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ON THE DYNAMICAL STRUCTURE OF THE TROJAN GROUP OF ASTEROIDS

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Abstract

Using a semi-analytical approach, domains of possible motion for Trojan asteroids have been established. It is shown that stable librating motion is possible for both high inclination and high eccentricity. frequency distributions have also been produced for real Trojan asteroids, against differing libration amplitudes and libration periods.

Using a semi-analytical model described by Yoshikawa (1990), we consider the possible motion of massless Trojan asteroids, that is asteroids situated close to the 1:1 resonance region with Jupiter, which is assumed to be moving on its actual elliptic orbit. The asteroids are allowed to move on non-circular orbits in three dimensions. For convenience we define the critical argument, σ^*, to be the mean longitude of Jupiter minus that of asteroid, and denote the longitude of perihelion of an asteroid minus that of Jupiter by $(\varpi-\varpi_J)^*$ .The variation of a*, the semi-major axis of an asteroid orbit as a function of σ^* are obtained for specified values of the other parameters of the test orbit, namely the proper eccentricity, e*, the proper inclination, i*, and $(\varpi-\varpi_J)^*$ .Figure 1 shows the resulting twenty five plots when i* is zero with five different values of $(\varpi-\varpi_J)^*$, 0^0, 90^0, 180^0, 270^0 and 360^0 and five different values of e*, 0.02, 0.05, 0.10, 0.15 and 0.20 .By inspecting these plots, it is evident that stable libration motion is possible in all cases provided the libration angle (maximum - minimum value of σ^*), D, is less than about 50^0. The results do not appear to be particularly dependent on either the value of e* or on $(\varpi-\varpi_J)^*$ throughout the ranges investigated. In Figure 2, the value of the proper inclination, i*, was increased to about 20^0, with minor variations about this value, the other parameters taking the same values as in Figure 1. As can be seen, the domain for stable libration is now generally larger than in Figure 1, especially at low values of the proper eccentricity and sometimes even horseshoe orbits are found (for example at e* = 0.1 and $(\varpi-\varpi_J)^* = 90^0$). It is also interesting to note that the size of the stable libration region is larger when e*=0.2 than when e*=0.02 and that some displacement of the libration centre from the Lagrangian equilibrium points ($\sigma^*=60^0$ and 300^0) has occurred. Space does not permit the inclusion of diagrams indicating areas of possible libration for all sets of parameters investigated, however, even when the proper inclination, i* is increased to 50^0, large domains of stable librating motion still remain, though in this case, zones are more sensitive to the value of $(\varpi-\varpi_J)^*$. The

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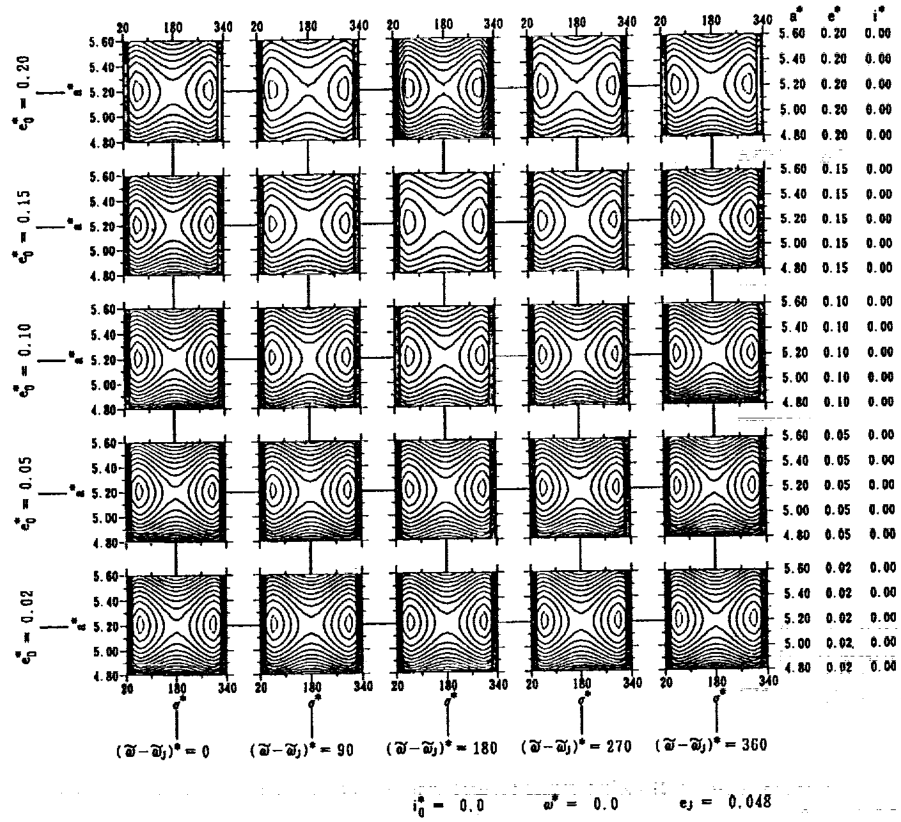

FIG.1. The 1:1 resonance. Model A for $e_J = 0.048$ and i^*=0°. On the right side, the values of e^* and i^* are shown as a function of a^*.

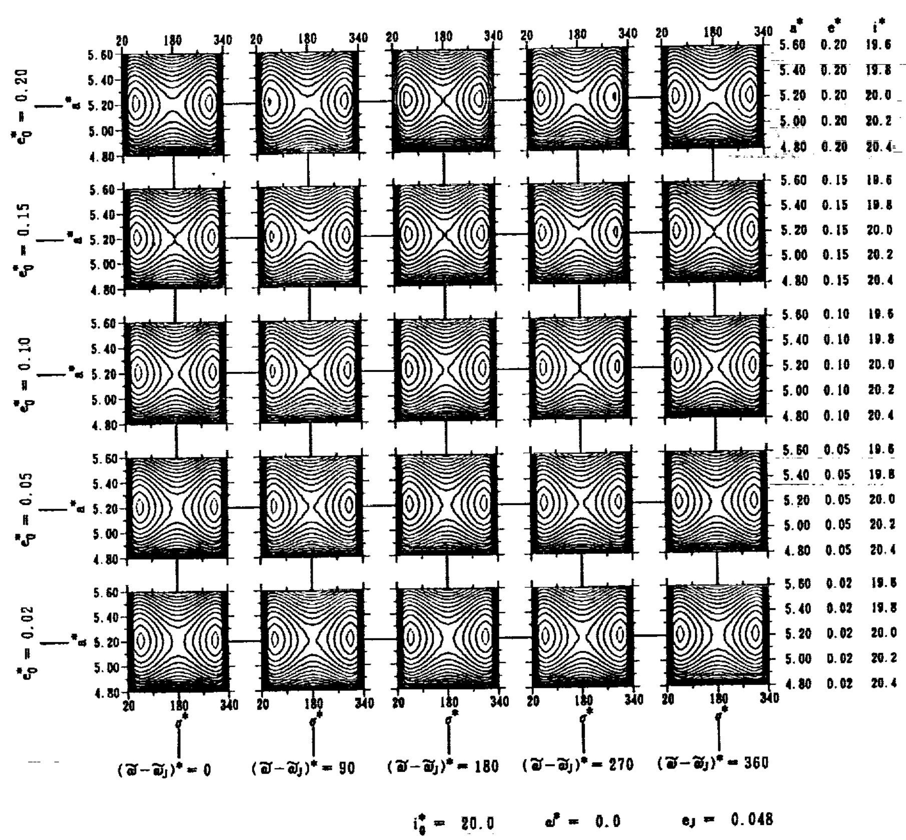

FIG.2. Model A for ω^*=0° and i^*=20°.

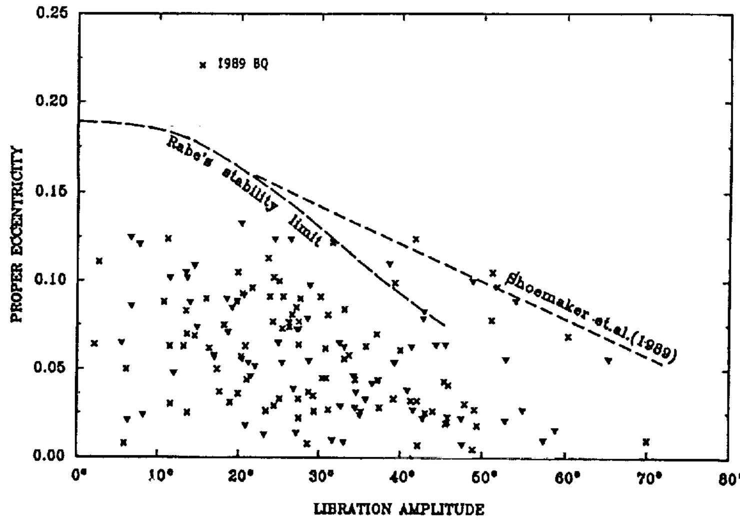

FIG.3. Distribution of proper eccentricity (e^*) vs libration amplitude (D) for 73 numbered (▾) and 88 unnumbered (×) Trojans.

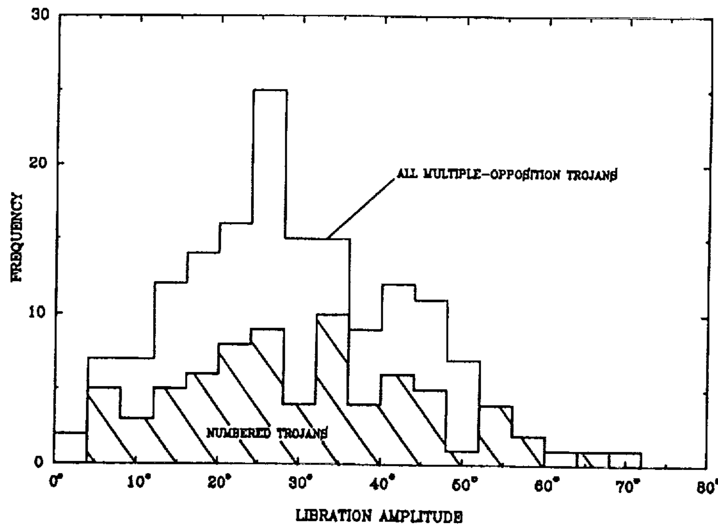

FIG.4. Frequency distribution of libration amplitude (D) of Trojans.

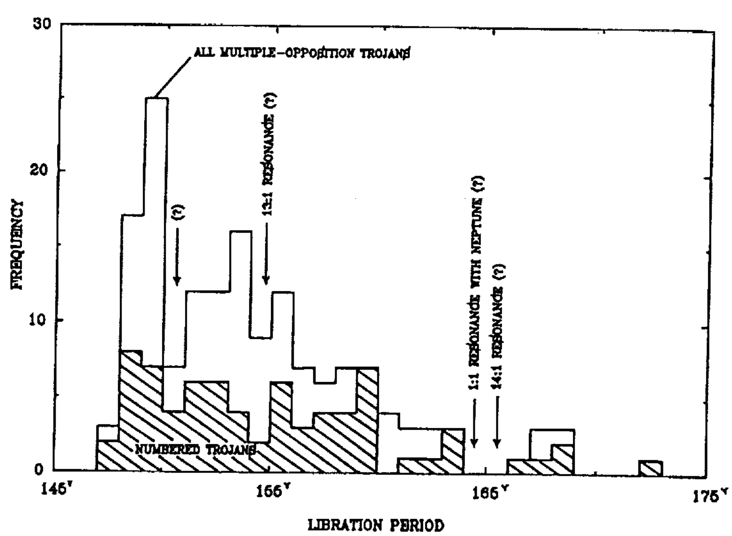

FIG.5. Frequency distribution of libration period of Trojans.

results do not appear to be sensitive to the value of the argument of perihelion. The dimensions of the domain for stable librating motion thus appears to be large, encompassing a wide range of values for eccentricity and inclination. It is clear that the real Trojan problem can't be meaningfully investigated in the context of the restricted circular planar three body problem.

Using an analytical theory (see Garfinkel, 1977, Zagretdinov, 1986) the amplitudes and periods of libration for 73 numbered and 88 unnumbered Trojan asteroids have been calculated. For this calculation, the proper inclination i* was estimated by referring the osculating asteroid orbit to the Jovian orbital plane. The proper eccentricity, e*, was obtained using the procedure described in Shoemaker et al (1989). Figure 3 displays some of these results, being a plot of proper eccentricity against libration amplitude.The curve defines the boundary of the stability region obtained by Shoemaker et al (1989). Also shown is an earlier limit to stability obtained by Rabe (1967) It is seen than while many asteroids lie in the unstable region as defined by Rabe, almost all of the 161 Trojan asteroids investigated lie in a region where Shoemaker et al (1989) had indicated that such motion was stable. Two lie just outside this region, (1989 UC5 and 1988 RNB) while 1989BQ (now numbered 4835) lies a very considerable distance into the forbidden zone.

Figure 4 shows the frequency with which Trojan asteroids with various libration amplitudes actually occur, the bin size being 4^0. The distribution for all Trojan asteroids shows a strong maximum at a libration amplitude of 30^0-40^0, but the corresponding peak does not appear to be present when only the numbered asteroids are considered.There is also evidence for a drop in the distribution when D exceeds 50^0. Eleven asteroids have a libration amplitude greater than 50^0, while three have an amplitude of over 60^0.

Finally, Figure 5 shows the frequency of occurrence of the Trojan asteroids with various libration periods, the bin size being one year.There is a clear peak in the distribution at a period of just under 150 years, when all multiple opposition Trojans are included, but the distribution is much flatter when only the numbered Trojans are included. Both figures 4 and 5 indicate the importance of obtaining a complete a sample of Trojans as soon as possible. there are gaps in the distribution corresponding to a 1:1 resonance with the orbital period of Neptune and also the 14:1 resonance with the orbital period of Jupiter, but the distribution in this region is so sparsely populated that it is not possible to say whether these gaps have any significance.There is also a suggestion of a gap coincident with the 13:1 orbital resonance with Jupiter, while a strong gap exists at 150 years which does not correspond to any obvious resonance.There are insufficient asteroids present in the sample for us to be able to comment positively on the hypothesis by Garfinkel (1977) that "internal" resonance may play a role in determining the distribution of Trojan asteroids

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