N93-19459

A Piecewise Continuous Timoshenko Beam Model for the Dynamic Analysis of Tapered Beam-like Structures

S7-39
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P.13

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Submitted to
The Journal of Aerospace Engineering
Aerospace Division, American Society of Civil Engineering

ABSTRACT

Distributed parameter modeling is being seen to offer a viable alternative to finite element approach for modeling large flexible space structures. The introduction of the transfer matrix method into the continuum modeling process provides a very useful tool to facilitate the distributed parameter model applied to some more complex configurations. A uniform Timoshenko beam model for the estimation of the dynamic properties of beam-like structures has given comparable

results. But many aeronautical and aerospace structures are of the non-uniform sections or sectional properties, such as aircraft wing, satellite antenna.

This paper proposes a piecewise continuous Timoshenko beam model which is used for the dynamic analysis of tapered beam-like structures. A tapered beam is divided into several segments of uniform beam elements. Instead of arbitrarily assumed shape functions used in finite element analysis, the closed-form solution of the Timoshenko beam equation has been used. Application of transfer matrix method relates all the elements as a whole. By corresponding boundary conditions and compatible conditions a characteristic equation for the global tapered beam has been yielded, from which natural frequencies can be derived. A computer simulation is shown in this paper, and compared with the results obtained from the finite element analysis. While piecewise continuous Timoshenko beam model decreases the number of elements significantly, comparable results to the finite element method are obtained.

SYMBOLS

A	sectional area, or characteristic matrix
a	parameter in the Timoshenko beam equation, $a^2 = EI/m$
C_1, C_2, C_3, C_4	mode shape coefficients
Det [A]	characteristic determinant
E	modulus of elasticity
G	shear modulus
I	second moment of area of the beam section
k	bending stiffness, $k = EI$
L_j	length of the jth beam segment
M	bending moment
m	mass per unit length of the beam
Q	shear force
r	radius of gyration of the beam section, $r^2 = I/A$
T	time function
t	time
x, y, z	Cartesian coordinates
y(z,t)	lateral deflection
$\bar{z}$	dimensionless z-coordinate, $\bar{z} = z/L$
Y(z)	spatial lateral deflection function
α, γ	dimensionless parameters
β	eigenvalue coefficient, $\beta^4 = \omega^2 / a^2$

ε	Timoshenko shear coefficient
ψ	slope of lateral deflection
ω	circular natural frequency
$[\Phi]$	transfer matrix for the global beam
$[\Phi]_j$	transfer matrix for the jth beam segment
Φ_{ki}	elements of the global transfer matrix
$\phi_{ki}^{(j)}$	elements of the jth beam transfer matrix

1. INTRODUCTION

Distributed parameter modeling is being seen to offer a viable alternative to finite element approach for modeling large flexible space structures. Continuum models have been made of several flexible space structures, which include the Spacecraft Control Laboratory (SCOLE) [1], Solar Array Flight Experiment [2], NASA Mini-Mast Truss [3], the Space Station Freedom [4]. Especially, the introduction of the transfer matrix method into the continuum modeling process provides a very useful tool to facilitate the distributed parameter model applied to some more complex configurations [5,6]. A uniform Timoshenko beam model for the estimation of the dynamic properties of beam-like structures has given comparable results [7]. But many aeronautical and aerospace structures are of the non-uniform sections or sectional properties, such as aircraft wing, satellite antenna.

This paper proposes a piecewise continuous Timoshenko beam model which is used for the dynamic analysis of tapered beam-like structures. A tapered beam is divided into several segments of uniform beam elements. Instead of arbitrarily assumed shape functions used in finite element analysis, the closed-form solution of the Timoshenko beam equation has been used. Application of transfer matrix method relates all the elements as a whole. By corresponding boundary conditions and compatible conditions a characteristic equation for the global tapered beam has been yielded, from which natural frequencies can be derived. A computer simulation is shown in this paper, and compared with the results obtained from the finite element analysis. While piecewise continuous Timoshenko beam model decreases the number of elements significantly, comparable results to the finite element method are obtained.

2. TRANSFER MATRIX OF A TIMOSHENKO BEAM

Timoshenko beam model accounts for both rotary inertia and shear deformation of the beam. Usually, Timoshenko beam model produces more accurate estimation of the modal natural frequencies compared with the Bernoulli-Euler beam equation, especially for the range of higher frequencies [8]. In this section, a transfer matrix for Timoshenko beam model has been derived. The Timoshenko beam is represented by the equation,

$$\frac{\partial^4 y}{\partial z^4} + \frac{m}{EI}\frac{\partial^2 y}{\partial t^2} - \frac{m}{EA}\left(1 + \frac{E}{\varepsilon G}\right)\frac{\partial^4 y}{\partial z^2 \partial t^2} + \frac{m^2}{\varepsilon EGA^2}\frac{\partial^4 y}{\partial t^4} = 0 \tag{2.1}$$

For harmonic motion, y(x,t) can be expressed as

$$y\,(x,t) = Y(x)\, e^{j\omega t}$$

then, Eq.(2.1) will become

$$Y'''' + \frac{m}{EA}\left(1 + \frac{E}{\varepsilon G}\right)\omega^2 Y'' + \left(\frac{m^2\omega^2}{\varepsilon EGA^2} - \frac{m}{EI}\right)\omega^2 Y = 0 \tag{2.2}$$

Defining $\beta^4 = \omega^2/a^2$, where $a^2 = EI/m$, Eq.(2.2) becomes

$$Y'''' + \beta^4 r^2 \left(1 + \frac{E}{\varepsilon G}\right) Y'' + \beta^4 \left[\beta^4 r^4 \left(\frac{E}{\varepsilon G}\right) - 1\right] Y = 0 \tag{2.3}$$

where, $r^2 = I/A$, the radius of gyration of the section. If we use the following dimensionless parameters,

$$\bar{z} = \frac{z}{L}, \qquad \alpha = \frac{1}{\varepsilon}\left(\frac{r^2}{L^2}\right)\frac{E}{G}, \qquad \gamma = \frac{r^2}{L^2}$$

the Timoshenko equation may finally be written as

$$Y'''' + (\beta L)^4 (\alpha + \gamma) Y'' + (\beta L)^4 \left[(\beta L)^4 \alpha\gamma - 1\right] Y = 0 \tag{2.4}$$

Assuming that the solution is

$$Y(\bar{z}) = A\, e^{(BL)\bar{z}}$$

which, when substituted into Eq.(2.4), leads to

$$(BL)^4 + (\beta L)^4 (\alpha + \gamma) (BL)^2 + (\beta L)^4 [(\beta L)^4 \alpha\gamma - 1] = 0 \tag{2.5}$$

The solution to the Eq.(2.5) is as follows.

$$\begin{cases} (BL)_{1,2} = \pm (\eta L) = \pm \frac{\sqrt{2}}{2} [- (\beta L)^4(\alpha + \gamma) + \sqrt{(\beta L)^8(\alpha - \gamma)^2 + 4(\beta L)^4}]^{1/2} \\ (BL)_{3,4} = \pm j (\theta L) = \pm j \frac{\sqrt{2}}{2} [(\beta L)^4(\alpha + \gamma) + \sqrt{(\beta L)^8(\alpha - \gamma)^2 + 4(\beta L)^4}]^{1/2} \end{cases}$$

Then, the solution to the Eq.(2.4) can be expressed as

$$Y(\bar{z}) = C_1 \sin (\theta L\bar{z}) + C_2 \cos (\theta L\bar{z}) + C_3 \sinh (\eta L\bar{z}) + C_4 \cosh (\eta L\bar{z}) \tag{2.6}$$

Similarly, for the bending slope ψ the differential equation has the same form as the Eq.(2.4),

$$\Psi'''' + (\beta L)^4 (\alpha + \gamma) \Psi'' + (\beta L)^4 [(\beta L)^4 \alpha\gamma - 1] \Psi = 0 \tag{2.7}$$

The solution to Eq.(2.7) will be

$$\Psi(\bar{z}) = \sigma_1 C_1 \cos (\theta L\bar{z}) - \sigma_1 C_2 \sin (\theta L\bar{z}) + \sigma_2 C_3 \cosh (\eta L\bar{z}) + \sigma_2 C_4 \sinh (\eta L\bar{z}) \tag{2.8}$$

where,

$$\sigma_1 = \frac{1}{L}\left[(\theta L) - \frac{\alpha}{(\theta L)} (\beta L)^4\right] \quad \text{and} \quad \sigma_2 = \frac{1}{L}\left[(\eta L) + \frac{\alpha}{(\eta L)} (\beta L)^4\right]$$

For the Timoshenko beam model, the shear force is

$$Q(z,t) = k \frac{\partial^3 y}{\partial z^3} - k \frac{m}{\varepsilon GA} \frac{\partial^3 y}{\partial z \partial t^2} - J \frac{\partial^2 \psi}{\partial t^2} \tag{2.9}$$

or, equivalently,

$$Q(z) = k Y''' + k \frac{m\omega^2}{\varepsilon GA} Y' + J\omega^2 \psi \tag{2.10}$$

And the bending moment is

$$M(z,t) = k \frac{\partial^2 y}{\partial z^2} - k \frac{m}{\varepsilon GA} \frac{\partial^2 y}{\partial t^2} \tag{2.11}$$

or, equivalently,

$$M(z) = k\,Y'' + k\,\frac{m\omega^2}{\varepsilon GA}\,Y \tag{2.12}$$

Eqs. (2.10) and (2.12) can be written in dimensionless format as,

$$Q(\bar{z}) = \frac{k}{L^3}Y''' + \frac{k}{L^3}\alpha(\beta L)^4\,Y' + \frac{k}{L^2}\gamma(\beta L)^4\,\psi \tag{2.13}$$

$$M(\bar{z}) = \frac{k}{L^2}Y'' + \frac{k}{L^2}\alpha(\beta L)^4\,Y \tag{2.14}$$

Substituting the solutions of Y(z)(Eq.2.6) and Ψ(z)(Eq.2.8) into Eqs.(2.13) and (2.14), we derive

$$Q(\bar{z})=-k\sigma_{11}C_1\cos\theta L\bar{z}+k\sigma_{11}C_2\sin\theta L\bar{z}+k\sigma_{21}C_3\cosh\eta L\bar{z}+k\sigma_{21}C_4\sinh\eta L\bar{z} \tag{2.15}$$

$$M(\bar{z})=-k\sigma_{12}C_1\sin\theta L\bar{z}-k\sigma_{12}C_2\cos\theta L\bar{z}+k\sigma_{22}C_3\sinh\eta L\bar{z}+k\sigma_{22}C_4\cosh\eta L\bar{z} \tag{2.16}$$

where,

$$\sigma_{11} = \theta^3 - \alpha L^2\beta^4\theta - \gamma L^2\beta^4\sigma_1\,, \qquad \sigma_{12} = \theta^2 - \alpha L^2\beta^4$$

$$\sigma_{21} = \eta^3 + \alpha L^2\beta^4\eta + \gamma L^2\beta^4\sigma_2\,, \qquad \sigma_{22} = \eta^2 + \alpha L^2\beta^4$$

For the jth beam element, the displacement Y(0), slope Ψ(0), shear Q(0), and the bending moment M(0) at the end of z=0 can then be written in matrix form as,

$$\begin{Bmatrix} Y_0 \\ \Psi_0 \\ Q_0 \\ M_0 \end{Bmatrix}_j = \begin{bmatrix} 0 & 1 & 0 & 1 \\ \sigma_1 & 0 & \sigma_2 & 0 \\ -k\sigma_{11} & 0 & k\sigma_{21} & 0 \\ 0 & -k\sigma_{12} & 0 & k\sigma_{22} \end{bmatrix}_j \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix}_j \tag{2.17}$$

Thus,

$$\begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix}_j = [\lambda]_j \begin{Bmatrix} Y_0 \\ \Psi_0 \\ Q_0 \\ M_0 \end{Bmatrix}_j \tag{2.18}$$

where,

$$[\lambda]_j=\begin{bmatrix} 0 & 1 & 0 & 1 \\ \sigma_1 & 0 & \sigma_2 & 0 \\ -k\sigma_{11} & 0 & k\sigma_{21} & 0 \\ 0 & -k\sigma_{12} & 0 & k\sigma_{22} \end{bmatrix}_j^{-1}=\begin{bmatrix} 0 & \frac{\sigma_{21}}{\sigma_1\sigma_{21}+\sigma_2\sigma_{11}} & -\frac{\sigma_2}{k(\sigma_1\sigma_{21}+\sigma_2\sigma_{11})} & 0 \\ \frac{\sigma_{22}}{\sigma_{12}+\sigma_{22}} & 0 & 0 & -\frac{1}{k(\sigma_{12}+\sigma_{22})} \\ 0 & \frac{\sigma_{11}}{\sigma_1\sigma_{21}+\sigma_2\sigma_{11}} & \frac{\sigma_1}{k(\sigma_1\sigma_{21}+\sigma_2\sigma_{11})} & 0 \\ \frac{\sigma_{12}}{\sigma_{12}+\sigma_{22}} & 0 & 0 & \frac{1}{k(\sigma_{12}+\sigma_{22})} \end{bmatrix}_j$$

Similarly, at the end of z=L, the corresponding quantities are, if written in matrix form,

$$\begin{Bmatrix} Y \\ \Psi \\ Q \\ M \end{Bmatrix}_j^L=[\zeta]_j\begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix}_j \tag{2.19}$$

where,

$$[\zeta]_j=\begin{bmatrix} \sin\theta L & \cos\theta L & \sinh\eta L & \cosh\eta L \\ \sigma_1\cos\theta L & -\sigma_1\sin\theta L & \sigma_2\cosh\eta L & \sigma_2\sinh\eta L \\ -k\sigma_{11}\cos\theta L & k\sigma_{11}\sin\theta L & k\sigma_{21}\cosh\eta L & k\sigma_{21}\sinh\eta L \\ -k\sigma_{12}\sin\theta L & -k\sigma_{12}\cos\theta L & k\sigma_{22}\sinh\eta L & k\sigma_{22}\cosh\eta L \end{bmatrix}_j$$

Substituting Eq.(2.18) into Eq.(2.19) we obtain

$$\begin{Bmatrix} Y_L \\ \Psi_L \\ Q_L \\ M_L \end{Bmatrix}_j=[\Phi]_j\begin{Bmatrix} Y_0 \\ \Psi_0 \\ Q_0 \\ M_0 \end{Bmatrix}_j \tag{2.20}$$

where, $[\Phi]_j$ is the transfer matrix of the Timoshenko beam,

$$[\Phi]_j=[\zeta]_j[\lambda]_j \tag{2.21}$$

and the elements of the transfer matrix $[\Phi]_j$ are as follows.

$$\varphi_{11}=\frac{1}{\sigma_{12}+\sigma_{22}}(\sigma_{22}\cos\theta L+\sigma_{12}\cosh\eta L)$$
$$\varphi_{12}=\frac{1}{\sigma_1\sigma_{21}+\sigma_2\sigma_{11}}(\sigma_{21}\sin\theta L+\sigma_{11}\sinh\eta L)$$

$$\varphi_{13} = \frac{1}{k(\sigma_1\sigma_{21}+\sigma_2\sigma_{11})}(-\ \sigma_2\sin\theta L + \sigma_1\sinh\eta L)$$
$$\varphi_{14} = \frac{1}{k(\sigma_{12}+\sigma_{22})}(-\ \cos\theta L + \cosh\eta L)$$

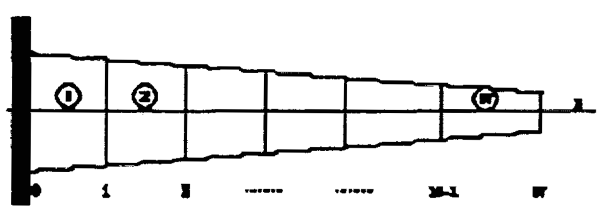
$$\varphi_{21} = -\frac{1}{\sigma_{12}+\sigma_{22}}(\sigma_1\sigma_{22}\sin\theta L - \sigma_2\sigma_{12}\sinh\eta L)$$
$$\varphi_{22} = \frac{1}{\sigma_1\sigma_{21}+\sigma_2\sigma_{11}}(\sigma_1\sigma_{21}\cos\theta L + \sigma_2\sigma_{11}\cosh\eta L)$$
$$\varphi_{23} = \frac{\sigma_1\sigma_2}{k(\sigma_1\sigma_{21}+\sigma_2\sigma_{11})}(-\ \cos\theta L + \cosh\eta L)$$
$$\varphi_{24} = \frac{1}{k(\sigma_{12}+\sigma_{22})}(\sigma_1\sin\theta L + \sigma_2\sinh\eta L)$$

$$\varphi_{31} = \frac{k}{\sigma_{12}+\sigma_{22}}(\sigma_{11}\sigma_{22}\sin\theta L + \sigma_{12}\sigma_{21}\sinh\eta L)$$
$$\varphi_{32} = -\frac{k\sigma_{11}\sigma_{21}}{\sigma_1\sigma_{21}+\sigma_2\sigma_{11}}(\cos\theta L - \cosh\eta L)$$
$$\varphi_{33} = \frac{1}{\sigma_1\sigma_{21}+\sigma_2\sigma_{11}}(\sigma_2\sigma_{11}\cos\theta L + \sigma_1\sigma_{21}\cosh\eta L)$$
$$\varphi_{34} = \frac{1}{\sigma_{12}+\sigma_{22}}(-\ \sigma_{11}\sin\theta L + \sigma_{21}\sinh\eta L)$$

$$\varphi_{41} = -\frac{k\sigma_{12}\sigma_{22}}{\sigma_{12}+\sigma_{22}}(\cos\theta L - \cosh\eta L)$$
$$\varphi_{42} = -\frac{k}{\sigma_1\sigma_{21}+\sigma_2\sigma_{11}}(\sigma_{12}\sigma_{21}\sin\theta L - \sigma_{11}\sigma_{22}\sinh\eta L)$$
$$\varphi_{43} = \frac{1}{\sigma_1\sigma_{21}+\sigma_2\sigma_{11}}(\sigma_2\sigma_{12}\sin\theta L + \sigma_1\sigma_{22}\sinh\eta L)$$
$$\varphi_{44} = \frac{1}{\sigma_{12}+\sigma_{22}}(\sigma_{12}\cos\theta L + \sigma_{22}\cosh\eta L)$$

3. PIECEWISE CONTINUOUS MODEL FOR A TAPERED BEAM

A tapered beam can be considered as a piecewise continuous step beam consisting of N uniform beam elements as shown in the figure. Using the transform matrix (Eq.2.21) to describe



each beam element, then the state vectors at the two ends of the global beam will be related by the global transfer matrix [Φ], that is,

$$\begin{Bmatrix} Y \\ \Psi \\ Q \\ M \end{Bmatrix}_N = [\Phi] \begin{Bmatrix} Y \\ \Psi \\ Q \\ M \end{Bmatrix}_0 \tag{3.1}$$

where, the global transfer matrix

$$[\Phi] = \prod_{j=N}^{1} [\Phi]_j$$

Without loss of generality, let us consider the case of N=3. As N=3, the global transfer matrix will be

$$[\Phi] = \begin{bmatrix}
\sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{1k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i1}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{1k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i2}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{1k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i3}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{1k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i4}^{(1)} \\
\sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{2k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i1}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{2k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i2}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{2k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i3}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{2k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i4}^{(1)} \\
\sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{3k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i1}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{3k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i2}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{3k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i3}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{3k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i4}^{(1)} \\
\sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{4k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i1}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{4k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i2}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{4k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i3}^{(1)} & \sum_{i=1}^{4}\left(\sum_{k=1}^{4} \phi_{4k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i4}^{(1)}
\end{bmatrix} \tag{3.2}$$

where, the superscripts (j) represent the jth beam, and $\phi_{ki}^{(j)}$ is the elements of the transfer matrix for the jth beam.

If we consider a cantilevered beam fixed at the end of z=0, we have the following boundary conditions: at the fixed end: Y(0)=0 and $\psi(0)=0$; at the free end: Q(L)=0 and M(L)=0. Applying the boundary conditions to the global equation (3.1), we will have

$$\begin{Bmatrix} Y \\ \Psi \\ 0 \\ 0 \end{Bmatrix}_3 = [\Phi] \begin{Bmatrix} 0 \\ 0 \\ Q \\ M \end{Bmatrix}_0 \tag{3.3}$$

Rearranging the state vector, Eq.(3.3) can be written as

$$[A]\begin{Bmatrix} Y_3 \\ \Psi_3 \\ Q_0 \\ M_0 \end{Bmatrix} = [0] \tag{3.4}$$

where, the coefficient matrix

$$[A] = \begin{bmatrix} -1 & 0 & \Phi_{13} & \Phi_{14} \\ 0 & -1 & \Phi_{23} & \Phi_{24} \\ 0 & 0 & \Phi_{33} & \Phi_{34} \\ 0 & 0 & \Phi_{43} & \Phi_{44} \end{bmatrix}$$

and Φ_{ij}'s are the elements of the global transfer matrix. The condition for Eq.(3.4) to have a non-trivial solution is that the determinant of the coefficient matrix equals zero, that is

$$\mathrm{Det}[A] = \mathrm{Det}\begin{bmatrix} -1 & 0 & \Phi_{13} & \Phi_{14} \\ 0 & -1 & \Phi_{23} & \Phi_{24} \\ 0 & 0 & \Phi_{33} & \Phi_{34} \\ 0 & 0 & \Phi_{43} & \Phi_{44} \end{bmatrix} = 0 \tag{3.5}$$

Eq.(3.5) is the so-called characteristic equation. Solving for the roots of the characteristic equation, we obtain the natural frequencies ω's. Expanding the determinant in Eq.(3.5) we can simplify the characteristic equation as

$$\Phi_{33}\,\Phi_{44} - \Phi_{34}\,\Phi_{43} = 0 \tag{3.6}$$

or, expressing Eq.(3.6) in terms of the elements of each sub-transfer matrices, we have

$$\left[\sum_{i=1}^{4}\left(\sum_{k=1}^{4}\phi_{3k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i3}^{(1)}\right]\left[\sum_{i=1}^{4}\left(\sum_{k=1}^{4}\phi_{4k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i4}^{(1)}\right] - \left[\sum_{i=1}^{4}\left(\sum_{k=1}^{4}\phi_{3k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i4}^{(1)}\right]\left[\sum_{i=1}^{4}\left(\sum_{k=1}^{4}\phi_{4k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i3}^{(1)}\right] = 0 \tag{3.7}$$

If we consider a free-free beam, then the boundary conditions will become as $Q_0=M_0=0$ and $Q_3=M_3=0$ at the both ends. Thus the characteristic equation (Eq.3.5) should be

$$\mathrm{Det}[A] = \mathrm{Det}\begin{bmatrix} \Phi_{31} & \Phi_{32} & 0 & 0 \\ \Phi_{41} & \Phi_{42} & 0 & 0 \\ \Phi_{11} & \Phi_{12} & -1 & 0 \\ \Phi_{21} & \Phi_{22} & 0 & -1 \end{bmatrix} = 0 \tag{3.8}$$

or,

$$\Phi_{31}\,\Phi_{42} - \Phi_{32}\,\Phi_{41} = 0 \tag{3.9}$$

Expressing Eq.(3.9) in terms of the elements of each sub-transfer matrices, we have

$$\left[\sum_{i=1}^{4}\left(\sum_{k=1}^{4}\phi_{3k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i1}^{(1)}\right]\left[\sum_{i=1}^{4}\left(\sum_{k=1}^{4}\phi_{4k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i2}^{(1)}\right]-\left[\sum_{i=1}^{4}\left(\sum_{k=1}^{4}\phi_{3k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i2}^{(1)}\right]\left[\sum_{i=1}^{4}\left(\sum_{k=1}^{4}\phi_{4k}^{(3)}\phi_{ki}^{(2)}\right)\phi_{i1}^{(1)}\right]=0 \tag{3.10}$$

4. COMPUTER SIMULATION

The computer simulation is designed to analyze the natural frequency for a tapered beam with 15-meter length (Fig.4.1). The modulus of elasticity is assumed to be $E=200*10^9$ N/m^2. To

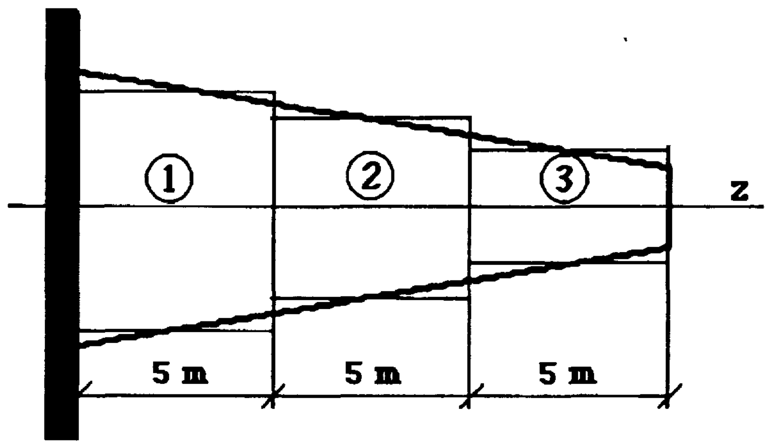


Fig.4.1 A Tapered Beam

simplify the calculation, the change of the sectional foil is specified by the changes of the second moment of area of the beam section and the mass of the beam segments along the longitudinal axis z, that is, assuming

$$I = (0.02z^2-0.6z+5.375)*10^{-6}\ (m^4) \tag{4.1}$$

and

$$m = 0.0112z^2-0.495z+7.708\ (kg.) \tag{4.2}$$

In so doing, we may readily determine the sectional parameters necessary for the element stiffness and mass matrices when we divide the beam as any desired number of segments. For example, if we use three uniform beam elements to represent the global tapered beam, then we use $z_1=2.5$, $z_2=7.5$ and $z_3=12.5$ to calculate the I_i and m_i for each beam element according to Eqs.(4.1) and (4.2). They are

$I_1=4*10^{-6}$, $I_2=2*10^{-6}$, $I_3=1*10^{-6}$ (m^4) and $m_1=6.54$, $m_2=4.03$, $m_3=3.27$ (kg)

In the computer simulation, three-segment piecewise continuous Timosenko beam model has been applied. For finite element analysis, the commonly used two-node and four-degree-of-freedom plane beam element has been selected. Table 4.1 exhibit the comparison of the frequency results calculated by both the piecewise continuous Timosenko beam model and the finite element model. The results show that at least ten beam elements are needed for the finite element analysis to achieve the comparable frequency values while the piecewise continuous Timosenko beam model uses only three beam segments. The advantage is clear in decreasing the number of elements by using the piecewise continuous Timosenko beam model to analyze large flexible tapered beam-like structures.

Table 4.1 The comparison of the results obtained from Finite Element Method & Piecewise Continuous Timoshenko Beam Model

(Circular Natural Frequency, rad/sec)

Order of Mode			1	2	3	4	5
Finite Element Analysis	Number of Elements N	3	15.274	70.066	179.455	415.646	775.990
		4	13.364	61.444	159.541	305.038	577.351
		5	12.210	55.790	143.917	278.314	455.985
		6	11.208	51.205	131.761	253.865	419.438
		7	10.411	47.566	122.267	234.973	387.374
		8	9.760	44.592	114.562	219.847	362.437
		9	9.223	42.108	108.144	207.421	340.663
		10	8.748	39.989	102.689	198.878	323.080
Piecewise Continuous Model (N=3)			8.776	39.993	101.758	204.243	329.268

5. CONCLUDING REMARKS

This paper proposed a piecewise continuous Timoshenko beam model which is to be used for the dynamic analysis of large flexible tapered beam-like structures. The procedure for establishing natural frequency has been described in detail. A tapered beam is divided into several segments of uniform beam elements. Instead of arbitrarily assumed shape functions used in finite element analysis,-the elosed-form solution of the Timoshenko beam equation has been used. Application of transfer matrix method relates all the elements as a whole. By corresponding boundary conditions and compatible conditions a characteristic equation for the global tapered beam has been yielded. Through the root-searching process to the characteristic equation the natural frequencies have been derived. A computer simulation is shown in this paper, and compared with the results obtained from the finite element analysis. While the comparable results is obtained, piecewise continuous Timoshenko beam model decreases the number of elements significantly.

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