

17302473-1
49501
12P
37
140279

P. 17N93-10016

Mathematical Modeling of Bent-Axis Hydraulic Piston Motors

R. D. Bartos

Ground Antennas and Facilities Engineering Section

Each of the DSN 70-m antennas uses 16 bent-axis hydraulic piston motors as part of the antenna drive system. On each of the two antenna axes, four motors are used to drive the antenna and four motors provide counter torque to remove the backlash in the antenna drive train. This article presents a mathematical model for bent-axis hydraulic piston motors. The model was developed to understand the influence of the hydraulic motors on the performance of the DSN 70-m antennas' servo control system.

I. Introduction

Hydraulic motors are often selected to drive mechanical devices because they are able to provide large output torque over a wide range of speeds while being relatively compact and lightweight. When a hydraulic motor is selected to drive a device under precision control conditions, it is necessary to understand how a particular motor will affect the overall system performance so that system components and control algorithms can be chosen to achieve optimum performance. The effect of a particular motor on system performance is best evaluated through computer simulations. This article develops a mathematical model, which can be implemented within any standard computer simulation package, for a bent-axis hydraulic piston motor. A glossary is provided in the Appendix.

II. Theoretical Model Development

The configuration of a typical bent-axis hydraulic piston motor has been provided in Figs. 1 [1], 2 [1], 3, and 4.

This section develops mathematical models which describe the output torque and fluid flow rate through a bent-axis hydraulic piston motor.

A. Hydraulic Motor Torque

Based upon the geometrical relationships indicated in Fig. 3 and Fig. 4, the instantaneous torque generated by the fluid pressure forces is found to be given by

$$T_P = \sum_{i=1}^n ARP_i \sin \gamma \cos \theta_i \quad (1)$$

where

$$\theta_i = \theta_1 + \frac{2\pi(i-1)}{n} \quad (2)$$

$$P_i = P_A \quad \text{if } \cos \theta_i > 0 \quad (3)$$

$$P_i = P_B \quad \text{if } \cos \theta_i < 0 \quad (4)$$

$$k = 0, \pm 1, \pm 2, \pm 3, \dots \quad (5)$$

The variables used in Eqs. (1) through (5) are defined as

A = area of one piston

n = the number of pistons

P_A = pressure at motor port A

P_B = pressure at motor port B

P_i = pressure acting on piston i

R = pitch circle radius of the cylinder block bores

γ = motor housing angle

θ_i = angle of the i th piston with respect to the valve plate, as shown in Fig. 4

Note that Eq. (1) only expresses the torque generated by the pressure forces and does not model the net torque output of the motor since losses due to viscous forces and Coulomb friction are not considered. Substituting the motor displacement per shaft revolution given by

$$D = 2nAR \sin \gamma \quad (6)$$

into Eq. (1) yields

$$T_P = \frac{D}{2n} \sum_{i=1}^n P_i \cos \theta_i \quad (7)$$

where θ_i and P_i are as defined in Eqs. (2) through (5).

$$T_P \approx \frac{D\Delta P}{2\pi} \left[C_1 + C_2 \left| \cos \left(\frac{n\theta_1}{2} \right) \right| \right] \quad \text{if } n = 4k - 2 \quad (9)$$

$$T_P \approx \frac{D\Delta P}{2\pi} \left[C_1 + C_2 \left| \sin \left(\frac{n\theta_1}{2} \right) \right| \right] \quad \text{if } n = 4k \quad (10)$$

$$\Delta P = P_A - P_B \quad (11)$$

$$k = 1, 2, 3, \dots \quad (12)$$

The nondimensional constants C_1 and C_2 are computed by means of least-squares estimation using Eq. (7) to obtain the actual motor torque. Parameter values are presented in Table 1 through Table 3, along with normalized standard deviation of the error between the approximations given in Eqs. (8) through (12) and the actual values given by Eq. (7). The estimation of the parameters was performed using 500 data points evenly spaced over a single ripple. The constant C_2 physically represents the fraction of the mean torque equivalent to the peak to peak amplitude of the ripple because

$$\int_0^{2\pi} [C_1 + C_2 |\cos(n\theta_1)|] d\theta_1 = 1 \quad (13)$$

$$\int_0^{2\pi} \left[C_1 + C_2 \left| \cos \left(\frac{n\theta_1}{2} \right) \right| \right] d\theta_1 = 1 \quad (14)$$

$$\int_0^{2\pi} \left[C_1 + C_2 \left| \sin \left(\frac{n\theta_1}{2} \right) \right| \right] d\theta_1 = 1 \quad (15)$$

$$\text{mean torque} = \frac{D\Delta P}{2\pi} \quad (16)$$

Some of the torque generated by the pressure forces act-

plate used to regulate the fluid pressure in the valve block bores. Figures 1 and 2 illustrate the interaction of the valve plate and cylinder block. The shear stress of the fluid in the clearance between the cylinder block and the valve plate is given by

$$\tau = \frac{\mu\omega r}{h} \quad (18)$$

where the variables are defined as

h = clearance between the cylinder block and the valve plate

r = radius from the center of the cylinder block

μ = absolute fluid viscosity

ω = angular velocity of the cylinder block

The torque due to viscous forces is found by integrating the shear stress over the area of the cylinder block to valve plate interface shown in Fig. 5 by using the equation

$$T = - \int_{R_1}^{R_2} \int_{\psi_1}^{\psi_2} (r\tau) (r\partial\psi\partial r) \quad (19)$$

Combining Eqs. (18) and (19) gives

$$T = - \int_{R_1}^{R_2} \int_{\psi_1}^{\psi_2} \frac{\mu\omega r^3}{h} \partial\psi\partial r \quad (20)$$

which after integration yields

$$T = - \frac{\mu\omega}{4h} (\psi_2 - \psi_1) (R_2^4 - R_1^4) \quad (21)$$

To compute the total viscous torque of the valve plate to cylinder block interface, the results gathered from Eq. (21) for all surfaces where viscous forces exist must be added together. Viscous forces also exist in the clearance between the cylinder block and the motor housing. The resisting torque created by these forces can be found by integrating the shear stress given by Eq. (18) over the lateral area of the cylinder block. By approximating the lateral surface of the cylinder block as a cylinder and integrating the shear stress over the lateral area of the cylinder, the torque created by viscous forces is found to be

$$T = - \frac{2\pi\mu L r^3 \omega}{h} \quad (22)$$

where

$$h \ll r \quad (23)$$

and where the variables are defined as

h = clearance

L = length of the cylinder block

r = outside diameter of the cylinder block

From Eqs. (21) and (22), it is evident that the torque due to viscous friction is proportional to both viscosity and angular velocity. Hence, the torque due to viscous friction can be expressed as

$$T_V = -K_V D \mu \omega \quad (24)$$

where K_V is a constant of proportionality for a given motor. The motor displacement, D , is included as part of the proportionality constant in Eq. (24) for mathematical convenience when one is experimentally estimating model parameters.

An examination of Fig. 3 reveals that the five bearings of a hydraulic motor are loaded by the pressure forces acting upon each of the motor pistons. A forces and moment analysis on the shaft and piston subassembly shows that the bearings will be loaded proportionally to the hydraulic pressures in motor ports A and B, assuming that the pressure in the motor case is negligible. Since the motor bearings are not ideal and have Coulomb friction, there exists an average frictional torque acting on the motor shaft according to the equation

$$T_{F1} = -D(P_A + P_B)(K_F + K_{FS} |\text{sgn}(\omega)|) \text{sgn}(\omega) \quad (25)$$

where

$$\text{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases} \quad (26)$$

and where the variables are defined as

K_F = a proportionality constant under non-stick-slip conditions

K_{FS} = a proportionality constant under stick-slip conditions

Shaft torques resulting from Coulomb friction are produced from the internal oil seals and from the piston motion within the barrel of the hydraulic motor. This internal friction is not related to the fluid pressures existing within the motor and is given by the relationship

$$T_{F2} = -T_{FC} (1 + K_S |\text{sgn}(\omega)|) \text{sgn}(\omega) \quad (27)$$

where T_{FC} is the constant friction torque at high speed and K_S is the proportionality constant under stick-slip conditions.

The torque output available to the hydraulic motor load is the sum of the torque effects due to the piston forces, motor inertia, viscous damping, and Coulomb friction. The net motor output torque to the load is therefore given by the equation

$$T_L = T_P + T_J + T_V + T_{F1} + T_{F2} \quad (28)$$

B. Hydraulic Motor Flow

Hydraulic motors are actuated by a control element such as a valve or pump which provides fluid flow to the motor. Hence, consideration of the fluid flow rate through the hydraulic motor is important from a system-level analysis point of view. By computing the rate at which volume is swept by the pistons using the geometrical relationships shown in Fig. 3 and Fig. 4, the theoretical flow rate through the motor, ignoring leakage effects, is found to be

$$Q_A = \sum_{i=1}^n K_{Ai} AR \sin \gamma \cos \theta_i \omega \quad (29)$$

$$Q_B = \sum_{i=1}^n K_{Bi} AR \sin \gamma \cos \theta_i \omega \quad (30)$$

$$K_{Ai} = 1 \quad \text{if } \cos \theta_i > 0 \quad (31)$$

$$K_{Ai} = 0 \quad \text{if } \cos \theta_i < 0 \quad (32)$$

$$K_{Bi} = 1 \quad \text{if } \cos \theta_i < 0 \quad (33)$$

The variables are defined as

A = area of one piston

K_{Ai} = a constant

K_{Bi} = a constant

Q_A = flow rate into motor port A

Q_B = flow rate into motor port B

R = pitch circle radius of the cylinder block bores

γ = motor housing angle

ω = angular velocity of the motor shaft

Substituting the motor displacement given by Eq. (6) into Eqs. (29) and (30) yields

$$Q_A = \frac{D\omega}{2n} \sum_{i=1}^n K_{Ai} \cos \theta_i \quad (36)$$

$$Q_B = \frac{D\omega}{2n} \sum_{i=1}^n K_{Bi} \cos \theta_i \quad (37)$$

It is clear that Eqs. (36) and (37) are difficult to implement within a computer simulation package because they involve very intensive computations. These equations also do not give any indication of the magnitude of the flow ripple relative to the mean value. By simulating the fluid flow through the motors using Eqs. (36) and (37), it is evident that the fluid flow rate through the motor can be approximated by the relations [2]

$$Q_A \approx \frac{D\omega}{2\pi} [C_1 + C_2 |\cos(n\theta_1)|] \quad \text{if } n = 2k + 1 \quad (38)$$

$$D\omega [C_1 + C_2 |\sin(n\theta_1)|] \quad \text{if } n = 4k \quad (39)$$

$$Q_A \approx \frac{D\omega}{2\pi} [C_1 + C_2 \left| \sin\left(\frac{n\theta_1}{2}\right) \right|] \quad \text{if } n = 4k \quad (40)$$

The nondimensional constants C_1 and C_2 are computed by means of least-squares estimation using Eqs. (36) and (37) to obtain the actual motor flow rates. It turns out that the nondimensional constants have the same values as those for the torque expressed in Eqs. (8) through (10). As a result of Eqs. (13), (14), and (15), the constant C_2 physically represents the fraction of the mean flow equivalent to the peak to peak amplitude of the ripple where

$$\text{mean flow rate} = \frac{D\omega}{2\pi} \quad (43)$$

An evaluation of the flow rate through a hydraulic motor must consider the internal leakage of the motor. The leakage past the hydraulic motor pistons from the pressure and return ports to the case drain can be modeled as laminar flow in an annulus between a circular shaft and a cylinder. The leakage flow rate is given by the equation [3]

$$Q = \frac{\pi r c^3}{6\mu L} \left[1 + \frac{3}{2} \left(\frac{e}{c} \right)^2 \right] \Delta P \quad (44)$$

where

$$c \ll r \quad (45)$$

The variables are defined as

c = radial clearance

e = eccentricity of the piston

L = length of the leakage path

ΔP = the pressure difference between the motor chamber and the case

r = radius of the cylinder bore

During the operation of a hydraulic motor, fluid leaks between the valve plate and the cylinder block. The leakage occurs from the case drain as shown by the curved arrows in Fig. 6. Also shown in Fig. 6 are the variables for one of the four leakage paths from the motor ports to the case drain. The leakage flow rate is derived as follows [4]: The leakage flow rate through a finite slot is given by

$$Q = \frac{\Delta P b h^3}{12\mu l} \quad (46)$$

where the variables are defined as

b = slot width

h = slot thickness

l = slot length

ΔP = differential pressure

μ = absolute fluid viscosity

Applying Eq. (46) to a differential slot yields

$$Q = -\frac{2\pi r h^3}{12\mu} \frac{\partial P}{\partial r} \quad (47)$$

where the negative sign is used because flow occurs in the opposite direction of $\partial P/\partial r$. By rearranging Eq. (47) and integrating, the following expressions are obtained:

$$P = \int -\frac{6\mu Q}{\pi h^3 r} \partial r \quad (48)$$

$$P = -\frac{6\mu Q}{\pi h^3} \ln r + C \quad (49)$$

Applying boundary conditions and solving for the constant of integration, C , one obtains the relationship

$$Q = \frac{(P_1 - P_2) \pi h^3}{6\mu \ln \left(\frac{R_2}{R_1} \right)} \quad (50)$$

Since Eq. (50) gives the radial leakage over an angle of 2π radians while the motor leakage only occurs over an angle ϕ , Eq. (50) is adjusted by a factor of $\phi/2\pi$ to express the leakage from the motor port to the case drain as

$$Q = \frac{(P_1 - P_2) h^3 \phi}{12\mu \ln \left(\frac{R_2}{R_1} \right)} \quad (51)$$

where the variables are defined as

P_1 = pressure at radius 1

P_2 = pressure at radius 2

R_1 = radius 1

R_2 = radius 2

ϕ = angular length of kidney-shaped motor port in radians

Using Eqs. (44) and (51), it is clear that the internal leakage from the port to the case is proportional to the pressure difference between the port and the case and inversely proportional to the fluid viscosity. Because the length of the leakage path found in Eq. (44) changes with the angular shaft position, the leakage proportionality constant will be a function of the motor shaft angle. The internal leakage variation with shaft angle has been observed by Johnson [5]. Hence, the internal leakage from the motor ports to the case drain is expressed as

$$Q_{LA} = \frac{K(\theta)_{LA}(P_A - P_C)}{\mu} \quad (52)$$

$$Q_{LB} = \frac{K(\theta)_{LB}(P_B - P_C)}{\mu} \quad (53)$$

where

$K(\theta)_{LA}$ = constant of proportionality for leakage from port A to case

$K(\theta)_{LB}$ = constant of proportionality for leakage from port B to case

P_C = case pressure

Because the valve plate is symmetrical across the line dividing the A and B motor chambers, it is expected that

$$K(\theta)_{LA} = K(\theta - \varphi)_{LB} \quad (54)$$

where φ is a phase angle given by

$$\varphi = \frac{2\pi}{n} \quad (55)$$

and n is the number of pistons.

Internal hydraulic motor leakage exists between the two motor ports. The leakage flow between motor ports occurs across the valve plate to cylinder block sealing surface as illustrated in Fig. 7. Assume that the leakage path between the motor ports is modeled as a rectangular leakage path. The leakage flow rate for a single cross port leakage path is given by [6]

$$Q = \frac{bh^3}{12\mu l} \Delta P \quad (56)$$

Since there are two identical cross port leakage paths, the total cross port leakage is expressed as

$$Q = \frac{bh^3}{6\mu l} \Delta P \quad (57)$$

where the variables are defined as

b = width of leakage path

h = clearance

l = length of leakage path

ΔP = pressure difference across the motor

Equation (57) shows that the cross port leakage is proportional to the pressure difference between the motor ports and inversely proportional to the viscosity of the fluid. The leakage from port A to port B can be expressed as

$$Q_{AB} = \frac{K_{AB}(P_A - P_B)}{\mu} \quad (58)$$

where K_{AB} is a constant of proportionality determined through experiments.

The net flow passing into port A, port B, and the case of a hydraulic motor is determined by summing the effects of the rate at which volume is swept by the pistons and the effects of internal leakage. The net flow to port A and port B is

$$Q_{AN} = Q_A + Q_{LA} + Q_{AB} \quad (59)$$

$$Q_{BN} = Q_B + Q_{LB} - Q_{AB} \quad (60)$$

respectively, and the net leakage to the case is

$$Q_{CN} = Q_{LA} + Q_{LB} \quad (61)$$

Positive flow is defined as flow into the motor port or case.

III. Conclusion

This article has presented a mathematical model for a bent-axis hydraulic piston motor; the motor can be used to perform dynamic system simulations which predict system performance under various conditions. The model parameters must be experimentally determined for a particular motor before the model can be used effectively because motor manufacturers do not possess this type of information

or are unwilling to provide it. A discussion of the experimental test procedures required to determine the model parameters is beyond the scope of this article but can be obtained: see Bartos,¹ Merritt [3], and the *Hydraulic Fluid Power* documents [7,8].

¹ R. D. Bartos, *Statement of Work 332-SW-1015: Hydraulic Motor and Relief Valve Package Experimental Evaluation* (internal document), Jet Propulsion Laboratory, Pasadena, California, January 7, 1992.

References

- [1] *Operations and Maintenance Information*, Vickers Hydraulic Piston Type Motors for Industrial Equipment Operation and Maintenance Information, I-1424-S, Vickers Incorporated, a TRINOVA company, Troy, Michigan, January 1, 1955.
- [2] J. Watton, *Fluid Power Systems: Modeling, Simulation, Analog, and Microcomputer Control*, New York: Prentice Hall, 1989.
- [3] H. E. Merritt, *Hydraulic Control Systems*, New York: John Wiley and Sons, 1967.
- [4] D. D. Fuller, *Theory and Practice of Lubrication for Engineers*, New York: John Wiley and Sons, 1956.
- [5] J. L. Johnson, "Predicting Low Speed Performance Of Hydraulic Motors By Digital Computer Simulation," *Proceedings of the National Conference on Fluid Power*, vol. 34, pp. 247-261, Cleveland, Ohio, October 28-30, 1980.
- [6] J. Halling, *Principles of Tribology*, London: Macmillan Education Ltd., 1978.
- [7] *Hydraulic Fluid Power: Determination of Characteristics of Motors, Part 1: At Constant Low Speed and At Constant Pressure*, ISO 4392-1, Global Engineering Documents, Irvine, California, August 15, 1989.
- [8] *Hydraulic Fluid Power: Determination of Characteristics of Motors, Part 2: Startability*, ISO 4392-2, Global Engineering Documents, Irvine, California, August 15, 1989.

Table 1. Nondimensional parameter values for $n = 2k + 1$.

Number of pistons	C_1	C_2	$2\pi\sigma/D\Delta P$ or $2\pi\sigma/D\omega$
3	9.1331e-01	1.3617e-01	2.1080e-03
5	9.6932e-01	4.8183e-02	8.0199e-04
7	9.8442e-01	2.4468e-02	4.1507e-04
9	9.9059e-01	1.4773e-02	2.5254e-04
11	9.9371e-01	9.8797e-03	1.6955e-04
13	9.9550e-01	7.0697e-03	1.2159e-04
15	9.9662e-01	5.3083e-03	9.1425e-05
17	9.9737e-01	4.1318e-03	7.1228e-05
19	9.9789e-01	3.3072e-03	5.7050e-05

C_1 = a dimensionless constant.

C_2 = a dimensionless constant.

σ = standard deviation of the approximation error.

$2\pi\sigma/D\Delta P$ = standard deviation of the approximation error normalized with respect to the average torque.

$2\pi\sigma/D\omega$ = standard deviation of the approximation error normalized with respect to the average flow rate.

Table 2. Nondimensional parameter values for $n = 4k - 2$.

Number of pistons	C_1	C_2	$2\pi\sigma/D\Delta P$ or $2\pi\sigma/D\omega$
2	2.9905e-16	1.5708e+00	6.5742e-16
6	9.1331e-01	1.3617e-01	2.1080e-03
10	9.6932e-01	4.8183e-02	8.0199e-04
14	9.8442e-01	2.4468e-02	4.1507e-04
18	9.9059e-01	1.4773e-02	2.5254e-04

C_1 = a dimensionless constant.

C_2 = a dimensionless constant.

σ = standard deviation of the approximation error.

$2\pi\sigma/D\Delta P$ = standard deviation of the approximation error normalized with respect to the average torque.

Table 3. Nondimensional parameter values for $n = 4k$.

Number of pistons	C_1	C_2	$2\pi\sigma/D\Delta P$ or $2\pi\sigma/D\omega$
4	7.9791e-01	3.1740e-01	4.2129e-03
8	9.5176e-01	7.5756e-02	1.2421e-03
12	9.7874e-01	3.3390e-02	5.6651e-04
16	9.8808e-01	1.8727e-02	3.2147e-04
20	9.9238e-01	1.1970e-02	2.0657e-04

C_1 = a dimensionless constant.

C_2 = a dimensionless constant.

σ = standard deviation of the approximation error.

$2\pi\sigma/D\Delta P$ = standard deviation of the approximation error normalized with respect to the average torque.

$2\pi\sigma/D\omega$ = standard deviation of the approximation error normalized with respect to the average flow rate.

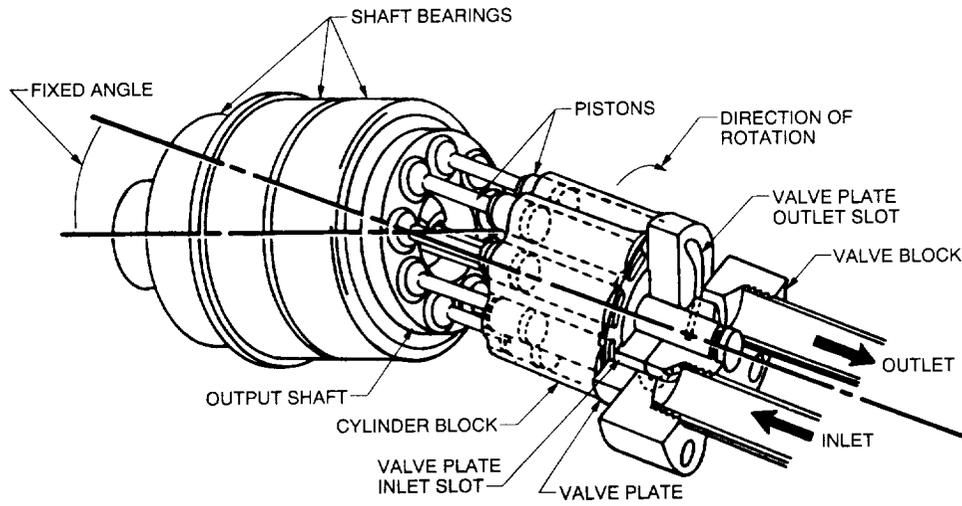
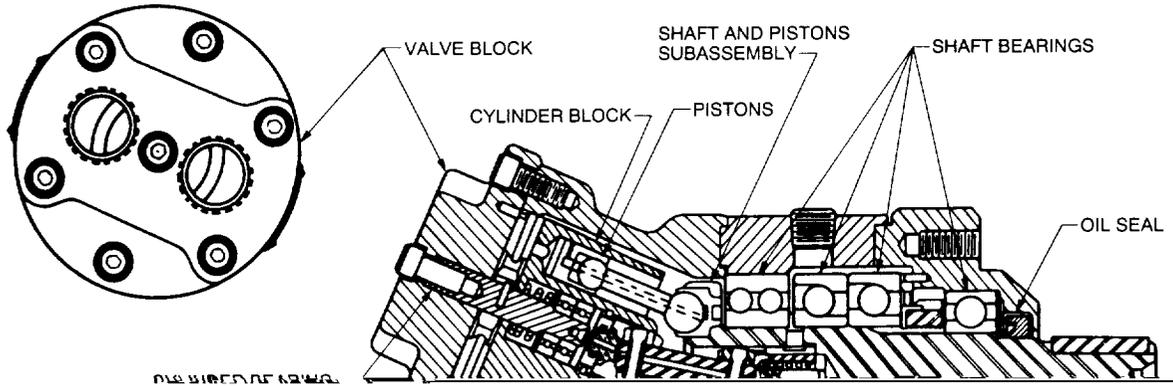
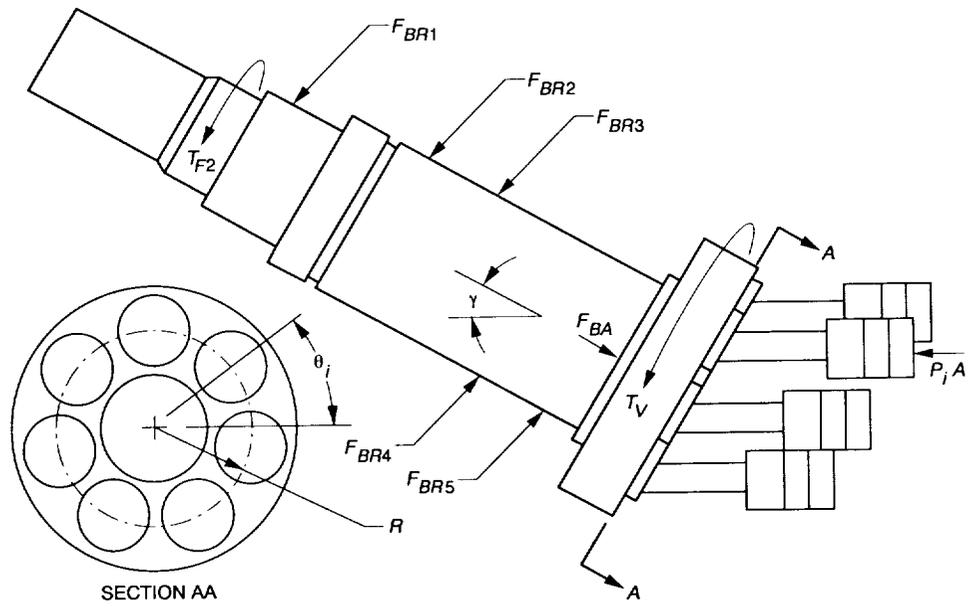


Fig. 1. Basic bent-axis hydraulic piston motor operation [1].





- | | | | |
|-----------|--------------------------------------|------------|--|
| A | PISTON AREA | T_{F2} | TORQUE DUE TO OIL SEAL AND PISTON FRICTION |
| F_{BA} | AXIAL REACTION FORCE OF BEARINGS | T_V | TORQUE DUE TO VISCOUS FORCES |
| F_{BRi} | RADIAL REACTION FORCE OF BEARING i | γ | MOTOR HOUSING ANGLE |
| P_i | PRESSURE ACTING ON PISTON i | θ_i | PISTON ANGLE FROM REFERENCE OF PISTON i |
| R | PITCH CIRCLE RADIUS | | |

Fig. 3. Shaft and piston subassembly of a bent-axis hydraulic piston motor.

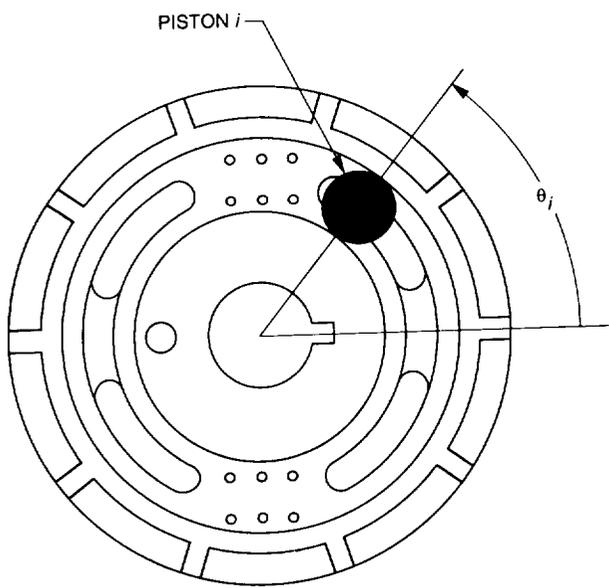


Fig. 4. Valve plate sealing surface of a bent-axis hydraulic piston motor and piston angle relative to the valve plate.

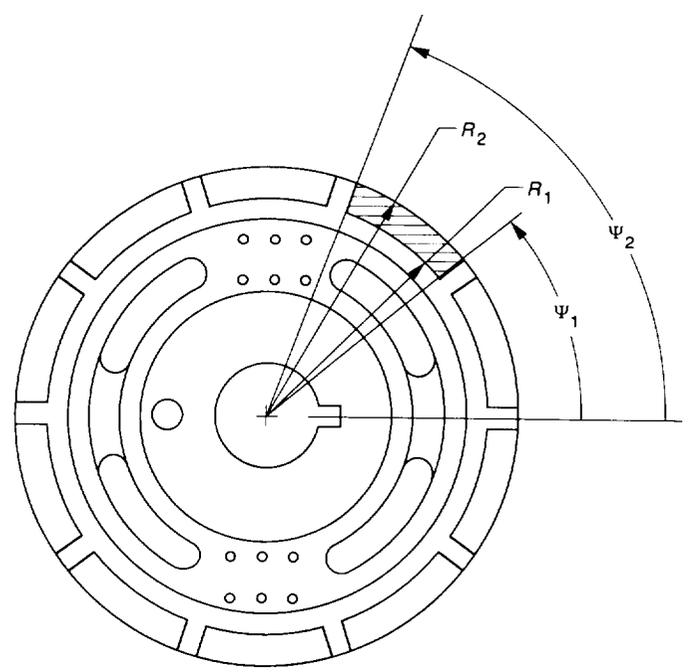


Fig. 5. One region of fluid shear stress acting between the valve plate to motor cylinder block interface.

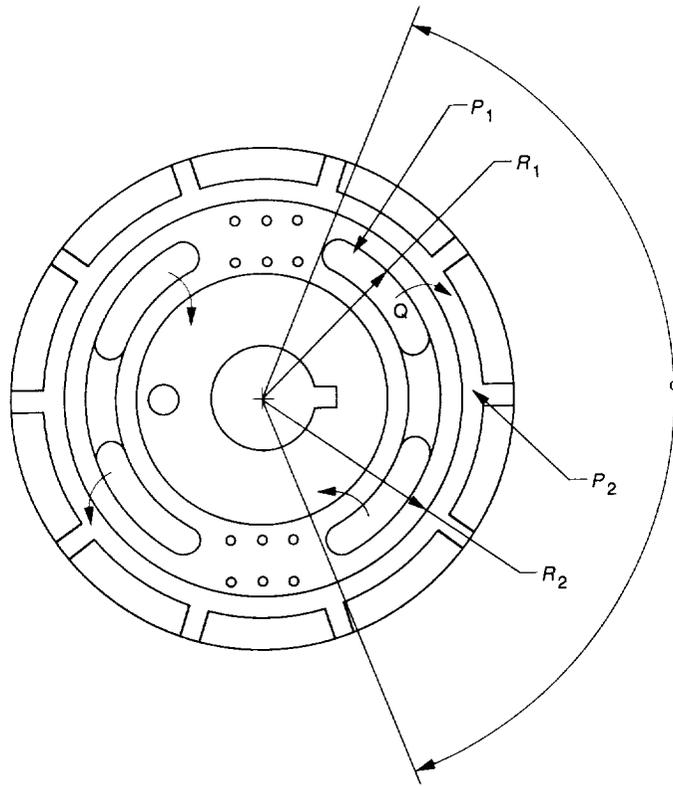


Fig. 6. Leakage flow paths across the valve plate sealing surfaces to the motor case.

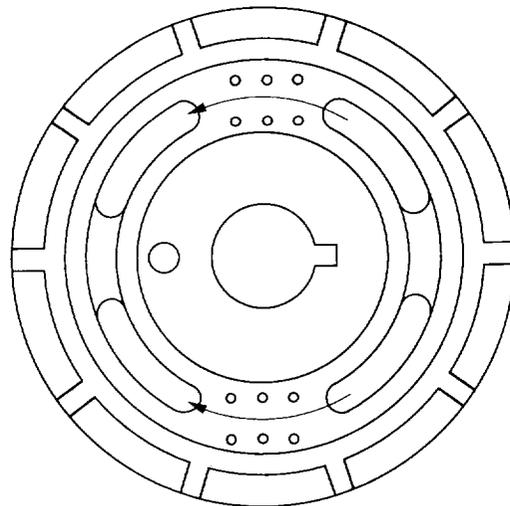


Fig. 7. Leakage flow path across the valve plate sealing surfaces between the two motor ports.

Appendix

<p>A = area of one piston</p> <p>b = slot width, width of leakage path</p> <p>c = radial clearance</p> <p>C = a constant of integration</p> <p>C_1 = a dimensionless constant</p> <p>C_2 = a dimensionless constant</p> <p>D = motor displacement per revolution</p> <p>e = eccentricity of the piston</p> <p>h = clearance, slot thickness</p> <p>i = piston number</p> <p>J = motor inertia</p> <p>k = integer</p> <p>K_{Ai} = a constant</p> <p>K_{Bi} = a constant</p> <p>$K(\theta)_{AB}$ = a constant of proportionality related to the leakage between motor ports</p> <p>K_F = a proportionality constant under kinetic friction conditions</p> <p>K_{FS} = a proportionality constant under static friction conditions</p> <p>$K(\theta)_{LA}$ = a constant of proportionality for leakage from port A to case</p> <p>$K(\theta)_{LB}$ = a constant of proportionality for leakage from port B to case</p> <p>K_S = a proportionality constant under static friction conditions</p> <p>K_V = a constant of proportionality related to viscous friction</p> <p>l = slot length, length of leakage path</p> <p>L = cylinder block length</p> <p>n = number of pistons</p> <p>P = pressure</p> <p>P_A = pressure of motor port A</p> <p>P_B = pressure of motor port B</p> <p>P_C = case pressure</p> <p>P_i = pressure acting on piston i, pressure at boundary i</p>	<p>ΔP = differential pressure across the motor</p> <p>Q_A = theoretical fluid flow rate into motor port A</p> <p>Q_{AB} = leakage flow rate from port A to port B</p> <p>Q_{AN} = net fluid flow rate into motor port A</p> <p>Q_B = theoretical fluid flow rate into motor port B</p> <p>Q_{BN} = net fluid flow rate into motor port B</p> <p>Q_{CN} = net leakage flow to the case drain</p> <p>Q_{LA} = internal leakage from port A to case</p> <p>Q_{LB} = internal leakage from port B to case</p> <p>r = radius</p> <p>R = pitch circle radius of the cylinder block</p> <p>R_i = radius at boundary i</p> <p>T = torque</p> <p>T_{FC} = constant friction torque at high speed independent of port pressures</p> <p>T_{F1} = torque due to Coulomb friction originating from motor bearings</p> <p>T_{F2} = torque due to Coulomb friction originating from oil seals and piston motion</p> <p>T_J = torque to accelerate the inertial load</p> <p>T_L = torque delivered to the load</p> <p>T_P = torque generated by fluid pressure</p> <p>T_V = torque required to overcome viscous friction</p> <p>x = a variable</p> <p>α = angular acceleration of the motor shaft</p> <p>γ = motor housing angle</p> <p>θ_i = angle of piston i</p> <p>μ = absolute fluid viscosity</p> <p>ρ = fluid density</p> <p>τ = shear stress</p> <p>φ = phase angle</p> <p>ϕ = angular length of kidney-shaped port</p> <p>Ψ = angle</p> <p>ω = angular velocity of the motor shaft</p>
---	--