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Stability of Mixing Layers

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1. Summary of Research Work

The research program for the first year of this project (see the original research proposal) consists of developing an explicit marching scheme for solving the parabolized stability equations (PSE). Performing mathematical analysis of the computational algorithm including numerical stability analysis and the determination of the proper boundary conditions needed at the boundary of the computation domain are implicit in the task. Before one can solve the parabolized stability equations for high-speed mixing layers the mean flow must first be found. In the past instability analysis of high-speed mixing layer has mostly been performed on mean flow profiles calculated by the boundary layer equations. In carrying out this project it is believed that the boundary layer equations might not give an accurate enough nonparallel, nonlinear mean flow needed for parabolized stability analysis. A more accurate mean flow can, however, be found by solving the Parabolized Navier-Stokes equations. The advantage of the Parabolized Navier-Stokes equations is that its accuracy is consistent with the PSE method. Furthermore, the method of solution is similar. Hence the major part of the effort of the work of this year has been devoted to the development of an explicit numerical marching scheme for the solution of the Parabolized Navier-Stokes equation as applied to the high-seed mixing layer problem.

The Dispersion-Relation-Preserving finite difference scheme developed recently by one of the principal investigators has been identified to be a suitable method for solving the Parabolized Navier-Stokes equations and the PSE. The numerical stability limit of the scheme when applied to the mixing layer problem has been determined. Analysis reveals that radiation boundary conditions are needed at the boundaries of the computation domain. They can easily be incorporated into the marching scheme. Numerical results of the mean flow of high-speed mixing layers based on the Parabolized Navier-Stokes equations and on the boundary layer equations are found. The u-velocity and temperature profiles calculated by the two sets of equations are almost identical. The v-velocity component (very small in amplitude) computed by the two systems of equations is, however, qualitatively very different. That given by the boundary layer equations appears to be unphysical (the fast stream is entraining fluid from the mixing layer instead of the other way around) whereas that calculated by the Parabolized Navier-Stokes equations is physically acceptable. This result suggests that boundary layer solutions of the mixing layer should not be used for nonparallel flow instability computation.

The parabolized stability equation was originally formulated for incompressible boundary layer flows. Recently it has been extended by a number of investigations to compressible boundary layer flows. When applied to compressible flows especially for mixing layers the best way to formulate the equations becomes not obvious. We have explored this question and found that the best way could be different depending on one's objective. The question ties closely with mathematical and numerical instability and boundary conditions. We plan to study this further in the forthcoming year.

2. The Parabolized Navier-Stokes Equations and the Dispersion-Relation-Preserving Scheme

At high temperature the viscosity and thermal conductivity of a gas are temperature dependent. On using a power law relation i.e. $\mu \sim T^n$ and $k \sim T^m$ the dimensionless Parabolized Navier-Stokes equations may be written in the form

$$\begin{aligned}
\frac{\partial \rho}{\partial x} &= \frac{1}{u}\left[\frac{\rho v}{u}\frac{\partial u}{\partial y} - \frac{1}{Ru}\frac{\partial \tau}{\partial y} - \frac{\partial \rho v}{\partial y}\right] \\
\frac{\partial u}{\partial x} &= \frac{1}{\rho u}\left[-\rho v\frac{\partial u}{\partial y} + \frac{1}{R}\frac{\partial \tau}{\partial y}\right] \\
\frac{\partial v}{\partial x} &= \frac{1}{\rho u}\left[-\rho v\frac{\partial v}{\partial y} - \frac{1}{\gamma M_1^2}\frac{\partial \rho T}{\partial y} + \frac{1}{R}\frac{\partial \sigma}{\partial y}\right] \\
\frac{\partial T}{\partial x} &= \frac{1}{\rho u}\left[-\rho v\frac{\partial T}{\partial y} - \frac{1}{RP_r}\frac{\partial q}{\partial y} + \frac{(\gamma-1)M_1^2}{R}\left(\frac{\partial u}{\partial y}\right)^2\right]
\end{aligned} \tag{1}$$

where $\tau = T^n \frac{\partial u}{\partial y}$, $\sigma = 2T^n \frac{\partial v}{\partial y}$, $q = T^m \frac{\partial T}{\partial y}$

$$\begin{aligned}
R \text{ (Reynolds number)} &= \frac{\rho_1 u_1 L}{\mu_1} \\
P_r \text{ (Prandtl number)} &= \frac{\mu_1 c_p}{k_1} \\
M_1 \text{ (Mach number)} &= \frac{u_1}{a_1}
\end{aligned}$$

(subscript 1 refers to the ambient condition of the high-speed stream).

According to the Dispersion-Relation-Preserving (DRP) scheme the y-derivatives are to be discretized as (7 point scheme)

$$\left(\frac{\partial f}{\partial y}\right)_m = \frac{1}{\Delta y}\sum_{j=-3}^{3} a_j f_{m+j} \tag{2}$$

where the subscript m denotes the index in the y-direction and Δy is the mesh size. If equation (1) is rewritten in a vector form e.g.

$$\frac{\partial U}{\partial x} = K \tag{3}$$

then the marching scheme in the x-direction is discretized as

$$U_m^{(n+1)} = U_m^{(n)} + \Delta x \sum_{j=0}^{3} b_j \, K_m^{(n-j)} \tag{4}$$

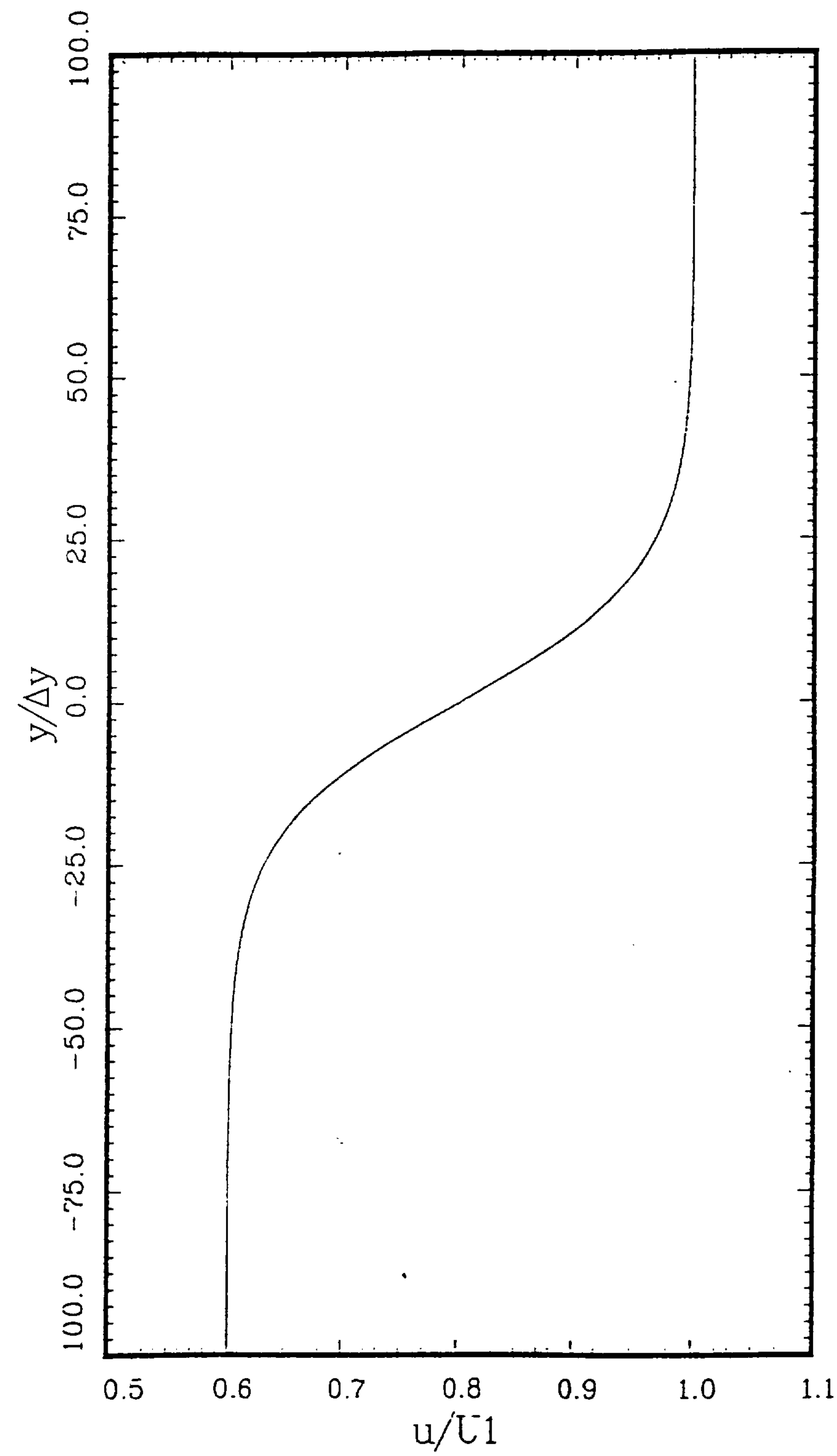
where the superscript denotes the index in the x-direction and Δx is the marching step size.

It is easy to show within the long wave range that the Fourier-Laplace transform of the partial differential equation (1) and that of the finite difference equations (2) and (3) are identical. Thus as long as the profiles are sufficiently smooth (no short waves) the discretized marching scheme (4) and (2) will provide accurate solution of the Parabolized Navier-Stokes equations (1).

3. Numerical Results

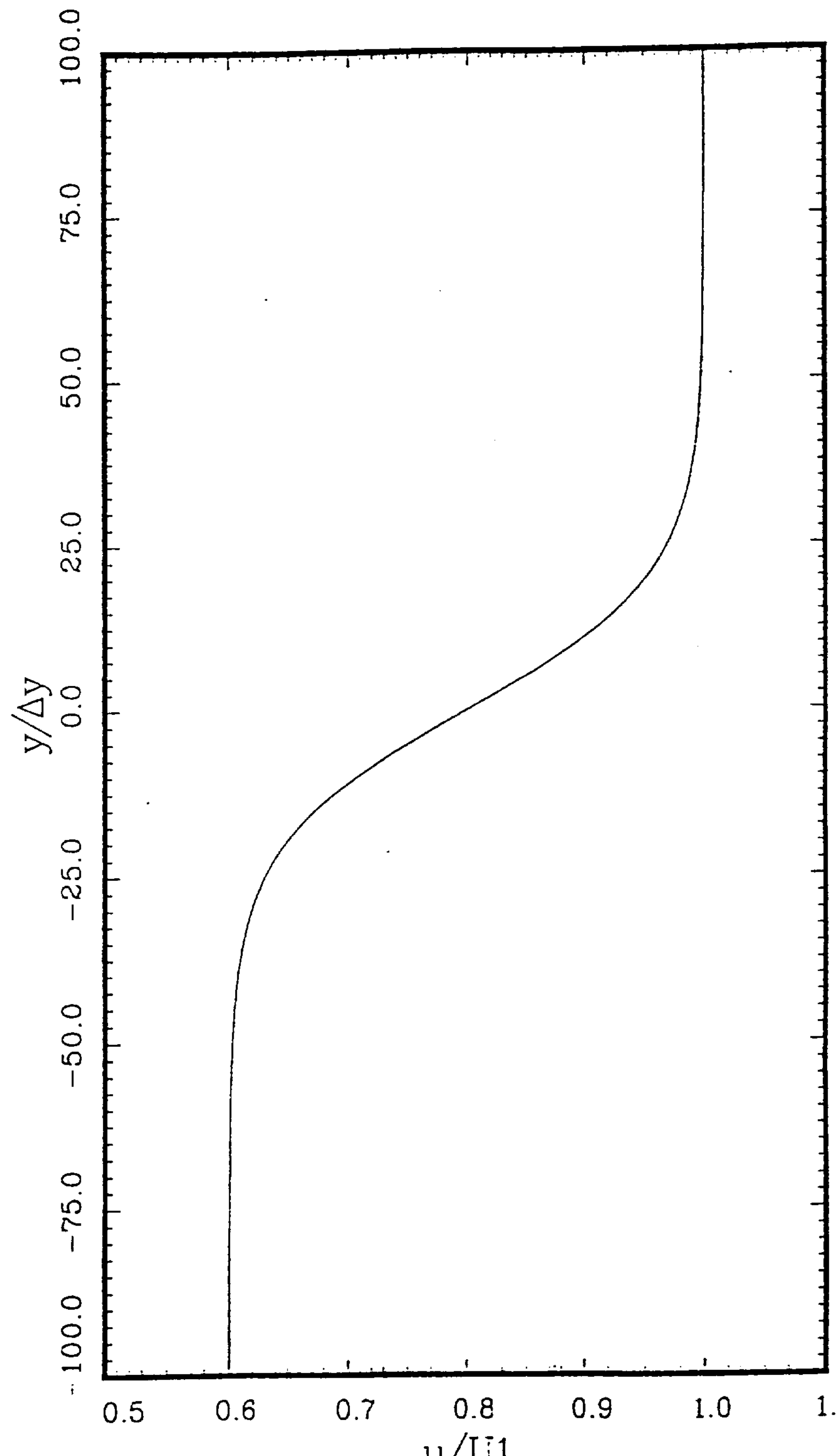
To illustrate the feasibility of using the DRP scheme to solve the Parabolized Navier-Stokes equations, numerical results of equations (4) and (2) are provided below. The case under consideration consists of two parallel streams. The fast stream with a Mach number of 2.0 is on top. The slow stream has a Mach number of 1.2. The static temperature is assumed to be the same in both streams. The initial velocity, u, is given by a hyperbolic tangent profile with momentum thickness Θ and v is zero. In the numerical computation Δy is taken to be $\Theta/40$. the calculated velocity and temperature profiles at $x = 1.6\Theta$, 8.0Θ and 16Θ are provided. It can easily be seen that the u-velocity and temperature profiles given by the boundary layer equations and the Parabolized Navier-Stokes equations are, for all intents and purposes, identical. The y-velocity component, v, (which is very small) is, however, very different. The solution of the Parabolized Navier-Stokes equations is negative in the upper layer and positive in the lower layer indicating that fluid is entrained into the mixing layer from the upper as well as the lower stream. The solution of the boundary layer equations (with boundary condition $v = 0$ at $y = 0$) is positive for all values of y. The implication is that fluid flows from the bottom uniform stream into the mixing layer. But at the same time, fluid flows from the mixing layer into the uniform upper stream. This is, of course, unphysical and must be rejected. This example suggests that one should use the solution of the Parabolized Navier-Stokes equations as mean flow in performing nonparallel flow instability computation of mixing layers.

$X = 1.6\,\Theta$



solid line: Boundary layer approximation for mean flow
dot line: Parabolized Navie–Stokes approximation for mean flow
($\Theta = 40\Delta y$)

X = 8.0 Θ



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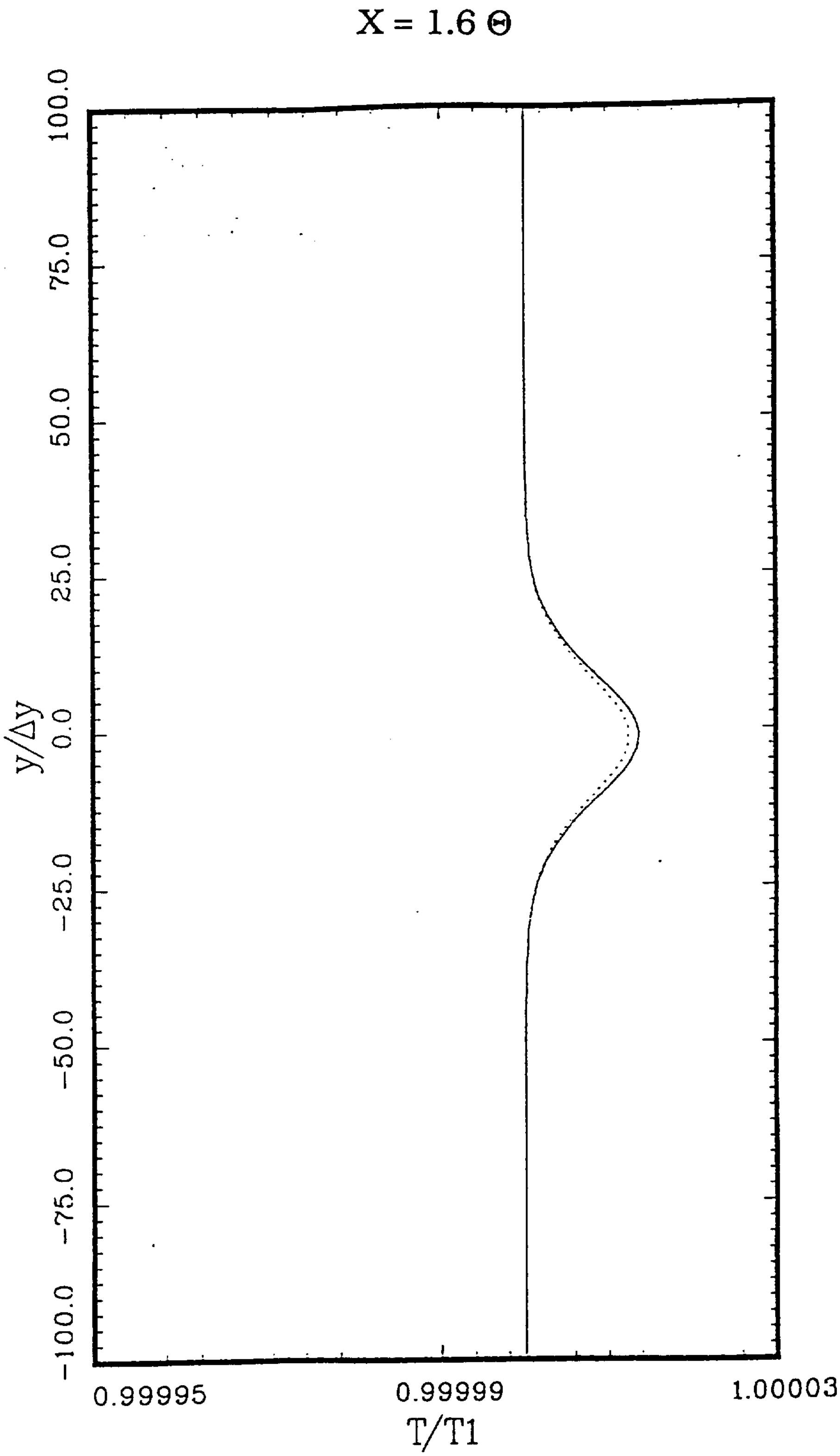

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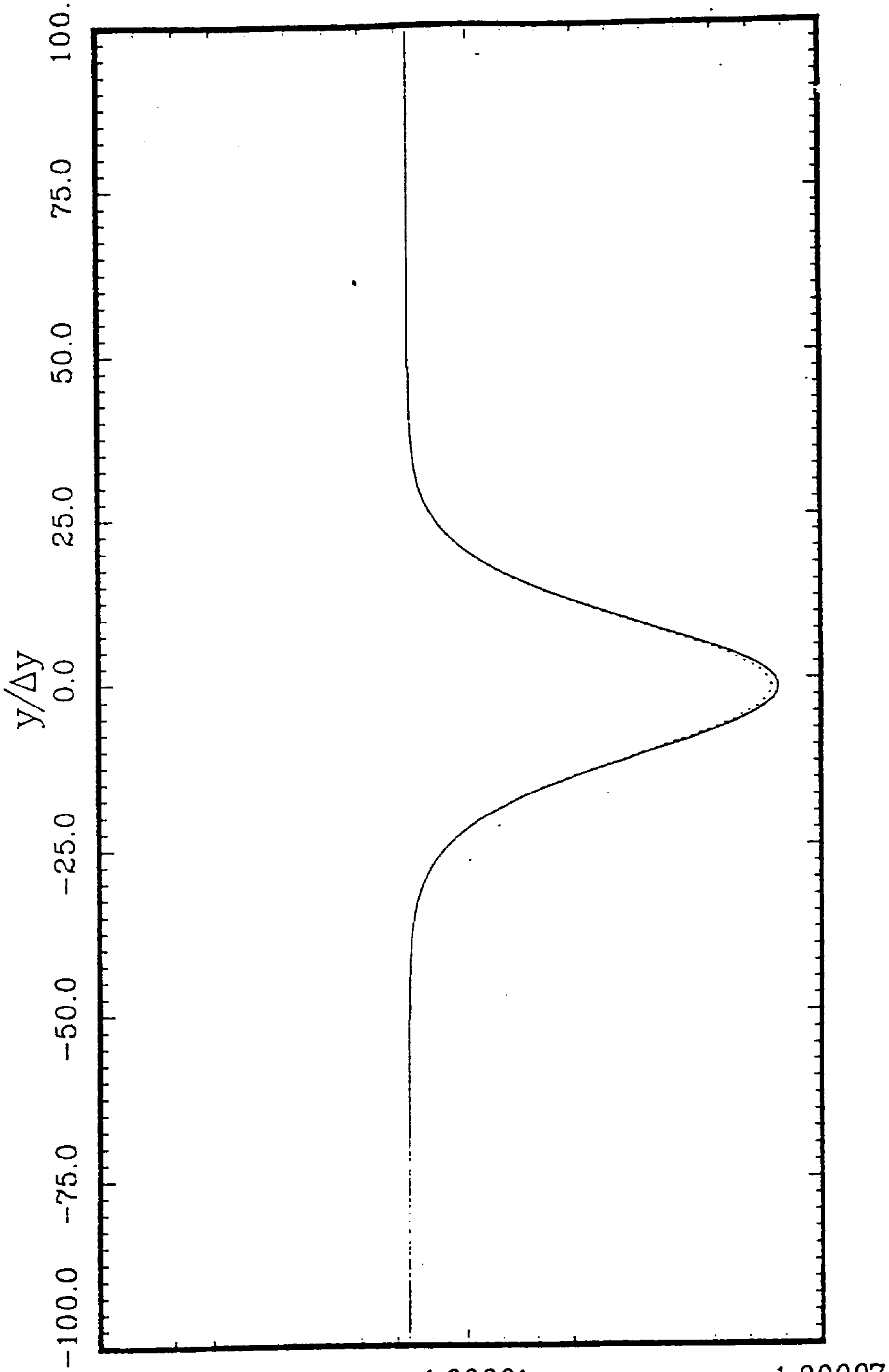
$X = 8.0\,\Theta$



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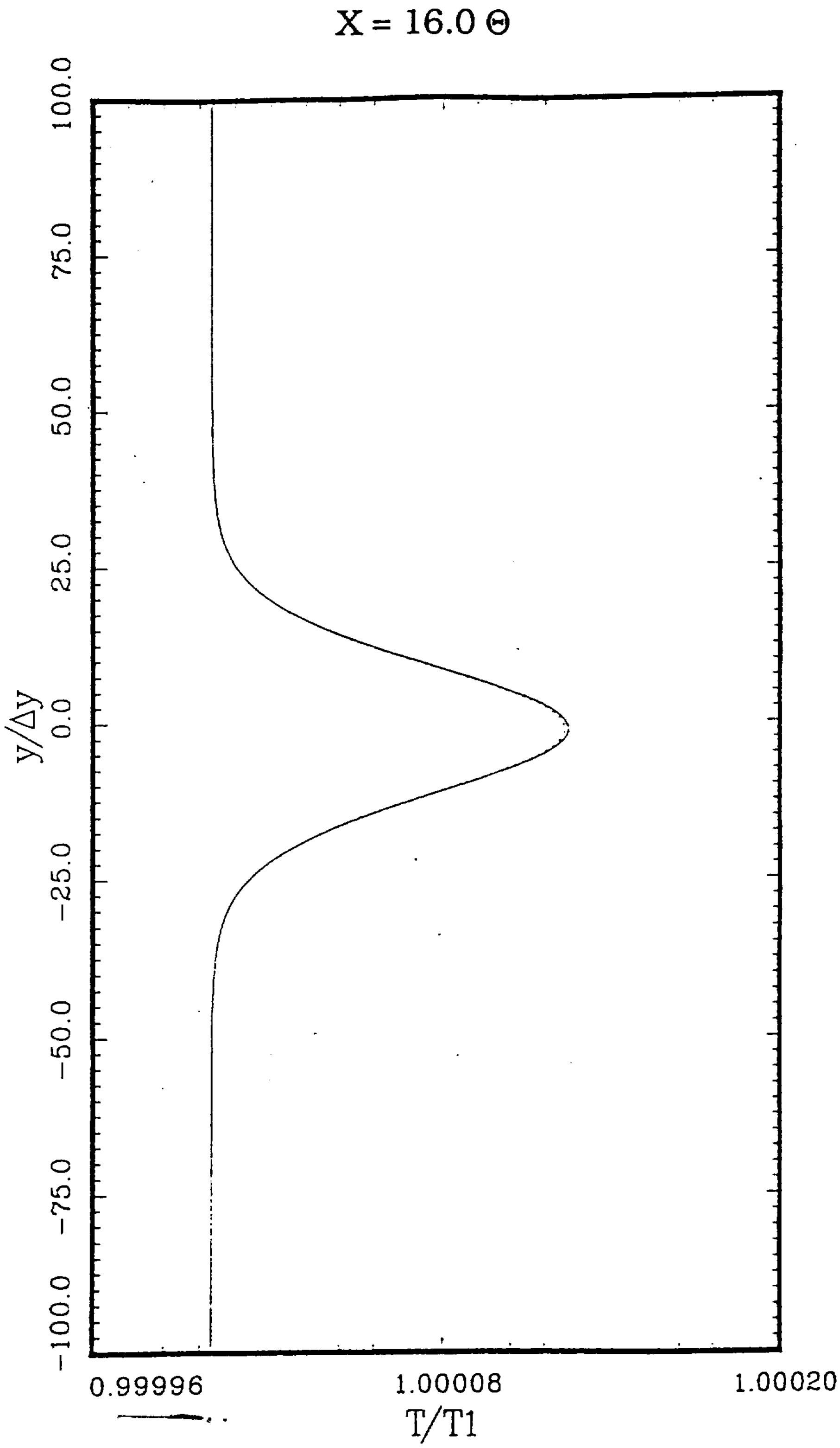

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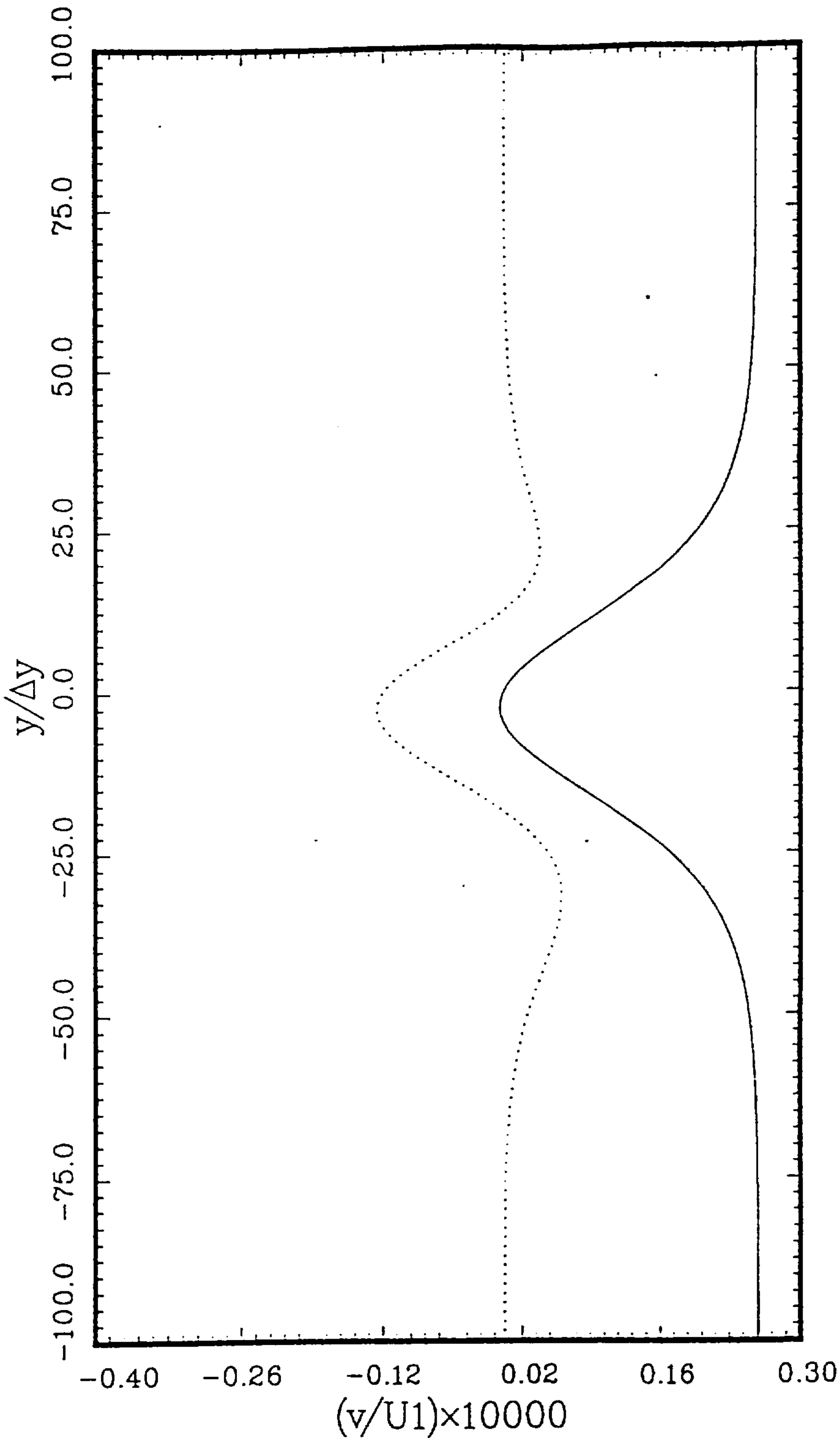

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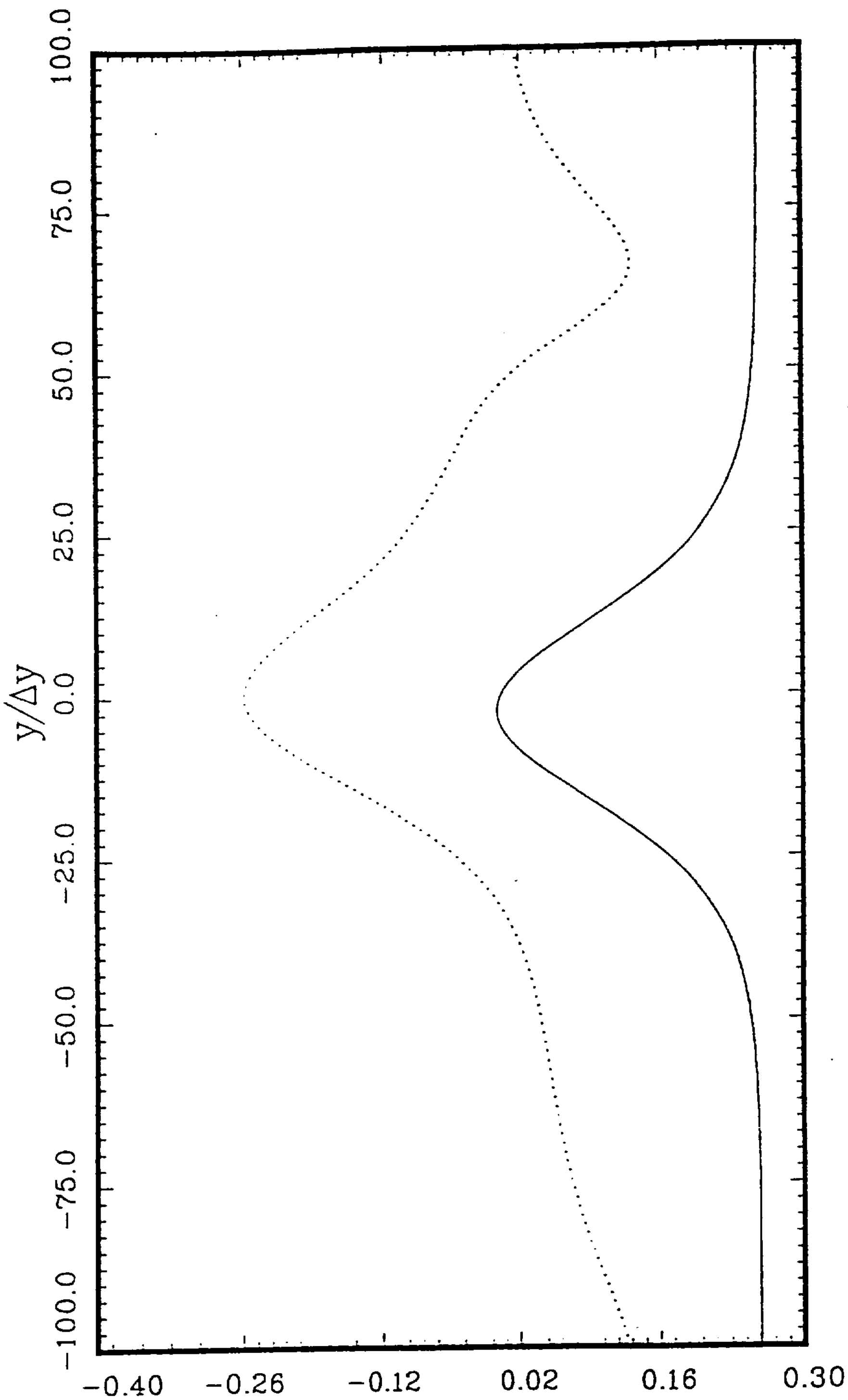

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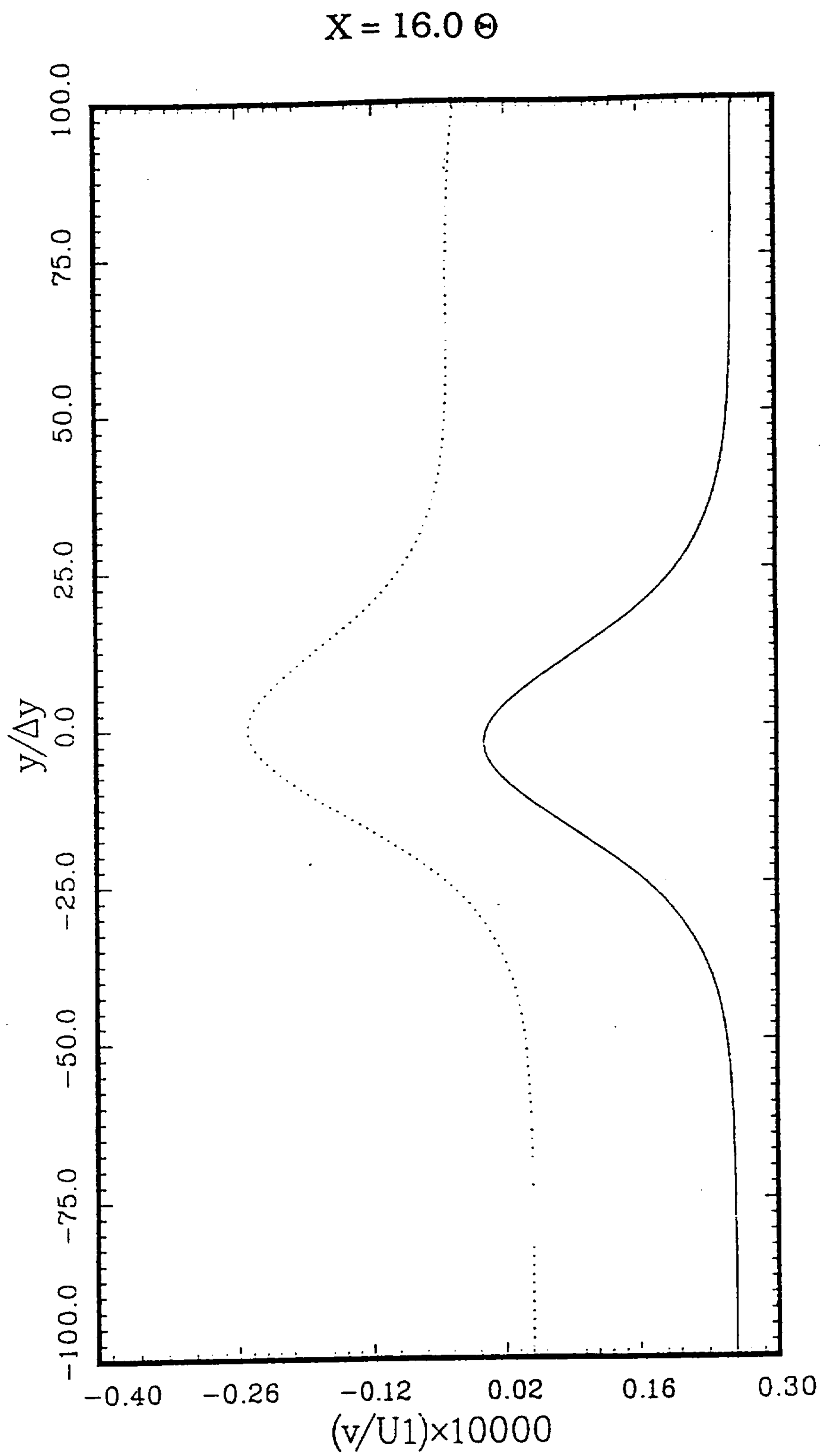

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