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Robust Motion Tracking Control of Robotic Arms Based on the Generalized Energy Accumulation Principle

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1. Objective

Consider a rigid-link robot with the dynamic model

$$\tau = H(q;p)\ddot{q} + C(q,\dot{q};p)\dot{q} + G(q;p) + \mathcal{N}(t)$$

where $\mathcal{N}(.)$ denotes a bounded external disturbance (definitions of other terms and variables can be found in the literature). The objective addressed herein is to find a control strategy that exhibits the following features: (1) simple to implement, (2) easy to code for program and (3) robust to possible time-varying uncertainties

2. Results

Define tracking error ϵ as $\epsilon = q - q^*$, where q^* denotes the desired trajectory. Also let $W = \dot{\epsilon} + D\epsilon$, $y_s = \dot{q}^* - D\epsilon$, and $x_s = \ddot{q}^* - D\dot{\epsilon} - \dot{\kappa}\kappa^{-1}W$, where $D = D^T > 0$ and $\kappa(t)$ is one of the rate functions which is introduced to adjust rate-of-convergence (ROC) (see [4]).

Definition 1

Let $\nu(t)$ be defined on $[t_0, \infty)$. $\nu(t)$ is in the class $\mathcal{V}_I$ if $\nu(t)$ is nonnegative constant or in $\mathcal{V}_{II}$ if $\nu(t)$ is bounded, positive, and decreasing for all $t \in [t_0, \infty)$.

The robust control law is given by

$$\tau = H_s(q^*;p^*)x_s + C_s(q^*,\dot{q}^*;p^*)y_s + G_s(q^*;p^*) - KW + U_a, \quad (1)$$

where $K = K^T > 0$, and $H_s(.)$, $C_s(.)$, and $G_s(.)$ are simplified versions of H, C, and G, respectively. U_a is an auxiliary control defined by

$$U_a = -\frac{W\eta^2}{||W||\eta + \nu(t)}, \quad (2)$$

where $\nu(t) \in \mathcal{V}_I$ or $\mathcal{V}_{II}$ and η is a nonnegative scalar nonlinear function defined as

$$\begin{aligned} ||H_s - H|| ||x_s|| &+ ||C_s - C|| ||y_s|| + ||G_s - G - \mathcal{N}|| \\ &\leq \alpha_0 ||x_s|| + \alpha_1 ||\dot{q}|| ||y_s|| + \alpha_2 \triangleq \eta. \end{aligned}$$

In this equation, α_i are constants representing bounds on the modelling errors. There are many possible choices for $\nu(t)$ in U_a, and different choices leads to different tracking properties. The $\nu(t)$ defined by

$$\nu(t) = \frac{1}{2}v_1(1 + t^m)^{\rho} e^{-v_2 t^n} \quad (3)$$

are in $\mathcal{V}_I$ or $\mathcal{V}_{II}$ if m, n, and ρ are chosen properly (v_1 and v_2 are appropriate positive constants). Specifically, if $\rho = -1$, $m, n = 0$, one gets $\nu(t) = v_1 e^{-v_2} \triangleq \mu_0$, which gives, $U_a = -\frac{W\eta^2}{||W||\eta + \mu_0}$. This is called saturation (or boundary layer) controller [1] and has been widely used to achieve bounded stability. Another specific choice for $\nu(t)$ ($\rho = -1$, $m = 0$, $n = 1$) is $\nu(t) = v_1 e^{-v_2 t}$, which gives the strategy proposed by Dawson, et al. [2], $U_a = -\frac{W\eta^2}{||W||\eta + v_1 e^{-v_2 t}}$. An extreme case, $n \to \infty$, $m \to \infty$ and $\rho = -1$, gives $\nu(t) = 0$, which corresponds to the variable structure control [3], $U_a = -\frac{W}{||W||}\eta$. As is shown later, $\nu(t) = 0$ gives the fastest convergence, while $\nu(t) = \mu_0$ gives the slowest. However, due to physical limitations, "too fast" could lead to chattering. Hence, the choice of $\nu(t)$ depends on the requirements for ROC, transient response, and steady state performance.

Tracking stability results based on the so-called generalized energy accumulation principle [4] are given next.

Theorem 1: *Given (1) and (2), if $\nu(t)$ and $\kappa(t)$ are chosen such that*

$$\int_{t_0}^{t} \nu(\tau)\kappa^2(\tau)d\tau \leq C_2^2 < \infty \qquad \forall t \in [t_0, \infty), \quad (4)$$

then stable path tracking is ensured and the rate-of-convergence is at least $\kappa^{-1}(t)$.

Proof (outline): The closed-loop model is governed by

$$\begin{aligned} &H(q;p)(\dot{W} + \dot{\kappa}\kappa^{-1}W) + C(q,\dot{q};p)W = -KW \\ &+ \delta H(q;p)x_s + \delta C(q,\dot{q};p)y_s + \delta\mathcal{G}(q,t;p) + U_a. \end{aligned}$$

where $\delta H(.) = H_s - H, \delta C(.) = C_s - C, \delta\mathcal{G}(.) = G_s - G - \mathcal{N}$. Introducing the transformation $\Psi = \kappa W$ gives

$$\begin{aligned} H(q;p)\dot{\Psi} &+ C(q,\dot{q};p)\Psi = -K\Psi + \delta H(q;p)x_s\kappa \\ &+ \delta C(q,\dot{q};p)y_s\kappa + \delta\mathcal{G}(q,t;p)\kappa + U_a\kappa. \end{aligned}$$

According to the criteria in [4], boundedness of the accumulated generalized energy, $\int_{t_0}^{t} \Psi^T K \Psi d\tau$, proves tracking stability. In fact,

$$
\begin{aligned}
J^c &= \int_{t_0}^{t} \Psi^T K \Psi d\tau \\
&= -\int_{t_0}^{t} \Psi^T H(q;p)\dot{\Psi} d\tau - \int_{t_0}^{t} \Psi^T C(q,\dot{q};p)\Psi d\tau \\
&+ \int_{t_0}^{t} \Psi^T \{\delta H(q;p)x_s\kappa + \delta C(q,\dot{q};p)y_s\kappa + \delta \mathcal{G}(q,t;p)\kappa\} \\
&+ \int_{t_0}^{t} \Psi^T U_a \kappa d\tau.
\end{aligned}
$$

The symmetric positive definite property of $H(.)$ and the skew-symmetric property of $\dot{H}(.) - 2C(.)$ yields,

$$
J^c \leq C_v^2 + \int_{t_0}^{t} ||\Psi||\eta\kappa d\tau + \int_{t_0}^{t} \Psi^T U_a \kappa d\tau \tag{5}
$$

where $C_v^2 = \frac{1}{2} W^T H W \kappa^2 |_{t=t_0}$. Inserting (2) into (5),

$$
\begin{aligned}
J^c &\leq C_v^2 + \int_{t_0}^{t} ||\Psi||\eta\kappa d\tau - \int_{t_0}^{t} \Psi^T \kappa \frac{W\eta^2}{||W||\eta + \nu(\tau)} d\tau \\
&= C_v^2 + \int_{t_0}^{t} \nu(\tau)\kappa^2 \frac{||W||\eta}{||W||\eta + \nu(\tau)} d\tau \\
&\leq C_v^2 + \int_{t_0}^{t} \nu(\tau)\kappa^2(\tau) d\tau.
\end{aligned} \tag{6}
$$

Condition (4) implies J^c is bounded. The result follows [4].

Theorem 2: *Given (1) and (2), if $\nu(t)$ and $\kappa(t)$ are chosen such that*

$$
\limsup_{t\to\infty} \frac{1}{t-t_0} \int_{t_0}^{t} \nu(\tau)\kappa^2(\tau) d\tau \leq C_2^2 < \infty \qquad \forall t \in [t_0, \infty),
$$

then stable path tracking is also ensured.

Proof: The proof follows the approach used in [4].

Corollary *If $\kappa(t)$ and $\nu(t)$ are chosen such that*

$$
\kappa^2(t)\nu(t) \leq C_s^2 < \infty, \tag{7}
$$

then stable path tracking is ensured.

Proof: Under the condition of the Corollary, it is seen that $\int_{t_0}^{t} \nu(\tau)\kappa^2(\tau) d\tau \leq C_s^2(t - t_0)$. Therefore, $\limsup_{t\to\infty} \frac{J^c}{t-t_0} = \limsup_{t\to\infty} \frac{C_v^2}{t-t_0} + C_s^2$.

Several observations are made. First H_s, C_s and G_s are not based on q, $\dot{q}$ and p, but on the desired path $\{q^*, \dot{q}^*\}$ and parameters p^* which can be precomputed off-line. Second, one does not need to re-organize the robotic dynamics (so as to isolate unknown parameters) before calculating the control torque. Also a simple way to get $H_s(.)$, $C_s(.)$, and $G_s(.)$ is to set, $H_s = 0$, $C_s = 0$, and $G_s = 0$, the control torque reduces to $\tau = -KW + U_a$. This gives the same structure as in [2]. However, since $||U_a|| \leq \eta$, $H_s = 0, C_s = 0$, and $G_s = 0$ leads to a larger U_a which could require more control energy.

3. Synthesis Examples

Example 1: (Natural Rate-of-Convergence)

Assume that a natural ROC is sufficient ($\kappa(t) = 1$). Then

$$
J^c \leq C_v^2 + \int_{t_0}^{t} \nu(\tau) d\tau.
$$

Suppose that $\nu(t)$ is chosen such that $J^c \leq C_v^2 + \int_{t_0}^{t} \nu(\tau) d\tau \leq J^*$, where J^* is a design specification. If $\nu(t) = v_1 e^{-v_2 t}$, with $v_1 > 0, v_2 > 0$, then $J_A \leq C_v^2 + \frac{v_1}{v_2} e^{-v_2 t_0}$. In order to meet the specification, v_1 and v_2 are determined such that $C_v^2 + \frac{v_1}{v_2} e^{-v_2 t_0} \leq J^*$. Suppose $t_0 = 0$ and the initial condition is such that $C_v^2 = 10$. It the performance specification is $J^* = 12$, then $\frac{v_1}{v_2} \leq 2$. So by choosing $v_2 > 0$ and $0 < v_1 \leq 2v_2$, $J_A \leq J^*$.

Example 2: (Variable Structure Control)

For any $\kappa(t)$, J^c is ensured to be less than or equal to C_v^2 (see (6)) by choosing $\nu(t) = 0$. This implies that the ROC can be arbitrarily fast and the accumulated tracking error is smaller than any other choice of ν. So one might conclude that variable structure control gives the best control performance and the greatest ROC. However, it is its fast speed that causes chattering. So from a practical point of view, one should not require too large a ROC over the entire period of operation. A piecewise ROC may be useful. This can be achieved by methods similar to those given in [4].

4. Comment

Clearly ν plays an interesting role in these robust control strategies. First, ν is related to the overall tracking performance in that the bound on J^c depends on the choice of ν. Second, ν specifies the boundary layer in the strategies. Since ν is time varying, the boundary layer is also varying. This property can be used to retain the merits of the VSC strategy and avoid the problem of chattering.

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System Stability and Performance Analysis Based on Generalized Energy Accumulation: Part II — Applications

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Abstract

Based on the generalized energy accumulation principle, criteria for system stability and performance analysis are established in the first part of this work [8]. These criteria are of immediate use in many systems. The main purpose in this part of the work is to apply these criteria to robotic systems. Both adaptive and robust control are investigated.

1. Introduction

The concept of stability concerning a dynamic system is always important to system engineers. Motivated by the generalized energy accumulation principle, certain criteria for testing system stability are proposed in [8]. As a continuation to that work, this paper demonstrates the applications of the established criteria to a practical system — robotic system. Specifically, the path tracking control problem of robotic systems is considered. By introducing the concept of rate transformation, new adaptive and robust control strategies are developed which achieve stable path tracking and provide a priori information about how fast the tracking errors will converge. With these strategies, different rates-of-convergence (ROCs) can be obtained by simply choosing a different scalar rate function $\kappa(t)$ (to be defined later). It turns out that the conventional adaptive control scheme is a special case of the proposed strategies (i.e., with $\kappa(t) = 1$). Global exponential path tracking is easily achieved by simply setting $\kappa(t) = e^{\lambda t}$, where $\lambda > 0$. Moreover, one may obtain other types of path tracking than asymptotic and exponential tracking by choosing a proper κ.

2. Review of Stability Criteria

For convenience and continuity, stability criteria established in [8] are summarized first.

By introducing a rate function $\kappa(t)$ (see §3 for definition), stability criteria represented by integral inequalities in [8] can be unified as follows,

$$J^c = \int_{t_0}^{t} \kappa^2(\tau) G_c[x(\tau)] d\tau \leq C_A^2 < \infty,$$

where $G_c(.)$ is a generalized energy function of the system and C_A is a real number. To analyze system stability and performance, one only needs to verify these equalities (see [8] for more details). It is seen that asymptotic stability corresponds to $\kappa(t) = 1$, while exponential stability corresponds to $\kappa(t) = e^{\lambda t}$.

3. A Useful Lemma [8]

The objective is to derive new adaptive and robust control strategies, which ensure stable path tracking and allow adjustable ROCs. To this end, the concepts of rate function and rate transformation are introduced.

Definition 3.1 Rate Function

A real function of time, $\kappa(t)$, is a rate function (denoted by $\kappa(t) \in \mathcal{S}$), if it satisfies the following conditions:
(1) $\kappa(t)$ is positive for all $t \in [t_0, \infty)$,
(2) $\kappa(t_0)$ is bounded,
(3) $\kappa(t)$ is increasing, and
(4) $\dot{\kappa}(t)$ is well-defined for $t \in [t_0, \infty)$.

Note that under these conditions, such a $\kappa(t)$ is invertible and $\kappa^{-1}(t)$ is upper bounded and decreasing. Obvious examples for such a rate function include $\kappa = 1$, $\kappa = 1+t$, $\kappa = e^{\lambda t}$, $(1+t)e^{\lambda t}$, (1+t)+$e^{\lambda t}$ ($\lambda > 0$) etc. (see [8] for other types).

Definition 3.2 Rate Transformation

The rate transformation is defined as

$$\Psi = \kappa(t)\chi, \tag{3.1}$$

where $\kappa(t)$ is a rate function as defined before.

The terminology "rate transformation" is motivated by the fact that such a transformation affects the rate of convergence of the system, as is shown in the following.

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Consider a nonlinear system described by

$$\dot{\chi} = f(\chi; p), \qquad \chi(t_0) = \chi_0, \tag{3.2}$$

where $\chi \in R^n$ is the system state vector, $p \in R^s$ is the system parameter vector, and $f \in R^n$ is a nonlinear vector function of χ and p. Applying the rate transformation (3.1) to (3.2) leads to

$$\dot{\Psi} = \dot{\kappa}(t)\kappa^{-1}\Psi + \kappa f(\Psi\kappa^{-1}; p) \triangleq F(\kappa, \dot{\kappa}, \Psi; p), \tag{3.3a}$$

$$\Psi(t_0) = \kappa(t_0)\chi(t_0) \tag{3.3b}$$

where $F(\kappa, \dot{\kappa}, \Psi, p)$ is a new nonlinear function. It should be noted that the transformed system (3.3) is not equivalent to the original system (3.2) in the sense that the stability of (3.2) does not necessarily guarantee the stability of (3.3). However the stability of (3.3) absolutely guarantees the stability of (3.2). This is specified by the following lemma.

Lemma 3.3

Let κ be a rate function as defined before. Consider the systems (3.2) and (3.3), related by (3.1). If system (3.3) is stable (bounded or asymptotically stable), the system (3.2) converges to its equilibrium point with a certain ROC.

Proof:

Noting that $\chi = \kappa^{-1}\Psi$ and κ^{-1} is decreasing, the result can be easily obtained. §

Based on this lemma, it is seen that if the transformed system (3.3) is at least bounded stable, then the original one exhibits enhanced stability. This result which is an extension of [6] (where a special choice for κ is utilised, i.e., $\kappa = e^{\lambda t}$) is used in the solution of the tracking problem. Note that since χ could either be tracking error, state estimation error, regulation error, or model following error, the idea behind this lemma could also be used in these cases. Investigation of this possibility, beyond the scope of this work, represents an interesting further research effort.

4. Application to Robotic Systems

The formulation for the dynamics of a serial-link robot with n joints is

$$H(q;p)\ddot{q} + C(q,\dot{q};p)\dot{q} + G(q;p) = \tau, \tag{4.1}$$

where

$\tau \in R^n$	control torque,
$q, \dot{q}, \ddot{q} \in R^n$	joint positions/velocities/accelerations,
$p \in R^s$	equivalent system parameters,
$H(q;p) \in R^{n\times n}$	symmetric, positive, definite inertia matrix,
$C(q,\dot{q};p)\dot{q} \in R^n$	joint torques and forces due to Coriolis and centrifugal effects, and
$G(q;p) \in R^n$	torques and forces due to gravity.

Concerning this model, the following structural property is useful.

Property 4.1

For a rigid robot with the dynamics (4.1) in which $C(q,\dot{q};p)$ is defined as in [5], then

$$H(q;p)x + C(q,\dot{q};p)y + G(q;p) = \Phi(q,\dot{q},y,x)p \tag{4.2}$$

and

$$v^T\left\{\frac{d}{dt}H(q;p) - 2C(q,\dot{q};p)\right\}v = 0,\ \forall v \in R^n, \tag{4.3}$$

where $x \in R^n, y \in R^n, \Phi = [\Phi_1, \Phi_2, ..., \Phi_s] \in R^{n\times s}$ and $\Phi_i \in R^n$. It is worth mentioning that in (4.2) $\dot{y} = x$ is not required. This restriction is typically assumed in the literature. Equation (4.3) represents the well-known skew-symmetric property.

Note that this equation represents a highly nonlinear and strongly coupled system. In general, not all the information concerning the system parameters is available a priori. Thus the path tracking control problem is stated as follows:

Given the desired path, $\{q^*, \dot{q}^*, \ddot{q}^*\}$, find a control torque, $\tau(q, \dot{q}, q^*, \dot{q}^*, \ddot{q}^*)$, which depends only on the joint positions and velocities, such that the actual path, $\{q, \dot{q}\}$, automatically tracks the desired path with a controllable ROC in the presence of uncertain system parameters represented by p.

Note that there is an essential difference between this problem statement and that which is typically seen in the literature. This problem statement not only requires convergence of the tracking errors, but also imposes the requirement of an adjustable ROC. This is of particular interest in many practical applications.

4.1 Adaptive Tracking with Adjustable ROC

The criteria proposed in Part I of this work are applied to synthesize new adaptive control algorithms for robotic systems that achieve stable path tracking with controllable ROCs. First let the tracking error be defined as

$$\epsilon \triangleq q - q^* \tag{4.4}$$

and let W represent an auxiliary variable defined by

$$W \triangleq \dot{\epsilon} + D\epsilon. \tag{4.5}$$

Also let

$$z_r \triangleq \ddot{q}^* - D\dot{\epsilon} - \dot{\kappa}(t)\kappa^{-1}(t)W \tag{4.6}$$

and

$$y_r \triangleq \dot{q}^* - D\epsilon, \tag{4.7}$$

where $D \in R^{n\times n}$ is a symmetric positive definite matrix and $\kappa(t)$ is a rate function. The controller structure for path tracking is given by

$$\tau = H(q;\hat{p})z_r + C(q,\dot{q};\hat{p})y_r + G(q;\hat{p}) - KW, \tag{4.8}$$

where $K = K^T > 0$ is a controller gain matrix, and $\hat{p} \in R^s$ is the vector of parameter estimates as determined by the following algorithms.

Estimation Algorithm 1 (I-Estimate)

$$\begin{aligned}\hat{p}_i &= -\alpha_i \int_{t_0}^{t} W^T \Phi_i(q,\dot{q},y_s,x_s)\kappa^2(\tau)d\tau + \hat{p}_i(t_0), \\ &\qquad i = 1,2,\dots,s, \qquad (4.9)\end{aligned}$$

where $\alpha_i > 0$ is an adaptation gain (design parameter), $\hat{p}_i(t_0)$ is the initial parameter estimate, and Φ_i is the i-th element of the vector Φ defined by Equation (4.2).

Estimation Algorithm 2 (PI-Estimate)

$$\begin{aligned}\hat{p}_i &= \hat{p}_i(t_0) - \alpha_i \int_{t_0}^{t} W^T \Phi_i(q,\dot{q},y_s,x_s)\kappa^2(\tau)d\tau \\ &- \beta_i W^T \Phi_i(q,\dot{q},y_s,x_s)\kappa^2(t),\ i = 1,2,\dots,s, \qquad (4.10)\end{aligned}$$

where $\beta_i \geq 0$ is one other adaptation gain.

Theorem 4.1

Consider the robot dynamics (4.1) with the controller (4.8). If the parameters are estimated by (4.9) or (4.10), then stable path tracking control is ensured. The rate-of-convergence is specified by $\kappa^{-1}(t)$, which can be chosen by the designer.

Proof: Combining (4.1) and (4.8) yields the closed-loop system dynamics,

$$\begin{aligned}H(q;p)(\dot{W} &+ \dot{\kappa}\kappa^{-1}W) + C(q,\dot{q};p)W = -KW \\ &+ \sum_{i=1}^{s}(\hat{p}_i - p_i)\Phi_i(q,\dot{q},y_s,x_s) \qquad (4.11)\end{aligned}$$

where Property 4.1 was used. Introducing the rate transformation, $\Psi = \kappa W$, (4.11) becomes

$$H(q;p)\dot{\Psi} + C(q,\dot{q};p)\Psi = \sum_{i=1}^{s}(\hat{p}_i - p_i)\Phi_i(q,\dot{q},y_s,x_s)\kappa - K\Psi.$$

According to the criteria in [8], it is only necessary to show the boundedness of the accumulated generalized energy $J^c = \int_{t_0}^{t} \Psi^T K \Psi d\tau$. Using the above equation, Property 4.1 and integration by parts, yields

$$\begin{aligned}J^c &= -\int_{t_0}^{t} \Psi^T H(q;p)\dot{\Psi}d\tau - \int_{t_0}^{t} \Psi^T C(q,\dot{q};p)\Psi d\tau \\ &+ \sum_{i=1}^{s}\int_{t_0}^{t}(\hat{p}_i - p_i)W^T\Phi_i\kappa^2(\tau)d\tau \\ &\leq C_\Psi^2 + \sum_{i=1}^{s}\int_{t_0}^{t}(\hat{p}_i - p_i)W^T\Phi_i\kappa^2(\tau)d\tau\end{aligned}$$

where $C_\Psi^2 = \frac{1}{2}W^T H W \kappa^2|_{t=t_0}$. Inserting (4.9) and applying the following relation

$$\int_{t_0}^{t}\int_{t_0}^{\gamma}\Phi(\tau)d\tau\Phi(\gamma)d\gamma = \frac{1}{2}\left[\int_{t_0}^{t}\Phi(\gamma)d\gamma\right]^2, \qquad (4.12)$$

J^c reads

$$\begin{aligned}J^c &\leq C_\Psi^2 - \sum_{i=1}^{s}\frac{\alpha_i}{2}\left[I_i - \frac{\hat{p}_i(t_0) - p_i}{\alpha_i}\right]^2 \\ &\quad + \sum_{i=1}^{s}\frac{[p_i - \hat{p}_i(t_0)]^2}{2\alpha_i} \\ &\leq C_\Psi^2 + \sum_{i=1}^{s}\frac{[p_i - \hat{p}_i(t_0)]^2}{2\alpha_i} < \infty, \qquad (4.13)\end{aligned}$$

where $I_i = \int_{t_0}^{t} W^T\Phi_i\kappa^2(\tau)d\tau$. The boundedness of J implies that Ψ is at least L_2. Note that $W = \kappa^{-1}\Psi$, the result follows. (The result for Estimation Algorithm 2 can be shown in the same way.) §

4.2 Illustrative Examples

To make the foregoing concepts clear, three examples are presented in this section.

Example 1 Asymptotic Convergence

Suppose the control torque is of the same structure as in (4.8). If $\kappa = 1$, then ϵ, y_s, W are defined as before and

$$x_s = \ddot{q}^* - D\dot{\epsilon}.$$

Estimation Algorithm 1 becomes

$$\hat{p}_i = -\alpha_i \int_{t_0}^{t} W^T\Phi_i(q,\dot{q},y_s,x_s)d\tau + \hat{p}_i(t_0), i = 1,2,\dots,s.$$

Considering the proof of Theorem 4.1, convergence for this case is asymptotic.

Example 2

Choosing $\kappa(t)$ as $\kappa(t) = 1 + t$ gives

$$x_s = \ddot{q}^* - D\dot{\epsilon} - \frac{1}{1+t}W,$$

and

$$W = \dot{\epsilon} + D\epsilon = \frac{\Psi}{1+t}.$$

Estimation Algorithm 2 is now

$$\begin{aligned}\hat{p}_i &= -\alpha_i \int_{t_0}^{t} W^T\Phi_i(q,\dot{q},y_s,x_s)(1+\tau)^2 d\tau \\ &- \beta_i W^T\Phi_i(q,\dot{q},y_s,x_s)(1+t)^2 + \hat{p}_i(t_0),\end{aligned}$$

$i = 1,2,\dots,s$. In this case, convergence is stronger than asymptotic due to the choice of κ.

Example 3 Exponential Convergence

Let $\kappa(t)$ be the exponential function $\kappa(t) = e^{\lambda t}$. In this case

$$x_s = \ddot{q}^* - D\dot{\epsilon} - \lambda W$$
$$W = \dot{\epsilon} + D\epsilon = \Psi e^{-\lambda t}.$$

The parameter estimation algorithm is

$$\begin{aligned}\hat{p}_i &= -\alpha_i \int_{t_0}^{t} W^T \Phi_i(q,\dot{q},y_s,x_s)e^{2\lambda\tau}d\tau \\ &- \beta_i W^T \Phi_i(q,\dot{q},y_s,x_s)e^{2\lambda t} + \hat{p}_i(t_0), i = 1,2,...,s.\end{aligned}$$

This corresponds to the exponential tracking.

It is observed from the above examples that for different $\kappa(t)$, different ROCs for the filtered tracking errors are achieved. It is interesting to note that $\kappa(t) = 1$ corresponds to the conventional adaptive control [1-2][4-5]. Note that in this case, $W = \Psi$ and the ROC of W is not adjustable. As for the exponential tracking, one only needs to choose $\kappa(t) = e^{\lambda t}$, where $\lambda > 0$. Also it is possible to change the ROC over different time intervals. This can be done by the technique shown in [8].

4.3 Robust Tracking with Adjustable ROC

Robust control of robotic systems has been extensively investigated recently [1]. Most of the strategies are based on upper bounds of the uncertain model. Obtaining such bounds, however, is not trivial because H, C, and G are complicated matrices depending on $q, \dot{q}$ and p. Improper determination of such bounds may lead to instability. A strategy based on the maximum absolute value of each element of H, C, and G is suggested as follows.

Let $H_s(.)$, $C_s(.)$ and $G_s(.)$ represent simplified versions of $H(.)$, $C(.)$ and $G(.)$, respectively. Also let p^* represent the nominal system parameters and q^* and $\dot{q}^*$ represent the desired trajectory.

For the following development, let

$$\begin{aligned}\delta H = [\delta h_{ij}] &= H(q;p) - H_s(q^*;p^*), \\ \delta C = [\delta c_{ij}] &= C(q,\dot{q};p) - C_s(q^*,\dot{q}^*;p^*), \\ \delta G = [\delta g_i] &= G(q;p) - G_s(q^*;p^*).\end{aligned}$$

The robust control torque is given by

$$\tau = H_s(q^*;p^*)x_s + C_s(q^*,\dot{q}^*;p^*)y_s + G_s(q^*;p^*) - KW + U_a, \quad (4.14a)$$

where $K = K^T > 0$ and U_a is an auxiliary control defined by

$$\begin{aligned}U_a &= \sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij}^H U(i,j)x_s \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij}^C U(i,j)y_s + \sum_{i=1}^{n} s_i^G U(i), \quad (4.14b)\end{aligned}$$

In equation (4.14b), $U(i,j)$ are '$0-1$' matrices [9] and s_{ij}^H, s_{ij}^C and s_i^G are scalars to be defined later. The tracking stability of the system is now addressed by the following result.

Theorem 4.2

Let the control strategy be defined by (4.14). If

$$s_{ij}^H = -\frac{W_i x_{s_j}\bar{h}_{ij}^2}{|W_i x_{s_j}|\bar{h}_{ij} + \nu(t)},$$

$$s_{ij}^C = -\frac{W_i x_{s_j}\bar{c}_{ij}^2}{|W_i y_{s_j}|\bar{c}_{ij} + \nu(t)}$$

$$s_i^G = -\frac{W_i \bar{g}_i^2}{|W_i|\bar{g}_i + \nu(t)},$$

where $\bar{h}_{ij} = \max|\delta h_{ij}|$, $\bar{c}_{ij} = \max|\delta c_{ij}|$, $\bar{g}_i = \max|\delta g_i|$, *and* κ *and* ν *satisfy*

$$\int_{t_0}^{t} \kappa^2 \nu d\tau \leq C_{1s} < \infty \quad (4.15)$$

or

$$\kappa^2 \nu \leq C_{2s} < \infty \quad (4.16)$$

then stable path tracking is achieved.

Proof (outline):

Note that with the control (4.14a), the closed-loop model becomes

$$\begin{aligned}&H(q;p)(\dot{W} + \dot{\kappa}\kappa^{-1}W) + C(q,\dot{q};p)W = -KW \\ &+ \delta H(q;p)x_s + \delta C(q,\dot{q};p)y_s + \delta G(q;p) + U_a.\end{aligned}$$

Introducing the transformation $\Psi = \kappa W$ and using U_a in (4.14b) gives

$$\begin{aligned}H(q;p)\dot{\Psi} &+ C(q,\dot{q};p)\Psi = -K\Psi \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}[s_{ij}^H + \delta h_{ij}]U(i,j)x_s\kappa \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}[s_{ij}^C + \delta c_{ij}]U(i,j)y_s\kappa \\ &+ \sum_{i=1}^{n}[s_i^G + \delta g_i]U(i)\kappa.\end{aligned}$$

Considering the performance index $J^c = \int_{t_0}^{t} \Psi^T K \Psi d\tau$, it is not difficult to show that

$$\begin{aligned}J^c &= \int_{t_0}^{t} \Psi^T K \Psi d\tau \\ &\leq C_v^2 + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t_0}^{t}\frac{|W_i x_{s_j}|\bar{h}_{ij}^2}{|W_i x_{s_j}|\bar{h}_{ij} + \nu}\kappa^2\nu d\tau \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t_0}^{t}\frac{|W_i y_{s_j}|\bar{c}_{ij}^2}{|W_i y_{s_j}|\bar{c}_{ij} + \nu}\kappa^2\nu d\tau \\ &+ \sum_{i=1}^{n}\int_{t_0}^{t}\frac{|W_i|\bar{g}_i^2}{|W_i|\bar{g}_i + \nu}\kappa^2\nu d\tau \\ &\leq C_v^2 + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t_0}^{t}\kappa^2\nu d\tau \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t_0}^{t}\kappa^2\nu d\tau + \sum_{i=1}^{n}\int_{t_0}^{t}\kappa^2\nu d\tau\end{aligned}$$

With the choices for κ and ν as in (4.15) or (4.16), either the index is bounded or its time average is bounded and the result follows. §

This strategy is easy to apply since the upper bound for each element, $|\delta h_{ij}|_{max}$, $|\delta c_{ij}|_{max}$, and $|\delta g_i|_{max}$, can be easily obtained by using the facts that $|\sin(.)| \leq 1$ and $|\cos(.)| \leq 1$.

Note that there are many possible choices for ν (see the table below, where $\mu_0 \geq 0, v_1 > 0$ and $v_2 \geq 0$). The impact of ν (and κ) on system performance is discussed later.

Table 4.1 Possible Choices for ν

μ_0	$v_1 e^{-v_2 t}$	$v_1 e^{-v_2 t^2}$	$\frac{v_1(1+t)}{e^{v_2 t}}$
$\frac{v_1 e^{-v_2}}{1+t}$	$\frac{v_1 e^{-v_2 t}}{1+t}$	$\frac{v_1 e^{-v_2 t^2}}{1+t}$	$\frac{v_1(1+t^2)}{e^{v_2 t}}$
$\frac{v_1 e^{-v_2}}{1+t^2}$	$\frac{v_1 e^{-v_2 t}}{1+t^2}$	$\frac{v_1 e^{-v_2 t^2}}{1+t^2}$	$\frac{v_1(1+t)}{e^{v_2 t^2}}$
$\frac{v_1 e^{-v_2}}{1+t^3}$	$\frac{v_1 e^{-v_2 t}}{1+t^3}$	$\frac{v_1 e^{-v_2 t^2}}{1+t^3}$	$\frac{v_1(1+t^2)}{e^{v_2 t^2}}$

The strategy presented herein exhibits the following features. The structure is simple and most of the required computations can be performed prior to real-time operation. As for the computation of H_s, C_s and G_s, one may choose them to be constant matrices/vector (or diagonal matrices for H_s and C_s), or simply zero. Additionally, time varying uncertainties can be easily handled by the strategy. Again since the rate function is utilized, the ROC is adjustable.

4.4 Tracking Performance Analysis

In addition to the tracking stability, it is important to explore the tracking performance that the strategies can achieve. The criteria for testing stability can also serve this purpose. The following is a brief discussion of this issue. Only adaptive control is considered. Referring to the proof of Theorem 4.1, it is found that the performance index for both the Estimation Algorithms 1 and 2 can be computed as

$$J^c(I, II) = \eta_0 + \eta_1 + \eta_2$$

where

$$\begin{aligned} \eta_0 &= W^T H(q(\tau); p) W \kappa^2(\tau)|_{\tau=t_0} \\ \eta_1 &= \sum_{i=1}^{s} \int_{t_0}^{t} (\hat{p}_i - p_i) W^T \Phi_i \kappa^2(\tau) d\tau \\ \eta_2 &= -W^T H(q(\tau); p) W \kappa^2(\tau)|_{\tau=t}. \end{aligned}$$

It is seen that only η_1 changes for the different estimation algorithms. When $\hat{p}_i$ is estimated by the I-Estimate (4.9),

$$\begin{aligned} \eta_1 &= -\sum_{i=1}^{s} \int_{t_0}^{t} (\hat{p}_i - p_i) W^T \Phi_i \kappa^2 d\tau \\ &= -\sum_{i=1}^{s} \left\{ \frac{\alpha_i}{2} I_i^2 + [p_i - \hat{p}_i(t_0)] I_i \right\} \triangleq \eta_1(I). \end{aligned}$$

For the PI-Estimate, since $\hat{p}_i$ is updated by (4.10),

$$\begin{aligned} \eta_1 &= -\sum_{i=1}^{s} \int_{t_0}^{t} (\hat{p}_i - p_i) W^T \Phi_i \kappa^2(\tau) d\tau \\ &= -\sum_{i=1}^{s} \left\{ \frac{\alpha_i}{2} I_i^2 + [p_i - \hat{p}_i(t_0)] I_i \right\} \\ &- \beta_i \int_{t_0}^{t} (W^T \Phi)^2 \kappa^2 d\tau \triangleq \eta_1(II). \end{aligned}$$

Thus

$$\eta_1(II) = \eta_1(I) - \beta_i \int_{t_0}^{t} (W^T \Phi)^2 \kappa^2(\tau) d\tau \leq \eta_1(I).$$

Correspondingly it is indicated that

$$J^c(II) \leq J^c(I),$$

which implies that better tracking performance can be achieved by using the PI-Estimate. This conclusion agrees with the comment made in [3]. Simulation results presented in Figures 4.1-4.2 also verify this point (see [7] for more details).

At this point, we are also able to address the effect of the initial estimation on tracking performance. Traditionally, it is suggested that the initial estimate may be chosen arbitrarily (zero in general). This is because the stability is global and the initial estimate does not affect tracking stability. However, as clearly shown in (4.13), the initial estimate affects the overall tracking performance in the sense that a "better" initial estimate results in a tighter bound J^c. Simply choosing $\hat{p}_i = 0$, as suggested typically in the literature, is among the "worst" choices. Choosing the nominal value of p_i as the initial estimate results in a smaller J^c, implying better performance. This is also confirmed by simulation results (see [7]). These points, however, are not directly evident from the Lyapunov stability method.

Finally, the impact of ν, $\kappa(t)$, K, and D on system performance is discussed. It is noted that K and D are required to be symmetric positive definite. Their choices are related to the desired robustness, speed of response, and disturbance rejection properties. The roles of κ and ν are related to the rate of convergence. Note that since the control torque τ is defined as in (4.8), if κ and ν are chosen such that ϵ and $\dot{\epsilon}$ rapidly tend to zero, then the control torque also rapidly tends to the desired value, τ^*. However, if they are chosen so that the convergence of rate is too fast, the control torques in the transition state may exceed the admissible values. Hence some trade-offs between ROC and control energy have to be made in practice.

5. Concluding Remarks

This paper has demonstrated the application of the criteria established in [8] to robotic systems. Performance analysis based on these criteria was also given. Additional applications of these results can be found in [7]. Note that the criteria and their applications are based on continuous systems. Given that discrete time systems are extensively encountered in practice, extensions of these results to discrete-time systems represent an important research effort. Due to the limited space, results concerning this aspect are omitted. Interested readers are referred to [7] for details.

Acknowledgement

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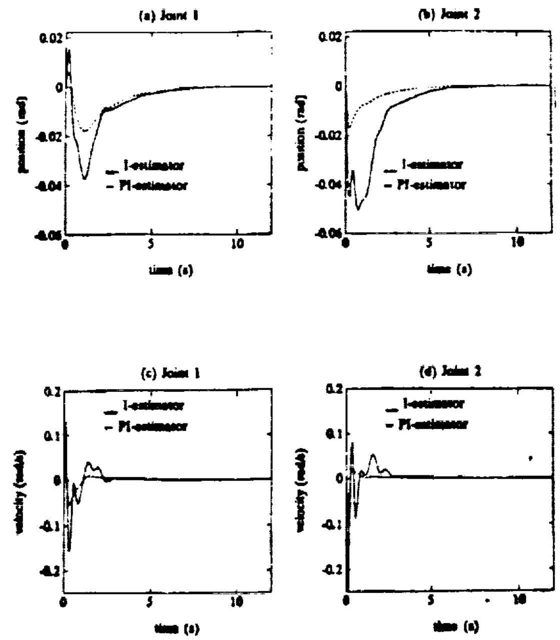
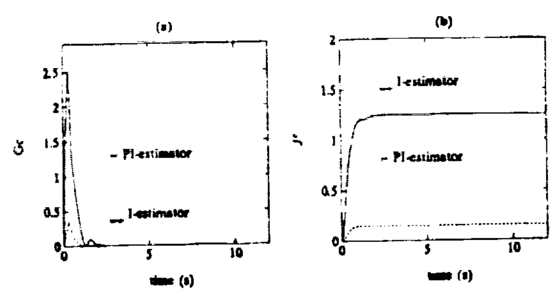


4.1 Tracking Errors with Adaptive Control



4.2 Performance Index with Adaptive Control