A COMPARISON BETWEEN FAMILIES OBTAINED FROM DIFFERENT PROPER ELEMENTS

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Abstract. Using the hierarchical method of family identification developed by Zappalà et al. [Astron. J., **100**, 2030 (1990)] we compare the results coming from the data set of proper elements computed by Williams (about 2100 numbered + about 1200 PLS II asteroids) and by Milani and Knežević (5.7 version, about 4200 asteroids). Apart from some expected discrepancies due to the different data sets and/or low accuracy of proper elements computed in peculiar dynamical zones, a good agreement was found in several cases. It follows that these high reliability families represent a sample which can be considered independent on the methods used for their proper elements computation. Therefore, they should be considered as the best candidates for detailed physical studies.

It is well known that asteroid dynamical families are a fascinating, but until recently puzzling subject. In fact, in spite of a noticeable effort carried out for several years by many authors, it has not long been possible to achieve a satisfactory agreement among the different proposed family lists. In particular, substantial disagreement exists for what concerns both the amount of existing families and the number and identity of their members. A detailed review of this subject can be found in Valsecchi et al. (1989).

Moreover, a review by Chapman et al. (1989) emphasizes another difficulty, in a physical sense, affecting many of the proposed families: in particular, only a few of them (mainly the most populous ones) are self-consistent from a cosmochemical point of view. In other words, most of the smaller clusterings identified in the space of proper elements by different authors, are composed by asteroids whose taxonomic types conflict with each other, in the sense that no plausible process of collisional origin from a single plausible parent body could explain the observed variety of taxonomic types of their supposed members.

On the other hand, the most recent advances in the studies of the overall collisional evolution of the asteroid belt (see, for a review, Davis et al., 1989) require some reliable estimates of the number and physical properties of the presently existing dynamical families, since they can provide some crucial constraints for the general evolutionary models proposed. At the same time, a knowledge of the main physical properties of the family members could shed some light on the general problem of the physics of catastrophic impacts and their collisional outcomes.

There are many reasons for the inconsistencies found in the past among the different proposed family lists. The main reasons are: different data sets used; different adopted methods of proper element computations; different procedures for clustering identification.

Recently, Zappalà et al. (1990) have carried out a new analysis, in which they used the biggest data set (about 4100 objects) ever analyzed for family identification purposes, as well as a new identification method. For what concerns the adopted proper elements, they used those computed by Milani and Knežević (1990), on the basis of a second-order, forth degree secular perturbation theory.

The new identification method is based on hierarchical clustering techniques of multivariate data analysis. The basic idea is to compute all the mutual distances among the analyzed objects, on the basis of some definition of a metric in the proper elements space (in particular, the adopted distance definition is related to the incremental velocity needed for orbital change after ejection from a fragmented parent body). In this way, it is possible to build some *dendrograms*, containing all necessary information about the existing clusterings of objects, having mutual distances smaller than any given value d. They are then compared with analogous dendrograms obtained for fictitious populations of quasi-randomly generated objects, in order to find a crytical value of distance ("quasi-random level") for which the clusterings found cannot be due to pure chance. These clusterings are defined as asteroid families in the Zappalà et al. (1990) paper.

In this way a new list of 21 families has been obtained, which includes, in addition to the well known and already firmly established Eos, Themis and Koronis families, at least 12 other families which appear to be highly reliable. These results must be compared with those obtained by other studies on the same subject available in the literature. In particular, one of the outstanding previous analyses is that carried out by Williams (1979), who used his own theory of proper elements computation (Williams 1969), and analyzed a large data set of 1796 numbered objects, in addition to a sample of the best orbits from the PLS survey. By applying a subjective method of clustering identification, Williams recognized 104 families. More recently, Williams (1989) has redone his analysis including a larger sample of numbered asteroids (2065), and found again a large number of families: 117.

It is interesting to infer whether the discrepancies between the Williams (1979, 1989) and the Zappalà et al. (1990) family lists are mostly due to the different identification methods used, or to the different sets of adopted proper elements.

An obvious procedure to discriminate among these two possibilities is to apply the same method of clustering identification to the two different samples of asteroid proper elements. In this paper, we present the results of such an exercise, in which the Zappalà et al. (1990) identification method has been applied separately to the extended data set (4258 objects) of the 5.7 version of asteroid proper elements by Milani and Knežević, as well as to the Williams (1989) proper elements lists, including 1968 numbered objects and 1227 PLSs.

The overall results of the present analysis are shown in Table 1, which shows, respectively, the number of resulting clusterings found using the Williams data set, the corresponding number of clusterings coming from the Milani and Knežević data set, and the number of "intersection clusterings", i.e., those who are found to be composed (at least partially) by the same objects in both the samples. Table 1 shows separately the cases corresponding to clusterings composed by a number $N \geq 5$ and $N \geq 10$ of objects. As can be seen, the agreement is quite good, mainly in the case of the most populous ($N \geq 10$) groupings. This can indicate that the global differences between the Williams (1979, 1989) and the Zappalà et al. (1990) classifications are probably mostly due to the different adopted methods of clustering identification, than to a substantial difference of the proper elements data sets.

Table 2 shows in a more detailed way the results of the present analysis for what concerns the clusterings having $N \geq 10$ members. In particular, for each of them, the number of Williams numbered objects (complete down to the number 2065), the number of PLS objects in the Williams data set, the number of the corresponding Milani and Knežević numbered asteroids (i.e., those having number $\leq$ 2065), and the Milani and Knežević numbered objects beyond the 2065 are shown, respectively, as well as the number of common objects present in both classifications. It is evident that the agreement can be considered quite good, in the sense that for these clusterings the intersection is generally very close to the number of common objects having number $\leq$ 2065, which is the maximum possible intersection of the two data sets. The next step of this analysis will be obviously to compute Milani and Knežević proper elements for the PLS asteroids having the best determined osculating elements, and to use them

Table I

	Williams	MK57	Intersection
$N \geq 5$	26	20	16
$N \geq 10$	15	14	14

Table II

Family [Zappalà et al. (1990)] list	Williams		MK 57		Intersection	remarks
	Num.	PLS	Num	H.Num.		
31 + 35	4	5	5	12	4	Vesta
32	5	3	9	32	5	F-type
33	4	29	2	21	2	
41	17	9	25	77	16	Eunomia
44	8	7	8	12	7	Adeona (Alexandra)
45	14	2	17	21	13	Maria
46	7	5	6	14	6	
48	5	5	3	20	3	
51	54	29	54	101	54	Koronis
61	89	9	80	127	79	Eos
71	75	38	72	165	71	Themis
1183 (*)	3	10	2	12	1	
135 (*)	1	18	1	16	1	Hertha
808 (*)	4	7	3	4	3	

(*) Families not found in the Zappalà et al. (1990) paper, due to the smaller data set used with respect to the present analysis. Here, they are identified by their least-numbered asteroid.

for carrying out a new updated list of asteroid dynamical families (Zappalà et al., in preparation).

Table 1 shows that the method for clustering identification by Zappalà et al. (1990), even in the case of smaller permitted clusterings, can allow us to identify less than 30 "families" by making use of the Williams' data set. This number has to be compared with the high number of families found by Williams (117) using the same data set. This discrepancy can be explained, in our opinion, if we admit that the Williams' cryterion of family identification was too liberal.

Figure 1 shows a general tentative classification of asteroid families; for a specific discussion of this subject, see the Farinella et al. paper published elsewhere in the present book. The figure shows a typical *stalactite diagram* for the asteroids having proper orbital semiaxes ranging between 2.500 and 2.825 *AU* (corresponding, respectively, to the 1/3 and the 2/5 mean motion resonances with Jupiter). The stalactite diagram shows, for different levels of the distance, the resulting clusterings of objects. The width of a stalactite at each level is given by the number of objects belonging to the clustering at that distance level. The dashed level is the "quasi-random level", or the distance level under which no clusterings are found in the case of fictitious populations of quasi-randomly generated objects (for a detailed explanation of this, see the Zappalà et al., 1990 paper).

As can be seen in the Figure, there are fundamentally three kinds of stalactites. The type II is sharp and deep, and corresponds to the families which are well defined and have been found by both Williams (1979, 1989) and Zappalà et al. (1990), with a large intersection of common objects. Type III are less sharp, and their members grow gradually as long as the distance level increases, so that the level at which one "cuts" the stalactite in order to define the family membership is crucial. Numerical simulations (see Bendjoya et al. in this book) show that in such cases one has to make a choice between two main possibilities: (1) to cut the stalactite at a low level, losing probably a significant amount of family members; this is in general the choice performed in the Zappalà et al. (1990) paper. (2) to be "liberal", cutting the

stalactite at higher levels, but taking into account that in this way a very large percentage of interlopers can be included in the assumed family; this has been generally done by Williams (1979, 1989). Finally, type IV stalactites are those that do not reach at present the rank of possible families on the basis of the Zappalà et al. (1990) cryterion. Instead, they have often been considered as real families by Williams (1979, 1989).

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Figure 1