Asteroids, Comets, Meteors 1991, pp. 245-248
Lunar and Planetary Institute, Houston, 1992

ASTEROID–TYPE ORBIT EVOLUTION NEAR THE 5:2 RESONANCE.

S.I. Ipatov,
M.V.Keldysh Institute of Applied Mathematics, Moscow, USSR

In the case of the 5:2 commensurability with the motion of Jupiter an asteroid can reach the orbits of Mars, Earth and Venus when eccentricity e is greater than 0.41, 0.65 and 0.74, respectively. For individual fictitious asteroids Ipatov [1] and Yoshikawa [6] obtained a growth in e from 0.15 to 0.74–0.76. Rates of changes in orbital orientations are different for Mars, Earth, Venus and the asteroid. Therefore, for corresponding values of e, the asteroid could encounter these planets and leave the gap at those encounters. In order to investigate this hypothesis of the 5:2 Kirkwood gap formation, Ipatov [2] studied the regions of initial data for which the eccentricities of asteroids located near the 5:2 commensurability exceeded 0.41 during evolution. The orbit evolution for 500 fictitious asteroids was investigated by numerical integration of the complete (unaveraged) equations of motion for the three–body problem (Sun–Jupiter–asteroid). The equations of motion were integrated in the time intervals $T\geq5\cdot10^3 t_J$ (t_J is the heliocentric orbital period of Jupiter) in the planar model, $T\geq10^4 t_J$ at initial inclination $5^o\leq i_o\leq20^o$ and $T=10^5 t_J$ at $i_o=40^o$. The larger interval T was taken at $i_o=40^o$ because in this case for the majority of runs maximum values of e and i were reached in the time $\Delta t>2\cdot10^4 t_J$.

Various initial orientations of orbit of the asteroid and its location in orbit were considered when initial value of asteroidal semimajor axis a_o was equal to the resonance value $a_{5/2}$. It was obtained that maximum value e_{max} of asteroidal eccentricity during evolution exceeded 0.41 and 0.65, respectively, for 2/3 and 1/3 of all investigated asteroids with the initial asteroidal eccentricity $e_o=0.15$, $i_o\leq20^o$, $a_o=a_{5/2}$ and the present–day value of Jupiter's eccentricity e_J. If at $a_o=a_{5/2}$ and some starting values of e, i, the argument of perihelion ω, the longitude of the ascending node Ω and the true anomaly ν it was obtained that $e_{max}\geq0.41$, then with the same starting values of e, i, ω, Ω and ν we have $e_{max}\geq0.41$ also for the values a_o located in some vicinity of $a_{5/2}$. For all the fictitious asteroids examined with $i_o=40^o$ we obtained $e_{max}\geq0.6$. Since earlier e_J exceeded 0.06, we took $e_J=0.06$ when determining the maximum region of a_o and e_o for which fictitious asteroids with some initial orbital orientations were Mars crossers. It was shown that the outer boundaries of this region coincided with the boundaries of the 5:2 Kirkwood gap. For $e_o\leq0.2$ and $i_o\leq20^o$ the regions of initial data for which fictitious asteroids were Earth crossers and Mars crossers are close to each other. Since the radius of Earth is approximately twice that of Mars, it is likely that meteorites, after having migrated from the 5:2 gap, impacted Earth more often than Mars. Both the range free of real asteroids for which $i>5^o$ and the range of a_o and e_o with $5^o\leq i_o\leq20^o$ for which $e_{max}\geq0.41$ are larger to the right of the resonance (for $a>a_{5/2}$)

than to the left (particularly for $e \geq 0.2$).

Analytical studies of the circular three–body (Sun–Jupiter–asteroid) problem, carried out by Williams [5], showed that e reached a maximum when $\Delta\tilde{\omega}=\tilde{\omega}-\tilde{\omega}_J=0$ or $\Delta\tilde{\omega}=180^o$ where $\tilde{\omega}$ is the longitude of asteroidal perihelion and its value for Jupiter is designated by subscript "J". This relation between e and $\Delta\tilde{\omega}$ was obtained for most of the fictitious asteroids. Different cases of the interrelation of the time variations in e and $\Delta\tilde{\omega}$ have been examined repeatedly [1–3, 6–7]. Our calculations showed that asteroids are Mars crossers and Earth crossers as a rule for certain types of interrelations of variations in e and $\Delta\tilde{\omega}$. For the most of considered fictitious asteroids the osculating elements changed in time almost periodically. We can distinguish several periodical components in the plotted time dependence of e and $\Delta\tilde{\omega}$ with different amplitudes which may vary during evolution. Let us denote by T_e the period of variation in e with the largest amplitude, and by $T_{\tilde{\omega}}$ the period of long–term variations in $\Delta\tilde{\omega}$ if $\Delta\tilde{\omega}$ librate around some constant usually equal to 0 or 180^o, or the time during which $\Delta\tilde{\omega}$ change by 360^o if $\Delta\tilde{\omega}$ circulate. It was obtained for most runs that $T_{\tilde{\omega}}=T_e\approx$const. In this case we can distinguish several types of interrelations $N_{\tilde{\omega}}$ for variations in $\Delta\tilde{\omega}$ and e. They are characterized by the formula $\Delta\tilde{\omega}=360^o(t-t_+)S_{\tilde{\omega}}/T_e+\xi+\Delta\tilde{\omega}_e^+$ and by the values of $S_{\tilde{\omega}}$, $\Delta\tilde{\omega}_e^+$, $S_{\tilde{\omega}}^+$, $\Delta\tilde{\omega}_e^-$, $S_{\tilde{\omega}}^-$ and N_a presented in Table I. We have $N_a=R$ if a value of asteroidal semimajor axis reaches $a_{5/2}$ during evolution and $N_a=N$ if it doesn't reach $a_{5/2}$. The first term in the right–hand side of this formula is the monotonic component in variation of $\Delta\tilde{\omega}$, with $S_{\tilde{\omega}}$ denoting the sign of this component. There is no monotonic component when $S_{\tilde{\omega}}=0$. The term $\Delta\tilde{\omega}_e^+$ denotes the value of $\Delta\tilde{\omega}$ at time $t=t_+$ when $e(t_+)\approx e_{max}$, and $\Delta\tilde{\omega}_e^-$ is the value of $\Delta\tilde{\omega}$ at time $t=t_-$ when $e(t_-)\approx e_{min}$. The ξ is the oscillatory component in the variation of $\Delta\tilde{\omega}$. It was calculated that during evolution $|\xi|<180^o$ and $\xi\approx0$ for $t=t_+\pm k\cdot T_e/2$, where k=0, 1, 2, 3, ... We introduce the plot of $\overline{\tilde{\omega}}(t)$ with $\tilde{\omega}(t)$ averaged over the shot–periodic oscillations. It was considered that $S_{\tilde{\omega}}^+=1$ if $\partial\overline{\tilde{\omega}}/\partial t>0$ at $t=t_+$ and $S_{\tilde{\omega}}^+=-1$ if $\partial\overline{\tilde{\omega}}/\partial t<0$. The values of $S_{\tilde{\omega}}^-$ for $t=t_-$ are denoted in the same way. For $N_{\tilde{\omega}}=T$ the value of $\Delta\tilde{\omega}$ almost didn't changed with time and it was considered that $S_{\tilde{\omega}}^-=S_{\tilde{\omega}}^+=0$. When $\Delta\tilde{\omega}=0$ for $N_{\tilde{\omega}}=B$ or $N_{\tilde{\omega}}=H$ the eccentricity has, as a rule, a local maximum which is several times lower than the main maximum obtained at $\Delta\tilde{\omega}=180^o$. This local maximum is higher than the minimum by 0.05–0.1 for $N_{\tilde{\omega}}=B$ and by 0.02–0.05 for $N_{\tilde{\omega}}=H$.

In some cases at $i_0=40^o$ the eccentricity doesn't reach maximum at $\Delta\tilde{\omega}=0$ or $\Delta\tilde{\omega}=180^o$. In these cases some types $N_{\tilde{\omega}}$ shown in Table II strongly depend on N_a, S_Ω and S_ω. If $\Delta\omega=\omega-\omega_J$ librated during evolution about 0 or 180^o with the amplitude $\delta\omega\leq20^o$, then it was assumed that $S_\omega=0$. For all other investigated cases the $\Delta\omega$ increases during evolution (neglecting short-periodical variations) and $S_\omega=1$. The variable Ω decreased during evolution if $i<90^0$ and increased if $i>90^0$. The values of S_Ω show the sign of changes in Ω. For all types $N_{\tilde\omega}$ presented in Table I it was obtained that $S_\omega=1$ and $S_\Omega=-1$. The type $N_{\tilde\omega}=Y_1$ is a particular case of the type $N_{\tilde\omega}=Y$. Subtypes $N_{\tilde\omega}=U^-$ and $N_{\tilde\omega}=U^+$, $N_{\tilde\omega}=W^-$ and $N_{\tilde\omega}=W^+$ are the parts of types $N_{\tilde\omega}=U$ and $N_{\tilde\omega}=W$, respectively. Each of these subtypes is replaced by other subtype when i reaches 90^0. It was obtained for types $N_{\tilde\omega}=U$ and $N_{\tilde\omega}=W$ that $e_{max}>0.99$ and the maximum value of i during evolution $i_{max}\approx160^0$. We obtained for one asteroid that $e_{max}=0.9999$. Types *J*, *T*, *E*, Y_1 as well as all the types presented in Table II were not obtained by Yoshikawa [7].

TABLE I

Types of interrelations $N_{\tilde\omega}$ for the long-term variations in e and $\Delta\tilde\omega$

$N_{\tilde\omega}$	*B*	*C*	*D*	*H*	*J*	*Q*	*T*	*Z*	*O*	*I*	*E*	Y_1
N_a	R	R	R	R	R	R	R	R	N	N	N	N
$S_{\tilde\omega}$	1	0	−1	0	0	1	0	−1	1	0	0	−1
$\Delta\tilde\omega_e^+$	180^0	180^0	0	0	0	0	180^0	180^0	0	0	90^0 or 180^0	180^0
$S_{\tilde\omega}^+$	1	1	−1	−1	1	1	0	−1	1	1	1	−1
$\Delta\tilde\omega_e^-$	0	180^0	180^0	0	0	180^0	180^0	0	180^0	0	90^0 or 180^0	0
$S_{\tilde\omega}^-$	1	−1	−1	1	−1	1	0	−1	1	−1	−1	−1

TABLE II

Types $N_{\tilde\omega}$ when e does not reach a maximum at $\Delta\tilde\omega=0$ or $\Delta\tilde\omega=180^0$.

$N_{\tilde\omega}$	*X*	W^-	W^+	*Y*	U^-	U^+
N_a	*R*	*R*	*R*	*N*	*N*	*N*
S_Ω	−1	−1	1	−1	−1	1
S_ω	0	1	1	0	1	1

Let us consider interrelations of variations in orbital elements with periods less than T_e for the three-dimensional model. The times during which $\Delta\omega$ and Ω change by 360^0 are denoted by T_ω and T_Ω. For the types $N_{\tilde\omega}=X$ and $N_{\tilde\omega}=Y$ the value of T_ω is the period of libration of $\Delta\omega$. Periods of long-term variations in i are denoted by T_i and T_i^* ($T_i\le T_i^*$). For most of the considered asteroids $T_i^*=T_e$. Computer simulation results showed that for variations in i with the period T_i the local maximum of e always corresponds to the minimum of i and the local minimum of e – to the maximum of i. It was obtained that $2T_i=T_\omega$ for most computer runs with $i_0\neq0$. For this equality the minimum of i was reached at $\Delta\omega=0$ or at $\Delta\omega=180^o$. The greater are the values of e_o and i_o, the greater are the variations in i and e with period $T_i=T_\omega/2$. Two other equalities between T_i and T_ω : $T_i=T_\omega$ and $T_i=2T_\omega$ were obtained more rarely. For both cases i reached minimum at $\Delta\omega=180^o$.

The relation $T_i^*=T_\Omega$ was fulfilled as a rule; besides for the considered runs i reached minimum at $\Omega=180^o$ and maximum at $\Omega=0$ for those cases when we observed no variations in e and i with the period $T_i=T_\omega/2$. Interrelations of periods of variations for four orbital elements (i, e, Ω and ω) were obtained mainly for $5^o \le i_o \le 10^o$ and $e_o \le 0.15$. For some asteroids with $N_{\tilde{\omega}}=J$ the relation $T_i^*=T_i=T_\omega=T_e=T_\Omega=T_{\tilde{\omega}}$ was fulfilled and in this case the minimum i and the maximum e were reached at $\Omega=180^o$ and $\Delta\omega=180^o$, while the maximum i and the minimum e – at $\Omega=0$ and $\Delta\omega=0$. If $N_{\tilde{\omega}}=O$, it was obtained for most of the considered asteroids that $T_i^*=T_i=2T_\omega=T_e=T_\Omega$ when $i_o=10^o$ and $e_o \le 0.05$.

For some fictitious asteroids the limits Δa of variation in a exceeded the width of the gap and the asteroids could migrate from one side of the gap to the other side. Maximum values of $\Delta a/a_J$ for types $N_{\tilde{\omega}}$ denoted by C, T, W and U reached 0.01 and for $N_{\tilde{\omega}}=B$ and $N_{\tilde{\omega}}=Q$ – 0.013 and 0.016 respectively.

In the three-dimensional space (e_{max}, a_o, e_o), the surface of values of e_{max} has a number of "plateaus" separated by steep "cliffs". Each plateau corresponds to a certain type $N_{\tilde{\omega}}$. The cliffs were obtained at the boundaries of initial data corresponding to different types $N_{\tilde{\omega}}$ (excluding the transitions between types $N_{\tilde{\omega}}=Q$ and $N_{\tilde{\omega}}=O$ as well as $N_{\tilde{\omega}}=J$ and $N_{\tilde{\omega}}=I$) and were almost absent in a number of cases with $e_o=0.3$. With initial data near these boundaries the transitions between the types were obtained for six asteroids in the planar model and for twenty asteroids in the three-dimensional model. Šidlichovsky [4] obtained that an orbit was chaotic if initially small starting eccentricity exceeded 0.3 during evolution. At types $N_{\tilde{\omega}}$ denoted by B, C, D, H, W, Z, E and U for all considered asteroids it was obtained that $e_{max} \ge 0.41$ and $\Delta e=e_{max}-e_{min} \ge 0.24$. Therefore, the number of chaotic orbits may be considerably larger than the number of asteroids for which such transitions were obtained.

References:

[1] Ipatov, S.I. (1988) *Kinematics Phys. Celest. Bodies*. **4**, *N* 4, 49.
[2] Ipatov, S.I. (1989) *Sov. Astron. Lett.* **15**, 324.
[3] Scholl, H., and C.F. Froeschlé (1975) *Astron. Astrophys.* **42**, 457.
[4] Šidlichovsky, M. (1987) In *"Figure and dyn. Earth, Moon, and planets" Proc. Int. Symp.* Pt. **2**, 571, Prague.
[5] Williams, J.G. (1969) Ph.D. dissertation, University of California, Los Angeles.
[6] Yoshikawa, M. (1989) *Astron. Astrophys.* **213**, 436.
[7] Yoshikawa, M. (1990) *Icarus* **87**, 78.