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LONG-TERM ORBITAL EVOLUTION OF SHORT-PERIOD COMETS FOUND IN PROJECT "COSMO-DICE"

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ABSTRACT. Orbital evolutions of about 160 short-period (SP) comets are numerically integrated for 4400 years in the framework of a realistic dynamical model. By the round-trip error in closure test, a reliable time span of the integrated orbits is estimated for each comet. Majority of the SP comets with their Tisserand's constant(J) between 2.8 and 3.1 are found to evolve within the past 1000-2000 years from the orbits whose perihelia are near the Jovian orbit to the orbits with perihelia of 1-2 AU. This evolution is much more rapid than that expected from Monte Carlo simulations based on symmetric distribution of planetary perturbations, thus suggesting that asymmetry of perturbation distribution play an important role in cometary evolution. Several comets are shown to evolve from the near-Saturn orbits and then to be handed over under the control of Jupiter. We also find that a few comets were captured from long-period orbits (a = 75-125 AU) via only a few close encounters with Jupiter. It is confirmed that the captured SP comets of low-inclination with 2.8 < J < 3.1 show more or less strong chaotic behavior. On the other hand, comets with longer orbital period and/or of high inclination reveal slow or quasi-periodic orbital evolution.

1. Introduction

"Cosmo-DICE" is a project, named after Dynamical Investigation of Cometary Evolution and after the erratic nature of SP comet motions just like casting dice. By this project we intend to provide reliable results of long-term orbital evolution of SP comets in the framework of a dynamical model as realistic as possible both for comets and planets. Although this work may look like only a successor of Carusi et al. (1985a), there are some important differences between theirs and ours. First, the time span covered by our calculations is 8.5 times toward the past and 2.5 times toward the future longer than those by Carusi et al. (1985a), which are both 400 years. For this achievement, some specific devices described below were necessary in terms of choice of integrators and calculation accuracy. As a result of this, we could find substantial orbital evolution of many SP comets, whereas, in Carusi et al. (1985a), only very limited cases of orbital evolution can be seen. Second, our calculations are characterized by an elaborate error check of integration for each orbit. This enables us to have a reliable time interval of integration for each comet.

Of course, considering the errors of the adopted orbital elements and possible non-gravitational effects which are not taken into account here, we cannot claim that every orbit in our calculations represents the true evolution for the 4400-year full time span. The main purpose of this project is, rather, to provide a standard database for statistical studies of long-term orbital evolution of SP comets, such as chaotic nature of thier orbits, capture into and ejection out of resonant motions, interrelation between asteroids and comets, and so on.

In section 2 we describe briefly the dynamical model and method of integration. Section 3 discusses the reliable time span of our integration in relation to the error growth rate. In section 4 are given the general characteristics of the orbital evolution of SP comets found in this project. The details of orbital evolution for each comet have been published in Nakamura and Yoshikawa (1991). A FITS-formatted magnetic tape including all the results is also available on request from the authors.

2. Dynamical Model and Method of Integration

Orbital elements as the initial conditions for numerical integration are taken for the most part from the Catalogue of Cometary Orbits, 6th edition (Marsden 1989) and the Minor Planet Circular. Planetary positions from Mercury through Pluto are not integrated simultaneously with comets, but are taken from the JPL numerical ephemerides DE102 (Newhall et al. 1983), which covers from BC 1411 through AD 3002.

In order to overcome the orbital instability inherent in SP comets, we adopted a quadruple-precision version of a variable-step extrapolation-type integrator (Bulirsch and Stoer 1966). The truncation parameter is set to 10^{-22} at r = 1 AU, which is a best compromise between CPU time and the accuracy requirement from orbital instability. Because of the variable-step control, regularization techniques are not used.

3. Error Estimate

After several trials we found that the round-trip error in closure test (Carusi et al. 1985b) is best as an error checker of integration for our purposes. Namely, integration is started from the present epoch toward the past up to BC 1411, and then this final state is integrated back to the present. The difference of these two orbital paths at each time step is expected to be a good measure of the error in the numerical integration. This is because the integration toward the present should reproduce exactly the original initial state if there is no error in the integration.

Fig. 1 shows some examples in the time history of the absolute value of the round-trip error in true longitude. General trend of these curves is always of linearly growing nature on the semi-log scale, though the error growth rate can be different by several order of magnitude from comet to comet. The error growth rate is found to have intimate relations with the orbital instability. This is reasonable because the Lyapunov characteristic exponent for a chaotic orbit should be positive (Lichtenberg and Lieberman 1983) and thus the linear error growth like Fig. 1 is expected.

We adopt here as a practical measure of orbital instability the elapsed time since BC 1411 in the backward integration within which the absolute error in true longitude attains one deg arc. This limit of time (Tlim) is calculated for each comet and the correlation of Tlim with J value is plotted in Fig. 2. We see that the mean Tlim for low-inclination SP comets is about 2000 years, and the low-inclination comets with their J near 3.0 show chaotic motions whereas the high-inclination ones behave more regularly. The Tlim is also useful to estimate the reliable time interval of the orbit calculation for a given error of adopted orbital elements. The error growth rate for the mean Tlim of 2000 years is about 10 per century. The relative error of best orbital elements determined from observations, on the other hand, is currently 10^{-7} or so. This implies that for an orbital longitude error of one deg arc the reliable time span of integration is about 700 years.

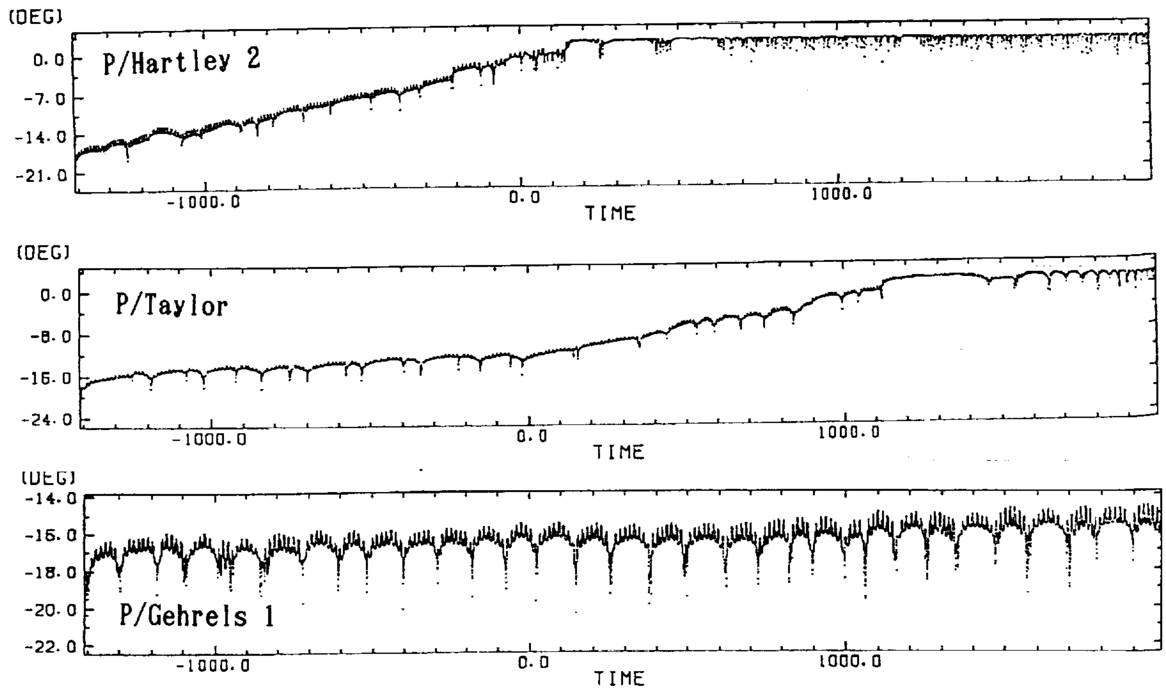

Fig.1: Some examples of the time history of the absolute value of the round-trip error in true longitude. The ordinate is in the scale of log(deg).

Table 1: q-change for 159 SP comets

Group	J	Number	Fraction
A	2.9 ≤ J	41	85%
A'		3	
a		6	15
B	2.8≤ J <2.9	11	57%
B'		2	
b		8	43
C	2.7≤ J <2.8	13	52%
C'		8	
c		12	48
D	2.5≤ J <2.7	6	33%
D'		1	
d		13	67
E	2.5 > J	1	3%
e		34	97

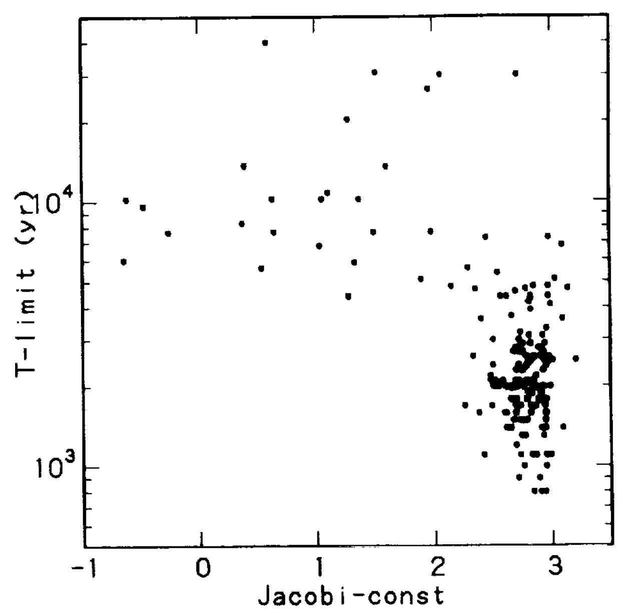

Fig.2: Tisserand(Jacobi) const. vs. T-limit

4. General Trends of Orbital Evolution

The following is some overall characteristics of orbital evolution of the SP comets found in this project. We have to keep in mind that these conclusions are more or less inevitablly statistical and the orbital evolution of individual comets should not always be taken at their face value.

(1) Majority of the SP comets that show drastic change of perihelion distance (q) have the Tisserand's constant (J) of 2.7~3.1:

Since the solar radiation makes comets luminous, the heliocentric distance or equivalently q is the most important factor for observability conditions. Therefore we concentrate our concern on the orbital evolution of q and classify the degree of q-change according to the value of J. In Table 1 group A is defined by the comets whose J is larger than 2.9 and whose q undergoes the change bewteen q_l=1~2AU and q_u=4~5AU during the 4400 years. We designate, on the other hand, as group a the comets whose J is in the same range but whose q shows the variation of no more than 1~2 AU. Similarly, other groups are also defined as given in Table 1. This classification of course cannot be so strict, because considerable number of calculated comets showed the J-change of 0.05~0.1. Some comets are classified with an apostrophed letter as critical cases of large q change, where q_l=1~2 AU but q_u=~3.5 AU.

When percentage fractions are calculated for each group in Table 1, a half number of the apostrophed comets are included in the upper-case group and another half in the lower-case group. It is remarkable to see that 85% of the comets with $2.9 \leq J < 3.1$ belongs to group A. This is partially explained by an approximate equation $U^2 = 3 - J$, where J is an expression of Tisserand's invariant and U is the total encounter velocity with Jupiter under the assumption of two-body interaction (Öpik 1963); for group A, U becomes nearly zero, so that large perturbations can be induced.

(2) The number of the large q-change comets whose q, semi-major axis, and aphelion distance decrease globally in their orbital evolution is much more than that of the comets that evolve in the opposite sense:

Among the 85 capital-letter comets, 59 (69%) showed the decreasing evolution, 9(11%) the increasing one, and 17 (20%) no specific trend. The decreasing evolution takes place mostly in the past, while the increasing one in the future. Therefore, in order to make unbiased comparison between the decreasing and increasing evolutions, the time span ratio for the future and the past (3400/1000) must be corredted to the number of the increasing evolution comets. However, the corrected number of the increasing evolution comets is 3.4 x 9 = 31, which is still only a half of the dereasing evolution comets. This means that capture process is more favoured than ejection in the evolution of SP comets. This finding is consistent with the reported asymmtery in the distribution of energy perturbation (Nakamura 1981; Manara and Valsecchi 1991) which prefers the paths toward shoter-period orbits. Interestingly, on the contrary, Everhart(1969) and Carusi et al.(1990) report the opposite tendency. This discrepancy will thus be a subject of further investigations.

(3) Most of the orbits before capture into Jupiter family have q ~ 5 AU:

All of the 59 captured SP comets mentioned in the previous paragraph were originated from the orbits of q ~ 5 AU. These comets correspond to the ones in the "capture region" by Jupiter, first confirmed numerically by Everhart (1972). The orbits before capture are also characterized by the very flat nature of q. Eight out of the 59 captured comets are shown to evolve from the orbits near Saturn and then to be relayed under the control of Jupiter; this is the first manifestation of the multiple-stage capture mechanism proposed by Everhart (1977). Some of the examples are given in Fig. 3. P/Helin-Roman-Alu 2, P/Honda-Mrkos-Pajdusakova, and P/Barnard 3 are found to be captured from the long-period orbits of a =75~125 AU into SP orbits through a few close encounters with Jupiter (Fig. 4).

(4) The evolutionary time scale for capture is much shorter than that predicted by

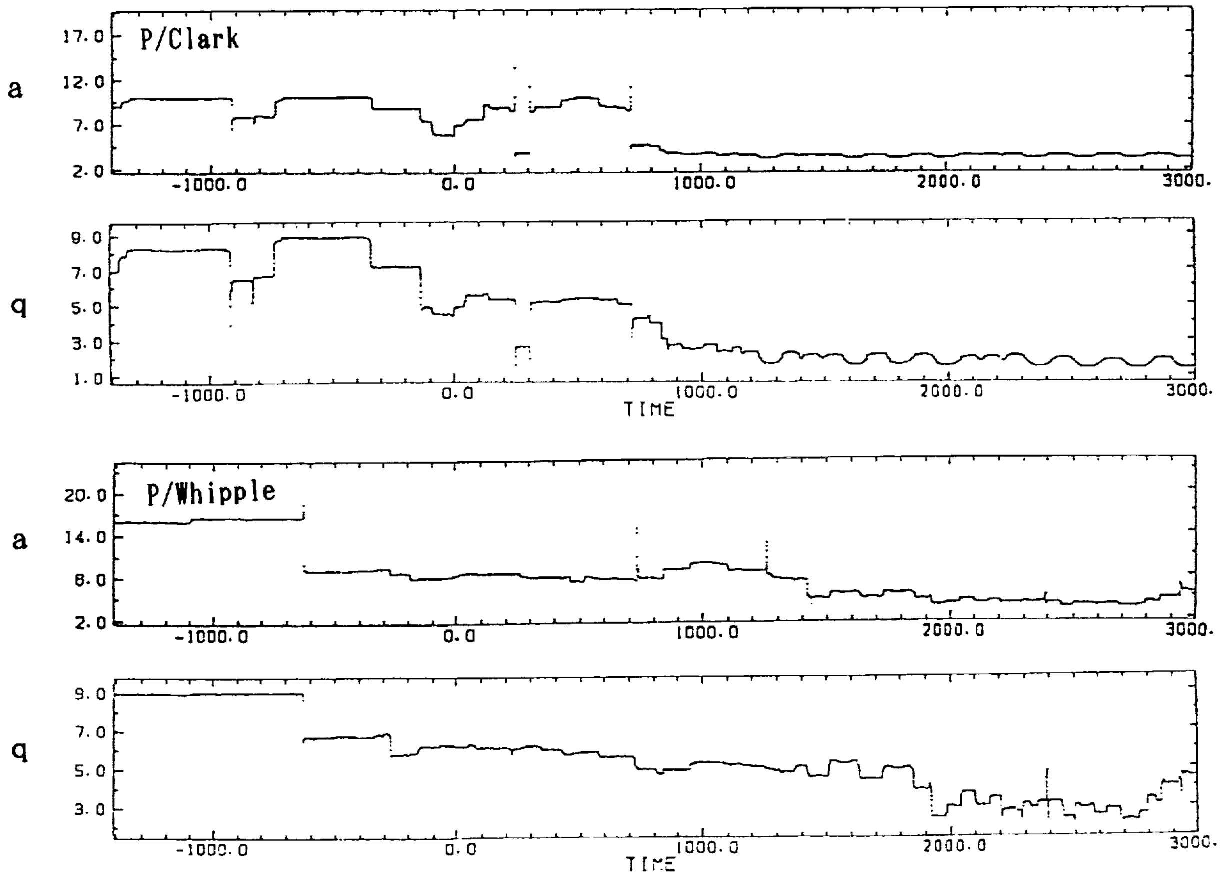

Fig. 3: Examples of relayed capture. The ordinate is in AU.

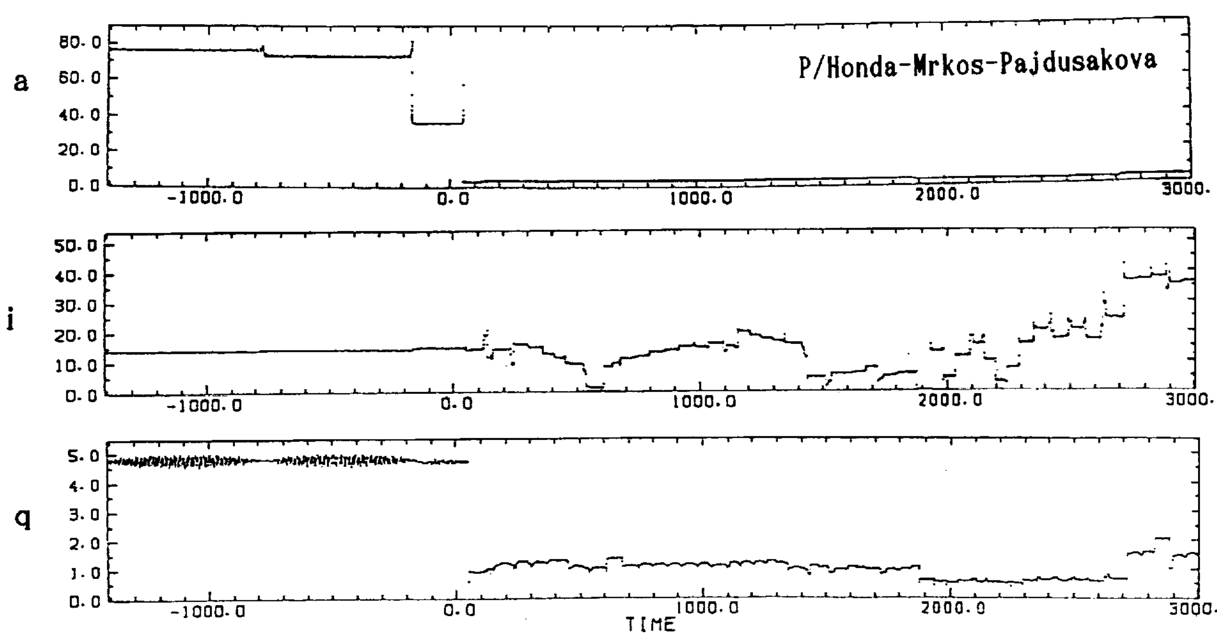

Fig. 4: An example of violent capture from a long-period orbit. a and q are in AU and inclination (i) is in degrees.

Monte Carlo simulations of simplified dynamical models:

We found many cases of the orbital evolution in which the orbital period (P) and q jumped from P=12~20 yr and q~5 AU into P=6~7 yr (typical of SP comets) and q=1~2 AU respectively via only one or two strong encounters with Jupiter. In analytical theory and Monte Carlo simulations based on symmetric distribution of perturbation, drastic orbital changes in such a short-time scale are very rare (e.g., Yabushita 1980; Nakamura 1981). This suggests that the asymmetric structure in the tail of perturbation distribution play an important role for the capture into SP comets.

Table 2 is a distribution of how many years ago an abrupt capture took place for the 59 comets analyzed above. It is seen that nearly one third of the comets were captured very recently (within the past 200 years) and they have been discovered soon after their capture. This may suggest that overlooked comets in observations be fewer than used to be anticipated as observational selection.

Table 2: Distribution of capture time

Interval (yr)	Number
0 ~ -200	17
-200 ~ -500	9
-500 ~ -1000	5
-1000 ~ -1500	11
-1500 ~ -2000	9
-2000 ~ -3400	8

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