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DETERMINATION OF ORBITS OF COMETS: P/KEARNS-KWEE, P/GUNN, INCLUDING NONGRAVITATIONAL EFFECTS IN THE COMETS' MOTION

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ABSTRACT

To improve the orbits we collected all the positional observations of the comets. The observations were selected and weighted according to objective mathematical criteria and the mean residuals *a priori* were calculated for both comets. We took into account non-gravitational effects in the comets' motion using the Marsden's method applied in two ways: either determining the three constant parameters A_1, A_2, A_3 or the four parameters A, η, I, ϕ connected with the rotating nucleus of the comet. To link successfully all the observations we had to assume for both comets that $A(t) = A_0 \exp(-B \cdot t)$ where B was an additional nongravitational parameter.

METHOD FOR NONGRAVITATIONAL EFFECTS

According to the known Marsden's method a nogravitational force acting on a comet is defined by its three components in orbital coordinates: F_1 – the radial component, F_2 – the transverse component in the orbit plane, F_3 – the component normal to the orbit plane (Marsden 1969). It is assumed that $F_i = A_i g(r)$, and the nongravitational parameters A_i, $i = 1, 2, 3$ are to be determined as constant quantities from observations for an individual comet; the analytical form of the function $g(r)$ is known (Marsden *et al.* 1973).

If the nongravitational force acts on the comet owing to the rocket effect of ejection of material from the rotating comet's nucleus, then:

$$A_i = AC_i(\eta, I, \lambda = v + \phi)$$

where $A = (A_1^2 + A_2^2 + A_3^2)^{1/2}$, η is the lag angle, I – the equatorial obliquity, v – the true anomaly of the comet, ϕ – the cometocentric solar longitude at perihelion; the direction cosines C_i have the following form:

$$\begin{aligned} C_1 &= \cos\eta + (1 - \cos\eta)\sin^2 I \sin^2\lambda \\ C_2 &= \sin\eta\cos I + (1 - \cos\eta)\sin^2 I \sin\lambda\cos\lambda \\ C_3 &= -[\sin\eta\cos\lambda - (1 - \cos\eta)\cos I \sin\lambda]\sin I \end{aligned}$$

It appears that A, η, I, ϕ can be determined from observations by the least squares method (Sitarski 1990). To link successfully all the observations of P/Kearns-Kwee and of P/Gunn as well, we had to assume that the nongravitational parameter A depended on time. We accepted that $A(t) = A_0 \exp(-B \cdot t)$ where t was counted in days from the epoch of osculation. Thus the five nongravitational parameters: A_0, B, η, I, ϕ have been determined from observations along with corrections to the six orbital elements.

OBSERVATIONAL MATERIAL

Observations of the comets were selected and weighted (each apparition separately) according to objective mathematical criteria (Bielicki 1972). A mean residual *a priori* was calculated as follows: *mean residual* $= [\Sigma_j^k n_j w_j \mu_j^2 / \Sigma_j^k n_j]^{1/2}$ where k—number of apparitions, n_j—number of residuals of the j-th apparition, w_j—weight of the j-th apparition, μ_j—value of the mean residual of the j-th apparition.

Characteristics of observations of Comet P/Kearns-Kwee:

Apparition	Number of observations	Number of residuals	Mean residual	Weight of apparition
1963.08.17 – 1965.04.24	60	115	1.″33	0.64
1971.07.26 – 1973.04.04	96	188	1.26	0.71
1981.06.29 – 1982.02.26	46	83	1.41	0.56
1989.09.10 – 1991.03.21	64	123	0.74	2.07

Observation interval: 1963 Aug. 17 – 1991 Mar. 21
Number of observations: 266
Number of residuals used for the orbit improvement: 509
Mean residual *a priori*: 1.″06

In case of Comet P/Kearns-Kwee we detected a displacement of the photometric center from the center of mass of the comet's nucleus (Sitarski 1984). We assume that the displacement of both centers along the radius-vector r is expressed by $\Delta r = Dr^{-3}\exp(-r^2/2)$ where the value of D is to be determined from the observational equations by the least squares method along with other parameters of the comet's motion.

Characteristics of observations of Comet P/Gunn:

Interval	Number of observations	Number of residuals	Mean residual	Weight of interval
1954.08.08 – 1954.08.08	2	4	–	1.00
1970.10.27 – 1971.11.18	13	26	0.″74	0.98
1972.12.03 – 1975.07.03	42	83	1.07	0.46
1976.04.24 – 1978.10.28	24	46	1.04	0.49
1980.12.07 – 1983.09.28	20	40	0.81	0.81
1984.08.24 – 1987.01.04	20	38	0.39	3.50
1988.01.15 – 1990.09.24	78	153	0.78	0.88

Observation interval: 1970 Oct. 27 – 1990 Sep. 24
Number of observations: 197
Number of residuals used for the orbit improvement: 386
Mean residual *a priori*: 0.″73

RESULTS

Orbit of Comet P/Kearns-Kwee:

(Epoch of osculation: 1963 Nov. 27.0 ET, Equinox: 1950.0)

1. With nongravitational parameters A_1, A_2, A_3:

$$T = 1963 \text{ Dec. } 7.00738 \text{ ET} \qquad \omega = 131^{\circ}\!.19146$$
$$q = 2.21317774 \text{ a.u.} \qquad \Omega = 315.43935$$
$$e = 0.48653298 \qquad i = 8.99170$$
$$A_1 = (+0.59457 \pm 0.13152) \times 10^{-8}$$
$$A_2 = (-0.38585 \pm 0.00154) \times 10^{-8}$$
$$A_3 = (-0.15898 \pm 0.20130) \times 10^{-8}$$
$$D = (+0.95212 \pm 0.17450) \times 10^{-2}$$

Mean residual *a posteriori*: $4''\!.67$

2. With $A = A_0 e^{-B \cdot t}$ and angular parameters η, I, ϕ:

$$T = 1963 \text{ Dec. } 7.00735 \text{ ET} \qquad \omega = 131^{\circ}\!.19250$$
$$q = 2.21318442 \text{ a.u.} \qquad \Omega = 315.43918$$
$$e = 0.48653473 \qquad i = 8.99151$$
$$A_0 = (+1.47950 \pm 0.06238) \times 10^{-8}$$
$$B = (+0.11971 \pm 0.00151) \times 10^{-3}/\text{day}$$
$$\eta = 33^{\circ}\!.30 \pm 2^{\circ}\!.68$$
$$I = 143.22 \pm 8.55$$
$$\phi = 334.45 \pm 17.01$$
$$D = (+0.36178 \pm 0.04559) \times 10^{-2}$$

Mean residual *a posteriori*: $1''\!.24$

Before the discovery of P/Kearns-Kwee a close aprroach of the comet to Jupiter to within 0.033 a.u. occured in November 1961. That approach considerably changed the comet's orbit (before the approach: $q = 4.302$, $e = 0.691$, $i = 2^{\circ}\!.88$).

Orbit of Comet P/Gunn:

(Epoch of osculation: 1989 Oct. 1.0 ET, Equinox: 1950.0)

1. With nongravitational parameters A_1, A_2, A_3:

$$T = 1989 \text{ Sep. } 24.96595 \text{ ET} \qquad \omega = 196^{\circ}\!.93927$$
$$q = 2.47155291 \text{ a.u.} \qquad \Omega = 67.86640$$
$$e = 0.31439342 \qquad i = 10.37272$$
$$A_1 = (+2.52440 \pm 0.00545) \times 10^{-8}$$
$$A_2 = (+0.50284 \pm 0.00050) \times 10^{-8}$$
$$A_3 = (-0.14875 \pm 0.15015) \times 10^{-8}$$

Mean residual *a posteriori*: $1''\!.52$

2. With $A = A_0 e^{-B \cdot t}$ and the angular parameters η, I, ϕ:

$$
\begin{array}{ll}
T = 1989 \text{ Sep. } 24.97004 \text{ ET} & \omega = 196^\circ\!.94062 \\
q = 2.47155081 \text{ a.u.} & \Omega = 67.86623 \\
e = 0.31439419 & i = 10.37270
\end{array}
$$

$$
\begin{aligned}
A_0 &= (+1.15530 \pm 0.07298) \times 10^{-8} \\
B &= (+0.12002 \pm 0.00751) \times 10^{-3}/\text{day} \\
\eta &= 53^\circ\!.45 \pm 12^\circ\!.91 \\
I &= 73.29 \pm 3.67 \\
\phi &= 84.84 \pm 4.37
\end{aligned}
$$

Mean residual *a posteriori*: $1''\!.15$

Comet P/Gunn approached Jupiter to within 0.353 a.u. in July 1965, and it changed the comet's orbit (before the approach: $q = 3.290$, $e = 0.178$, $i = 10^\circ\!.87$).

CONCLUSIONS

We can see that accepting the constant nongravitational parameters A_1, A_2, A_3 it was impossible to find resonable solutions for orbits of the considered comets: the mean residuals *a posteriori* were much greater than those *a priori* (especially for P/Kearns-Kwee). Assuming an expotential time dependence for the nongravitational parameter $A(t)$ we got the acceptable solutions for both comets.

Both comets approached Jupiter, and the approaches changed the comets' orbits in the same manner: perihelion distances diminished. The expotential decreasing of non-gravitational effects, as detected in the motion of both comets (the positive values of the parameter B), can be explained by the diminishing of the comets' activity with time after a rapid increase of activity caused by the changes of orbits due to approaches of the comets to Jupiter.

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