An Analysis of I/O Efficient Order-Statistic-Based Techniques for Noise Power Estimation in the HRMS Sky Survey's Operational System

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Noise power estimation in the High-Resolution Microwave Survey (HRMS) sky survey element is considered as an example of a constant false alarm rate (CFAR) signal detection problem. Order-statistic-based noise power estimators for CFAR detection are considered in terms of required estimator accuracy and estimator dynamic range. By limiting the dynamic range of the value to be estimated, the performance of an order-statistic estimator can be achieved by simpler techniques requiring only a single pass of the data. Simple threshold-and-count techniques are examined, and it is shown how several parallel threshold-and-count estimation devices can be used to expand the dynamic range to meet HRMS system requirements with minimal hardware complexity. An input/output (I/O) efficient limited-precision order-statistic estimator with wide but limited dynamic range is also examined.

I. Introduction

The purpose of this article is to examine I/O efficient methods for dynamically estimating noise power in the High Resolution Microwave Survey (HRMS) sky survey element. One I/O efficient estimation method was suggested during the development of the HRMS sky survey prototype system (SSPS) signal-processing subsystem by Dr. Bernard Oliver at NASA Ames Research Center.[1] At that time this technique, now called the "threshold-and-count" estimator, was labeled the "truncated data" method. It was examined during the SSPS development;[2,3,4] however, several relevant system param-

[1] B. Oliver, private communication, Deputy Chief of HRMS Office, NASA Ames Research Center, Moffett Field, California, summer 1991.

[2] R. Brady, "An Alternative to Using Order Statistics for Determining the Mean Noise Power Estimate in the EDM," Interoffice Memorandum, Jet Propulsion Laboratory, Pasadena, California, April 14, 1988.

[3] W. Deich, "Truncated Data and Background Estimation," Interoffice Memorandum, Jet Propulsion Laboratory, Pasadena, California, May 3, 1988.

[4] M. F. Garyantes, "Search For Extraterrestrial Intelligence Microwave Observing Project Sky Survey Element Subsystem Functional Requirements and Design BECAT 1 Processor (Prototype)," JPL D-7116, (internal document), Jet Propulsion Laboratory, Pasadena, California, December 1, 1989.

eters have since changed, and in an effort to save special-purpose hardware complexity and cost, a new analysis of this technique was undertaken. This article describes the analysis and results.

HRMS sky survey signal detection will operate on the output of a real-time, 640-MHz, 32-megachannel polyphase discrete Fourier transform (DFT) spectrum analyzer [1]. In normal operation, the spectrum analyzer will accumulate the power of from 2 to 10 spectra for input to the signal-detection assembly. The signal-detection assembly applies gain normalization and a five-tap finite impulse response (FIR) matched filter to each accumulated spectral channel. The matched filter outputs are then thresholded at a probability of false alarm, P_{FA}, of approximately 10^{-5} to reduce the data rate for data input to a general-purpose computer. Using adjacent scans across the sky, the computer excises interference and combines data. After the excision of interference, the data are passed through a second threshold at a much lower P_{FA}, and a relatively small number of sky positions passing the threshold are re-observed. Because of the large number of channels and limited re-observation time, the false alarm rate due to noise must be tightly controlled. False-alarm-rate control is accomplished by normalizing the accumulated power spectra with estimates of the noise power as a function of frequency.

Dynamic estimation of the noise environment is required for many signal-processing algorithms. Noise power estimates obtained from order statistics are robust in the presence of interference, and they can be adapted to variations in the statistical distribution of the noise samples [2,3,4,5]. In general, computation of a fixed order statistic requires that a sample population be sorted. Sorting techniques require multiple passes on a data set and can be I/O intensive. The advantages of the order-statistic estimation approach can be obtained by fixing the order statistic's value rather than fixing the rank of the order statistic, provided that the value remains within certain bounds. This article presents an analysis of the estimator requirements and introduces an additional performance requirement called the estimator dynamic range. A simple order-statistic-based estimator is analyzed, and simple modifications to extend the limited dynamic range of the estimator are presented. An efficient estimator with a wide dynamic range is formed by combining this simple estimator with a fixed-order-statistic technique. The requirements for noise power estimation and the solutions are discussed in the context of NASA's HRMS sky survey, although the approach is applicable to other constant false-alarm-rate (CFAR) detection applications.

II. Summary of Requirements

There are two performance requirements on the estimator. The first requirement is the obvious one of estimate accuracy. For interference immunity, methods under consideration in this article use an order statistic to estimate the noise power. As a result, in the limit of large population sizes (populations >1000), the error in the estimate will be normally distributed and zero mean [4]. When the estimate is used to control detection thresholds, as in HRMS, errors in noise power estimates change the P_{FA} for the detection algorithms. By specifying the allowable variation in the P_{FA}, the required estimator accuracy can be determined. A new requirement considered here is the estimator dynamic range. This is defined as the variation in noise power over which the estimator will perform within the specified error. For example, if the estimator can meet the accuracy requirement as the noise power varies over a factor-of-two range, the estimator has a dynamic range of 3 dB. This requirement depends on the variability in the detection environment and the underlying probability distribution. Thus, determination of the dynamic range requirement is dependent both on the observations to be performed and on the estimate accuracy requirement. Dynamic range is discussed following the discussion of estimate accuracy.

A. Estimate Accuracy

The noise power estimator in the HRMS system is used to control the data rate by setting detection thresholds. It is not intended to be used for radio-astronomy-continuum-measurement calibration. As a result, errors in the estimate will only change the data rate out of the sky survey signal-processing hardware, and will ultimately change the P_{FA} for the HRMS detection algorithms. The effect of errors in the noise power estimate, $\hat{T}$, on the data rate at the output of the hardware has been previously addressed.[5] However, because an error in $\hat{T}$ changes where the data rate–reducing threshold is located relative to the distribution function of the noise, the sensitivity of the data rate to estimate error is dependent on the position of the threshold. At the time of W. Deich's memorandum,[5] the hardware threshold was to be positioned to provide a P_{FA} of 10^{-3}; however, more recent analysis[6] has indicated that this threshold should be set at a P_{FA} ranging from 10^{-5} to 10^{-6}. Therefore, a new look at the accuracy requirement is needed.

[5] W. Deich, "Baseline Ripple, Estimation Error in $\hat{T}$, and False Alarms," Interoffice Memorandum, Jet Propulsion Laboratory, Pasadena, California, February 14, 1989.

[6] S. Levin, personal communication, Member of Technical Staff, Space Physics and Astrophysics Section, Jet Propulsion Laboratory, Pasadena, California, spring 1991.

When the data can come from more than one statistical distribution, it is useful to identify the limiting distribution as a case for analysis. Because the number of spectra per accumulated power spectral output is selectable in HRMS sky survey operations, the power spectral samples for white Gaussian noise input will come from χ^2_ν distributions with varying degrees of freedom (ν). For χ^2_ν data, the limiting case to consider in the analysis of the effect of estimate error on P_{FA} is that of the maximum number of degrees of freedom expected. Normal-rate operations in the HRMS sky survey require the greatest control over the P_{FA}, and these will have a maximum of 10 spectra per accumulated power spectral output.

At 10 spectra per accumulated output, each spectrum will consist of samples from the χ^2_{20} distribution. The value to be thresholded is a weighted sum of five of these samples weighted approximately by $\{0.64, 0.89, 1.00, 0.89, 0.64\}$ in a five-tap FIR filter matched to the antenna beam passing over a point source. The weighted sum can be approximated as samples of the χ^2_{81} distribution. For such a large number of degrees of freedom, it is appropriate to use the following approximation [Eq. (1)] for the probability that a sample is greater than a value x (see [6], Eq. 26.4.14).

$$\begin{aligned} \mathrm{Prob}[\chi^2_\nu > x] &= Q(x|\nu) \approx Q(x_2) \\ x_2 &= \frac{(x/\nu)^{1/3} - (1 - 2/(9\nu))}{\sqrt{2/(9\nu)}} \end{aligned} \tag{1}$$

Note that in the region of interest where $P_{FA} \leq 10^{-5}$, the following approximation [Eq. (2)] is valid:

$$Q(x_2) \approx \frac{e^{-x_2^2/2}}{x_2\sqrt{2\pi}} \tag{2}$$

Let the estimated threshold be related to the ideal threshold by $\hat{T} = (1+\alpha)T$; i.e., the fractional error is α. The probability of false alarm, given a fractional estimate error α, is:

$$\begin{aligned} \mathrm{Prob}[\chi^2_{81} > (1+\alpha)T] &\approx Q(\hat{x}_2) \approx \frac{e^{-\hat{x}_2{}^2/2}}{\hat{x}_2\sqrt{2\pi}} \\ \hat{x}_2 &\approx 19.092[(1+\alpha)^{1/3}(T/81)^{1/3} \\ &\quad + 0.99726] \end{aligned} \tag{3}$$

Figure 1 shows the effect of errors in the estimate on the probability of false alarm for 10 spectra per accumulation.

It is important to note that the noise power estimates are obtained on a per-spectrum basis, and it is the effective combined error of the five estimates that must meet the accuracy requirement. The errors in the methods under consideration are normally distributed, zero mean, and independent for each spectrum. It can be shown that the combined fractional error is approximately normally distributed, zero mean, with standard deviation

$$\sigma\sqrt{\frac{0.64^2 + 0.89^2 + 1 + 0.89^2 + 0.64^2}{0.64 + 0.89 + 1 + 0.89 + 0.64}} = 0.454\sigma \tag{4}$$

where σ is the standard error of an individual estimate. This results in a reduction of the fractional error by 54.6 percent. For example, a 1-percent error in the final effective estimate corresponds to an error of 2.2 percent in the noise power estimates for the individual spectra.

Conservative estimates of false alarms due to interference suggest that the data rate out of the special-purpose hardware will be a factor of 10 greater than the target P_{FA} of 10^{-5}. This implies that maintaining the data rate out of the special-purpose hardware requires that the noise power estimate be effectively accurate to within 6 percent. If such an error were specified as a 3-standard-error event, the effective estimator accuracy requirement for one standard error would be only 2 percent.

The significantly smaller P_{FA} required to select candidates for re-observation would tend to imply a stricter requirement on the noise power than that driven by the hardware data rate. Additional candidates translate directly into additional re-observation time, and as a result, it is unlikely to think that any more than a factor-of-2 variation would be tolerable. A 3-standard-error event corresponding to a factor-of-2 change would imply an effective standard error for the five estimates of about 0.37 percent. This corresponds to a standard error of 0.82 percent in the individual estimates. It is important to note, however, that determination of the P_{FA} for re-observation candidate selection uses many individual noise power estimates. Improvement of the noise power estimates or rejection of obviously bad data may be possible at this stage. Taking this possibility into account, and considering that a 3-standard-error event occurs with approximately 10^{-3} probability, the required error in the noise power estimate is taken to be less than 1 percent.[7]

[7] The error in the false alarm rate, defined as the difference between the target and the actual false alarm rate, will not be zero mean and will not, in general, be symmetrically distributed about the mean. However, this can be easily compensated for.

B. Dynamic Range of the Estimator

Once an estimate accuracy requirement has been defined, a dynamic range within which the estimator meets its accuracy requirement can be determined. In almost all estimation applications, the dynamic range of the estimated parameter can be limited. It is shown below that by limiting the dynamic range, one can construct simple, efficient order-statistic-based estimators.

A noise power estimator must be flexible enough to cover natural variations in received noise power. Unnatural variations that would result in unusable data, e.g., significant drops in receiver gain, need not be within the performance limits of the estimator. For HRMS or radio astronomy purposes, natural variations are mainly caused by four sources: changes in air mass as a scan changes elevation angle, increased water vapor as the antenna beam passes through a cloud or water vapor bubble in the atmosphere, the contribution from galactic or extragalactic radio sources at $\lambda = 21$ cm due to Doppler-broadened atomic hydrogen hyperfine emission, and the contributions from common strong astrophysical radio sources. For all of these sources, increases in system temperature will be a function of frequency. Compensating for the variations due to the passage of the beam across rare strong astrophysical sources is not considered a requirement, but is a goal.

Since the underlying statistical distribution affects the dynamic range of the estimator, the statistics under which the dynamic range must be met also need to be defined. The HRMS sky survey will vary the number of spectra per accumulation inversely as the sky frequency of the observations changes from 1 to 10 GHz. The nominal range is from 10 to 2 spectra per accumulated power spectral output. To accommodate slower scan rates, the nominal accumulation may be increased by a factor of 10. For this application, the system should also be capable of allowing the number of spectra per accumulation at 1 GHz to vary from 10 to 100 and the number of spectra per accumulation at 10 GHz to vary from 2 to 20.

The observing environment ultimately defines the required dynamic range. Conditions at Canberra, Australia, will be worse than those at Goldstone, California.[8] The HRMS system will have a nominal (i.e., cold, dry sky at zenith) system temperature of 25 K. The atmospheric contribution to the system temperature scales as the square of the frequency.

[8] Based on the models for the atmospheric contribution to system temperature in *DSN Standard Flight Project Interface Design Project Handbook* 810-5 (internal document), Revision D, Jet Propulsion Laboratory, Pasadena, California, September 15, 1991.

Dynamic range requirements for from 20 to 40, 40 to 60, 60 to 80, and 80 to 100 spectra per accumulation can be derived by using the translations from number of spectra per accumulation to RF center frequency that are given by Olsen,[9] and the planned slowdown factor of 10. Dynamic range calculations due to atmosphere alone are given in Table 1, and consideration of strong astrophysical sources is presented in Table 2. As shown in Table 3, the dynamic range requirement is for 7.1 dB of dynamic range for from 2 to 20 spectra per accumulated output, with a goal of 9.0 dB, and 3.8 dB of dynamic range for from 20 to 100 spectra, with a goal of from 4.2 to 4.8 dB.

III. Description of the Threshold-and-Count Estimator

The threshold-and-count estimator consists of a threshold applied to a population and a count of the number of points in the population that do not exceed the threshold.[10] Modifications may be made to this method to dynamically adjust the threshold, if necessary, or to apply multiple thresholds and pick the best one, but the basic method remains the same. Provided that the population is sufficiently dense around the threshold, this method is equivalent to choosing an order statistic for the noise power estimate.[11] However, the value of the order statistic is now fixed, as it is the threshold value, and the rank of the order statistic is the random variable. The result is an estimate of the value of the cumulative distribution function at the threshold, i.e., the probability that a sample is less than the threshold value. The noise power estimate can then be obtained from a look-up table, interpolating from the cumulative distribution function to recover the mean noise power of the population. A block diagram of the threshold-and-count estimator is shown in Fig. 2.

IV. Estimator Performance

As expected, and verified by simulation,[12] the performance of the threshold-and-count estimator is equivalent to choosing a true order statistic, given a sufficiently dense population of points around the threshold. Simulations

[9] E. T. Olsen, "Time Required to Complete the All Sky Survey Campaign," Interoffice Memorandum 1720-6025-3280, Jet Propulsion Laboratory, Pasadena, California, February 27, 1991.

[10] B. Oliver, personal communication, Deputy Chief of HRMS Office, NASA Ames Research Center, Moffett Field, California, summer 1991.

[11] R. Brady, op. cit.

[12] Ibid.

demonstrated[13] that 1 percent of 8192, or 82 points below the threshold, was a sufficient number for the variance of the order statistic to mask the threshold error due to finite sample density around the threshold. In the limit of large population size, order statistics are normally distributed, with a mean and variance for the rth smallest order statistic

$$E[x_{(r)}] = \xi = F^{-1}(r/N) \quad \mathrm{Var}[x_{(r)}] = \frac{r/N\left(1-\frac{r}{N}\right)}{N f(\xi)^2} \tag{5}$$

where $F()$ is the cumulative distribution function of the population, $f()$ is the population's probability density function, and N is the number of samples in the population. For accumulated power spectra of white Gaussian input voltage samples, the populations will be drawn from the χ^2_ν distribution with an even number of degrees of freedom with $\nu = 2\times$the number of spectra per accumulated output ($\nu = 2n$), with the probability density function

$$f(\xi) = \frac{\xi^{n-1}\, e^{-\xi/T_s}}{T_s^n\,(n-1)!} \tag{6}$$

Since the mean value of the order statistic is proportional to the mean noise power, and

$$E\left[\frac{r}{N}\right] = \mathrm{Prob}\left[\chi^2_{2n} \le \xi\right] = 1 - e^{-\beta}\sum_{i=0}^{n-1}\frac{\beta^i}{i!} \tag{7}$$

the fractional standard error is:

$$\frac{\sigma}{\xi} = \sqrt{\frac{r/N\,(1-r/N)}{N}\,\frac{T_s^n e^{\xi/T_s}(n-1)!}{\xi^n}}$$

$$= \sqrt{\frac{r/N\,(1-r/N)}{N}\,\frac{e^{\beta}(n-1)!}{\beta^n}} \;;\; \beta = \frac{\xi}{T_s} \tag{8}$$

In a true order-statistic estimate, the operating point, r/N, is fixed. As a result, establishment of a given error at a chosen operating point requires a minimum population size, $N_{\min}$:

$$N_{\min} = \left(\frac{r}{N}\right)\left(1-\frac{r}{N}\right)\left[\frac{e^{\beta}(n-1)!}{\sigma\beta^n}\right]^2 \tag{9}$$

[13] Ibid.

Table 4 gives $N_{\min}$ for some likely operating points. Order-statistic-based methods similarly require populations larger than the $N_{\min}$ at their operating points. Table 4 shows that an order-statistic-based approach can achieve a 1-percent standard error with a population of 16K (16,384) samples, a 0.5-percent standard error with 64K (65,536) samples, and a 0.33-percent standard error with 128K (131,072) samples. Since the threshold-and-count estimator is equivalent to a true order-statistic estimate at the given values of r/N, it achieves the same standard errors with the same population sizes.

Because r/N remains fixed in a true order-statistic algorithm, it will always produce the same standard error, provided that the statistics of the data do not change. Thus, if it can operate within the estimate accuracy requirement, its dynamic range will only be limited by the dynamic range of the numerical representation. Approximate order-statistic methods, like the threshold-and-count method, have more limited dynamic range. The dynamic range of a single, fixed-threshold threshold-and-count estimator is examined first below. Modifications may be made to the single, fixed-threshold estimator to increase its dynamic range, such as automatically adjusting the threshold over time to track slow variations, or using multiple thresholds based on different a priori values of the noise power, but the performance of these modifications can be analyzed easily through the single fixed-threshold case.

Consider the factor β in the probability density and distribution functions. The factor β reflects the relative position of the threshold within the probability distribution of the samples. Increasing the noise power by a factor α is equivalent to dividing β by α. When the noise temperature changes, the new value of r/N must be calculated using the cumulative distribution function with the new value of β. The accuracy of the estimator can then be computed as before. Figure 3 shows the standard error as a function of system temperature change.

The shape of the probability density function for the χ^2 distribution and the quadratic form in r/N in the expression for the estimate variance guarantee that if the population size is greater than $N_{\min}$, the estimator will perform within the accuracy requirement within an isolated, continuous range of noise power values. The ratio of the maximum noise power value at which the estimator accuracy is within specification to the minimum such value is the estimator's dynamic range.

The single, fixed-threshold threshold-and-count estimator was evaluated with 64K samples for the dynamic range defined by a standard error of less than 1 percent. Table 5 shows that the dynamic range of the single, fixed-threshold threshold-and-count estimator is limited by the behavior of the estimator at high numbers of spectra per accumulation. As the number of degrees of freedom, ν, increases, the χ_ν^2 distribution of the samples approaches the Gaussian distribution, with the mean approaching ν, and the standard deviation approaching $\sqrt{\nu}$. This indicates that the probability density is being concentrated about the mean, collapsing the range over which it has non-negligible values. As a result, the threshold-and-count estimator becomes more sensitive to changes in the noise power. Hence, the dynamic range of the threshold-and-count estimator is limited by the performance with the maximum specified number of degrees of freedom.

It can be observed from Table 5 that a 1-percent error usually corresponds to the fairly loose requirement of the threshold, falling between the 0.1 and the 99.9 percentage points of the distribution. However, in order to preserve the assumption of a dense population of points near the threshold, required for the equivalence of the threshold-and-count method to a true order-statistic noise power estimator, the operating range will be limited to the 1-percent to the 99-percent points of the distribution.

It has been asserted,[14] perhaps incorrectly, that as the upper limit of the range increases past the median of the distribution, the interference robustness of the algorithm is lost. This is based on a generalization of the effect of interference on high-level order statistics in a true order-statistic algorithm. Since the threshold is fixed in the threshold-and-count algorithm, as opposed to the threshold in a true order-statistic technique, depletion of the population due to interference can always be corrected for. This is different from using a high-level order statistic, e.g., the 99-percent point, where high levels of interference might give an erroneously high value for the order statistic. With a fixed threshold, high levels of interference do not change the threshold. The interference reduces the effective population size, the same effect that it has on order statistics from below the median of the distribution [4]. In both a true order statistic and the threshold-and-count methods, this reduction of the population biases the estimate. Correction for this bias can easily be performed in the threshold-and-count estimator by adjusting the contents of the lookup table.

Therefore, consider the dynamic range of the threshold-and-count estimator to be the ratio of the 99-percent to the 1-percent point of the distribution, guaranteeing both a dense population around the threshold and a standard error less than 1 percent for all but 2 or 4 spectra per accumulation. For 2 or 4 spectra per accumulation, a 95- to 5-percent range achieves the required accuracy, and demonstrates dynamic ranges still in excess of the 99- to 1-percent ranges achieved when there are more than 8 spectra per accumulation. Standard errors at the 1-, 5-, 50-, 95-, and 99-percent points of the distributions are shown in Table 6. The dynamic range for the 1- to 99-percent points is shown in Table 7, as well as the dynamic range defined by the more conservative 5- to 95-percent and 1- to 50-percent levels.

None of the estimators will cover the 7.1-dB dynamic range required at 20 spectra per accumulated output with 1 percent of the points remaining above or below the threshold. This problem is not insurmountable. One previously explored solution [15,16] is to adapt the threshold to track a slowly time-varying noise power. This assumes that the noise power will not vary outside the dynamic range of the estimator from accumulation to accumulation. This assumption can be difficult to maintain at high numbers of spectra per accumulation, suggesting adaptation of the threshold during the accumulation. Such a solution would have high dynamic range but might be difficult to implement, since the number of degrees of freedom in the data increases with each spectrum added to the accumulation. Such a scheme would require a look-up table to invert several different cumulative distribution functions, at least until the distribution becomes sufficiently Gaussian. Evaluation of adaptation methods requires knowledge of the dynamics of the noise power, which could be obtained from field experiments with the SSPS. Time-varying thresholds will not be considered further here.

Another, simpler modification to increase the dynamic range of the estimator would be to have multiple thresholds arranged so that one would always be within the estimator's dynamic range. An estimator with a dynamic range of γ dB can be extended to cover a dynamic range of μ dB by replicating it μ/γ times, placing the operating points such that the lower edge of one threshold's range was at the upper edge of the previous threshold's range. Table 7 shows that only two such combined thresholds produce a dynamic range of 9.15 dB at 20 spectra per accumulated output and a dynamic range of 4.0 dB at 100 spectra, exceeding the dynamic range requirements and meeting the dynamic range goals at 20 spectra. The dynamic range

[14] Ibid.

[15] Ibid.

[16] W. Deich, "Truncated Data and Background Estimator," Interoffice Memorandum, Jet Propulsion Laboratory, Pasadena, California, May 3, 1988.

requirements at all numbers of spectra per accumulation up to 100 can be met with from 2 to 4 threshold estimators, depending on how conservative one is in choosing the percentage points defining the dynamic range. All the dynamic range goals can be met with from 4 to 6 estimators, with the number again depending on how conservatively one chooses the percentage points. Since the threshold does not need to be settable to a high precision, e.g., 1 percent is sufficient, a floating-point threshold comparison can be performed with fewer than 16 bits compared. Given the simplicity of the algorithm, 6 threshold units is not an unwieldy number, and the additional threshold counts could be used as a check on how well the data fit the assumed distribution. Furthermore, all the threshold-and-count units operate on the same input data, making this architecture an ideal candidate for implementation as integrated logic, e.g., a field-programmable gate array or application-specific integrated circuit (ASIC). The drawback of this approach is that the dynamic range of the noise power estimator is limited at design time by the number of thresholds built into special-purpose hardware. This can limit flexibility for possible long accumulation studies.

V. A Hybrid Threshold-Order-Statistic Estimator

By combining a threshold with the current order-statistic technique, it is possible to produce an estimator that is significantly more hardware-efficient than the two-pass order-statistic estimator in the SSPS, yet with a much wider dynamic range than the threshold method alone. The key to this method is to use an a priori threshold to determine a range of numeric values to histogram to obtain an order statistic of a desired rank.

First, consider that the required estimate accuracy is 1 percent. One should then make the fractional numeric precision of the estimate small relative to the required estimate accuracy. In floating-point arithmetic, the fractional numeric precision defines the number of bits required to represent the mantissa. A precision of ± 0.1 percent will be achieved with 9 bits of mantissa. Hence, if one were to perform a one-pass histogram, one would require 9 bits of mantissa plus 8 bits of exponent for a total of 17 bits, or 128K histogram bin values. An efficient hardware implementation used in the SSPS requires that the population size be at least twice the histogram length, and hence, the current approach could not support an expansion to 17 bits with a population size of 64K points. As a result, a one-pass reduced-precision histogram, while attractive, would either require a more complex hardware implementation, or a significantly larger minimum population size of 256K points.

If only 3 bits of exponent were supported in addition to the 9 bits of mantissa required for 0.1-percent precision, a dynamic range of almost a factor of 2^8, or 24 dB, would be obtained. This far exceeds the dynamic range requirements for the estimator, and, if desired, bits of exponent can be traded for greater precision. Two bits of exponent give a dynamic range of a factor of almost 16, or 12 dB, while 1 bit of exponent reduces the dynamic range to a factor of almost 4, or 6 dB. Two bits of exponent meet all the HRMS requirements and goals. Such a hybrid noise power estimator would consist of the following: (1) a threshold value determined by the most significant 6 exponent bits of the desired order statistic, (2) a comparator that determines if the 6 most significant exponent bits of a data point are less than, greater than, or equal to the threshold, (3) a counter for the data points less than the threshold, and (4) an 11-bit histogram unit, as in the SSPS, for data points with the most significant 6 bits of exponent equal to the threshold. The desired order statistic is then obtained by adding the count below the threshold to the ascending values in the histogram until the desired level is reached, producing an order statistic of the desired rank. This procedure is of approximately the same complexity as that implemented in the SSPS on a single 14- by 14-in. wire-wrap board.

VI. Conclusions

At the cost of limited dynamic range, estimation equivalent to sampling fixed order statistics can be performed with a single pass of the data. The requirements and goals for the HRMS sky survey's signal detection are met with a noise-power-estimator accuracy requirement of 1 percent and a dynamic range of up to 7.1 dB for most observations.

Under normal operating conditions, a fixed-threshold threshold-and-count estimator is equivalent to a true order-statistic estimator in accuracy, with a limited dynamic range. If a population size of 64K is used, the estimator will have better than 1-percent accuracy, but less than the required 7.1 dB dynamic range at 20 spectra per accumulation. Multiple thresholds may be used to increase the dynamic range. The HRMS requirements are met with from 2 to 4 thresholds, and the goals are met with from 4 to 6 thresholds. This technique provides a single-pass noise power estimator, as opposed to the current two-pass order-statistic technique, at the cost of a fixed dynamic range. In summary, an acceptable noise power estimator could be constructed with the threshold-and-count technique, and this estimator might occupy less than half of a single board in the sky survey operational system. In contrast, the current true order-statistic estimator implementation would occupy from 2 to 3 boards.

A combination of the threshold-and-count estimator and the current implementation of the true order-statistic estimator also produces a single-pass, single-board noise power estimator for floating-point systems with superior dynamic range. This new, hybrid threshold-histogram estimator would produce an order statistic of a desired rank, provided that the desired order statistic is within its dynamic range. This new, hybrid estimator would require only one threshold, would be based on the design in the SSPS, and while slightly more complicated than the raw threshold-and-count, offers the advantage of constant estimate accuracy.

Based on this analysis, it is recommended that the following research activities be performed with the SSPS, currently deployed at the Goldstone Deep Space Communications Complex, California: (1) monitoring the dynamic range of the noise power estimates to confirm the requirements derived in this article, (2) testing the interference robustness of the true order-statistic estimator in the SSPS, (3) testing the performance and interference robustness of the threshold-and-count estimator, and (4) analyzing the estimator bias induced by interference in all the candidate estimators, and if necessary, (5) analyzing and testing an algorithm for automatically correcting significant estimator biases.

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Table 1. Dynamic range constraints due to atmosphere, calculated by using the Slobin model, a 99-percent cumulative, 20-deg elevation, 25-K system, for the Canberra Deep Space Communications Complex.

RF, GHz	Maximum system temperature, K	Dynamic range needed, dB
1–10	128	7.1
1–3.8	49	2.9
1–2.25	38	1.8
1–1.63	35	1.5
1–1.32	34	1.3

Table 2. High-temperature radio sources considered in conjunction with atmosphere.

Atmosphere 20-deg elevation, K	Source temperature, K	Dynamic range needed, dB	Source and frequency
35	10	2.6	Galactic plane background (λ 21 cm, 1.4 GHz)
35	25	3.8	Common strong sources (L-band, 1–1.55 GHz)
34 to 128	70	9.0	Rare strong sources (all frequencies)

Table 3. Dynamic range requirements and goals.

Number of accumulations	Dynamic range requirement, dB	Dynamic range goal, dB
2–20	7.1	9.0
20–40	3.8	6.8
40–60	3.8	6.4
60–80	3.8	6.2
80–100	3.7	6.2

Table 4. Minimum population size versus fractional error.

Error, σ, %	$r/N = 0.25$	$r/N = 0.375$	$r/N = 0.5$
1.00	15,058	10,931	9035
0.50	60,231	43,726	36,140
0.33	135,520	98,383	81,316

Table 5. Fixed threshold, threshold-and-count estimator performance with system temperature change to T'_{sys} from T_{sys}. N = 65536 samples in the population.

Number of accumulations	β factor	T_{sys}/T'_{sys} 0.5		T_{sys}/T'_{sys} 0.75		T_{sys}/T'_{sys} 1		T_{sys}/T'_{sys} 1.25		T_{sys}/T'_{sys} 1.5		T_{sys}/T'_{sys} 1.75		T_{sys}/T'_{sys} 2		T_{sys}/T'_{sys} 2.25	
Operating point, $r/N_{,}$ = 0.25		r/N, %	Standard error, %	r/N, %	Standard error, %	r/N, %	Standard error, %	r/N, %	Standard error, %	r/N, %	Standard error, %	r/N, %	Standard error, %	r/N, %	Standard error, %	r/N, %	Standard error, %
2	0.96	57.19	0.358	36.61	0.413	24.95	0.479	17.98	0.548	13.52	0.619	10.53	0.690	8.42	0.761	6.88	0.832
4	2.54	74.60	0.246	43.87	0.261	25.10	0.310	14.87	0.373	9.22	0.448	5.97	0.534	4.02	0.630	2.79	0.735
6	4.22	84.58	0.217	49.27	0.205	25.01	0.245	12.64	0.308	6.63	0.392	3.64	0.498	2.09	0.626	1.25	0.780
8	5.96	90.69	0.211	53.96	0.174	25.05	0.208	11.03	0.272	4.96	0.366	2.33	0.494	1.15	0.664	0.59	0.882
10	7.73	94.37	0.217	57.98	0.155	25.05	0.183	9.72	0.249	3.77	0.354	1.53	0.509	0.65	0.730	0.29	1.035
20	16.83	99.56	0.375	72.54	0.113	25.00	0.126	5.67	0.202	1.13	0.375	0.23	0.738	0.05	1.472	0.01	2.897
MAX error			0.375		0.413		0.479		0.548		0.619		0.738		1.472		2.897
Operating point, $r/N_{,}$ = 0.375																	
2	1.31	73.65	0.344	52.11	0.367	37.67	0.409	28.19	0.456	21.78	0.506	17.29	0.557	14.03	0.609	11.61	0.661
4	3.1	86.58	0.266	59.21	0.246	37.52	0.273	23.82	0.315	15.51	0.367	10.42	0.427	7.21	0.495	5.13	0.569
6	4.95	92.90	0.255	64.53	0.199	37.53	0.218	20.86	0.259	11.71	0.316	6.76	0.389	4.04	0.477	2.49	0.582
8	6.82	96.15	0.265	68.69	0.174	37.45	0.186	18.51	0.228	9.05	0.292	4.52	0.379	2.34	0.493	1.26	0.638
10	8.71	97.90	0.290	72.22	0.157	37.44	0.166	16.63	0.208	7.13	0.278	3.10	0.382	1.40	0.528	0.66	0.727
20	18.29	99.89	0.647	83.88	0.123	37.47	0.115	10.52	0.163	2.45	0.275	0.55	0.504	0.13	0.948	0.03	1.786
MAX error			0.647		0.367		0.409		0.456		0.506		0.557		0.948		1.786
Operating point, $r/N_{,}$ = 0.5																	
2	1.6	84.86	0.357	65.51	0.348	50.05	0.371	38.87	0.404	30.83	0.441	24.95	0.479	20.57	0.518	17.22	0.558
4	3.67	93.43	0.308	71.97	0.245	49.96	0.254	33.84	0.281	23.11	0.318	16.08	0.362	11.44	0.412	8.32	0.467
6	5.67	96.94	0.319	76.51	0.204	50.00	0.205	30.32	0.231	18.15	0.271	11.00	0.324	6.82	0.388	4.34	0.463
8	7.67	98.52	0.356	79.95	0.182	50.01	0.176	27.49	0.202	14.55	0.247	7.73	0.309	4.19	0.390	2.34	0.493
10	9.67	99.27	0.413	82.70	0.168	50.02	0.157	25.12	0.183	11.81	0.233	5.51	0.306	2.62	0.408	1.29	0.546
20	19.67	99.97	1.163	91.02	0.141	50.00	0.110	16.96	0.140	4.58	0.218	1.15	0.372	0.29	0.664	0.07	1.198
MAX error			1.163		0.348		0.371		0.404		0.441		0.479		0.664		1.198

Table 6. Standard error versus percentage points of the distribution functions.

Number of accumulations	Standard error at percentage point, percent				
	1	5	50	95	99
2	2.01	0.96	0.37	0.43	0.67
4	1.15	0.58	0.25	0.33	0.53
6	0.86	0.44	0.20	0.28	0.45
8	0.71	0.36	0.18	0.25	0.41
10	0.61	0.32	0.16	0.22	0.37
20	0.39	0.21	0.11	0.16	0.28
50	0.23	0.13	0.07	0.11	0.19
100	0.17	0.09	0.05	0.08	0.12

Table 7. Estimator dynamic range.

Accumulations	Percentage point					Dynamic range percentage, dB		
	1	5	50	95	99	1–99	5–95	1–50
2	0.30	0.71	3.36	9.49	13.28	16.5	11.3	10.5
4	1.65	2.73	7.34	15.51	20.09	10.9	7.54	6.48
6	3.57	5.23	11.34	21.02	26.22	8.66	6.04	5.02
8	5.81	7.96	15.34	26.30	32.00	7.41	5.19	4.22
10	8.26	10.85	19.34	31.41	37.57	6.58	4.61	3.69
20	22.16	26.51	39.34	55.76	63.69	4.58	3.22	2.50
50	70.06	77.93	99.33	124.3	135.8	2.88	2.04	1.52
100	156.0	167.9	199.5	233.8	248.3	2.01	1.43	1.07

a-5

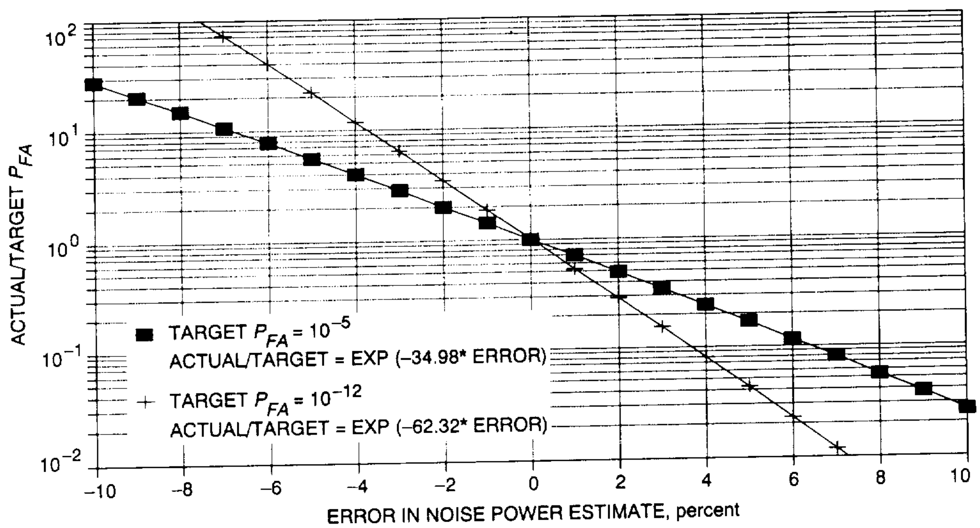

Fig. 1. Effect of noise power estimate error on P_{FA} 10 accumulations.

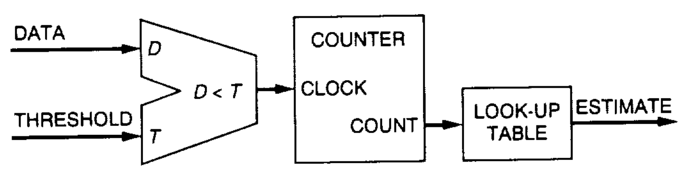

Fig. 2. Threshold-and-count estimator.

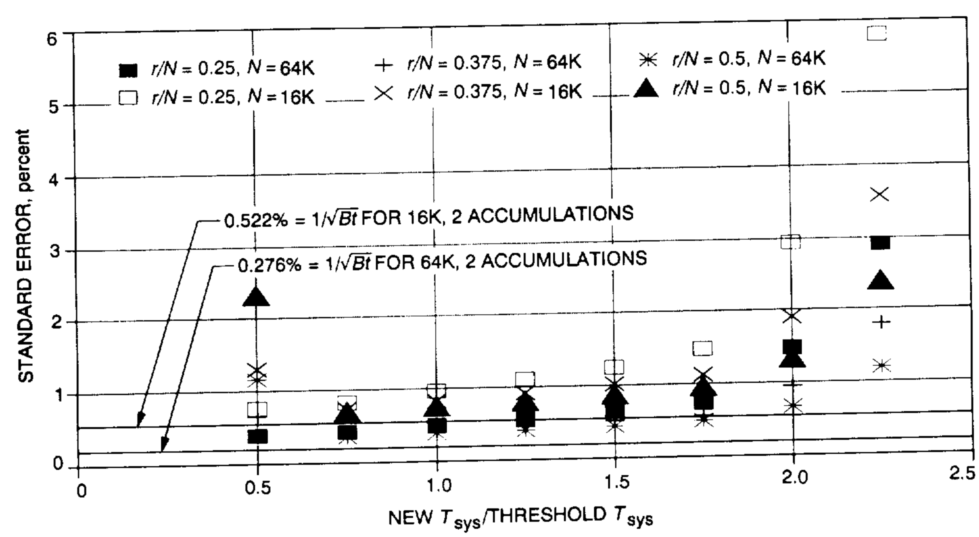

Fig. 3. Standard error versus system temperature change for a fixed-threshold estimator, showing the maximum error over 2 to 20 accumulations.