

ASTEROID PROPER ELEMENTS AND SECULAR RESONANCES: PROGRESS REPORT

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In a series of papers (e.g. Knežević, 1991; Milani and Knežević, 1990; 1991) we reported on the progress we were making in computing asteroid proper elements, both as regards their accuracy and long-term stability, and the efficiency and "intelligence" of our software. At the same time, we studied the associated problems of resonance effects, and introduced the new class of "nonlinear" secular resonances; we determined the locations of these secular resonances in proper-element phase space and analysed their impact on the asteroid family classification. Here we would like to summarize the current status of our work and possible further developments.

Improved proper elements

The basis for our studies is an analytical theory of asteroid secular perturbations developed by Yuasa (1973), corrected and completed by Knežević (1988, 1989), and further improved by ourselves (Milani and Knežević, 1990; 1991). This theory is incorporated in a fully automated and reasonably efficient software system that performs a complex sequence of operations, including an iterative procedure, and allows the production of large catalogues of proper elements. The proper elements are tested for accuracy by means of a conceptually simple, but technically delicate procedure based on their operational definition: we numerically integrate orbits of a number of representative asteroids (family heads, dynamically peculiar objects), compute the corresponding time series of proper elements, and measure the deviation of these elements from constancy. The validity of the tests for the entire asteroid population and for time spans much longer than those covered by numerical integrations cannot be rigorously proved, but the very existence of some accuracy estimates is already an essential improvement (see Zappalà et al., 1990 for the use of our accuracy estimates in assessing the reliability of family classification) and the results are already almost always good enough (see below) for family identification.

The current version (release 5.7) of our software differs from the previous one ((4.2), discussed in Milani and Knežević, 1990) in several respects. The basic theory is the same (for a discussion of the iterative procedure see Milani, 1990), but the dynamical model of the solar system used in the current version is significantly more complete: we have included all the terms we could compute by a straightforward generalisation of the procedure and which could be significant for at least some asteroids; the procedure is now flexible enough to accommodate different choices of the model for different applications. Laplace, Leverrier and Yuasa coefficients are computed by means of a more consistent and

reliable algorithm, an additional quality code based on the convergence of proper elements is defined, a secular resonance monitoring has been introduced, and the data set increased by $\simeq 500$ asteroids (up to No. 4722). The most important differences between the previous and current versions are summarized in Table 1. (note that version 5.6 pertains to the model used for the accuracy tests without the inner planets).

Table 1

Version	5.6	5.7	4.2
Planets affecting linear secular frequencies	5-8	2-8	5-6
Planets affecting forced terms	5-8	5-8	5-6
Planets included to second order	5-6	5-6	5
Planets included to degree 4	5-6	5-6	5
Secular frequencies in degree 4 (perihelia)	5,6	5,6	5,6
Secular frequencies in degree 4 (nodes)	6,7	6,7	6

Tests of the stability of the proper elements

A total of 22 asteroids representative of the most populated regions of the main belt have been chosen for tests of the accuracy of the theory, and their orbits, as a massless body attracted by the Sun and the four outer planets (solved a full 5-body gravitational problem), have been numerically integrated for a time span of 400,000 *yr*. For 10 test bodies results were found to be fully satisfactory, with rms changes in the proper elements less than 5×10^{-5} AU for a , less than 2.4×10^{-3} for e and less than 0.9×10^{-3} for $\sin I$. Although there is an obvious decrease of the accuracy with increasing inclination, the stability of proper elements was always satisfactory and in particular good enough for the identification of asteroid families. The most important unremoved oscillation of the proper elements in these cases was one with frequency $g + g_5 - 2g_6$ (in five cases it is responsible for more than 80% of the spectral power of the proper eccentricity changes), due to the fact that the forced term with this frequency was not accounted for by the theory (future versions of the theory will have to take account of this term and possibly for other similar ones).

For five test asteroids the results were inferior because of the nearby low-order mean motion resonances. Most of these objects have proper semimajor axes significantly less stable than the objects in the previous group, and poorly determined secular frequencies; nevertheless, their proper eccentricities and inclinations were of satisfactory, or even high, accuracy. From the point of view of family identification the most significant effect of the resonance pertains to the difficulty in identifying all the family members in the Themis family.

Four test asteroids, located in the inner part of the main belt ("Flora region"), are affected by the strongest secular resonance $g - g_6$. Therefore, the formal errors of the proper eccentricities and inclinations are substantial, although, for example, the error in the proper eccentricity of 8 Flora itself is much smaller than in the previous version of the theory

(due to the fact that the fundamental frequencies are determined with comparatively high precision). Typically, variations of all three proper elements are quite irregular, comprising several oscillations, some with periods in excess of 400,000 *yr*. We estimate that to be safe from the violent resonant perturbations and with proper elements reliable enough for the purpose of the asteroid family classification, the orbits must depart from the resonance (in terms of the frequency) by at least $\sim 4 \text{ arcsec/yr}$.

The last group of test asteroids (three of them) consists of bodies for which a nonlinear resonance warning was issued by our secular resonance monitoring routine. Small divisors representing combinations of four frequencies affect computation of proper elements in a significant way, although the slices in phase space in which these resonances are effective are very narrow. Removal of the resonant terms and the use of an “adapted” theory results in a remarkable improvement of the accuracy of proper elements.

On the basis of the above results we can conclude that over the time span covered by our integration and probably even somewhat longer (say, a few million years), the proper elements are in general stable to an accuracy which is better than required for family identification purpose. There are, in contrast, quite a few “exceptions” for which, unfortunately, this is not completely true or even not at all true: objects in the high inclination zones, slices near the main mean motion resonances, near the $g - g_6$ secular resonance, near the $g + g_5 - 2g_6$ resonance because of the missing forced term, and even near other nonlinear secular resonances. Furthermore, no rigorous conclusion can be proven on the stability of the proper elements over time spans much longer than that covered here, since the number of resonances concerned becomes much larger. The above list of problem cases determines the future directions of our work; all of them are understood and most of them can be solved, but at great effort.

Nonlinear secular resonances

To understand the nonlinear secular resonances and their impact to the family classification, we used our proper-element generation software to study the dynamical behaviour of orbits in a resonance of this new class: we modified the software so as not to remove the critical term, and manufactured “adapted” proper elements for all the asteroids within a box defining the family. For two families—one large (Eos) and one small (Lydia)—affected by the $g + s - g_6 - s_6$ secular resonance, the position of the libration region (as inferred from a time series of proper elements freed from the other important secular variations) has been compared with the position of the family.

In the case of the Eos family we have tested seven family members for 5.4 *Myr*, and found libration of the critical argument for some of them; for asteroid 339 Dorothea we found a maximum libration amplitude of about 108° , and proper eccentricity and proper sine of inclination oscillations of amplitudes 5.9×10^{-3} , and 2.6×10^{-3} , respectively. The family head—221 Eos—was found also to be in deep resonance, although amplitudes of libration and proper elements are significantly smaller than in the previous case. The critical argument of the $g + s - g_6 - s_6$ resonance slowly circulates for the other tested family members, for two of which, however (1075 Helina and 320 Katharina), the critical argument of the nearby $g + s - g_5 - s_7$ resonance librates. This libration of one critical argument is synchronized with the circulation of the other in a super resonance, or secondary resonance

between two resonances; the libration centre moves very little for 1075 Helina, slowly wanders for 320 Katharina. Note that, apart from these asteroids, this kind of resonance is known to occur for only one other body in the solar system: the planet Pluto (Milani et al., 1989). The overall effect of the secular resonances on the Eos family appears not to be very important since the widths of both resonances are significantly smaller than the spread of proper elements within the family; therefore, the classification of an asteroid as a family member is not vitiated by the resonance, unless the singularity occurring exactly at the resonance surface results in completely wrong proper elements (in the case of the Eos family, however, we found that only one of more than 200 family members is missing from the Zappalà et al., 1990 list for this reason).

The 110 Lydia family has been studied for the effect of the nonlinear resonance on a small family. According to Zappalà et al. (1990) this is a minor family, consisting of six members, which does not survive the reliability test against the error in proper elements. Because of the lower accuracy of the proper elements near the resonance, some family members are removed by the statistical reliability tests and too few were left for the family to be significant. However, the use of an adapted theory (with resonant term not removed from the osculating elements), quality codes and resonance warnings allowed us to correctly identify this family and to reliably reconstruct its membership. Thus, we found that at least two additional asteroids probably belong to the Lydia family, and that the family is statistically robust provided special care is taken because of the peculiar dynamical behaviour of its members.

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