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THE RECOVERY OF ASTEROIDS AFTER TWO OBSERVATIONS

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ABSTRACT

It is shown that a generalization of the use of "Väisälä orbits", briefly mentioned at the *Asteroids II* conference, can be very conveniently accomplished by means of an inversion of the "GEM" form of the Gauss method. The procedure can also be applied to Apollo objects and to indeterminate cases of normal three-observation orbit computation, and there is also a simple extension to situations involving four or more observations.

A popular procedure for planning and then identifying the third night's observations of an asteroid is to utilize sets of orbits fitted to positions on the first and second nights on the assumption that the object is then at perihelion—or aphelion, a circular solution being the transition between the two sets. In the proceedings of the *Asteroids II* conference *Bowell et al.* (1989) briefly considered that this procedure (Väisälä 1939) might be generalized to delineate the precise region of the sky in which a main-belt asteroid must lie. In a typical application the third night will be in the dark of the moon following that of the first and second nights, which—to avoid incorrect linkage—should themselves be separated by no more than about five days; and the two lunations will tend to surround or adjoin the asteroid's being at opposition. Here I shall discuss in more detail how the "generalized Väisälä procedure" can conveniently be carried out in practice.

If the asteroid had actually been identified on the third night, it would be reasonable to attempt a three-observation orbit computation from one position on each night. This could be accomplished by the Gauss (1809) method, preferably in the rigorous "GEM" (Gauss-Encke-Merton) form I described (Marsden 1985) in response to Taff's (1984) misguided claim that the Gauss method is not mathematically valid.

In the GEM form the orbit computation is made in Cunningham's (1946) coordinate system, where the xy plane passes through the observations at times t_1 and t_3 , the x axis being directed toward that at t_3 . For reasons that will shortly become apparent, I shall consider t_1 to refer to the first observation and t_3 to the *second* observation. The time t_0 will then refer to the hypothetical third observation. The Cunningham coordinate system has the great advantage that the usual vector equation of the Gauss method separates into

$$\begin{aligned}\rho_0 &= (Z_0 - c_1 Z_1 - c_3 Z_3)/n_0 \\ \rho_1 &= (\rho_0 m_0 - Y_0 + c_1 Y_1 + c_3 Y_3)/c_1 m_1 \\ \rho_3 &= (\rho_0 l_0 - c_1 \rho_1 l_1 - X_0 + c_1 X_1 + c_3 X_3)/c_3,\end{aligned}\tag{1}$$

where the (l_i, m_i, n_i) and (X_i, Y_i, Z_i) represent the components of the unit vectors from the observer to the asteroid and of the vectors from the observer to the sun at the times t_i . The scalar distances ρ_i from the observer to the asteroid at the t_i are initially unknown, as are the scalars c_1 and c_3 , the usual ratios of the areas of the triangles that are associated with the Gauss method. Eqs. (1) can simply be evaluated in turn as successive approximations to these scalars become available.

The first of Eqs. (1) is fundamental to the Gauss method, and standard to any evaluation of it is consideration of the analogous equation

$$\rho_0^0 = (Z_0 - c_1^0 Z_1 - c_3^0 Z_3)/n_0,\tag{2}$$

where c_1 and c_3 are approximated by the ratios of the areas of the corresponding sectors,

$$\begin{aligned}c_1^0 &= (t_3 - t_0)/(t_3 - t_1) \\ c_3^0 &= (t_0 - t_1)/(t_3 - t_1),\end{aligned}\tag{3}$$

and can be immediately computed. Migration from Eq. (2) to the first of Eqs. (1) requires use of the "sector-triangle ratios", and in view of the well-known power-series expansions for these it has been usual to establish this in the form

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$$\rho_0 = \rho_0^o(1 - \gamma_0/r_0^3), \quad (4)$$

where r_0 , the asteroid's heliocentric distance at time t_0 , is also related to ρ_0 geometrically, and γ_0 is another quantity that is modified during the iterative process.

Although not essential to the normal orbit-determination process, it is useful for the present purpose also to define a quantity

$$\alpha^o = m_0/(m_1 c_1^o), \quad (5)$$

and since m_0 and m_1 are the sines of the angles between, respectively, the observations at t_0 and t_3 and at t_1 and t_3 , in a short-arc orbit the ratio m_0/m_1 goes roughly as the time ratio c_1^o , in which case $\alpha^o \sim 1$.

Although Gauss used Eq. (4) and its geometric counterpart quite rigorously and computed the sector-triangle ratios in terms of the hypergeometric function, others have instead relied heavily on power-series expansions, and since these expansions can certainly diverge, Taff's criticism has been understandable. In the case of a main-belt asteroid observed for up to a month or so near opposition, $\gamma_0 \sim R_0^3$, where $R_0 = (X_0^2 + Y_0^2 + Z_0^2)^{1/2}$, and convergence is to be expected. Nevertheless, acknowledging Taff's comments, and also because Eq. (4) is in reality a pure contrivance, I recommended (Marsden 1985) that the orbit solution be carried out by incorporating successive iterates for the c_1 and c_3 directly into Eqs. (1).

In the particular situation being considered here, however, the unit vector (l_0, m_0, n_0) at the isolated third observation is unknown. This suggests that Eqs. (2) and (5) should be inverted and written

$$\begin{aligned} \bar{n}_0 &= (Z_0 - c_1^o Z_1 - c_3^o Z_3)/\bar{\rho}_0 \\ \bar{m}_0 &= \bar{\alpha} m_1 c_1^o. \end{aligned} \quad (6)$$

One can therefore assume suitable values of $\bar{\rho}_0$ and $\bar{\alpha}$ and compute the corresponding \bar{n}_0 , \bar{m}_0 and $\bar{l}_0 = (1 - \bar{m}_0^2 - \bar{n}_0^2)^{1/2}$. The sets of $(\bar{l}_0, \bar{m}_0, \bar{n}_0)$ can then, on the one hand, be transformed from the Cunningham system back to the equatorial system and unique sets of values of the right ascension and declination, and on the other hand, be substituted for the (l_0, m_0, n_0) in Eqs. (1) and each time subjected to the full GEM orbit-determination process. The region of the sky to be searched can then be restricted on the basis of those orbital solutions that give acceptable values of semimajor axes a and eccentricities e (and perhaps also the inclinations i , although this requires conversion back from the Cunningham coordinate system).

It is useful to attempt the computation of a circular orbit from the observations at t_1 and t_3 . The resulting position at t_0 can be substituted in Eqs. (2) and (5) to give ρ_0^c and α^c , say, and the trial values needed in Eqs. (6) are then taken in the vicinity of $\bar{\rho}_0 = \rho_0^c$ and $\bar{\alpha} = \alpha^c$. It is not clear *a priori* how extended that vicinity should be, although I note that, for a main-belt object, Eq. (4) implies that $\bar{\rho}_0$ is ~ 0.1 AU larger than ρ_0 , and since, near opposition, $\rho_0 \sim r_0 - 1$ AU, to meet the extremes of perihelion and aphelion distance considered by *Bowell et al.* (1989) requires $0.7 \lesssim \bar{\rho}_0 \lesssim 3.1$ AU.

For an example, I use that of 1985 FZ considered by *Bowell et al.* (1989), and Fig. 1 is an adaptation of their Figure. The times of the connected observations are $t_1 = \text{Mar. 21.301 UT}$, $t_3 = \text{Mar. 24.323}$, and the isolated plate was taken on $t_0 = \text{Apr. 14.287}$. The circular orbit from the March observations yields $a = 2.79$ AU, $\rho_0^c = 1.924$ AU, $\alpha^c = 0.952$, and Fig. 1 shows that all the acceptable solutions are contained within $0.8 < \bar{\rho}_0 < 2.8$ AU, $0.82 < \bar{\alpha} < 1.02$. The original Apr. 14 candidate, denoted by "A", has an orbit with $a = 2.66$ AU, $e = 0.206$, $i = 5^\circ.5$. However, *Williams* (1991) has shown that this observation, but not the March observations, refers to a different object. There is another candidate to the north, "B", and this has an orbit with $a = 2.36$ AU, $e = 0.253$, $i = 4^\circ.2$.

If the motion in ecliptic latitude is too high, it may not be possible to calculate a circular orbit. Near opposition, and if i is ignored, the radius of the orbit (in AU) can be approximated by

$$a = 1 + k(t_3 - t_1)(1 - as)/m_1, \quad (7)$$

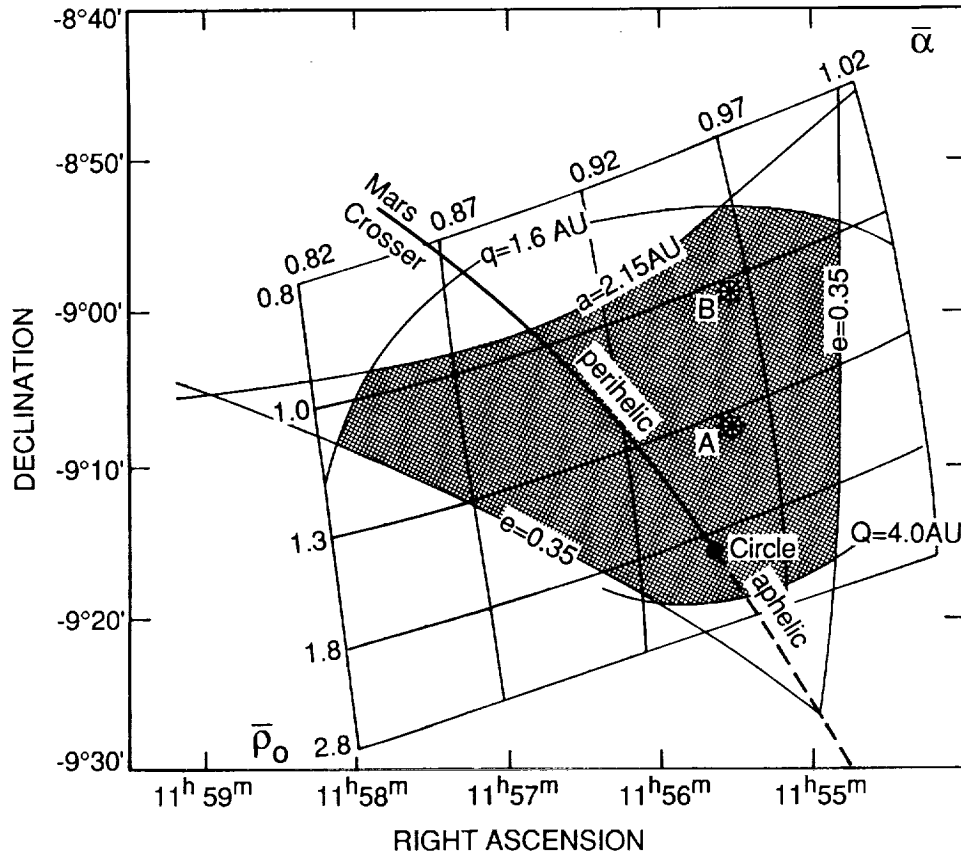


Fig. 1. The region of the sky (equinox 1950.0) in which 1985 FZ was located on 1985 Apr. 14, showing the positions in terms of the quantities $\bar{\rho}_0$ and $\bar{\alpha}$. The shaded region refers to acceptable orbits. The Väisälä locus of perihelic and aphelic orbits (including a circle) is shown. "A" is the original candidate, "B" an alternate candidate.

where the times are measured in days and the object's mean daily motion $s = a^{-\frac{3}{2}}$ is in units of the earth's mean daily motion k , i.e., the Gaussian constant 0.017202. For the 1985 FZ example, $m_1 = +0.01185$, and Eq. (7) can be solved iteratively to give $a = 2.73$ AU.

The quantity $\bar{\alpha}$ describes the foreshortening of the apparent motion, and for a circular orbit in the plane of the ecliptic with observations placed symmetrically about opposition it follows that $\bar{\alpha} = 1$. The foreshortening is greater than or less than unity according as to whether the observations are mainly before or mainly after opposition, the approximate time of which is given by

$$t_{\text{opp}} = t_3 + Y_3(t_3 - t_1)/(Y_1 - Y_3). \tag{8}$$

For 1985 FZ, $Y_1 = +0.09084$, $Y_3 = +0.03992$, so that $t_{\text{opp}} = \text{Mar. 26.692}$. The foreshortening can then be approximated by

$$\bar{\alpha} = (1 + \frac{1}{2}k^2P)^T, \tag{9}$$

where, with the time intervals in days,

$$T = (t_0 - t_1)(t_{\text{opp}} - t_{\text{mean}}), \tag{10}$$

i.e., -86.64 day^2 in the example, t_{mean} being the mean observation time Mar. 30.304, and

$$P = (1 - as^3)/(1 - as) + 3a[(1 - s)/(a - 1)]^2, \tag{11}$$

i.e., 4.12 for $a = 2.73$ AU, so that $1 + \frac{1}{2}k^2P = 1.00061$ and $\bar{\alpha} = 0.949$. The factor P increases between limits of 1 and $\frac{55}{4}$ as a decreases from ∞ to 1 AU, or between 3.5 and 5.2 over the main belt. Unless $|T|$ is very large, the expression 1.0006^T is therefore quite sufficient for establishing the initial trial value of $\bar{\alpha}$ for a main-belt object.

The new procedure can also be used in the case of Apollo asteroids, for which Väisälä orbits are troublesome because these objects can not simultaneously be at opposition and perihelion. Circular orbits are also meaningless for Apollo objects, so the relevant ranges of $\bar{\rho}_0$ and $\bar{\alpha}$ must be established by trial and error. The object 1990 MU was observed on two nights separated by six days in June 1990. The resulting search region for a night one month later extended some 1.5 hr in right ascension, its width increasing from as little as 1° at the eastern end to more than 5° at the western. The range of acceptable values of $\bar{\alpha}$ was ~ 0.3 - 1.3 , with $\bar{\rho}_0 \sim 0.9$ AU for small $\bar{\alpha}$, up to 0.8 - 1.5 AU for moderate $\bar{\alpha}$, then down to 0.6 - 0.9 AU for large $\bar{\alpha}$. Small values of $\bar{\rho}_0$ put the object near the earth and in earth-like orbits, and large $\bar{\rho}_0$ involved solutions that made the object distant with large e and i . For the largest $\bar{\alpha}$ the orbits degenerated into Aten type, and the search area became rather nebulous. The object was identified at $\bar{\rho}_0 = 1.28$ AU, $\bar{\alpha} = 0.89$ with $a = 1.62$ AU, $e = 0.66$, $i = 24^\circ$.

The case of the Apollo object 1991 JW is interesting because searches were to be made on the basis of observations spanning only a single 46-min exposure three weeks earlier. Given that the object was retrograding, however, the general character of the solutions was similar to that of the 1990 MU solutions, though with slightly larger ranges in $\bar{\rho}_0$ and $\bar{\alpha}$ not anticipated to be greater than 1.1. Although 1991 JW was in the mean time independently discovered by another astronomer, the first observer did succeed in recovering it right in a corner of the search area. With $\bar{\rho}_0 = 1.06$ AU, $\bar{\alpha} = 0.45$, the object had an exceptionally earth-like orbit with $a = 1.04$ AU, $e = 0.11$, $i = 8^\circ$.

Of course, the procedure outlined in this paper is equally valid however the three observations are ordered—if the isolated observation were made during the month *before* the others, it would be reasonable to reverse the order of the t_i —and a more detailed write-up in the Oct. 1991 issue of the *Astronomical Journal* discusses how the procedure can be used with advantage when a three-observation orbit solution is indeterminate, or where one might suspect that an observation is significantly in error.

The *Astronomical Journal* write-up also considers how the procedure can be extended to handle additional observations. The 1985 FZ example would have fared much better if there had been a second night of observations to verify the correct linkage of the object in April. Given such a “verification” observation, made at time t_{ver} , preferably only a few days from t_0 , it is not difficult to compute, *still for the time t_0* , a $\bar{\rho}_0 = \rho_0^v$ and an $\bar{\alpha} = \alpha^v$ that exactly satisfy the verification observation. If the linkage is correct, these should be very similar to ρ_0^o and α^o , and the residuals at t_0 and t_{ver} can then be better distributed by adopting the means $\rho_0^m = \frac{1}{2}(\rho_0^o + \rho_0^v)$, $\alpha^m = \frac{1}{2}(\alpha^o + \alpha^v)$. Obviously, further observations could also be included in these means, and the outcome resembles that of Herget’s (1965) method and “poor man’s” least-squares fit, the residuals of the observations at t_1 and t_3 remaining precisely zero. Herget’s example of 1935 QA, with the observations in the order $t_1 = \text{Aug. 30}$, $t_0 = \text{Sept. 6}$, $t_{\text{ver}} = \text{Sept. 23}$, $t_3 = \text{Oct. 21}$, results in $\rho_0^o = 1.78999$ AU, $\alpha^o = 0.95298$, $\rho_0^v = 1.78883$ AU, $\alpha^v = 0.95296$. The means ρ_0^m and α^m yield residuals (in arcsec in the equatorial system) of $+0.4$, $+0.1$ at t_0 and -1.1 , -0.6 at t_{ver} . These are a little larger than Herget’s values because of the wide separation of t_0 and t_{ver} . Residuals more comparable to Herget’s follow if ρ_0^m and α^m are weighted closer to ρ_0^v and α^v .

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