# NUMERICAL INVESTIGATION OF THE

# EARTH'S ROTATION DURING A

# COMPLETE PRECESSION CYCLE

David L. Richardson

Department of Aerospace Engineering
University of Cincinnati
Cincinnati, Ohio 45221

## 1 Overview

A theory for the long-term rotational motion of the quasi-rigid Earth has been constructed by numerical integration. The theory spans 72000 years centered about 1968 A.D., and provides accurate rotational and positional data for the Earth in the recent past and the near future. Details are provided in [6]. The physical model is termed *dynamically consistent* because developments for the active forces and torques are truncated based solely on their magnitudes regardless of their origin. The model includes all appropriate forces and torques due to the geopotential and tidal effects as well as lunisolar and planetary contributions. The elastic and inelastic deformations due to tidal action were too small to affect the mass properties of the Earth at the truncation level of the model. However, long-term dissipative effects of the tidal forces and torques were not negligible. These considerations gave the model its quasi-rigid characterization. The numerical output provided both rotational and orbital-element data. The data have been fitted throughout the 72000-year range using Chebyshev polynomial series. These series are quite portable and are available upon request.

The project was based on the desire to provide researchers with a data base of accurate orbital and rotational parameters which are needed as the astronomical input to (paleo)climatology theories. Current theories [2] use only a rigid-Earth model and are analytical in nature. In addition, they are not completely consistent in their physical models. To keep pace with the accuracy of current observational data, the numerical theory maintains an accuracy on the order of 0.01 mas. The physical model includes:

1. rigid-body torques produced by the Moon, Sun, Venus, Jupiter and Saturn

2. tidal torques due to the Moon and Sun

3. effects due to Earth rotation/lunar orbit coupling

4. point-mass effects arising from the orbital motion of a 10-body planetary system (counting the Sun and Moon with Pluto excluded)

5. a 4×4 geopotential field

The numerical integration was conducted using a $10^{\text{th}}$-order Adams-Moulton predictor with an $11^{\text{th}}$-order Adams-Bashforth corrector. This combination produced a local truncation error $E \approx h^{12}$. The stepsize $h$ was set to 1/50 day. The selection of $h$ was based on error comparisons with output generated by extended precision (32 place) integrations.

The major results are displayed graphically in [6]. The data give an average precession rate of $-50.45''$/year. The present adopted IAU value is $-50.29''$/year, and that of the best analytical theory ([2]) is $-50.41''$/year. The output for obliquity (relative to the invariable plane of the solar system) and the precession index[1] $e \sin \tilde{\omega}$ are shown on the following page.

## 2  Future Work

The data base is not nearly long enough to be of great use to paleoclimatologists. Part of what is needed as an accurate theory that extends backward approximately 1 million years. (The data would thus span at least two of the 400,000-year Milankovitch cycles). A hybrid numerical integration procedure would be required—one that effectively suppresses the growth of round-off error. Such a procedure has been developed by Panovsky and Richardson [4] and has been used successfully in a multi-million-year integration of the planetary system [5]. Data spanning a much longer time frame would allow the retrieval of an accurate (geologically-recent) history for the lunar retreat rate.

To produce a data base spanning a much greater time frame, the quasi-rigid model for the Earth in the present theory would have to be discarded in favor of some sort of radially-dependent viscoelastic model which includes core-mantle coupling (gravitational and pressure) and mass-property variations. The new model should include changes in continentality from plate motions as well as the long-term effects of the oceans and glaciation. It may be possible to devise a model that is accurate over the long-term by adjusting parameters so that the (concocted) tidal dissipation provides lunar retreat rates consistent with the various measurements that have been reported in the recent literature. The extremes of these measurements now span 650 million years (see [1] and [7]).

To push a numerically-integrated theory backwards through tens of millions of years would require the development and use of a semi-numerical theory applied to a much-improved physical model. Such a theory is based on the numerical integration of the equations of motion after they have been analytically averaged at least through second order. A similar process has been successfully implemented by Laskar [3] in his recent investigations of the long-term motion of the eight-body planetary system. The averaging process would remove all high-frequency effects and leave only the secular, long-period, and dominant non-linear effects. The numerical integration could then proceed with stepsizes on the order of weeks or months instead of hours.

---

[1]The parameters $e$ and $\tilde{\omega}$ are respectively, the orbital eccentricity of the Earth and the Earth's argument of perihelion measured from the moving Vernal equinox.
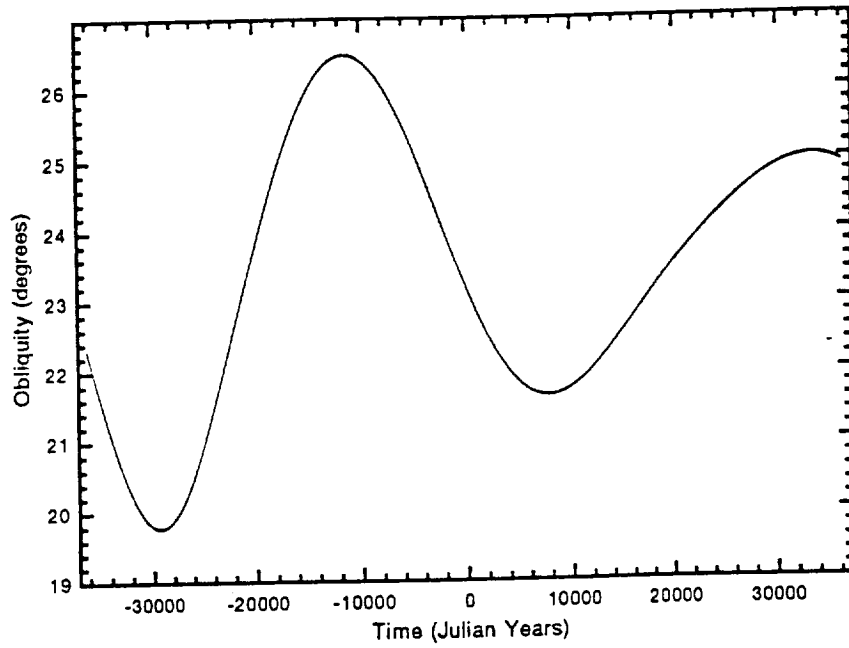
Obliquity for Full 72,000 Year Span.
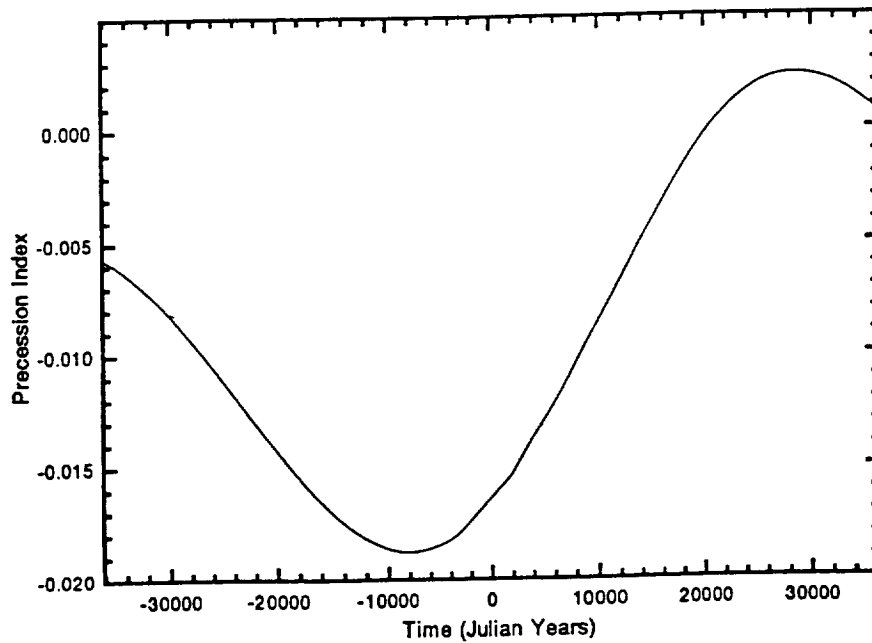


Precession Index $e \sin \bar{\omega}$.

# REFERENCES

[1] Dickey, J.O., Williams, J.G., and Newhall, X.X., "The impact of Lunar Laser Ranging on Geodynamics," *EOS. Trans. American Geophysical Union*, **71**, p. 475, 1990.

[2] Kinoshita, H. and Souchay. J., "The Theory of the Nutation of the Rigid Earth Model at Second Order," *Celestial Mechanics* **48**, 1990.

[3] Laskar, J., "Secular Evolution of the Solar System over 10 Million Years," *Astron. Astrophys.*, **198**, 1988.

[4] Panovsky, J. and Richardson, D.L., "A Method for the Numerical Integration of Ordinary Differential Equations Using Chebyshev Polynomials," AAS/AIAA Paper no. 85-405, 1985.

[5] Panovsky, J. and Richardson, D.L., "A Family of Implicit Chebyshev Methods for the Numerical Integration of Second-Order Ordinary Differential Equations," *J. of Computational and Applied Mathematics*, **23**, 1988.

[6] Walker, C.F. and Richardson, D.L., "A Self-Consistent Theory for the Earth's Rotational Motion," AAS/AIAA Paper no. 91-468, 1991.

[7] Williams, J.G., "Tidal Rhythmites: Key to the History of the Earth's Rotation and the Lunar Orbit," *J. Phys. Earth*, **38**, 1990.