Asteroids, Comets, Meteors 1991, pp. 359-362
Lunar and Planetary Institute, Houston, 1992

S85-90

Numerical Simulations of Cometary Dust

140940

David J. Lien
Bucknell University

p 4

N93-19198

Most observations of comets are done photometrically or spectrophotometrically. The interpretation of the aperature–averaged flux is relatively simple for an isotropic, radially expanding coma of infinite extent – the canonical model. However, the interpretation of the observations is not so clear when the the motion of the dust is affected by radiation pressure, or when the emission is time–varying and anisotropic. For example, in a sample of CCD images of 10 comets, Jewitt and Meech (1987, *Ap.J.* **317**, 992) found that the photometric profiles of only three comets were consistent, within the observational errors, with the profiles predicted from the canonical model. Photometric observations with large apertures, however, seem to suggest that the canonical model may be quite adequate (c.f. Osip, Schleicher, and Millis, 1992, *Icarus* **98**, 115).

The dust itself is characterized by a size distribution, with size dependencies on the expansion velocity, the scattered and thermal radiation, the response to radiation pressure, and probably the density. How good then are the approximations normally used in determining the production rates of the dust when these effects are present?

Given an isotropically expanding dust envelope (from a non-moving nucleus), the radial profile can be shown to decrease as the inverse of the projected cometcentric distance:

$$\sigma(\rho) = \int_{-\infty}^{\infty} \frac{Q}{4\pi(\rho^2 + l^2)v} dl = \frac{Q}{4\rho v} \tag{1}$$

where $\sigma(\rho)$ is the column density at the radial distance ρ, Q is the production rate $[s^{-1}]$, and v is the expansion velocity. If the limits are not $\pm\infty$, then

$$\sigma(\rho) = \int_{-R}^{R} \frac{Q}{4\pi(\rho^2 + l^2)v} dl = \frac{Q\pi}{2\rho v} \tan^{-1}\left(\frac{\sqrt{R^2 - \rho^2}}{\rho}\right) \tag{2}$$

where R is the edge of the envelope.

The total number of particles inside an aperture of projected radius s is

$$N(s) = \int_0^s 2\pi\rho\,\sigma(\rho) d\rho = \frac{Q}{v}\left[s \tan^{-1}\left(\frac{\sqrt{R^2 - s^2}}{s}\right) - \sqrt{R^2 - s^2} + R\right] = \frac{\pi Q s}{2v} \tag{3}$$

where the last equality is for $R = \infty$. Since the time over which the dust is emitted is finite, the total number of dust particles in the coma is just the product of the production rate and the interval of time over which the dust was emitted. Evaluating equation 3 for an aperture equal to the outer edge of the coma yields $N(R) = \frac{QR}{v} = Qt$, as expected.

As part of a program to better understand the dynamics of cometary dust and gas, a computer program has been developed which numerically simulates the emission of both dust and gas from the nucleus of a tilted rotating nucleus. Only the dust coma will be discussed here. The dust coma simulation includes the effects of the size dependencies on the expansion velocity, and scattering or thermal emissivity (based on either approximations or Mie theory calculations) and on the response to radiation pressure. Anisotropic emission is approximated by a gaussian jet centered at any latitude and longitude on a rotating nucleus of arbitrary rotation rate and obliquity.

The "image" of the gas or dust coma can be generated, as well as aperture– or annulus–averaged fluxes. An example of the annulus–averaged flux is presented below for P/Halley on 15 March 1986. Figure 1a shows $\sigma(\rho)$ from the numerical simulation of dust emission for 14 days at $v = 0.025\,\mathrm{km/s}$ with $\beta = 0$. This gives a maximum size of the coma of about 44". A least squares fit of equation (2) to the data is displayed as the solid line in Figure 1a. The solid line in this and subsequent figures corresponds to the theoretical fit, and the dotted line is derived from the numerical simulations. The aperture averaged flux is shown in Figure 1b, along with a least squares fit of equation (3) to the data.

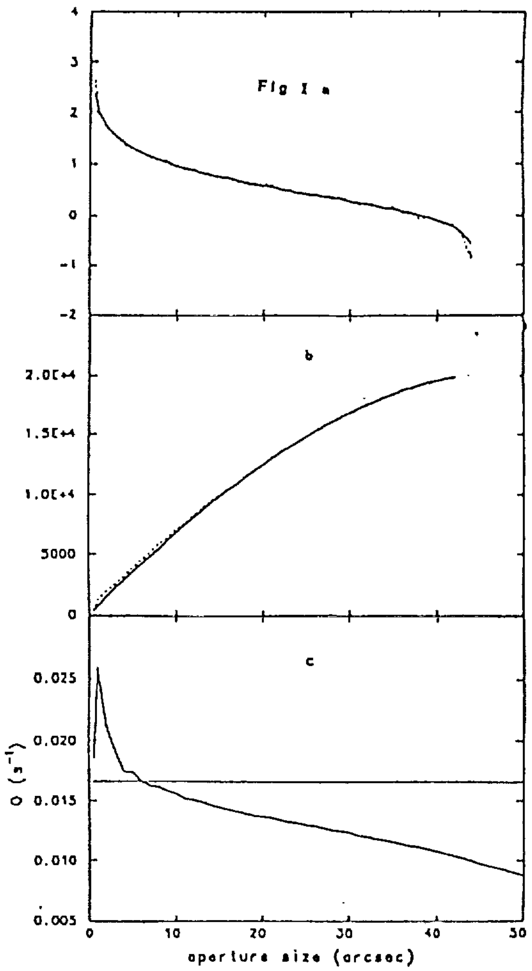
Since the production rate is constant and known from the parameters input into the numerical simulation, the $R = \infty$ solution can be solved for Q based on the results in Figure 1b. This is shown graphically in Figure 1c, where the production rate, Q, derived by assuming the $R = \infty$ solution to equation (3) is plotted as a function of the aperture size, s. The thin horizontal line indicates the true production rate. For this simple model, the $R = \infty$ approximation overestimates the production rate for small apertures, and underestimates Q for large apertures. As expected, the agreement between the $R = \infty$ approximation and the simulations inproves as R increases.

The dynamics of a large dust particle once well away from the nucleus depend primarily on the initial velocity vector of the particle and the ratio of the radiative pressure force to the force of gravity, β. Small charged dust grains are not considered here. As β increases, the coma becomes increasingly non-spherical, and one expects that the application of spherical approximations to the coma will become increasingly inaccurate. To test this statement, a series of models were run with $v = 0.025\,\mathrm{km/s}$ and a single β from the range $0.0001 < \beta < .5$. These parameters were chosen such that a spherically expanding, $\beta = 0$ envelope would occupy approximately $\frac{1}{2}$ the length of the image array. Figure 2 ($\beta = 0.0001$) and Figure 3 ($\beta = 0.1$) show both a projection against the sky of a subset of the total number of particles used in each simulation (typically 50,000), and the value of Q as a function of the aperture diameter. Again, the thin horizontal line indicates the input value of Q In these models, even a small β causes a significant change, with the predicted Q more than a factor of 2 too low for most aperture sizes. It is interesting to note that these simulations suggest that spectophotometric observations, with relatively small apertures, will yield higher dust production rates than the larger aperture photometric observations.

The question of a size distribution on the predicted Q is illustrated in Figure 4. To illustrate the effects of a size distribution, a β distribution which is rectangular in $log(\beta)$ (equal numbers of particles in equal size bins in $\log(\beta)$ with β within the range $0.0001 < \beta < 0.1$ was used. Each particle contributes an equal amount to the flux, and the velocity scales as $\sqrt{\beta}$. The scale factor is chosen so that the velocity of the center of the β distribution is $0.025\,km/s$. Figure 4 shows the results of this simulation.

To more realistically simulate the scattering or thermal emission properties of real grains, the same input parameters used in Figure 4 were modified by having each particle contribute a flux proportional to β^{-2}. This relationship is derived from the inverse relationship between the diameter of the particle and the value for β, which is valid for particles large compared with the wavelength of light. The results are shown in Figure 5. To determine the production rate, an "average" flux and velocity must be chosen. The values for the center of the β distribution was used to determine Q from the simulations. The production rate is overestimated by as much as 2 orders of magnitude ($Q = 0.041\,s^{-1}$), even with a large aperture. This result suggests that the derived production rate is very sensitive to the the choice of the optical properties of the "average" particle.

Although the results of most of the simulations run with a single *beta* are consistent with the statement that the production rate derived by assuming $R = \infty$ is correct to better than a factor of 5, this conclusion is misleading. As illustrated in Figure 5, when a broad size distribution with the intensity proportional to β^{-2} is used, the derived production rate is off by orders of magnitude. The main difficulty lies in determining the "average" velocity and β. Future work will concentrate on more realistic models: longer emission time, incorporation of the optical properties of spheres composed of potential cometary-dust materials, and a more realistic size or β distribution. Additionally, efforts will be focused on using the results of these simulations to aid in the interpretation of photometric and spectrophotometric observations. The results from the simulations presented here suggest that the canonical model, which assumes a radial outflow of dust, is inadequate in measuring the dust production rate. However, further simulations with more realistic input parameters are needed before any final conclusions can be drawn.



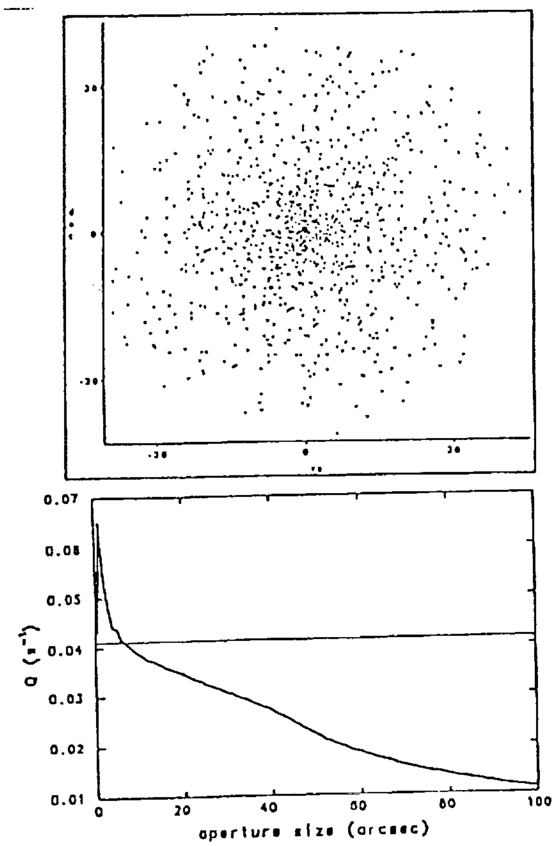


Figure 2

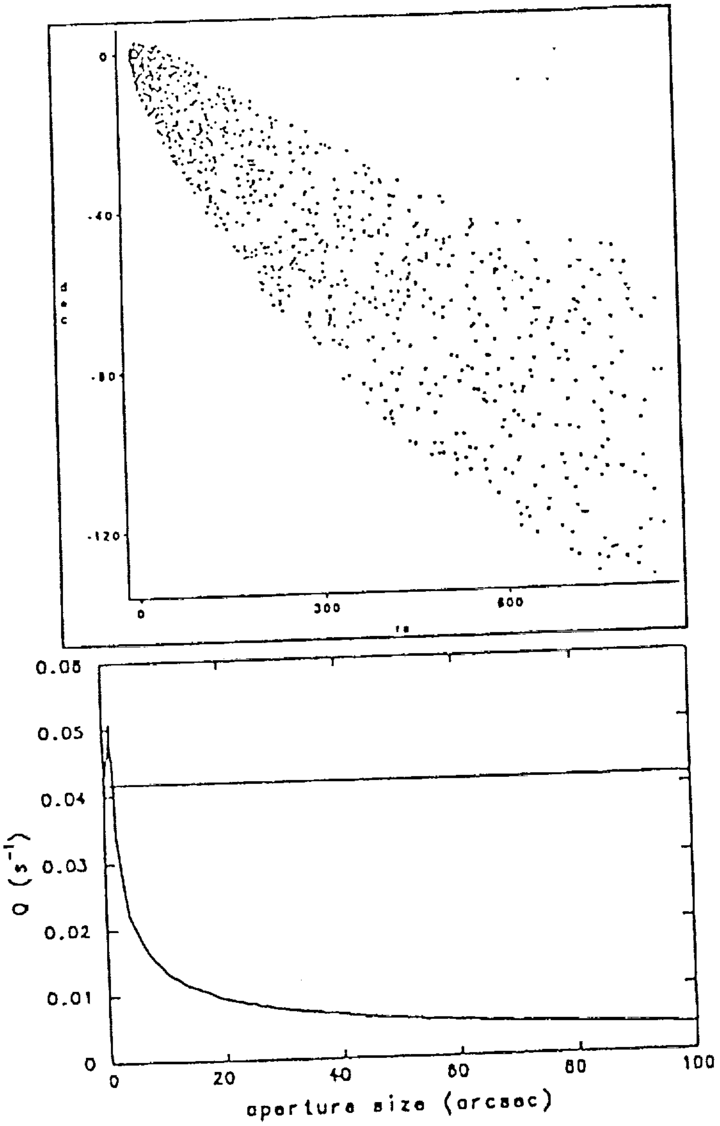


Figure 3

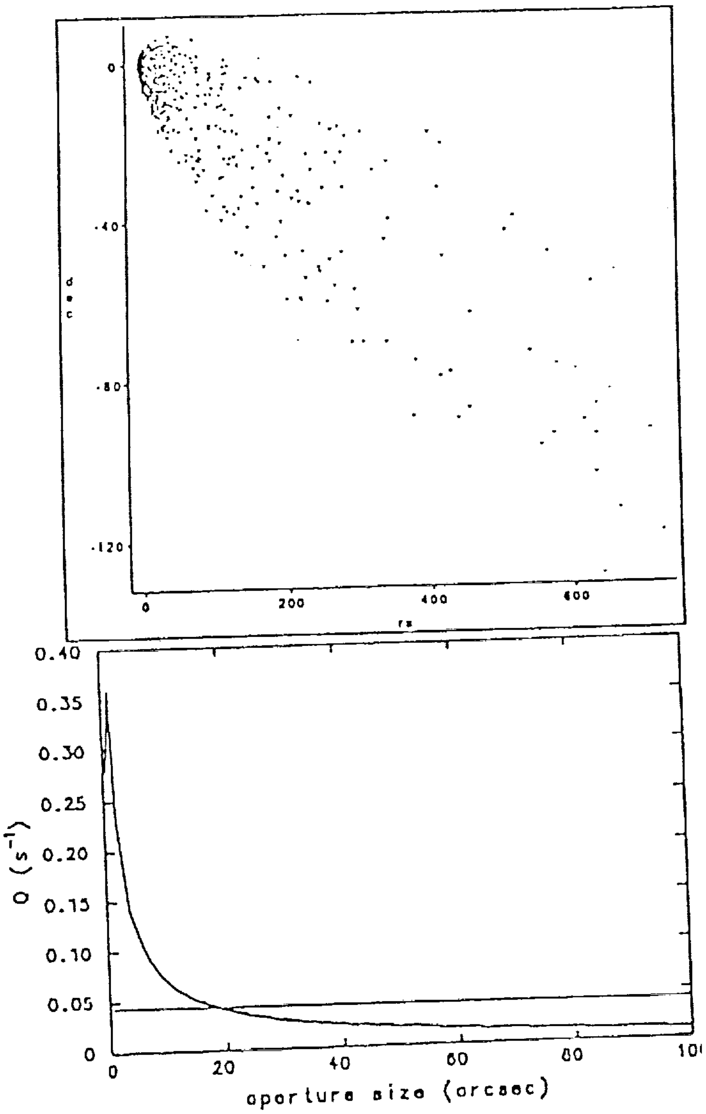


Figure 4

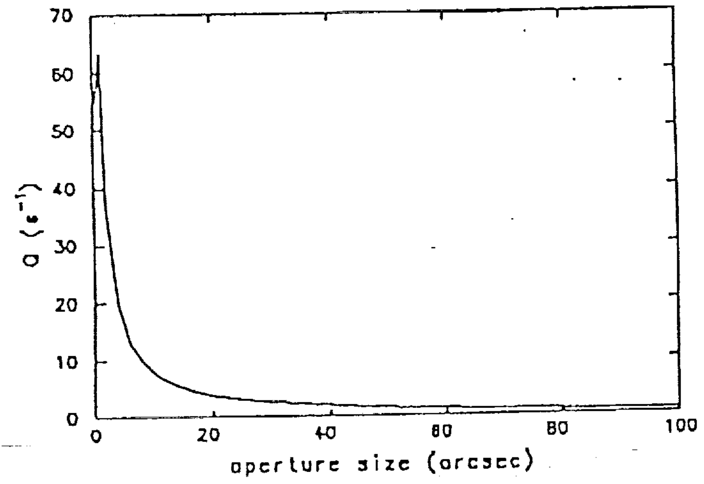


Figure 5