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Reducing the Net Torque and Flow Ripple Effects of Multiple Hydraulic Piston Motor Drives

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The torque and flow ripple effects which result when multiple hydraulic motors are used to drive a single motion of a mechanical device can significantly affect the way in which the device performs. This article presents a mathematical model describing the torque and flow ripple effects of a bent-axis hydraulic piston motor. The model is used to show how the ripple magnitude can be reduced when multiple motors are used to drive a motion. A discussion of the hydraulic servo system of the 70m antennas located within the Deep Space Network is included to demonstrate

the application of the concepts presented.

I. Introduction

Multiple bent-axis hydraulic piston motors are frequently used to drive mechanical devices because they are capable of providing high torque while being physically small and lightweight. The output torque and flow rate of a bent-axis hydraulic piston motor vary with the angular position of a hydraulic motor shaft. Because most controllers are designed without regard for these variations in output torque and fluid flow rates, the performance of some machines may be significantly less than expected. This article presents a mathematical model for the output torque and fluid flow rate of a bent-axis hydraulic piston motor. This model is subsequently used to derive a method of reducing the torque and flow rate variations in hydraulic systems where multiple hydraulic motors are used to drive a single device. A glossary is provided in the Appendix.

resultant magnitude of the torque and flow ripple experienced by the device is significantly affected by how the relative angle between the reference piston and the valve plate within each motor is phased with the corresponding relative angles of the other hydraulic motors. The convention in this article for defining the angle between the reference motor piston and the valve plate is shown in Fig. 1. This section describes how the resultant torque and flow ripple can be reduced by properly phasing the hydraulic motors when multiple motors are used to drive a device.

A. Motor Torque

The analysis of the effects of motor phasing on the net torque ripple delivered to a device begins with the equation for the torque generated by the pressure forces acting on the pistons of a single bent-axis hydraulic piston motor, which is given by [1],

where

$$\theta_i = \theta_1 + \frac{2\pi(i-1)}{n} \quad (2)$$

$$P_i = P_A \quad \text{if } \cos \theta_i > 0 \quad (3)$$

$$P_i = P_B \quad \text{if } \cos \theta_i < 0 \quad (4)$$

$$k = 0, \pm 1, \pm 2, \pm 3, \dots \quad (5)$$

The variables are defined as

D = motor displacement per revolution

k = integer

n = the number of pistons

P_A = pressure at motor port A

P_B = pressure at motor port B

P_i = pressure acting on piston i

θ_i = angle of the i th piston with respect to the valve plate, as shown in Fig. 1.

ω = angular velocity of the motor shaft

Based on Eq. (1), the total resultant torque generated by m motors driving a single device while operating at the same speed is given by

$$T_P = \sum_{j=1}^m \frac{Dm}{2n} \sum_{i=1}^n P_{ij} \cos \theta_{ij} \quad (6)$$

where

$$\theta_{ij} = \theta_{11} + \frac{2\pi(i-1)}{n} + \phi_{1j} \quad (7)$$

$$P_{ij} = P_A \quad \text{if } \cos \theta_{ij} > 0 \quad (8)$$

$$P_{ij} = P_B \quad \text{if } \cos \theta_{ij} < 0 \quad (9)$$

$$k = 0, \pm 1, \pm 2, \pm 3, \dots \quad (10)$$

and where the variables are defined as

j = motor index number

P_{ij} = the pressure acting on piston i in motor j

ϕ_{1j} = the phase angle of the first piston in motor j

θ_{ij} = the angle of piston i in motor j

By using Eqs. (6) through (10), the maximum torque ripple is determined to occur when

$$\phi_{1j} = 0 \quad j = 1, 2, 3, \dots, m \quad (11)$$

while the minimum torque ripple is found to occur when

$$\phi_{1j} = \frac{\pi(j-1)}{mn} \quad \text{if } n = 2k + 1 \quad (12)$$

$$\phi_{1j} = \frac{2\pi(j-1)}{mn} \quad \text{if } n = 2k \quad (13)$$

$$k = 1, 2, 3, \dots \quad (14)$$

If the motor phasing is given by Eqs. (12) through (14), the total instantaneous torque generated by the hydraulic motors is equivalent to the instantaneous torque generated by a single motor with a displacement of mD and $2mn$ pistons if the motor has an odd number of pistons, or a displacement of mD and mn pistons if the motor has an even number of pistons. The approximations for the generated torque given by [1],

$$T_P \approx \frac{D_e \Delta P}{2\pi} [C_1 + C_2 |\cos(n_e \theta_1)|] \quad \text{if } n_e = 2k + 1 \quad (15)$$

$$T_P \approx \frac{D_e \Delta P}{2\pi} \left[C_1 + C_2 \left| \cos \left(\frac{n_e \theta_1}{2} \right) \right| \right] \quad \text{if } n_e = 4k - 2 \quad (16)$$

$$T_P \approx \frac{D_e \Delta P}{2\pi} \left[C_1 + C_2 \left| \sin \left(\frac{n_e \theta_1}{2} \right) \right| \right] \quad \text{if } n_e = 4k \quad (17)$$

$$\Delta P = P_A - P_B \quad (18)$$

$$k = 1, 2, 3, \dots \quad (19)$$

can be used to analyze systems in which the ripple magnitude is either a maximum or a minimum in relationship to the motor phasing. In these equations, D_e represents the equivalent motor displacement, n_e represents the equivalent number of pistons, and C_1 and C_2 are nondimensional constants presented in Tables 1 through 3. The variables D_e and n_e are given by

$$D_e = mD \quad (20)$$

$$n_e = n \quad (21)$$

when the torque ripple magnitude is a maximum and

$$D_e = mD \quad (22)$$

$$n_e = mn \quad \text{if } n = 2k \quad (23)$$

$$n_e = 2mn \quad \text{if } n = 2k + 1 \quad (24)$$

when the torque ripple is a minimum. Notice that

$$n_e = 2n \quad \text{when } m = 1 \quad \text{and } n = 2k + 1 \quad (25)$$

Equation (25) shows why a manufacturer will not produce a motor with an even number of pistons, since a ripple of lower magnitude can be achieved using fewer pistons if there are an odd number of them.

B. Motor Flow

In applications where multiple hydraulic motors are actuated using a single control element, flow variations through the control element and pressure variations in the hydraulic plumbing can be reduced by phasing the motors properly. The flow rate derived from the rate at which volume is swept by the motor pistons is given by [1]

$$Q_A = \frac{D\omega}{2n} \sum_{i=1}^n K_{Ai} \cos \theta_i \quad (26)$$

$$Q_B = \frac{D\omega}{2n} \sum_{i=1}^n K_{Bi} \cos \theta_i \quad (27)$$

$$K_{Ai} = 1 \quad \text{if } \cos \theta_i > 0 \quad (28)$$

$$K_{Ai} = 0 \quad \text{if } \cos \theta_i < 0 \quad (29)$$

$$K_{Bi} = 1 \quad \text{if } \cos \theta_i < 0 \quad (30)$$

$$K_{Bi} = 0 \quad \text{if } \cos \theta_i > 0 \quad (31)$$

$$\theta_i = \theta_1 + \frac{2\pi(i-1)}{n} \quad (32)$$

where

D = motor displacement per revolution

K_{Ai} = a constant

K_{Bi} = a constant

n = the number of pistons

Q_A = flow rate into motor port A

Q_B = flow rate into motor port B

θ_i = angle of the i th piston with respect to the valve plate, as shown in Fig. 1

ω = angular velocity of the motor shaft

Under the conditions when multiple hydraulic motors are operated, the total instantaneous flow rate through the motors is found, using Eqs. (26) and (27), to be

$$Q_A = \sum_{j=1}^m \frac{D\omega}{2n} \sum_{i=1}^n K_{Aij} \cos \theta_{ij} \quad (33)$$

$$Q_B = \sum_{j=1}^m \frac{D\omega}{2n} \sum_{i=1}^n K_{Bij} \cos \theta_{ij} \quad (34)$$

where

$$\theta_{ij} = \theta_{11} + \frac{2\pi(i-1)}{n} + \phi_{1j} \quad (35)$$

$$K_{Aij} = 1 \quad \text{if } \cos \theta_{ij} > 0 \quad (36)$$

$$K_{Aij} = 0 \quad \text{if } \cos \theta_{ij} < 0 \quad (37)$$

$$K_{Bij} = 1 \quad \text{if } \cos \theta_{ij} < 0 \quad (38)$$

$$K_{Bij} = 0 \quad \text{if } \cos \theta_{ij} > 0 \quad (39)$$

The variables are defined as

- j = motor index number
- K_{Aij} = a constant
- K_{Bij} = a constant
- ϕ_{1j} = the phase angle of the first piston in motor j
- θ_{ij} = the angle of piston i in motor j

Based upon Eqs. (33) through (39), the maximum flow ripple through the control element occurs when

$$\phi_{1j} = 0 \quad \text{if } j = 1, 2, 3, \dots, m \quad (40)$$

and the minimum occurs when

$$\phi_{1j} = \frac{\pi(j-1)}{mn} \quad \text{if } n = 2k+1 \quad (41)$$

$$\phi_{1j} = \frac{2\pi(j-1)}{mn} \quad \text{if } n = 2k \quad (42)$$

$$k = 1, 2, 3, \dots \quad (43)$$

It is important to note that the motor phasing at which the maximum and minimum flow ripples occur is the same as the phasing at which the maximum and minimum torque ripples occur. If the motor phasing is given by Eqs. (41) through (43), the total instantaneous flow rate through the control element is equivalent to the instantaneous flow rate of a single motor with a displacement of mD and $2mn$ pistons if the motor has an odd number of pistons, or a displacement of mD and mn pistons if the motor has an even number of pistons. The approximations for the instantaneous flow rate given by [1],

$$Q_A \approx \frac{D_e \omega}{2\pi} [C_1 + C_2 |\cos(n_e \theta_1)|] \quad \text{if } n_e = 2k+1 \quad (44)$$

$$Q_A \approx \frac{D_e \omega}{2\pi} \left[C_1 + C_2 \left| \cos\left(\frac{n_e \theta_1}{2}\right) \right| \right] \quad \text{if } n_e = 4k-2 \quad (45)$$

$$Q_A \approx \frac{D_e \omega}{2\pi} \left[C_1 + C_2 \left| \sin\left(\frac{n_e \theta_1}{2}\right) \right| \right] \quad \text{if } n_e = 4k \quad (46)$$

$$Q_B = -Q_A \quad (47)$$

$$k = 1, 2, 3, \dots \quad (48)$$

can be used to analyze systems in which the ripple magnitude is either a maximum or a minimum in relationship to the motor phasing. In these equations D_e represents the equivalent motor displacement, n_e represents the equivalent number of pistons, and C_1 and C_2 are nondimensional constants presented in Tables 1 through 3. The variables D_e and n_e are given in Eqs. (20) through (24) since they take on the same values as they do in the torque equations.

III. An Application

The Deep Space Network 70-m antenna hydraulic drive system is considered here in order to apply the analysis techniques presented. The drive system of the 70-m antennas for each axis of motion consists of four hydraulic bent-axis piston motors which are simultaneously controlled by a single servovalve. In addition, there are four counter-torque motors which have a constant differential pressure maintained across the ports. The configuration of the system for a single axis is shown in Fig. 2. The objective of this analysis is to reduce the net torque variations experienced by the antenna at a given differential pressure across the motors in order to improve the performance of the position controller. On the 70-m antennas the following parameter values apply:

$$D = 3.949 \times 10^{-5} \text{ m}^3/\text{revolution} \\ (2.41 \text{ in.}^3/\text{revolution}) \quad (49)$$

$$n = 7 \text{ pistons} \quad (50)$$

$$m = 4 \text{ motors} \quad (51)$$

$$\text{Transmission line length between the servovalve} \\ \text{and motors 1 and 3} = 22.352 \text{ m (880 in.)} \quad (52)$$

$$\text{Transmission line length between the servovalve} \\ \text{and motors 2 and 4} = 11.430 \text{ m (450 in.)} \quad (53)$$

The analysis begins by first considering the phasing of the counter-torque motors. It is evident from Fig. 2 that two of the counter-torque motors create a torque on the antenna opposite the direction of the other two counter-torque motors. Since the differential pressure across all of the counter-torque motors is maintained at 4136.88 kilopascals (kPa), or 600 pounds per square inch differential (PSID), the instantaneous torque delivered to the antenna can be maintained at zero independent of the angular position of the antenna, if the counter-torque motors are all in

phase. This occurs because the instantaneous torque generated by two of the counter-torque motors exactly cancels the instantaneous torque generated by the other two counter-torque motors. The second consideration in reducing the torque variations with respect to the antenna angular position is the phasing of the four drive motors. All of the hydraulic motors have approximately the same pressure drop across the motor ports at any instant in time since they are connected in parallel to the same servovalve. Any differential pressure variations that exist among them are due to the pressure drops across the various lengths of pipe connecting the servovalve to each of the motors. These pressure drops are generally assumed to be negligible since the flow rates through the pipes under tracking conditions are very small in relation to the diameter of the pipes. Hence, the optimal phasing for the four motors with seven pistons each, as given by Eqs. (12) and (41), is

$$\phi_{11} = 0 \text{ radians} \quad (54)$$

$$\phi_{12} = \frac{\pi}{28} \text{ radians} \quad (55)$$

$$\phi_{13} = \frac{\pi}{14} \text{ radians} \quad (56)$$

$$\phi_{14} = \frac{3\pi}{28} \text{ radians} \quad (57)$$

These phases were selected such that the motor pairs (motor 1 and motor 3) and (motor 2 and motor 4) had the optimal phasing of a two-motor drive system because the length of the hydraulic line from the servovalve is the same for each motor of a given pair. [See Eqs. (52) and (53).] The total torque exerted on each antenna axis for both the best and worst phasing of the motors and counter-torque motors for a differential motor pressure of 4826.36

kPa (700 PSID) is presented in Fig. 3, and the total flow passing through the motors for the best and worst phasing of the motors and counter-torque motors for a rotational speed of one radian per second is presented in Fig. 4. Upon examination of Fig. 3, it is clear that the peak-to-peak magnitude of the torque ripple experienced by the antenna is approximately 1.5818×10^5 newton-meters (N-m) under the worst phasing of the hydraulic motors and counter-torque motors, while the peak-to-peak magnitude is approximately 5.3017×10^3 N-m under the best phasing conditions. The 1.5818×10^5 N-m peak-to-peak amplitude which occurs under the worst phasing conditions is considered significant since the amplitude is equivalent to the axis torque created by a 2.69 m/sec wind load [2]. Figure 4 shows that the flow variations experienced by the servovalve are 6.1416×10^{-7} m³/sec and 3.8243×10^{-8} m³/sec under the worst and best phasing conditions, respectively. The results obtained through this analysis without consideration of friction, leakage, or line capacitance indicate that the performance of the 70-m antennas may be improved by properly phasing the motors, thus reducing the magnitude of system nonlinearities.

IV. Conclusion

Frequently multiple bent-axis piston type hydraulic motors are used to drive a single motion of a mechanical device. With this type of system configuration, the phase of the pistons within each motor relative to the pistons of the other motors can significantly affect the magnitude of the torque ripple experienced by the device and the flow variations experienced by the control element which provides fluid to the motors. This article has described how the minimum torque and flow variations can be obtained. An example related to the hydraulic system of the 70-m antennas within the Deep Space Network illustrated how the concepts presented can be applied to improve system performance.

References

- [1] R. D. Bartos, "Mathematical Modeling of Bent-Axis Hydraulic Piston Motors," *TDA-Dynamics Report 10 111*, vol. July-September, 1992. Jet Propulsion Labo-

Table 1. Nondimensional parameter values for $n_g = 2k + 1$.

Number of pistons	C_1	C_2	$2\pi\sigma/D\Delta P$ or $2\pi\sigma/D\omega$	Number of pistons	C_1	C_2	$2\pi\sigma/D\Delta P$ or $2\pi\sigma/D\omega$
3	9.1331e-01	1.3617e-01	2.1080e-03	53	9.9973e-01	4.2478e-04	7.3442e-06
5	9.6932e-01	4.8183e-02	8.0199e-04	55	9.9975e-01	3.9444e-04	6.8199e-06
7	9.8442e-01	2.4468e-02	4.1507e-04	57	9.9977e-01	3.6725e-04	6.3498e-06
9	9.9059e-01	1.4773e-02	2.5254e-04	59	9.9978e-01	3.4277e-04	5.9267e-06
11	9.9371e-01	9.8797e-03	1.6955e-04	61	9.9980e-01	3.2066e-04	5.5445e-06
13	9.9550e-01	7.0697e-03	1.2159e-04	63	9.9981e-01	3.0062e-04	5.1981e-06
15	9.9662e-01	5.3083e-03	9.1425e-05	65	9.9982e-01	2.8241e-04	4.8832e-06
17	9.9737e-01	4.1318e-03	7.1228e-05	67	9.9983e-01	2.6580e-04	4.5961e-06
19	9.9789e-01	3.3072e-03	5.7050e-05	69	9.9984e-01	2.5061e-04	4.3335e-06
21	9.9828e-01	2.7069e-03	4.6717e-05	71	9.9985e-01	2.3669e-04	4.0929e-06
23	9.9856e-01	2.2564e-03	3.8956e-05	73	9.9986e-01	2.2390e-04	3.8717e-06
25	9.9878e-01	1.9097e-03	3.2979e-05	75	9.9986e-01	2.1212e-04	3.6680e-06
27	9.9896e-01	1.6372e-03	2.8279e-05	77	9.9987e-01	2.0124e-04	3.4799e-06
29	9.9910e-01	1.4191e-03	2.4516e-05	79	9.9988e-01	1.9118e-04	3.3060e-06
31	9.9921e-01	1.2418e-03	2.1457e-05	81	9.9988e-01	1.8185e-04	3.1448e-06
33	9.9930e-01	1.0958e-03	1.8936e-05	83	9.9989e-01	1.7319e-04	2.9950e-06
35	9.9938e-01	9.7415e-04	1.6835e-05	85	9.9989e-01	1.6514e-04	2.8558e-06
37	9.9945e-01	8.7166e-04	1.5065e-05	87	9.9990e-01	1.5763e-04	2.7260e-06
39	9.9950e-01	7.8454e-04	1.3560e-05	89	9.9990e-01	1.5063e-04	2.6049e-06
41	9.9955e-01	7.0985e-04	1.2270e-05	91	9.9991e-01	1.4408e-04	2.4916e-06
43	9.9959e-01	6.4535e-04	1.1156e-05	93	9.9991e-01	1.3795e-04	2.3856e-06
45	9.9962e-01	5.8925e-04	1.0187e-05	95	9.9992e-01	1.3220e-04	2.2862e-06
47	9.9966e-01	5.4017e-04	9.3383e-06	97	9.9992e-01	1.2681e-04	2.1929e-06
49	9.9968e-01	4.9697e-04	8.5918e-06	99	9.9992e-01	1.2174e-04	2.1052e-06
51	9.9971e-01	4.5875e-04	7.9313e-06				

C_1 = a dimensionless constant.

C_2 = a dimensionless constant.

σ = standard deviation of the approximation error.

$2\pi\sigma/D\Delta P$ = standard deviation of the approximation error normalized with respect to the average torque.

$2\pi\sigma/D\omega$ = standard deviation of the approximation error normalized with respect to the average flow rate.

Table 2. Nondimensional parameter values for $n_{\theta} = 4k - 2$.

Number of pistons	C_1	C_2	$2\pi\sigma/D\Delta P$ or $2\pi\sigma/D\omega$
2	2.9905e-16	1.5708e+00	6.5742e-16
6	9.1331e-01	1.3617e-01	2.1080e-03
10	9.6932e-01	4.8183e-02	8.0199e-04
14	9.8442e-01	2.4468e-02	4.1507e-04
18	9.9059e-01	1.4773e-02	2.5254e-04
22	9.9371e-01	9.8797e-03	1.6955e-04
26	9.9550e-01	7.0697e-03	1.2159e-04
30	9.9662e-01	5.3083e-03	9.1425e-05
34	9.9737e-01	4.1318e-03	7.1228e-05
38	9.9789e-01	3.3072e-03	5.7050e-05
42	9.9828e-01	2.7069e-03	4.6717e-05
46	9.9856e-01	2.2564e-03	3.8956e-05
50	9.9878e-01	1.9097e-03	3.2979e-05
54	9.9896e-01	1.6372e-03	2.8279e-05
58	9.9910e-01	1.4191e-03	2.4516e-05
62	9.9921e-01	1.2418e-03	2.1457e-05
66	9.9930e-01	1.0958e-03	1.8936e-05
70	9.9938e-01	9.7415e-04	1.6835e-05
74	9.9945e-01	8.7166e-04	1.5065e-05
78	9.9950e-01	7.8454e-04	1.3560e-05
82	9.9955e-01	7.0985e-04	1.2270e-05
86	9.9959e-01	6.4535e-04	1.1156e-05
90	9.9962e-01	5.8925e-04	1.0187e-05
94	9.9966e-01	5.4017e-04	9.3383e-06
98	9.9968e-01	4.9697e-04	8.5918e-06

C_1 = a dimensionless constant.

C_2 = a dimensionless constant.

σ = standard deviation of the approximation error.

$2\pi\sigma/D\Delta P$ = standard deviation of the approximation error normalized with respect to the average torque.

$2\pi\sigma/D\omega$ = standard deviation of the approximation error normalized with respect to the average flow rate.

Table 3. Nondimensional parameter values for $n_{\theta} = 4k$.

Number of pistons	C_1	C_2	$2\pi\sigma/D\Delta P$ or $2\pi\sigma/D\omega$
4	7.9791e-01	3.1740e-01	4.2129e-03
8	9.5176e-01	7.5756e-02	1.2421e-03
12	9.7874e-01	3.3390e-02	5.6651e-04
16	9.8808e-01	1.8727e-02	3.2147e-04
20	9.9238e-01	1.1970e-02	2.0657e-04
24	9.9471e-01	8.3062e-03	1.4376e-04
28	9.9612e-01	6.0998e-03	1.0576e-04
32	9.9703e-01	4.6688e-03	8.1039e-05
36	9.9765e-01	3.6882e-03	6.4067e-05
40	9.9810e-01	2.9871e-03	5.1916e-05
44	9.9843e-01	2.4684e-03	4.2919e-05
48	9.9868e-01	2.0740e-03	3.6072e-05
52	9.9887e-01	1.7671e-03	3.0742e-05
56	9.9903e-01	1.5236e-03	2.6511e-05
60	9.9915e-01	1.3271e-03	2.3096e-05
64	9.9926e-01	1.1664e-03	2.0301e-05
68	9.9934e-01	1.0332e-03	1.7985e-05
72	9.9941e-01	9.2155e-04	1.6043e-05
76	9.9947e-01	8.2709e-04	1.4399e-05
80	9.9952e-01	7.4643e-04	1.2996e-05
84	9.9957e-01	6.7703e-04	1.1788e-05
88	9.9961e-01	6.1687e-04	1.0741e-05
92	9.9964e-01	5.6439e-04	9.8280e-06
96	9.9967e-01	5.1833e-04	9.0263e-06
100	9.9970e-01	4.7769e-04	8.3188e-06

C_1 = a dimensionless constant.

C_2 = a dimensionless constant.

σ = standard deviation of the approximation error.

$2\pi\sigma/D\Delta P$ = standard deviation of the approximation error normalized with respect to the average torque.

$2\pi\sigma/D\omega$ = standard deviation of the approximation error normalized with respect to the average flow rate.

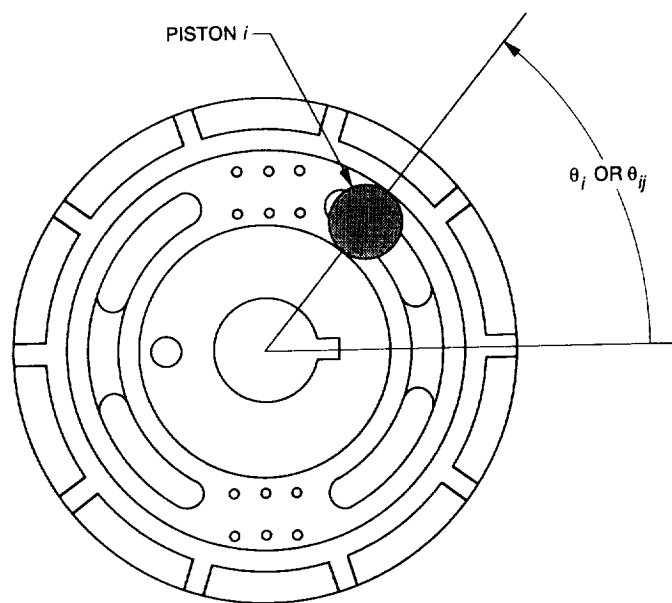


Fig. 1. The valve plate sealing surface of a bent-axis hydraulic piston motor and the piston angle relative to the valve plate.

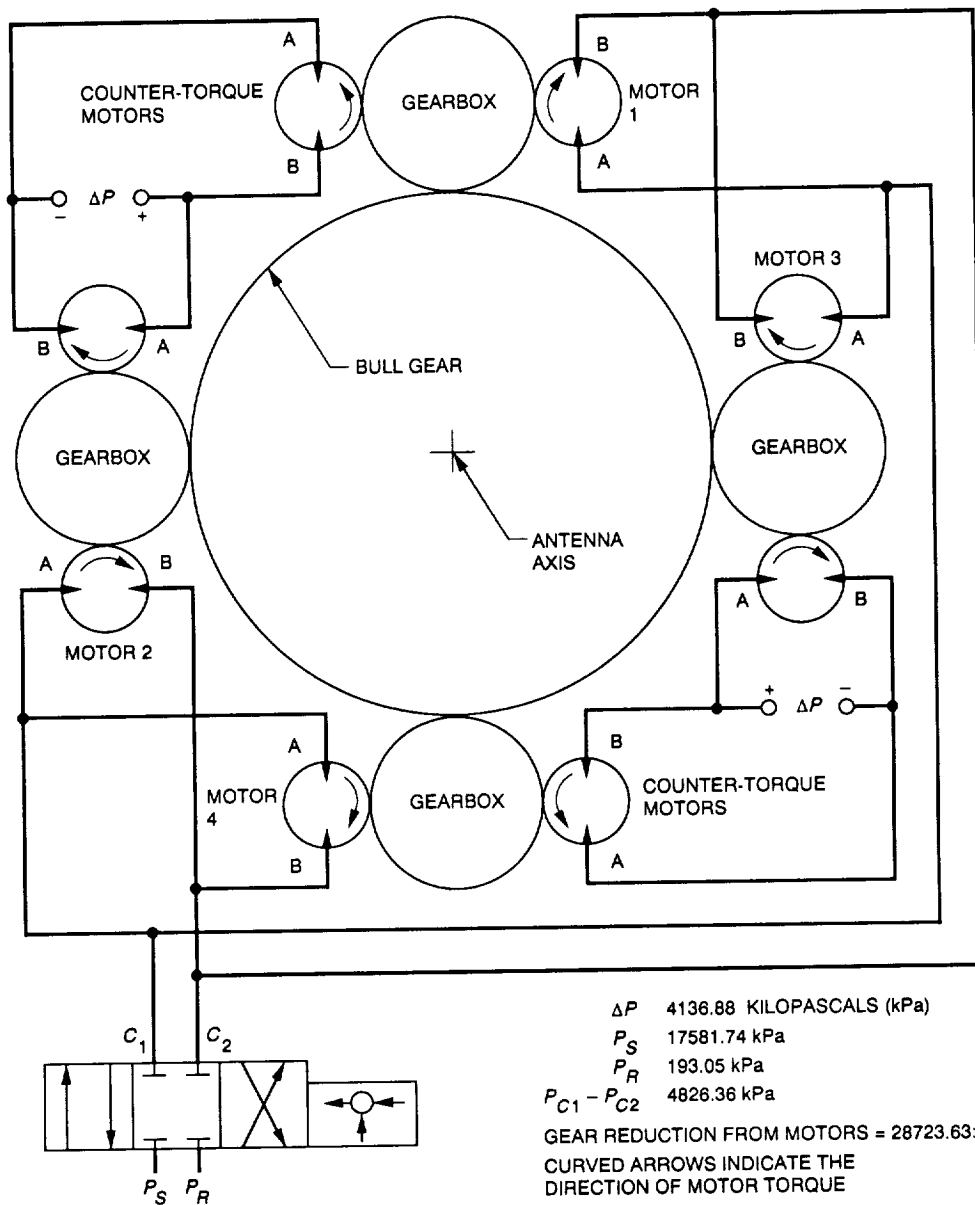


Fig. 2. System configuration of the 70-m antenna azimuth axis.

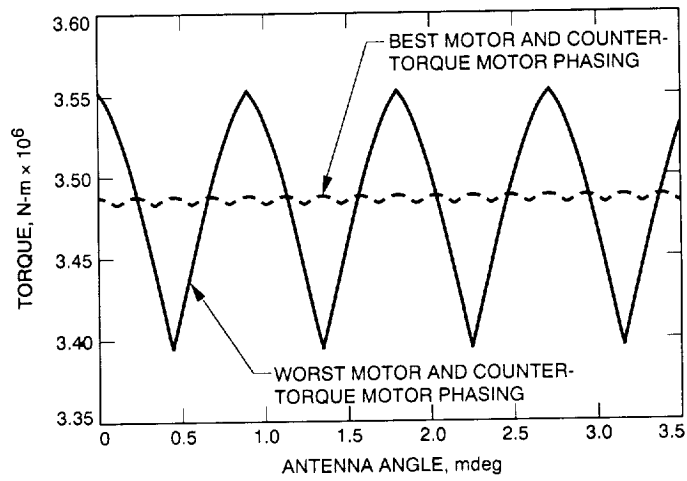


Fig. 3. The 70-m antenna azimuth axis torque as a function of the antenna angular position for the best and worst phasing of the hydraulic motors and counter-torque motors. The differential pressure is 4826.36 kilopascals (kPa) for the drive motors and 4136.88 kPa for the counter-torque motors.

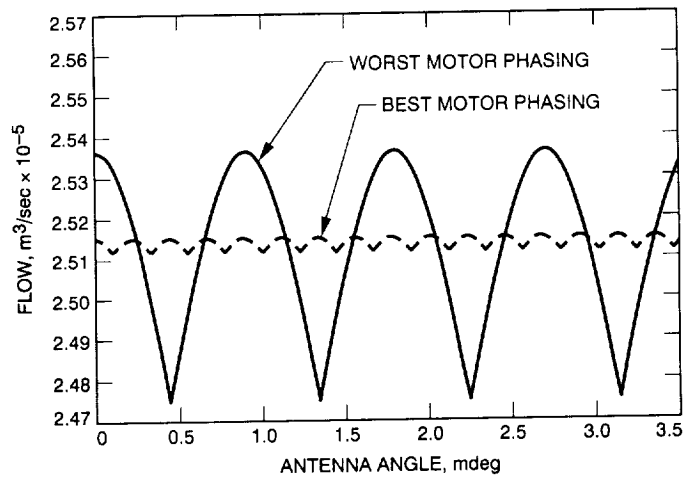


Fig. 4. Total instantaneous motor flow rate through the 70-m antenna azimuth hydraulic drive motors as a function of the antenna angular position for the best and worst phasing of the hydraulic motors. The angular velocity of the antenna is two millidegrees per second, which corresponds to a one radian per second angular velocity of the hydraulic motor shafts.