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STOCHASTICITY OF COMET P/SLAUGHTER-BURNHAM

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ABSTRACT

Three comets are now known to be at or near the 1/1 resonance with Jupiter: P/Slaughter-Burnham, P/Boethin and the newly discovered P/Ge-Wang. Although details of the individual orbits differ, the three comets have very similar general dynamical behaviour: their orbits show many transitions between the different types of resonant motion (satellite libration, anti-satellite libration and circulating motion).
The stochastic character of such cometary orbits, mainly due to encounters with Jupiter, is investigated using Lyapunov Characteristic Indicators. For each comet of the group, we study the influences on the stochasticity of initial eccentricity, inclination, longitude of node and $l - l_J$ (mean longitude of comet minus mean longitude of Jupiter).
We present here our first results for P/Slaughter-Burnham.

INTRODUCTION

Three comets are now known to be at or near the 1/1 resonance with Jupiter, namely their heliocentric period is the same as Jupiter's: P/Slaughter-Burnham (1981 XVIII), P/Boethin (1986 I) and P/Ge-Wang (1988 VIII). In a previous paper (Benest, 1990), their orbital evolutions were compared, using the classical elliptic three-dimensional restricted three-body model Sun - Jupiter - comet. Although details of the individual orbits differ, the three comets have very similar general dynamical behaviour, which confirms that they obviously belong to the same family.

The three cometary orbits show many transitions between the different types of resonant motion, classically named according to the evolution versus time of the simple argument $l - l_J$ (difference of mean longitudes between the comet and Jupiter): libration and circulation. *Libration* corresponds to an oscillation of $l - l_J$ around a given value l_0; when $l_0 = 0^0$, we have a ***satellite libration***, i.e. the comet is thus considered as a remote satellite of Jupiter, at least temporarily (as is P/Boethin now). A libration around $l_0 = 180^0$ is an ***anti-satellite libration***, as P/Slaughter-Burnham now is. Any other kind of libration may occur, as $l - l_J$ may oscillate around any given value of L_0, as examples $\pm 60^0$ (the well-known Trojans). *Circulation* corresponds to a monotonous variation of $l - l_J$, as do P/Ge-Wang nowadays (see Benest, 1990, for a detailed bibliography).

Generally speaking, low amplitude of libration prevent close encounters with Jupiter; on the other hand, when the amplitude of the libration becomes high enough, or during circulation, a close encounter with Jupiter will occur sooner or later, which is known to induce a more or less drastic change in the cometary orbital elements. As is now well-known, such close encounter with a planet introduces generally stochasticity in the long-term dynamical behaviour of the cometary orbit: namely, two orbits with very close initial conditions diverge exponentially and have very different ultimate fates.

Our aim now is to study the stochasticity of the orbits of the three comets known to be at or near the 1/1 resonance with Jupiter, P/Slaughter-Burnham, P/Boethin and P/Ge-Wang. For this purpose, we compute quantities related to the theory of Lyapunov Characteristic Exponents.

Two orbits initially close diverge either linearly or exponentially depending on whether the initial points lie in an integrable or in a stochastic region of the phase space. This classical property has been extensively used as an indicator of stochasticity, and theoretically developped by Lyapunov, whose Characteristic Exponents (hereafter called LCE) indicate quantitatively how fast nearby orbits diverge and thus the degree of unpredictability of such orbits. But as the LCE's are ***stricto sensu*** limiting values at $t \to \infty$, they are evidently impossible to compute practically. We then define the Lyapunov Characteristic Indicators (LCI's) as the truncated values of the LCE's for a finite time; these LCI's provide however a fairly good quantitative measure of the stochasticity for a given orbit (for more details, see e.g. Gonczi and Froeschlé, 1981).

Our dynamical model is the classical elliptic three-dimensional restricted three-body problem (Sun - Jupiter - comet). The equations of motion are integrated during 10^5 years, using a Bulirsch-Stoer method with variable step size. LCI's are computed every three steps. We plan to study both the evolution of the orbital elements and the LCI's for our three comets; thereafter, we will try to evaluate the respective influences of the initial eccentricity and the initial inclination of the cometary orbit and of the initial value of $l - l_J$ on the stochasticity of the orbits. Finally, we will undertake more precise studies, which could take into account other perturbations (e.g. from the other planets, or even the so-called non-gravitational forces known to act often on comets). We present here our first results for P/Slaughter-Burnham (hereafter called P/SB).

RESULTS

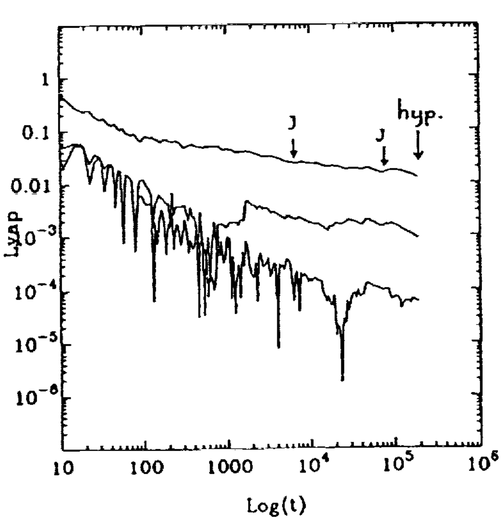

Figure 1. Evolution versus time of the 3 main LCI's (whose limits are identified with the LCE's) for comet P/SB; "J"= close approach with Jupiter; "hyp"= hyperbolic escape.

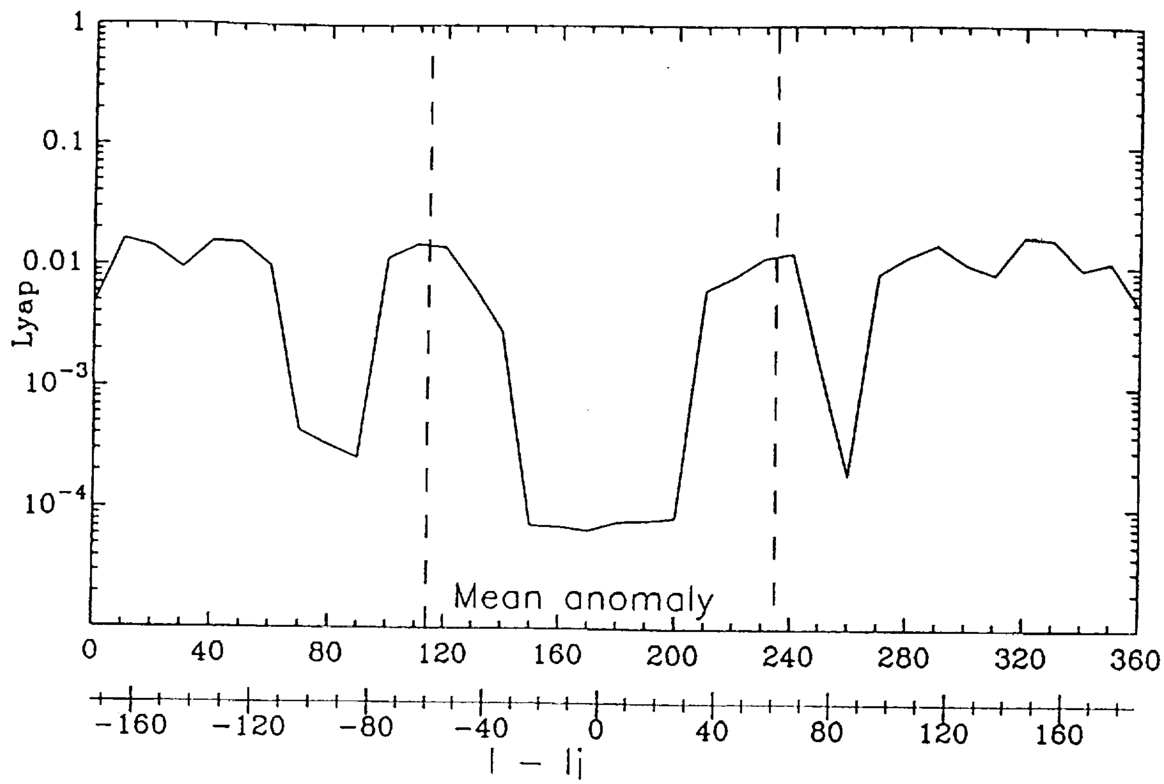

Figure 2. Evolution versus initial $l - l_J$ of the value of the largest LCI for fictitious P/SB-like comets.

Figure 1 shows that, as expected, the orbit of P/SB is stochastic. Now, all other elements remaining equal to those of the actual comet, we vary the initial value of $l - l_J$, i.e. we vary the initial value of M (the mean anomaly of the comet) from 0^0 to 360^0 every 10^0; figure 2 shows the evolution versus $l - l_J$ of the value of the largest LCI at 10^5 years for these fictitious comets ("P/SB-like comets") together with the actual comet P/SB (which has initial $l - l_J$=186.16^0, i.e. M=0^0); such a curve needs a mean computing time of 50 CPU hours on a SUN 4 at the Observatory of Nice. The curve reveals an interval of stability (i.e. for "Lyap" $< 10^{-4}$) between $l - l_J$=–20^0 and $l - l_J$=+20^0.

How does this curve evolve when we vary the other elements? Therefore we have varied independently (and again all other elements remaining equal to those of the actual comet) the initial eccentricity, inclination, longitude of node and longitude of perihelion; and for each such set of elements for fictitious P/SB-like comet, we vary $l - l_J$ as before.

We vary the initial eccentricity e from 0 to 0.9 every 0.1; for e=0, we observe two intervals of stability around $l - l_J$=–60^0 and $l - l_J$=+60^0, which correspond to the triangular points L_4 and L_5 (the Trojans); when e increases, these two intervals progressively shrink while the central interval appears and grows (see fig.2, where e=0.50395) up to e=0.8; for e=0.9, the curve is much more irregular and the orbits are probably all stochastic (see fig.3).

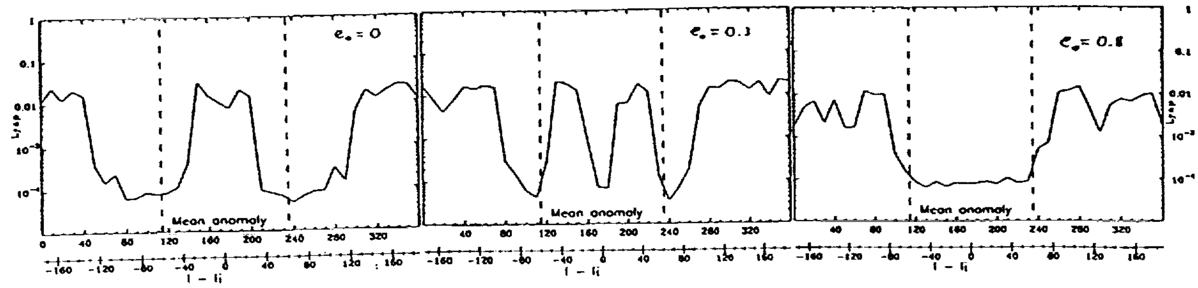

Figure 3. Evolution versus initial $l - l_J$ of the value of the largest LCI for fictitious P/SB-like comets for varied values of the initial eccentricity e.

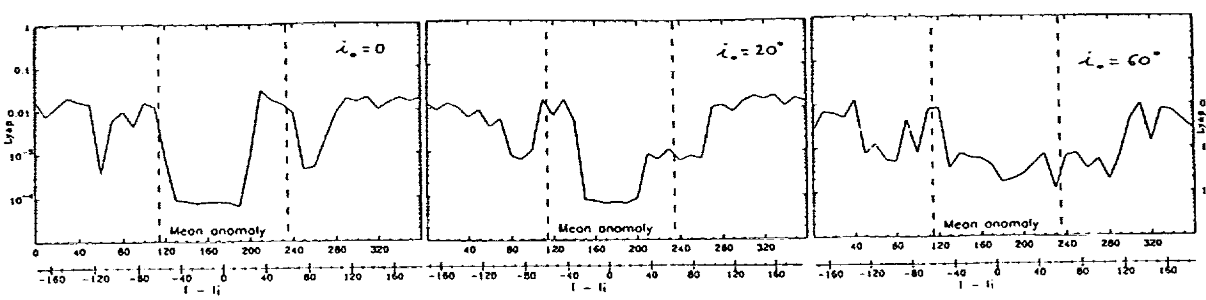

Figure 4. Evolution versus initial $l - l_J$ of the value of the largest LCI for fictitious P/SB-like comets for varied values of the initial inclination i.

We vary the initial inclination i from 0^0 to 90^0 every 10^0; between i=0^0 and i=40^0, the curve looks roughly like fig.2 (where i=8.1531^0), although becoming more and more irregular; for $i \geq 50^0$, the curve is completely irregular and the orbits are all stochastic (see fig.4).

When we vary the initial longitudes of node and of perihelion, we observe as expected only a shift of the curve.

We have now to do the same job for the two other comets (P/Boethin and P/Ge-Wang) and to compare the results in order to explain their dynamical behaviour.

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