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INJECTING ASTEROID FRAGMENTS INTO RESONANCES

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Abstract. We have quantitatively modeled the chance insertion of asteroid collisional fragments into the 3 : 1 and $g = g_6$ resonances, through which they can achieve Earth–approaching orbits. Although the results depend on some poorly known parameters, they indicate that most meteorites and NEAs probably come from a small and non–representative sample of asteroids, located in the neighbourhood of the two resonances.

Most meteorites and Near–Earth Asteroids (NEAs) are widely believed to be asteroidal fragments, coming from the asteroid belt through chaotic dynamical routes associated with mean motion and secular resonances (for recent reviews, see Wetherill and Chapman, 1988; Greenberg and Nolan, 1989). Numerical experiments carried out over the last decade have pointed out two specific source locations: the 3 : 1 mean motion resonance with Jupiter near 2.5 AU (Wisdom, 1983) and the inner edge of the main belt near 2.1 AU, where the dynamics is dominated by the $g = g_6$ (or ν_6) secular resonance (Scholl and Froeschlé, 1991). Continuing dynamical studies will hopefully provide accurate quantitative estimates of both the *width* of the resonances (i.e., of the strip in the phase space bordering the exact resonance surface where chaotic eccentricity jumps can occur) and their *effectiveness* (i.e., the probability and timescale with which the jumps actually occur at any given location in the phase space). On the other hand, we believe that some modeling effort is worthwhile today, by adopting some simplifying assumptions about the dynamical mechanisms and testing the sensitivity of the results to changes in these assumptions.

Therefore, we developed a numerical model for the first part of the meteorite/NEAs delivery process, namely the ejection of fragments from impact cratering or break–up of the existing asteroids, and the chance insertion of the escaping fragments into the "dangerous" regions of the phase space close to the 3 : 1 and $g = g_6$ resonances where chaotic behavior may arise. For every parent asteroid, the efficiency of this process depends on several factors: (i) the amount of ejected material per unit time; (ii) the mass vs. ejection velocity distribution of the fragments; (iii) the escape velocity of the parent body; (iv) the ΔV required to approach a resonance surface; (v) and the width of the "resonant strip". Finally, the overall yield depends on the size distribution of the asteroids. By varying some model parameters, we estimated the fraction of ejected fragments falling in the two resonances from a large number of main–belt asteroids.

Our model works in the following way. For all the 2355 numbered asteroids up to no. 4265, with semimajor axis $a < 2.8$ AU, eccentricity $e < 0.3$, inclination $i < 30°$ and perihelion distance > 1.1 AU (to exclude outer–belt, high–eccentricity/inclination and Earth–approaching objects), we simulated the isotropic ejection of a large number ($\approx 10^3$) of fragments. The distribution of the ejection velocity V_{ej} was derived from laboratory data on the outcomes of hypervelocity impacts: $dN(V_{ej}) \propto V_{ej}^{-\alpha}\, dV_{ej}$ for $V_{ej} > V_{min}$, $dN(V_{ej}) = 0$ for $V_{ej} < V_{min}$; for the exponent α we adopted the value 3.25, while for the lower–cutoff V_{min} we tested the three values 50, 100 and 200 m/s, which are consistent with the properties of asteroid families (Zappalà et al., 1990). For every asteroid, we then computed an escape velocity $V_{esc} = (120\ m/s)(R/100\ km)$, with the mean radius R taken from the data base of Cellino et al. (1991); fragments for which $V_{ej} > V_{esc}$ were assumed to escape "to infinity" with a velocity $V = (V_{ej}^2 - V_{esc}^2)^{1/2}$. For each escaped fragment, we used the Gauss formulae (see, e.g., Zappalà et al., 1990, eq.(1)) for computing its proper elements from those of the parent asteroid. The proper elements, derived according to the theory of Milani and Knežević (1990), were then tested using a numerical grid in the 3–dimensional proper elements space whether the fragment's apsidal secular frequency g was such that $|g - g_6| < \delta$ (with $g_6 = 28.2455$ $arcsec/yr$ and $\delta = 0.5$ or 1 $arcsec/yr$); if so, the fragment was assumed to lie in the $g = g_6$ secular resonance, where large eccentricity increases may occur (see Knežević et al., 1991, for further details about this procedure). A fragment was also assumed to lie within the $g = g_6$ chaotic region for $a < 2.10$ AU, as the whole zone bordering from inside the main belt is dominated by $g = g_6$ and other resonances (see Scholl and Froeschlé, 1991). On the other hand, a fragment was considered as injected in the chaotic zone associated with the 3 : 1 resonance whenever its osculating elements were such that $a > (2.497 - e/8.85)$ AU and $a < (2.510 + e/9.615)$ AU, in agreement with the results of numerical experiments on the dynamics of this resonance and the observed width of the corresponding Kirkwood gap (Wisdom, 1983; Yoshikawa, 1990). In this way, for each asteroid and for six parameter choices (two for δ times three for V_{min}), we could compute the fraction of escaped fragments falling into either of the two resonances.

The next issue is: for any given parent asteroid, can we compute a fragment production rate from both cratering and fragmentation events, taking into account the different probabilities of these events? Or can we at least estimate the relative *fragment production efficiencies* of asteroids of different sizes? Let us consider separately (1) impact fragmentation and (2) cratering, and assume that the projectile population has a cumulative size distribution close to a power law with (negative) exponent $-b$ (this assumption is not necessarily a good approximation, see Cellino et al., 1991, and later in the text). (1) For fragmentations, if we assume a size–independent impact strength, we get that the critical projectile–to–target size ratio is also independent of size. Then the number of projectiles capable of shattering a target of size R is $\propto R^{-b}$, and taking into account the target's cross–section the probability of fragmentation per unit time is $\propto R^{(2-b)}$. Since the mass ejected per fragmentation event is $\propto R^3$, the mass delivered per asteroid of size R per unit time is $\propto R^{(5-b)}$. However, since only fragments with $V_{ej} > V_{esc}$ escape reaccumulation, for $R > R_{min}$ (R_{min} being the size for which $V_{esc} = V_{min}$, namely the minimum size for which part of the fragments are reaccumulated), the fraction of escaping fragments is $(R/R_{min})^{1-\alpha}$, hence the mass delivered per unit time becomes $\propto R^{(6-b-\alpha)}$. (2) During cratering events, we assume that projectiles eject an amount of mass proportional to their own mass, up to a maximum value proportional to the target mass. Therefore, the projectile forming the largest possible crater has a size $R_{cr} \propto R$. The mass ejected by cratering events per unit time is proportional to the total impacting mass per unit time, which is proportional to the cross-section times $R_{cr}^{(3-b)}$, namely to $R^{(5-b)}$. Again, this is correct only for $R < R_{min}$, otherwise the escaping mass is $\propto R^{(6-b-\alpha)}$. Thus, within the simplifying assumptions described above, the same scaling rule applies to both cratering and fragmentation. For each target asteroid, we have therefore derived a *relative fragment delivery efficiency* by multiplying the fraction of ejected fragments falling into either resonance times a scaling factor proportional to $R^{(5-b)}$ for $R < R_{min}$, and to $R^{(6-b-\alpha)}$ for $R > R_{min}$. We have used as "nominal" values $\alpha = 3.25$ and $b = 2.5$, but we have tested the way results are affected when different values are adopted. The value $b = 2.5$ corresponds to an equilibrium size distribution for a collisionally evolving population, provided the parameters determining collisional outcomes are size–independent (Dohnanyi, 1969); however, for different zones of the asteroid belt and different size ranges, Cellino et al. (1991) have actually found that b varies in the range from ≈ 1 to ≈ 3. Notice that since for small bodies the scaling factor is $R^{(5-b)}$ and their cumulative number is also roughly proportional to R^{-b}, the choice of b may critically affect the overall fragment production efficiency of the different size ranges.

Some results of these computations are shown in Tables 1 and 2, referring to the "nominal" parameter values given above and to the case $\delta = 1$ $arcsec/yr$, $V_{min} = 100$ m/s (for a more detailed discussion and a parameter sensitivity analysis, see Gonczi et al., 1992, in preparation). Table 1 lists all the asteroids with nos. < 1000 that deliver at least 20% of their escaping fragments to one of the two resonances; it can also be considered a list of the asteroids lying closest to the "dangerous" resonant strips. Notice that some asteroids (nos. 6, 304, 631, 759, 907, 930) are found to be *inside* the $g = g_6$ strip, yielding fragment percentages $> 50\%$. Although this may be due to an overestimate of the resonance width or to its location being inaccurately determined near these bodies, at least in one case (759 Vinifera) this result has been confirmed by numerical integrations (Froeschlé and Scholl, 1987); Vinifera did not show chaotic behavior, but this may be due to a "lucky" dynamical configuration (Morbidelli and Henrard, 1991). In any event, we believe that the asteroids listed above are so close to the resonant surface that they should be considered as promising potential sources of Earth–approaching fragments. Table 2 gives the total fragment delivery efficiencies (in arbitary units) for some subsamples, sorted according to size and taxonomic type. As for size, we recall that according to Cellino et al. (1991) our overall asteroid sample is complete only for $D > 44$ km, and is increasingly affected by discovery selection effects for smaller sizes; similar selection effects certainly apply to taxonomic types also. An interesting finding was that in the S-type and 150 $km < D < 200$ km subsamples, some 90% of the total fragment delivery efficiciency to $g = g_6$ is contributed by the single large asteroid 6 Hebe.

Although the results depend on some parameters which are not well known (the width of the secular resonance, the mass vs. velocity distribution of the fragments, the mass distribution index of small asteroids), we summarize our results as follows: (1) both resonances are effective channels for fragment collection and delivery and their efficiencies are of the same order of magnitude, but they sample in a different way the orbital elements and the physical properties (size and taxonomic type) of the parent objects; (2) a large fraction of NEAs and meteorites can be generated by a small fraction of the overall asteroid population (mostly located in the vicinity of resonances), whose average properties do not necessarily coincide with those of the overall population; (3) as suggested by Knežević et al. (1991), the $g = g_6$ resonance is probably an effective fragment delivery channel in the moderate–inclination (15° to 20°) regions near 2.4 and 2.8 AU, where several relatively large asteroids are located (e.g., nos. 6, 304, 631, 907), as well as near the inner edge of the belt, populated by a larger number of small asteroids.

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Ast. n^o	a(AU)	e	i(deg)	$V_e(km/s)$	R(km)	% 3:1	% $g = g_6$
6	2.425	0.169	15.052	0.115	96.00	2.3	83.9
17	2.471	j0.134	4.865	0.056	46.60	30.[illegible]	1.6
46	2.526	0.124	2.493	0.079	65.50	39.8	0.3
198	2.459	0.183	10.673	0.035	29.35	21.4	2.6
304	2.404	0.091	15.480	0.041	34.25	0.8	94.4
330	2.470	0.226	6.281	0.008	6.30	44.8	1.5
335	2.475	0.161	4.681	0.056	46.80	40.9	1.3
355	2.539	0.121	4.870	0.015	12.85	26.1	0.3
421	2.539	0.255	7.550	0.011	8.80	42.8	0.3
475	2.594	0.240	19.767	0.019	15.50	1.1	76.9
495	2.487	0.126	2.550	0.025	20.85	48.7	1.4
518	2.535	0.189	7.418	0.011	8.80	43.1	0.4
556	2.465	0.120	6.090	0.024	19.75	28.3	1.2
603	2.542	0.209	8.158	0.009	7.60	34.1	0.2
619	2.520	0.025	13.851	0.023	19.00	23.9	3.6
623	2.460	0.125	14.797	0.028	23.00	18.0	48.0
631	2.793	0.060	19.487	0.036	30.25	0.0	64.9
649	2.549	0.240	11.947	0.009	7.20	33.2	0.4
660	2.535	0.105	15.035	0.027	22.10	30.2	1.6
695	2.539	0.097	15.035	0.031	25.60	20.7	0.5
714	2.535	0.086	15.135	0.025	20.50	25.0	1.2
724	2.455	0.248	11.660	0.005	4.10	29.4	4.8
759	2.618	0.196	19.597	0.032	26.35	0.6	72.5
765	2.547	0.245	6.690	0.009	7.25	32.4	0.4
787	2.540	0.085	15.058	0.018	15.15	20.6	1.1
797	2.536	0.090	5.394	0.020	16.95	27.2	0.5
877	2.487	0.148	3.382	0.024	19.80	53.5	1.4
879	2.531	0.095	14.851	0.009	7.85	31.9	2.0
900	2.472	0.120	11.759	0.014	11.35	30.5	8.6
907	2.801	0.168	19.548	0.039	32.90	0.0	69.9
908	2.475	0.197	12.240	0.017	14.00	49.6	8.9
930	2.431	0.112	15.391	0.023	19.55	1.8	87.4
969	2.463	0.176	3.405	0.012	10.25	28.1	1.1
974	2.534	0.077	4.485	0.015	12.40	24.5	0.5
994	2.530	0.068	15.129	0.016	13.60	29.3	1.6

Table 1

All the asteroids with nos. $<$ 1000 *yielding a fraction* $>$ 20% *of escaped fragments to either the 3:1 or the* $g = g_6$ *resonance. Parameters choice :* $V_{min} = 100\ m/s$, $\delta = 1\ arcsec/yr$, $\alpha = 3.25$, $b = 2.5$.

$Eff1\ (D < 50\ km) = 20.073$	$Eff2\ (D < 50\ km) = 6.182$
$Eff1\ (50 \leq D < 100\ km) = 24.528$	$Eff2\ (50 \leq D < 100\ km) = 18.167$
$Eff1\ (100 \leq D < 150\ km) = 29.358$	$Eff2\ (100 \leq D < 150\ km) = 2.360$
$Eff1\ (150 \leq D < 200\ km) = 12.513$	$Eff2\ (150 \leq D < 200\ km) = 57.577$
$Eff1\ (200 \leq D < 250\ km) = 12.069$	$Eff2\ (200 \leq D < 250\ km) = 0.281$
$Eff1\ (250 \leq D < 300\ km) = 0.591$	$Eff2\ (250 \leq D < 300\ km) = 0.296$
$Eff1\ (D \geq 300\ km) = 3.169$	$Eff2\ (D \geq 300\ km) = 0.989$
$Eff1\ (C) = 15.483$	$Eff2\ (C) = 12.443$
$Eff1\ (S) = 30.664$	$Eff2\ (S) = 62.333$
$Eff1\ (M) = 1.154$	$Eff2\ (M) = 1.156$
$Eff1\ (F) = 2.769$	$Eff2\ (F) = 0.050$
$Eff1\ (V) = 2.067$	$Eff2\ (V) = 0.689$
$Eff1\ (P) = 14.303$	$Eff2\ (P) = 0.104$
$Eff1\ (X) = 35.860$	$Eff2\ (X) = 10.125$

Table 2

Fragment delivery efficiencies (arbitrary units) to the 3:1 (Eff1) and $g = g_6$ (Eff2) resonances for different size ranges and taxonomic types. Parameters choice : $V_{min} = 100\ m/s$, $\delta = 1\ arcsec/yr$, $\alpha = 3.25$, $b = 2.5$.

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