546-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NASA TM-103993

11. SUPPLEMENTARY NOTES

Point of Contact: Parviz A. Bahrami, Ames Research Center, MS 229-1, Moffett Field, CA 94035-1000;
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12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified — Unlimited
Subject Category 34

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Plane compressible Poiseuille flow with uniform pressure (Couette flow with stationary boundaries) is revisited where the Lees two-stream method with the Enskog equation of change is applied. Single particle velocity distribution functions are chosen, which preserve the essential physical features of this flow with arbitrary but uniform plate temperatures and gas pressure. Lower moments are shown to lead to expressions for the parameter functions, molecular number densities, and temperatures which are entirely in agreement with those obtained in the analysis of Lees for compressible plane Couette flow in the limit of low Mach number and vanishing mean gas velocity. Important simplifications result, which are helpful in gaining insight into the power of kinetic theory in fluid mechanics. The temperature distribution, heat flux, as well as density, are completely determined for the whole range of Knudson numbers from free molecular flow to the continuum regime, when the pressure level is specified.

14. SUBJECT TERMS

Kinetic theory, Heat transfer, Fluid dynamics

15. NUMBER OF PAGES

16

16. PRICE CODE

A02

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified		