Precessional quantities for the Earth over 10 Myr

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Introduction

The insolation parameters of the Earth depend on its orbital parameters and of the precession and obliquity. Until 1988, the usually adopted solution for paleoclimate computation consisted in (Bretagnon, 1974) for the orbital elements of the Earth, which was completed by (Berger, 1976) for the computation of the precession and obliquity of the Earth. In 1988, I issued a solution for the orbital elements of the Earth, which was obtained in a new manner, gathering huge analytical computations and numerical integration (Laskar, 1988). In this solution, which will be denoted La88, the precession and obliquity quantities necessary for paleoclimate computations were integrated at the same time, which insure good consistancy of the solutions. Unfortunately, due to various factors, this latter solution for the precession and obliquity was not widely distributed (Berger, Loutre, Laskar, 1988). On the other side, the orbital part of the solution La88 for the Earth, was used in (Berger and Loutre, 1991) to derive an other solution for precession and obliquity, aimed to climate computations. I also issued a new solution (La90) which presents some slight improvements with respect to the previous one (Laskar, 1990). As previously, this solution contains orbital, precessional and obliquity variables. In order to make it widely available, it was distributed during this meeting on magnetic support and can also be obtain directly by request to the author at `laskar@friap51.bitnet`. In the present talk, I will discuss the main features of this new solution.

The orbital solution La90

The orbital solution is obtained by the numerical integration of an extended averaged system, which represents the mean evolution of the orbits of the planets. All the 8 main planets of the solar system are taken into account, as well as lunar and relativistic main perturbations. The use of numerical integration for the computation of the solution of the secular system is one of the reasons for the good quality of this solution, which can be checked by comparison with the available ephemeris over a short time scale (Laskar, 1985, 1986, 1988). In (Laskar, 1988), the solution La88 was represented in quasi-periodic form over 10 Myr, but these representations are slowly convergent, which prevents a good accuracy for the solution. Later on, I understood fully the reason of this slow convergence, which is due to the presence of multiple resonances in the secular system of the inner solar system. Due to these resonances, the motion of the solar system is chaotic, and not quasi-periodic, as was demonstrated by the computation of its Lyapunov exponents which reachs 1/(5 Myr) (Laskar, 1989). This implies that it is not possible to give any precise solution for the motion of the Earth over more than about 100 Myr, and most probably, ephemeris can only be given for about 10 Myr with good precision. Several integrations of the secular system of the solar system were made over 200 Myr and 400 Myr. The origin of the chaotic behaviour was identified, and is due to the presence of secular resonances in the inner solar system (Laskar, 1990). With a new numerical method, it was possible to show that the chaotic zones were the solar system belongs is large in the directions of the proper modes related to the inner planets. This is an indication that these results are stable against small changes of initial conditions or model.

One of the main consequence of interest for paleoclimate studies, is the fact that the main frequencies of the orbital motion of the planets can no longer be considered as constants, but are slowly evolving with time. The measured shift in frequency amount about 0.2 arcsec/year over 200 Myr for g_3 and g_4 while for g_5 it is of only 0.00002 arcsec/year. It should be stressed that, as the motion is chaotic, the computed solution cannot be considered as close to the real solution over more than a few 10 Myr, but it is reasonable to assume that the drift in frequency is of the same order of magnitude over the 200 Myr. One should nevertheless mention that over this time span, the change of the precession main frequency, due to tidal effects in the Earth-Moon system, are probably more important (Berger *et al.*, 1989).

Since, direct numerical integrations over 3Myr backward and forward were issued by (Quinn *et al.*, 1991), including also solutions for precession and obliquity (QTD6). The orbital solutions have been compared with La90. The two solutions present very small discrepancies over 3Myr, and the existence of the secular resonances in the inner solar system is confirmed (Laskar, Quinn, Tremaine, 1991). The very close agreement of the two orbital solutions over 3Myr gives the confirmation that the Earth parameters are now very well known over this time span, and insure that the orbital solution La90 can be used with confidence over 10 Myr for paleoclimate use. The precession and obliquity solutions present some small discrepancies which are probably due to the presence of the tidal effect in the Earth-Moon system in the QTD6 solution.

Precession and Obliquity in the La88 and La90 solutions

The precession quantities are completely determined by the two motions of the equatorial and ecliptic pole. The motion of the ecliptic is given by the secular theory La90 (Laskar 1990); the precession quantities are integrated at the same time, using the equations of the theory of the rigid Earth of Kinoshita (Kinoshita 1975, 1977, Laskar, 1986). The equations for the general precession in longitude p_A, and for the obliquity of the date ε are then

$$
\begin{aligned}
\frac{dp_A}{dt} &= R(\varepsilon) - \cot\varepsilon\left[\,\mathbf{A}(\mathbf{p},\mathbf{q})\sin p_A + \mathbf{B}(\mathbf{p},\mathbf{q})\cos p_A\right] - 2\,\mathbf{C}(\mathbf{p},\mathbf{q}) - p_g \\
\frac{d\varepsilon}{dt} &= - \qquad\qquad \mathbf{B}(\mathbf{p},\mathbf{q})\sin p_A + \mathbf{A}(\mathbf{p},\mathbf{q})\cos p_A
\end{aligned}
$$

with :

$$
\begin{aligned}
\mathbf{A}(\mathbf{p},\mathbf{q}) &= \frac{2}{\sqrt{1-\mathbf{p}^2-\mathbf{q}^2}}\;(\dot{\mathbf{q}} + \mathbf{p}(\mathbf{q}\dot{\mathbf{p}} - \mathbf{p}\dot{\mathbf{q}})) \\
\mathbf{B}(\mathbf{p},\mathbf{q}) &= \frac{2}{\sqrt{1-\mathbf{p}^2-\mathbf{q}^2}}\;(\dot{\mathbf{p}} - \mathbf{q}(\mathbf{q}\dot{\mathbf{p}} - \mathbf{p}\dot{\mathbf{q}})) \\
\mathbf{C}(\mathbf{p},\mathbf{q}) &= (\mathbf{q}\dot{\mathbf{p}} - \mathbf{p}\dot{\mathbf{q}})
\end{aligned}
$$

and :

$$
\begin{aligned}
R(\varepsilon) = {} & \frac{3k^2 m_M}{a_M^3\omega}\frac{2C-A-B}{2C}\Bigg[(\mathrm{M}_0 - \mathrm{M}_2/2)\cos\varepsilon + \mathrm{M}_1\frac{\cos 2\varepsilon}{\sin\varepsilon} \\
& \qquad - \mathrm{M}_3\frac{m_M}{m_E+m_M}\frac{n_M^2}{\omega n_\Omega}\frac{2C-A-B}{2C}(6\cos^2\varepsilon - 1)\Bigg] \\
& + \frac{3k^2 m_\odot}{a_\odot^3\omega}\frac{2C-A-B}{2C}\qquad [\mathrm{S}_0\cos\varepsilon]
\end{aligned}
$$

where $\mathbf{p} = \sin(i/2)\sin(\Omega), \mathbf{q} = \sin(i/2)\cos(\Omega)$, ($i$ is the inclination of the Earth with respect to a fixed ecliptic, and Ω the longitude of the node). $R(\varepsilon)$ is the secular term due to the direct lunisolar perturbations. The quantities $\mathrm{M}_0, \mathrm{M}_1, \mathrm{M}_2, \mathrm{M}_3, \mathrm{S}_0$, and S_2 depend only on the orbital elements of the Moon and the Sun. The pincipal moments of inertia of the Earth are denoted by A, B, and C, and the angular velocity of the Earth is ω . The masses of the Sun, the Earth, and the Moon are denoted by $m_\odot$, m_E, and m_M; the sideral motion of the Sun and of the Moon by $n_\odot$ and n_M ; and the mean motion of the node of the Moon by n_Ω. The other terms present in equation (23) represent the effects of the secular variation of the ecliptic, caused by the secular planetary perturbations. The numerical values of $\mathrm{M}_0, \mathrm{M}_1, \mathrm{M}_2, \mathrm{M}_3$ are given in (Kinoshita, 1977):

$$
\begin{aligned}
\mathrm{M}_0 &= 496303.3 \times 10^{-6} \\
\mathrm{M}_1 &= -20.7 \times 10^{-6} \\
\mathrm{M}_2 &= -0.1 \times 10^{-6} \\
\mathrm{M}_3 &= 3020.2 \times 10^{-6}
\end{aligned}
$$

and from (Laskar, 1986),

$$S_0 = \frac{1}{2}(1-e^2)^{-3/2} - 0.422 \times 10^{-6}$$

The following numerical values, were also used (see Laskar, 1986 for complete references) :

$$\begin{aligned}
\omega &= 474\,659\,981.597\,57\ ''/\mathrm{yr} \\
n_M &= 17\,325\,593.4318\ ''/\mathrm{yr} \\
n_\Omega &= -69\,679.193\,6222\ ''/\mathrm{yr} \\
a_M &= 384\,747\,980.645\ \mathrm{m} \\
k &= 0.017\,202\,098\,95 \\
m_\odot/(m_E+m_M) &= 328\,900.5 \\
m_\odot/m_E &= 332\,946.0
\end{aligned}$$

The quantity p_g is the geodesic precession due to the general relativity,

$$p_g = 0.019188''/\mathrm{yr}$$

The value of the dynamical ellipticity $E_D = (2C - A - B)/2C = 0.00328005$ is obtained by adjustment at the origine J2000 to the values of the speed of precession and obliquity give by the IAU (Grenoble, 1976) :

$$\begin{aligned}
p &= 50.290966''/\mathrm{yr} \\
\epsilon_0 &= 23°26'21''448
\end{aligned}$$

For $t = 0$, we have $p_A = 0$,$\varepsilon = \epsilon_0$, $i = \Omega = 0$,and thus :

$$\left.\frac{dp_A}{dt}\right|_{t=0} = R(\epsilon_0) - 2\,\dot{\mathrm{p}}_{t=0} - p_g \cot\epsilon_0$$

These formula for precession gives a solution for precession and obliquity in agreement with the requirements of high accurate ephemeris (Laskar, 1986) and are thus best suited for paleoclimate computations.

Descripion of the files of the solution La90

The different files which are distributed are

`ear0m5.dat`	`orbital elements 0 to -5 Myr`
`pre0m5.dat`	`obliquity and precession 0 to -5 Myr`
`ear5m10.dat`	`orbital elements -5 to -10 Myr`
`pre5m10.dat`	`obliquity and precession -5 to -10 Myr`

All the elements are referred to the ecliptic and equinox J2000. The starting date is J2000.
All angles are in radians

The `ear0m5.dat` and `ear5m10.dat` files contains $T, \mathbf{k}, \mathbf{h}, \mathbf{q}, \mathbf{p}$

$$
\begin{aligned}
T &= \text{time from J2000 in 1000yr} \\
\mathbf{k} &= e\cos(\varpi) \\
\mathbf{h} &= e\sin(\varpi) \\
\mathbf{q} &= \sin(i/2)\cos(\Omega) \\
\mathbf{p} &= \sin(i/2)\sin(\Omega)
\end{aligned}
$$

where e is the eccentricity of the Earth, i denotes the inclination of the Earth, ϖ the longitude of perihelion, and Ω the longitude of the node of the Earth with respect to the fixed ecliptic and equinox J2000.

The `pre0m5.dat` and `pre5m10.dat` files contains T, ε, p_A where

T is the time from J2000 in 1000yr, ε is the obliquity of the date (mean equator of date with respect to mean ecliptic of date), and p_A is the general precession in longitude.

All quantities generally used for climate studies are derived easily from these fundamental quatities. The eccentricity of the Earth is obtained by

$$
e = \sqrt{k^2 + h^2}
$$

And the longitude of perihelion from moving equinox is

$$
\omega^\star = \varpi + p_A
$$

The climatic precession $e\cos(\omega^\star)$ is thus equal to the real part of $z{*}\exp(ip_A)$, where $z = \mathbf{k}+i\mathbf{h}$.

Conclusions

The present solution La90 for orbital and precession elements for the Earth over can be used for as an ephemeris paleclimate computations over 10 Myr. On this time span, the fact that the motion of the solar system is chaotic is not perceptible on this level of precision. This new solution present some improvements from my previous solution La88 (Laskar 1988). The orbital solution is in very good agreement with the recent numerical integrations of (Quinn *et al.*, 1991) . Over the geological time scales exceeding 100 Myr, there is no hope to obtain such an ephemeris, due to the chaotic behaviour of the solar system, but one can assume that for a few 100 Myr, the slow diffusion of the astronomical frequencies observed during the 200 Myr integrations remains of about the same order. This should not be granted for billion years computations and more computations on the diffusion process in the solar system are clearly needed. More, due to the changes in frequencies, and presence of secular resonances in the orbital forcing, more extended studies of the resulting influence on the rotational evolution of the Earth should be done. Studies on the long term evolution of the Earth-Moon system are also needed in order to improve our knowledge of the rotational evolution of the Earth. The geological records, assuming that a good accuracy in the determination of the fundamental frequencies could be achieved, would then be the only possible observations for tracking the long term past evolution of the solar system for time span longer than 100 Myr.

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