Results of a study of high-resolution Viking Orbiter images of the northwestern slopes of Elysium Mons, utilizing a variety of image analysis techniques, provide striking evidence of an extended period of glaciation that involved a large ice sheet of greater than 1.2 km thickness and subice fluvial activity that contributed to the formation of the channels of Hrad Valles and Granicus Valles. These two unusual channel systems begin on the lower slopes of Elysium Mons and extend into Utopia Planitia in the region lying between 215° and 230°W and between 25° and 45°N. Key indicators are the presence of serrated volcanic constructs whose dimensions and morphology indicate an origin involving successive, localized, subice fissure eruptions. The channels visible in Hrad Valles bear a striking resemblance to subice fluvial features found in the dry valleys of Antarctica at the periphery of this massive continental ice sheet. Crater size distributions and crater morphologies are quite consistent with this interpretation, as are certain other topographic features suggesting the presence in the past of ice-rich permafrost that, while having undergone significant degradation, still remains.

**ORBITAL, ROTATIONAL, AND CLIMATIC INTERACTIONS: LESSONS FROM EARTH AND MARS.** Bruce G. Bills, Geodynamics Branch, NASA Goddard Space Flight Center, Greenbelt MD 20771, USA.

**Introduction:** Though variations in orbital and rotational parameters of the Earth and Mars are widely recognized as plausible sources of significant climatic variation on $10^3$-$10^6$-yr timescales, many aspects of the connection between orbital, rotational, and climatic variations remain poorly understood. In general, the orbital histories are very well known, the rotational histories are less well known, and the climatic histories (especially for Mars) are very poorly known. A brief review will be given of recent progress in computing orbital and rotational secular variations, and in connecting them to climatic change. The emphasis will be on highlighting those areas that limit our present understanding.

It is obvious that mass redistributions associated with climatic change (glaciations) are an important source of crustal deformation and geodynamic change on the Earth, and may have played a similar role on Mars in the distant past. It is much less widely appreciated, however, that rates, phases, and amplitudes of deformation of the deep interior of the planet can influence climate. The mantle and core, if completely decoupled, would precess at quite different rates, and even with plausible coupling strengths, some degree of differential precession is possible. Mass flow associated with glaciations can influence the precessional dynamics of the Earth or Mars, and changes in orbital and rotational parameters influence the latitudinal and seasonal pattern of insolation. Previous attempts to account for astronomically forced climatic change have usually only considered extremely simplistic models for the response of the solid planet to external torques and surface loads.

The **Traditional Perspective:** The latitudinal and seasonal pattern of incident solar radiation depends on the eccentricity of the planetary orbit and the orientation of the spin axis relative to both the orbit normal and the aspidal line. Unit vectors $s$ and $n$ characterize the directions of the spin axis and orbit normal respectively. Two angles completely characterize the relative orientation of the spin axis. The obliquity $\varepsilon$ is simply the angle between the orbit normal and the spin axis

$$\varepsilon = \cos^{-1}(n \cdot s) \tag{1}$$

The ascending node of the orbit plane on the instantaneous equator plane has an orientation given by $(s \times n)$, and the longitude of perihelion $\omega$ is just the angle in the orbit plane from that node to perihelion. It is widely appreciated that secular variations in these three parameters ($e, \omega, \sigma$) produce major climatic change [1,2]. In fact, spectral analyses of long, high-resolution marine sediment isotopic records show significant variance at periods near 100 kyr, 41 kyr, and 19-23 kyr, which are generally attributed to spectral lines in the radiative forcing fluctuations associated with $e, \varepsilon,$ and $\sin(\sigma)$ respectively.

The causes and effects of the orbital changes are quite well understood. Gravitational interactions with the other planets cause the shape and orientation of the orbit to change on timescales of $10^6$-$10^8$ yr. The inclination $I$ and nodal longitude $\Omega$ determine the orientation of the orbit plane. The eccentricity $e$ and perihelion longitude $\omega$ determine the shape of the orbit and its orientation within the plane. Note that $\omega$ is measured from an inertially fixed direction, rather than the moving node as is the case for $\sigma$. The secular evolution of the orbital element pairs $(I, \Omega)$ and $(e, \omega)$ can be conveniently represented in terms of Poisson series

$$p = \sin(I)\sin(\Omega) = \sum_{I} \sin(s_i + g_i) \tag{2}$$

$$q = \sin(I)\cos(\Omega) = \sum_{I} \cos(s_i + g_i)$$

$$h = e\sin(\omega) = \sum_{j} e\sin(r_j + f_j) \tag{3}$$

$$k = e\cos(\omega) = \sum_{j} e\cos(r_j + f_j)$$

In the lowest-order solution, there are as many frequencies $f_j$ and $s_j$ as there are planets. However, the frequencies $r_j$ and $s_j$ are characteristic modal frequencies (eigenvalues) of the coupled system of oscillators and are not each uniquely asso-
associated with a particular planet [3]. The frequencies \( r_j \) are all positive, indicating that the perihelia advance. In the lowest-order solution, the apsidal rates are all in the interval \((0.667 < r_j < 28.221 \text{ arsec/yr})\). The corresponding periods are 45.92 kyr to 1.943 m.y. One of the frequencies, \( s_j \), is zero, and all the others are negative, indicating that the nodes regress. In the lowest-order solution, the nonzero nodal rates are all in the interval \((0.692 < s_j < 26.330 \text{ arsec/yr})\). The corresponding periods are 49.22 kyr to 1.873 m.y. In higher-order solutions, variations in \((e, \omega)\) become coupled to variations in \((I, \Omega)\), but the solutions can still be cast in terms of Poisson series like equations (2) and (3).

Laskar [4] has recently published a secular variation theory that is complete to fifth order in eccentricity and inclination. Agreement between this secular variation model and strictly numerical computations [5,6] is much better than for any previous analytical model. The inclination and eccentricity series for Earth and Mars each contain 80 distinct terms.

In computing these secular orbital variations, the Earth, Moon, and planets can all be treated as point masses. No internal structure or processes are relevant to orbital evolution. The physics of the process is simple and well understood, though development of proper mathematical tools to represent the long-term evolution remains an area of active research [4,6,7]. On the other hand, the rotational evolution does depend rather sensitively on various aspects of the structure and dynamics of the interior.

Lunar and solar gravitational torques acting on the oblate figure of the Earth cause the spin axis \( s \) to precess about the instantaneous orbit normal \( n \). If the Earth is considered to be a rigid body, the evolution of the spin axis orientation is given by

\[
\frac{ds}{dt} = \alpha(n \times s)(s \times n)
\]

where

\[
\alpha = \frac{3(C - A)}{2Cn} \sum \frac{Gm}{b_i^3} \left[ 1 - 3\sin^2(i) \right]
\]

is a scalar rate factor that depends on intrinsic properties of the Earth, such as polar and equatorial moments of inertia \((C, A)\) and rotation rate \( n \), and on extrinsic influences, such as masses \( m \), orbital inclinations \( i \), and semiminor axes \( b \), of the Moon and Sun. The solar and lunar torques together produce a precession of the spin axis of the Earth at a rate of \( \alpha(n \times s) = 50.38 \text{ arsec/yr} \) [8,9].

Unfortunately, in the case of Mars, the precession rate is still rather poorly known. The best estimates at present come from analysis of the Viking Lander range data, which yield values of \( 9.6 \pm 0.6 \text{ arsec/yr} \). Much smaller relative errors are frequently cited, but all such optimistic estimates are directly dependent upon an assumed value for the moment of inertia of Mars, which is highly model dependent [10], precisely because the axial precession rate is not known. The uncertainty in this parameter is the largest single impediment to accurate reconstructions of the obliquity history of Mars [11]. Fortunately, the Mars Observer mission radio science investigations will significantly improve our knowledge of the precession rate of Mars within a few months after orbit insertion.

Once the present spin axis direction \( s \) is known and orbital element histories are given via equations (2) and (3), an obliquity history can be constructed from equation (4) in two different ways. The linear perturbation approach [12–17] involves deriving coefficients of a trigonometric series, similar to equations (2) and (3), which yields the obliquity and longitude of perihelion directly as functions of time. An alternative is to apply standard numerical algorithms for solving initial value problems to generate a vector time series \( s(t) \) and then compute the obliquity and longitude of perihelion directly [11,18,19]. The spectrum of obliquity variations, in the linear perturbation model, is simply obtained from the inclination spectrum by shifting each frequency \( s_j \) by the lunisolar precession rate \( s = 50.38 \text{ arsec/yr} \) for the Earth, or 8–10 \text{ arsec/yr} for Mars) and multiplying each amplitude \( N_j \) by the spectral admittance

\[
F_j = \cos(e) \left( \frac{s}{s_j} \right)
\]

There are several important features of this solution: (1) As \( a \) is positive, and \( s_j \) is negative in all cases, it is possible for the denominator to vanish; (2) near this resonance, the obliquity variations are large; and (3) at smaller or larger forcing frequencies, the obliquity variations are small.

Uncertainty in the obliquity history of Mars derives from two facts: the value of the precession rate is uncertain by 6% (compared to <0.04% for the Earth) and the obliquity history of Mars depends more sensitively on the precession rate than is the case for Earth because of a near resonance with some of the inclination forcing frequencies. To investigate the sensitivity of the computed martian obliquity history to assumed precession rate, a series of numerical integrations of the rigid body precession equations was made [11], covering a 20-m.y. interval centered on the present, using precession rates in the interval 8.5–9.5 \text{ arsec/yr}. The maximum obliquity encountered can be anywhere in the range 35°–50°, and the minimum value can be 8°–14°. Despite this large uncertainty in the particulars of the obliquity history, all the computed time series were characterized by two dominant features: large oscillation with a characteristic period of \( \approx 10^3 \text{ yr} \) and a significant modulation with a characteristic period of 2 \( 10^5 \text{ yr} \).
Fourier, Legendre, and Milankovitch: The influence of orbital and rotational variations on climate is operative through perturbations in the latitudinal and seasonal pattern of insolation. The diurnal average intensity of radiation at a point is inversely proportional to the squared solar distance and directly proportional to the diurnal average rectified solar direction cosine

$$F = \frac{(a/r)^2}{\|u \cdot u_s\|}$$  \hspace{1cm} (7)

where $a$ and $r$ are mean and instantaneous solar distance, and $u$ and $u_s$ are unit vectors from the center of the Earth to the surface point of interest and the subsolar point respectively. The insolation pattern, as a function of latitude $\theta$ and mean anomaly $M$, can be readily computed once values are specified for the orbital and rotational parameters $e$, $e$, and $\omega$ \cite{14,15,18,20,21}. This pattern can also be written in terms of a Fourier-Legendre series \cite{20,22-24}

$$F(\mu, M, e, e, \omega) = \sum F_n(\mu) \sum \exp(ipM) F_n, p(e, e, \omega)$$  \hspace{1cm} (8)

where $\mu = \cos(\theta)$ and $P_n$ is a Legendre polynomial. The number of terms in the Fourier summation required to obtain a good representation of the seasonal pattern is greater in the polar regions than in the tropics and mid latitudes. The primary difficulty in the polar regions is reproducing the abrupt change in slope of the insolation curve at times of transition to continual darkness or continual light. It is also true that the polar regions place the greatest demands on the Legendre summation, since the spatial pattern also has a discontinuous first derivative at the latitude where the transition occurs to continual darkness or light.

Precessional Dynamics with Variable Rate: All but the most recent reconstructions of the radiative forcing input to paleoclimate models have assumed that both the orbital and rotational dynamics could be readily and accurately reconstructed from their present configurations via the simple analyses mentioned in the introduction. These expectations seem well founded in the case of orbital evolution, though the possibility of chaotic dynamics in the inner solar system \cite{7,25} does seem to preclude confident extrapolation beyond $10^7$ yr. However, there are a number of processes, working in different locations and at different rates, that all serve to compounding the difficulty of accurately computing the precessional evolution.

On the longest timescales of interest ($10^7$--$10^9$ yr) the limiting uncertainty is variability in the tidal transfer of angular momentum from the rotation of the Earth to the orbit of the Moon. At present, these tidal torques are increasing the length of the day by $2.25 \times 10^{-6}$ s/yr and increasing the size of the lunar orbit by $3.88 \text{ cm/yr}$ \cite{26,27}. Berger et al. \cite{28} have made a useful first step toward including this effect in climatic time series. They computed the change in the major precession and obliquity frequencies due to lunar tidal evolution assuming that the present rate of tidal energy dissipation is representative of the past 500 m.y. However, the present rates are considerably higher than the long-term average \cite{29}, largely due to a near resonance between sloshing modes of ocean basins and the diurnal and semidiurnal tidal periods \cite{30}, and apparently compounded by a contribution from shallow seas \cite{31,32}. Sedimentary records that constrain lunar orbital evolution show some promise of resolving this problem \cite{33-36}, but the situation is definitely more complex than is suggested by Berger et al. \cite{28}.

Another parameter that can vary, on rather shorter timescales and in an equally irregular fashion, is the gravitational oblateness of the Earth (C-A)/C. Thomson \cite{37} has recently made three important contributions to the understanding of this source of variability. First, he pointed out that mass redistribution associated with major glaciations and compensating subsidence and crustal deformations \cite{38,39} can cause fractional changes in oblateness of order $10^{-3}$--$10^{-2}$. Second, he showed that high-resolution spectral analyses of several climatic time series appear to indicate fluctuations of the lunisolar precession rate of this magnitude, and with a dominant period near 100 kyr. Finally, Thomson pointed out that the best fit to the paleoclimate proxy data was obtained using a mean lunisolar precession rate 0.6 arcsec/yr less than the present observed value. He notes that the resulting value would correspond rather closely with that expected for a hydrostatic flattening \cite{40}. If these important results are corroborated, they will demonstrate that important feedback loops exist in the orbital-rotational-climatic interactions system, further "up-stream" in the presumed causal chain than has been previously recognized.

Differential Precession of the Mantle and Core: The hydrostatic figure of a planet represents a compromise between gravitation, which attempts to attain spherical symmetry, and rotation, which prefers cylindrical symmetry. Due to its higher mean density, the core of the Earth is more nearly spherical than the mantle. The direct lunisolar precessional torques on the core will thus be inadequate to make it precess at the same rate as the mantle. In fact, the core oblateness is only about three-fourths that required for coprecession with the mantle \cite{41}. However, it is clearly the case that the core and mantle precess at very nearly the same rate \cite{42}. A variety of different physical mechanisms contributes to the torques that achieve this coupling, but a purely phenomenological partitioning is useful. The net torque can be described as a sum of inertial torques, which are parallel to $(\chi_m \times \chi_c)$, and dissipative torques, which are parallel to $(\chi_m - \chi_c)$. Here, $\chi_c$ and $\chi_m$ are the rotation vectors of the core and mantle respectively. The two types of torques have qualitatively different results: Inertial torques cause the core and mantle axes to precess at fixed angular separations and on the opposite side of their combined angular momentum vector,
whereas the effect of dissipative torques is to reduce the angle between the axes.

On short timescales it is appropriate to consider the core to be an inviscid fluid constrained to move within the ellipsoidal region bounded by the rigid mantle [43–45]. The inertial coupling provided by this mechanism is effective whenever the ellipticity of the container exceeds the ratio of the precessional to rotational rates. If the mantle were actually rigid, or even elastic [46,47], this would be an extremely effective type of coupling. However, on sufficiently long timescales, the mantle will deform viscously and can accommodate the motions of the core fluid [48]. The inertial coupling torque exerted by the core on the mantle will have the form

\[ T_i = k_i [X_m \times X_c] \]  \hspace{1cm} (9)

A fundamentally different type of coupling is provided by electromagnetic or viscous torques [49–51]. The dissipative coupling torque exerted by the core on the mantle will have the form

\[ T_d = k_d [X_m \times X_c] \]  \hspace{1cm} (10)

This type of coupling is likely to be most important on longer timescales. In each case, the mantle exerts an equal and opposite torque on the core. The response of the coupled core-mantle system to orbital forcing is given by [52–54]

\[
\begin{align*}
\frac{d\alpha_m}{dt} &= \alpha_m (n \cdot \alpha_m) (\alpha_m \times n) - \beta_m (\alpha_m \times \alpha_c) - \gamma_m (\alpha_m \times \alpha_c)
\end{align*}
\]

\[
\begin{align*}
\frac{d\alpha_c}{dt} &= \alpha_c (n \cdot \alpha_c) (\alpha_c \times n) + \beta_c (\alpha_c \times \alpha_m) + \gamma_c (\alpha_c \times \alpha_m)
\end{align*}
\]  \hspace{1cm} (11)

where \( \alpha_m \) is similar to \( \alpha \) above, except that only mantle moments \( A_m \) and \( C_m \) are included, and

\[
\begin{align*}
\beta_m &= k_d / C_m \nu^2 \\
\gamma_m &= k_d / C_m \nu^2
\end{align*}
\]  \hspace{1cm} (12)

where \( \nu \) is the mean rotation rate.


**CONDENSATION PHASE OF THE MARTIAN SOUTH POlar CAP**

J. Capuano, M. Reed, and P. B. James, Department of Physics and Astronomy, University of Toledo, Toledo OH 43606, USA.

One type of database that can be useful in constraining models of the martian surface-atmosphere system is the time-dependent boundary of CO₂ frost for the polar caps. These