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M.I.T. SPACE ENGINEERING RESEARCH CENTER

Semi-Annual Report: January 1991

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OVERVIEW

The Space Engineering Research Center (SERC) at M.I.T., started in July 1988, has completed two and one-half years of research. This Semi-Annual Report presents annotated viewgraph material presented at the January 1991 Steering Committee and Technical Representative Review.

The objective of the Space Engineering Research Center is to develop and disseminate a unified technology of controlled structures. There has been continued evolution of the concept of intelligent structures (including in this past year the first successful embedding of a microelectronic component into a structural element). Nine faculty and thirty-five graduate students and a like number of undergraduates through the Undergraduate Research Opportunity Program (UROP) participate in the activities of SERC, drawn from the Departments of Aeronautics and Astronautics, Electrical Engineering and Computer Science, and Mechanical Engineering.

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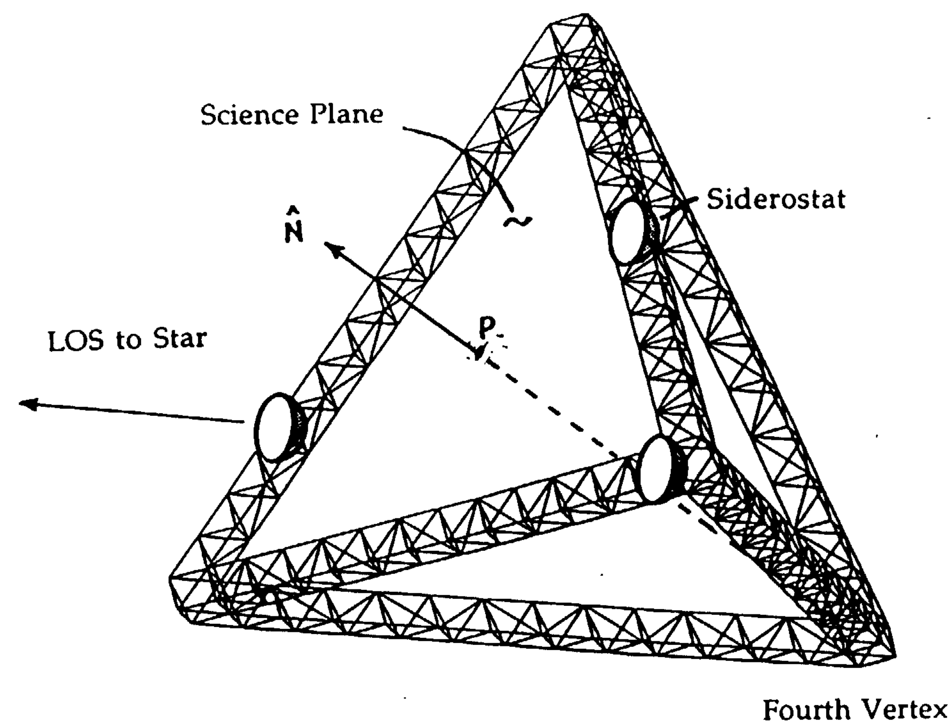
SERC TESTBED PROGRAM:

OPTICAL INTERFEROMETER

Dr. David W. Miller

OPTICAL INTERFEROMETER TESTBED (OVERVIEW)

- Testbed based on scientific mission for focus of graduate student theses.
- Testbed modelled on a 35 meter baseline earth-orbiting optical interferometer.
- Precision alignment requirement between three onboard optical elements is 50 nanometers RMS above 1/10th of a hertz.



OPTICAL INTERFEROMETER (PROGRESS IN PAST YEAR)

- Structural and control hardware

  3.5 meter on a side trusswork tetrahedron constructed.
  1000 Hz real time computer integrated to testbed.
  Active struts and precision accelerometers acquired.
  Active mirror mounts designed, calibrated and installed.

- Performance metric

  A performance metric which uses 6 optical measurements was selected. A subset are available for feedback. This is representative of an internal metrology system which cannot measure all error sources.
  One leg of on-board laser metrology system completed.

- Testing and modelling

  Damping measurements at nanostrain levels completed.
  Full modal system identification performed
  Finite element model developed.
  Disturbance model was selected which is flat to 100 Hz and followed by a 1st order rolloff. An additional narrowband disturbance is applied to the first set of modes.

OPTICAL INTERFEROMETER (COMPLETION SCHEDULE)

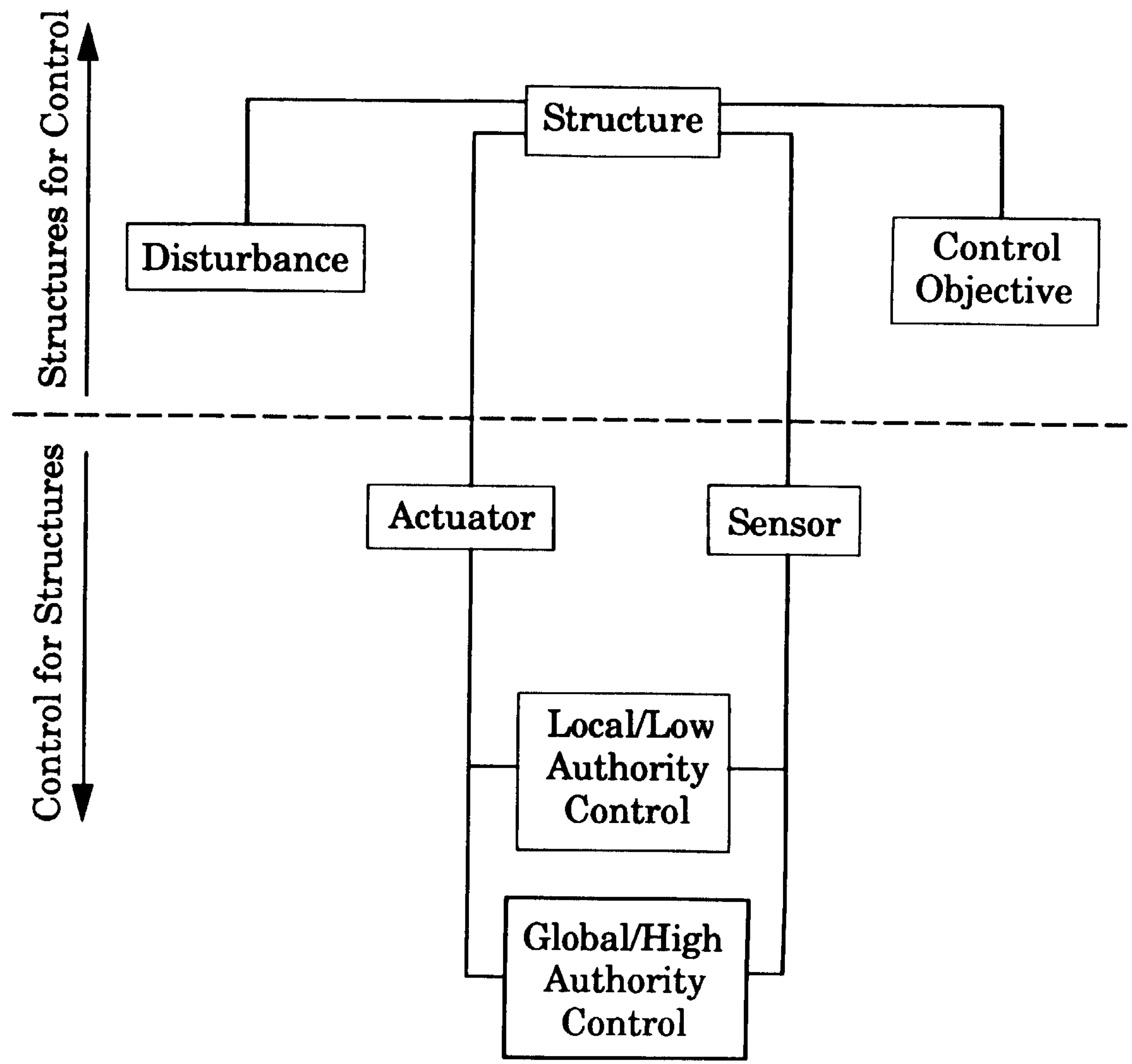
- Phase 1 testbed operational by end of January 1991.
- Tasks needing completion include:
  - Acquisition of cat's eye retroreflectors and operation of two legs.
  - Disturbance source hardware installation.
  - Integration of active struts and precision accelerometers.
  - ~1% damping augmentation via passive viscoelastic struts.
  - Correlation of modal identification and FEM model.
- Task needing completion for Phase 2 testbed.
  - Installation of two more cat's eye retroreflectors to provide six optical legs.

OPTICAL INTERFEROMETER (RESEARCH ACTIVITIES)

- Microdynamics
- Passive damping augmentation
- CST systems design
- Isolation
- Impedance matching
- Robustness methods for uncertain structures
- Control/structures optimization

A CST DESIGN METHODOLOGY

1. Design structure.
2. Design disturbances and control objective.
3. Design actuator and sensor.
4. Design local/low authority control.
5. Design high authority control.



INTERFEROMETER: EXAMPLE OF CST DESIGN

The interferometer will be used not only as a testbed for the elements of Controlled Structures Technology but also for the evolution of the design process.

Step 0 - Mission Requirement Specification

Disturbance selected from spacecraft experience.

Performance metric established which captures challenge of real spacecraft mission.

Step 1 - Design Structure

Structure chosen for "rigid" alignment of primary performance measures - optical elements of interferometer.

Placement of representative disturbance at representative location gives baseline performance.

Addition of passive damping to 1% gives performance improvement and robustness.

Step 2 - Design disturbances and control objective

Performance of spacecraft systems identifies means to reduce disturbances and place them at locations of lower disturbability giving performance improvement.

Isolation at disturbances and optical elements give performance improvement

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SERC TESTBED PROGRAM:

DYNAMIC ANALYSIS OF GRAVITY DEPENDENT NONLINEARITIES

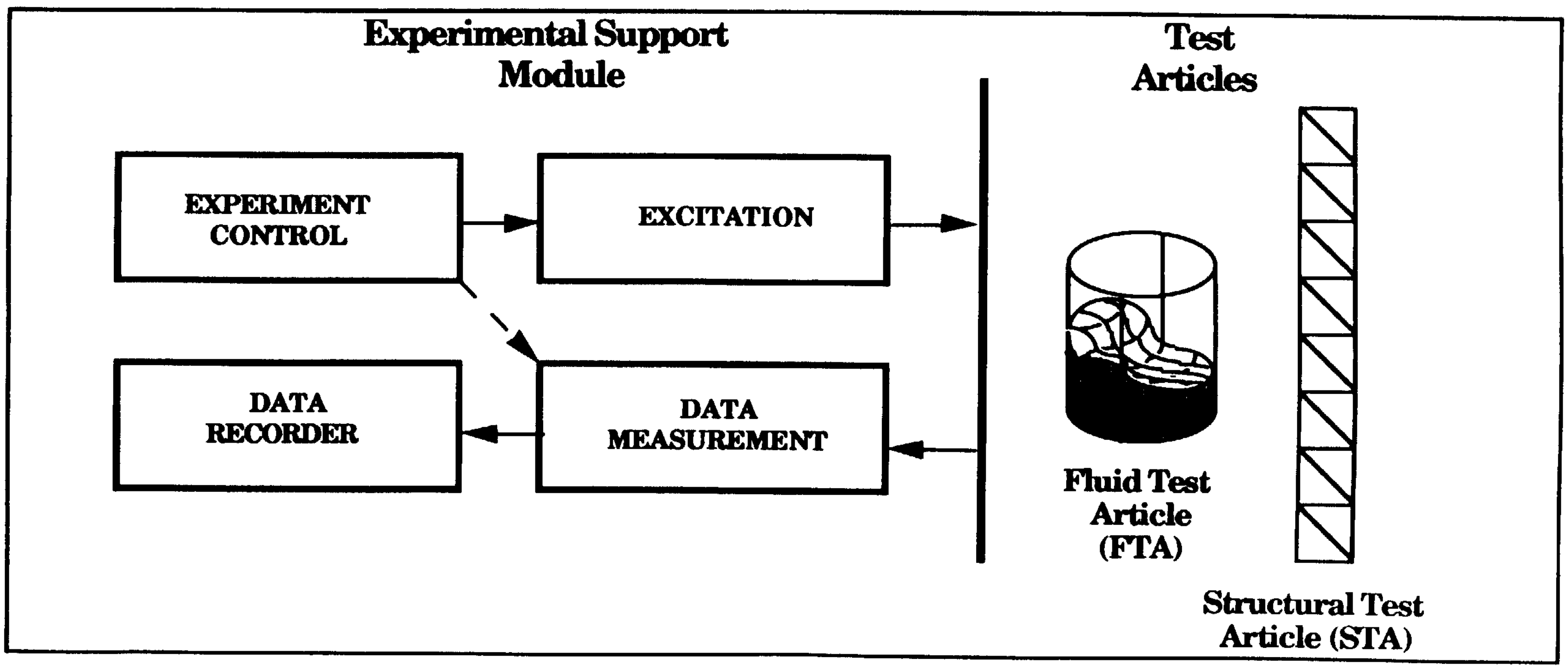
COMPANION GROUND PROGRAM TO THE

MIDDECK 0-GRAVITY DYNAMICS EXPERIMENT (MODE)

Dr. David W. Miller

MODE PROGRAM (OVERVIEW)

- In-Step program awarded in 1988.
- Objective is to study gravity dependent nonlinearities associated with fluid slosh and truss structure dynamics.
- MODE provides a reusable facility for on-orbit dynamic testing of small scale test articles in the shirt sleeve environment on the Shuttle middeck.



MODE PROGRAM
(PROGRESS IN PAST YEAR)

- Testbed progress
  - Structural test article (STA) designed, procured and delivered,
  - STA testing proceeding,
  - Design of 6-axis component tester completed,
  - Fluid slosh tests are ongoing.

- Flight experiment progress
  - Completed Critical Design Review in Spring of 1990,
  - 90% of flight hardware fabrication completed,
  - Phase 2 safety meeting completed.

MODE PROGRAM
(COMPLETION SCHEDULE)

- Flight hardware will be completed in February, 1991.
- Flight assignment at Flight Planning and Storage Review in February, 1991.
- Precursor flight to investigate fluid alignment issues scheduled for SLS 1 in May, 1991.
- Hardware delivery in Aug.-Sept., 1991.
- Tentatively scheduled for STS 48 in Oct.-Nov. 1991 with the UARS payload.

MODE PROGRAM
(RESEARCH ACTIVITIES)

- Gravity effects on structural nonlinearity.
- Describing function approach to structural nonlinearity.
- Component force state mapping.
- Modelling of fluid slosh in arbitrary tank geometries and under different gravity loads.

SERC Testbed Program:

Multibody Articulation and Control

Companion Ground Program to the

Middeck Active Control Experiment (MACE)

Dr. David W. Miller

THE MODE FAMILY OF EXPERIMENTS

Fluid Test Article (FTA)

Coupled Non-Linear Dynamics of Fluids and Structures in Zero Gravity

Structural Test Article (STA)

Non-Linear Dynamics of Jointed Truss Structures in Zero Gravity

Flight # 1:
August 1991

MACE Test Article

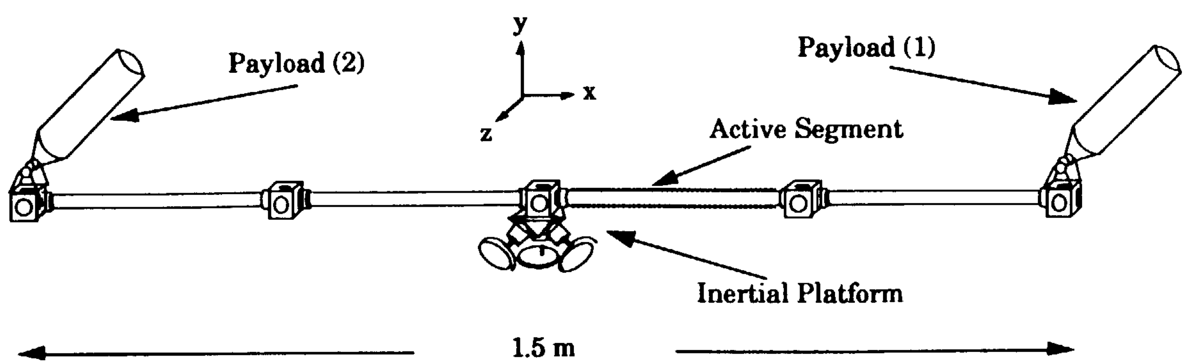
Influence of Gravity on the Active Control of a Multibody Platform

Flight #2:
September 1993

MACE is part of a logical sequence of cost-effective flight experiments designed to advance technology of interest to NASA in the area of controlled structures.

MULTIBODY TESTBED (OVERVIEW)

- Testbed based on scientific mission for focus of graduate student theses.
- Testbed modelled on a multi-payload platform generically representative of an Earth observing platform.
- Control objectives include single and multiple payload precision pointing and scanning.
- Precision pointing and scanning requirements are on the order of one arc minute RMS which is two orders of magnitude below open-loop response.
- Flight program objective (MACE) is to study gravity effects on the performance and stability of controlled structures.



Multibody Platform (Progress in Past Year)

- Tested progress

    Bus structure, attitude control torque wheel assembly and one two-axis gimbal have been acquired,

    Three-axis rate gyros, angular encoders, accelerometers, strain gauges and load cells have been acquired,

    Real time computer (AC-100), DEC Vax station, signal conditioning and power amplifiers have been acquired,

    Developing a test verified dynamic model,

    Control analysis ongoing using different complexity models,

    Pointing Sample Problem available.

- Flight experiment progress

    Awarded In-Step program in spring of 1990.

    Completed Conceptual Design Review in December 1990.

    Critical Experiment Support Module elements have been designed (e.g., realtime computer, up/down link, etc.)

    Testbed hardware and control analysis development satisfy various flight specific requirements.

MULTIBODY TESTBED (COMPLETION SCHEDULE)

- Testbed
  - Three CSA zero spring rate suspension devices to be installed in late January, 1991.
  - Two axis gimbal to be shipped from LMSC to MIT in late January, 1991.
  - Detailed dynamic model development completed by early February, 1991.
  - Realtime control capability available by mid February, 1991.
  - Testbed operational by end of February 1991.
- Flight experiment
  - Preliminary Design Review in early fall 1991.
  - Critical Design Review in early 1992
  - Launch in spring 1994.

MULTIBODY TESTBED
(RESEARCH ACTIVITIES)

- Influence of gravity and suspension effects on controlled structures.
- Command Input shaping for minimal excitation of residual flexible motion.
- Constrained feedback architecture optimal control.
- Large motion, nonlinear dynamics and control.
- Pointing and scanning control of rigid payloads mounted on a flexible sub-structure.

MATERIAL AND STRUCTURAL DAMPING FROM NANOSTRAIN TO MILLISTRAIN

Joseph Ting
Edward Crawley

RECENTLY, AN EFFORT HAS BEEN MADE TO DEVELOP LARGE, SPACE-BASED ASTRONOMICAL INSTRUMENTS SUCH AS THE HUBBLE SPACE TELESCOPE (HST) AND ITS POSSIBLE SUCCESSOR, A SPACE-BASED OPTICAL INTERFEROMETER. THE OPTICAL ELEMENTS OF THESE AND OTHER FUTURE OPTICAL INSTRUMENTS MUST BE HELD STABLE RELATIVE TO EACH OTHER WITHIN FRACTIONS OF A WAVELENGTH OF LIGHT. TO ACHIEVE THE RIGOROUS REQUIREMENTS FOR ALIGNMENT AND POSITIONING, THERE IS A NEED FOR PASSIVE DAMPING, PERHAPS SUPPLEMENTED BY ACTIVE CONTROL. IN EITHER A PASSIVE OR ACTIVE DESIGN, A KNOWLEDGE OF PASSIVE DAMPING ESTABLISHES LIMITS ON THE ACHIEVABLE PERFORMANCE. THE OPERATING WAVELENGTH AND PRECISION REQUIREMENTS OF THESE INSTRUMENTS IMPLIES THAT THE STRAIN AMPLITUDES AT WHICH THIS DAMPING IS IMPORTANT IS ON THE ORDER OF NANOSTRAIN TO MICROSTRAIN (IE. 10^{-9} TO 10^{-6}) -- ORDERS OF MAGNITUDE SMALLER THAN THE AMPLITUDE AT WHICH STRUCTURAL DYNAMIC CHARACTERIZATION COMMONLY OCCURS.

PREVIOUSLY, THERE WAS UNCERTAINTY IN THE AEROSPACE COMMUNITY ABOUT THE DAMPING OF STRUCTURES AT SUCH SMALL AMPLITUDES. ONE SCHOOL OF THOUGHT ARGUED THAT STRUCTURES WILL BEHAVE LINEARLY BY DESIGN AND THE PROPERTIES WILL BE CONSTANT WITH STRAIN LEVEL. THE OTHER SCHOOL OF THOUGHT ARGUED THAT THE MECHANISMS THAT NORMALLY PROVIDE STRUCTURAL DAMPING, ATOMIC AND THERMAL DIFFUSION, PLANE SLIPPAGE, JOINT FRICTION, ETC., WILL NO LONGER FUNCTION AT VERY SMALL STRAINS. THIS UNCERTAINTY LED TO AN EFFORT TO STUDY THE BEHAVIOR OF STRUCTURES AT NANOSTRAIN LEVELS OF MOTION.

BACKGROUND

- In a variety of optical instruments, the level of precision requires vibration levels in the 10 to 100 nanometer range
- For 10 meter baseline instruments, this implies strain in the 1 to 10 $\eta\varepsilon$ range
- Open loop jitter budgets or closed loop stability performance/robustness both require a knowledge of damping in this range

THE OBJECTIVE OF THE RESEARCH PRESENTED WAS TO CHARACTERIZE THE DAMPING OF MATERIALS AND STRUCTURES FROM NANOSTRAIN ($n\varepsilon$) LEVELS TO 1000 MICROSTRAIN ($\mu\varepsilon$), IE. A MILLISTRAIN ($m\varepsilon$). TO DO THIS, SPECIMENS OF TYPICAL SPACECRAFT MATERIALS AND PROTOTYPICAL SPACE STRUCTURAL TRUSS HARDWARE WERE TESTED. THE TEST RESULTS ARE USED TO DETERMINE WHETHER THERE ARE ANY CHANGES IN MATERIAL AND STRUCTURAL DAMPING OVER THIS STRAIN RANGE.

OBJECTIVE

- Characterize the damping of materials and a structure from nanostrain levels to hundreds of microstrain
- Identify the range over which damping is uniform, and the existance of any regions of sudden increase or decrease in damping

TO CHARACTERIZE DAMPING, SPECIMENS OF TYPICAL SPACECRAFT MATERIALS, ALUMINUM AND GRAPHITE/EPOXY, IN TYPICAL SPACE STRUCTURAL MEMBER CROSS-SECTIONS WERE TESTED. TO STUDY PURE MATERIAL DAMPING IN A SIMPLE GEOMETRY, RECTANGULAR CROSS SECTION BARS WERE CONSTRUCTED OF ALUMINUM AND GRAPHITE EPOXY. TO STUDY MATERIAL DAMPING IN A SLIGHTLY MORE COMPLEX FORM, TUBES OF ALUMINUM AND GRAPHITE/EPOXY WERE ALSO CONSTRUCTED. THE MATERIAL DAMPING SPECIMENS WERE FABRICATED WITH VARIOUS LENGTHS TO PERMIT THE STUDY OF DAMPING OVER A RANGE OF FREQUENCIES. STRUCTURAL DAMPING IN A BUILT-UP STRUCTURE WAS DETERMINED BY TESTING A TETRAHEDRAL SPACE TRUSS. THREE AND ONE-HALF METERS ON A SIDE, THE SIX ARM TETRAHEDRAL TRUSS WAS CONSTRUCTED FROM ALUMINUM BALL NODES AND ALUMINUM TUBES CONNECTED WITH TIGHT JOINTS. THE MATERIAL AND STRUCTURAL DAMPING SPECIMENS WERE TESTED FOR VALUES OF CRITICAL DAMPING RATIO OVER A RANGE OF STRAIN FROM 1 $n\varepsilon$ TO 1000 $\mu\varepsilon$ TO DETERMINE IF THERE WAS ANY DEPENDENCE OF DAMPING ON STRAIN LEVEL. THIS RANGE OF SPECIMENS ENCOMPASSES MANY OF THE PASSIVE LOSS MECHANISMS INHERENT IN STRUCTURES: THERMOELASTIC, DISLOCATION MOTION, VISCOELASTIC AND JOINT FRICTION. SINCE THESE SAME LOSS MECHANISMS UNDERLIE MANY PASSIVE DAMPING AUGMENTATION SCHEMES, THE MEASURED TRENDS OF DAMPING VS. STRAIN LEVEL CAN ALSO BE EXTENDED TO THESE DAMPING ENHANCEMENT SCHEMES.

Approach

- Test rectangular bar and cylindrical tube specimens of typical spacecraft materials to determine changes in material damping
    - Aluminum
    - Graphite/epoxy
- Test prototypical space structural truss hardware to determine changes in structural damping

Test Specimens

- Material damping specimens
  - Rectangular 6061-T6 aluminum bars
  - Rectangular $[0]_{24}$ AS4/3501-6 Gr/Ep bars
  - Rectangular $[\pm 15]_{6s}$ AS4/3501-6 Gr/Ep bars
  - 6061-T6 aluminum tubes
  - $[\pm 15]_{3s}$ AS4/3501-6 Gr/Ep tubes
- Interferometer testbed

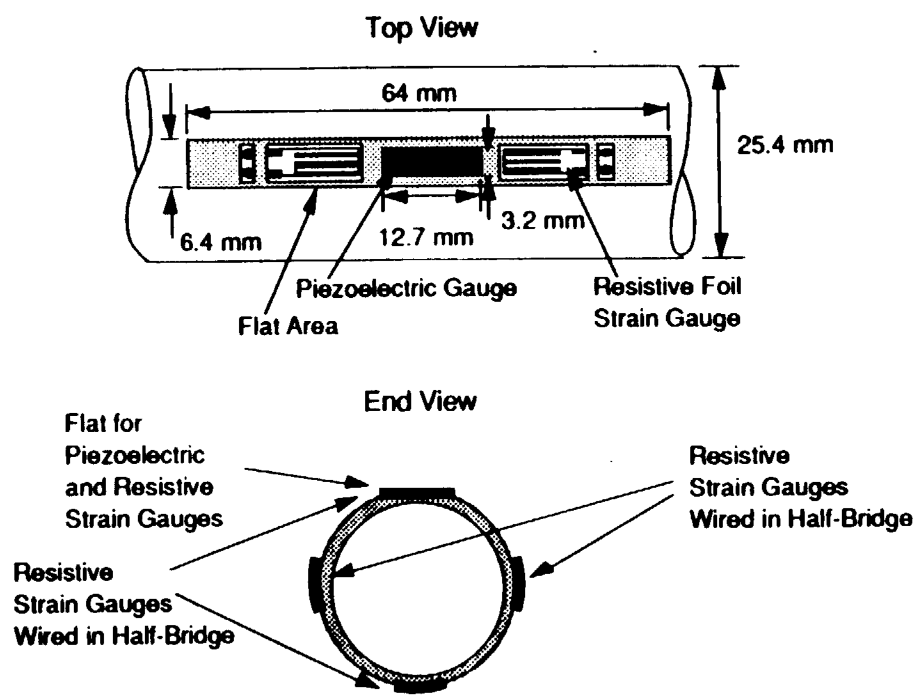
After instrumenting the material and structural damping specimens, tests were performed to obtain values of critical damping ratio. First, the material damping test specimens were suspended using wires and soft springs at the nodes of their free-free mode shapes. The structural damping test specimen were suspended using wires and soft springs at the vertices of the tetrahedral truss. Then, damping was measured using a sine sweep procedure. To reduce the effects of electrical and mechanical noise on the material damping measurements, tests for material damping were performed in the ASTROVAC, a metal vacuum chamber, 4.27 m high and 3.05 m in diameter. The ASTROVAC acted as a Faraday cage and acoustic isolator. To reduce mechanical noise, specimens were suspended using two soft spring suspension systems each consisting of three springs and three lead masses. The frequency of the highest mode of the system was 0.9 Hz with a mechanical isolation of 40 dB at the lowest specimen frequency tested, 6 Hz. To reduce the effects of air, measurements at large strain levels, from 10 $\mu\varepsilon$ to 1000 $\mu\varepsilon$, were performed in a vacuum of 10^{-2} Torr. Measurements at amplitudes below 10 $\mu\varepsilon$ were performed in air.

Test Procedure

- Material damping specimens suspended in ASTROVAC vacuum chamber using 3 mass 3 spring suspension system for mechanical noise isolation
- Material damping specimens suspended at nodes to minimize interaction with suspension
- Damping ratios obtained by performing sine sweep
- Tests performed in air and vacuum to determine effects of air damping

To sense and actuate vibrations in the material and structural damping test specimens, the specimens were instrumented with piezoceramic strain sensors and excited with piezoceramic proof mass actuators. These sensors and actuators were chosen because they were found to best meet the requirements defined by the objectives of the experiment: to be able to actuate strains and provide measurements from which strains could be inferred down to 1 nε with good signal to noise content. The strain sensors were small rectangular wafers manufactured from 0.25 mm thick sheets of a G-1195 piezoceramic material. Two different size sensors were applied to the test specimens of the two different geometries. The sensors used on the flat rectangular bar specimens were 6.4 mm wide by 12.7 mm long and were bonded directly to the surfaces of the bars using a cyanoacrylate adhesive. The sensors used on the tube specimens, as shown in the attached viewgraph, and also on the single, instrumented strut of the testbed were 3.2 mm wide by 12.7 mm long. These sensors were bonded to small flat areas milled into the surfaces of the tubes. All the piezoceramic sensors were calibrated in situ against resistive foil strain gages operating in the effective range of the foil strain gages (0.1 με to 100 με). For all the test specimens, the strain sensors were placed such as to allow the sensing of several mode shapes. Measured strain at these locations was then corrected using the spatial mode shapes to obtain values of maximum strain in the specimen.

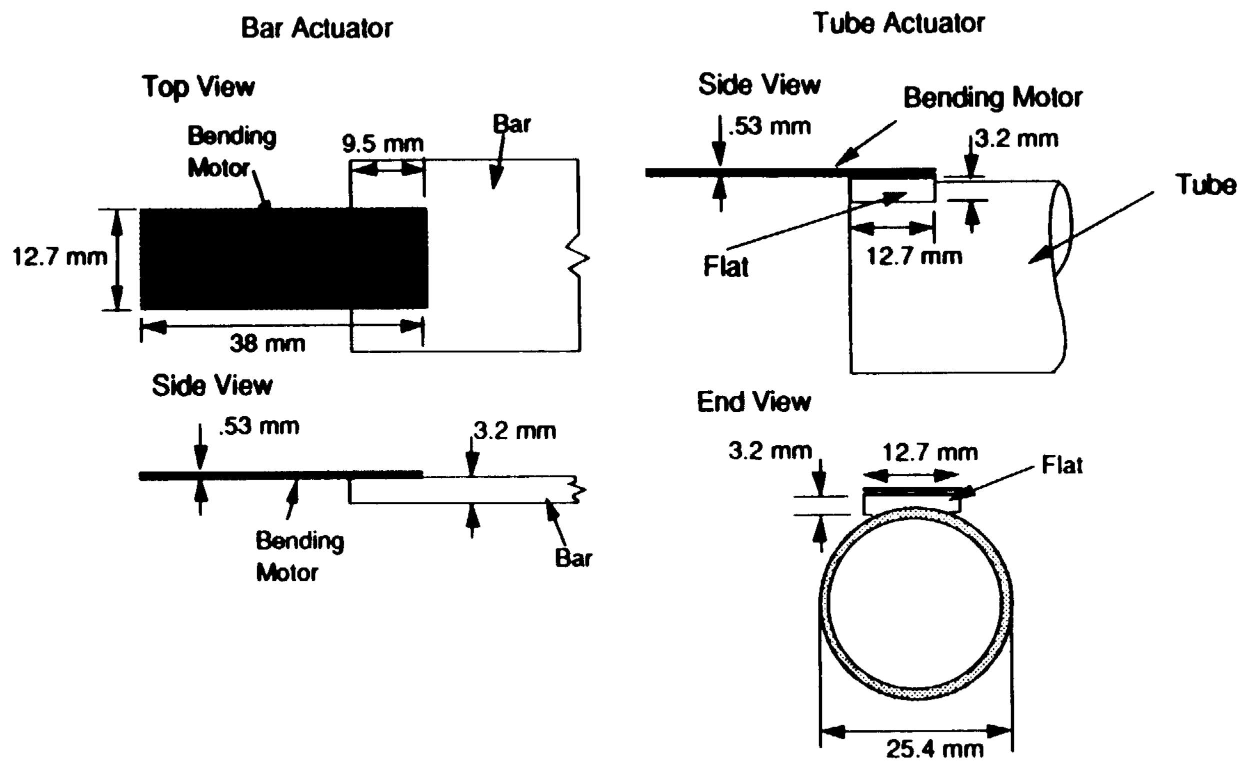
Instrumentation of Tubes



- Gauges bonded to flat milled into surface of tubes
- Smaller gauges used - 12.7 mm x 3.2 mm x 0.25 mm

THE SPECIMENS WERE ACTUATED WITH PIEZOCERAMIC PROOF MASS ACTUATORS AS SHOWN IN THE ATTACHED VIEWGRAPH. THE ACTUATORS, 12.7 MM X 38 MM X 0.53 MM RECTANGULAR PLATES, WERE CUT FROM SHEETS OF G-1195 PIEZOCERAMIC MATERIAL IN A BI-MORPH CONFIGURATION. IN A BI-MORPH CONFIGURATION, TWO OPPOSITELY POLARIZED SHEETS OF THE 0.25 MM THICK MATERIAL ARE BONDED TO EITHER SIDE OF A 0.03 MM THICK SHEET OF METALLIC SHIM SO THAT WHEN A VOLTAGE IS APPLIED ACROSS THE DEVICE, THE OPPOSING STRAINS CAUSE THE DEVICE TO BEND. THE BENDING MOTORS WERE CANTILEVERED FROM THE TIPS OF THE MATERIAL DAMPING TEST SPECIMENS. ON THE BAR SPECIMENS, THE ACTUATORS WERE BONDED DIRECTLY TO THE ENDS OF THE SPECIMENS, IE. IN AREAS OF LOW STRAIN. ON THE TUBE SPECIMENS, THE ACTUATORS WERE BONDED TO SMALL, RAISED FLATS MACHINED FROM ALUMINUM WHICH WERE BONDED TO THE ENDS OF THE TUBES. ON THE TESTBED, AN ACTUATOR WAS BONDED TO THE END OF A PLATFORM MADE OF A SMALL PIECE OF ALUMINUM BOLTED TO A NODE. IN ORDER TO EXCITE STRAINS ABOVE 10 $\mu\varepsilon$ IN THE MATERIAL DAMPING TEST SPECIMENS, SMALL WEIGHTS WERE ADDED TO THE TIPS OF THE ACTUATORS TO INCREASE THE PROOF MASS. FOR LARGE STRAIN TESTS OF THE STRUCTURAL DAMPING TEST SPECIMEN, THE INTERFEROMETER TRUSS, A DC MOTOR PROOF MASS ACTUATOR WAS USED.

Actuators



- Proof mass actuators manufactured from piezoelectric bending motors - 38.1 mm x 12.7 mm x 0.53 mm
- Actuators bonded directly to tips of material damping specimens using 3M 2215 adhesive

THE SOURCES OF IMPRECISION IN THE EXPERIMENT WERE THE RESOLUTION OF THE DIGITALLY CONTROLLED FREQUENCY GENERATOR AND THE SIGNAL NOISE, THE LATTER DOMINANT. THE PRECISION OF THE DAMPING MEASUREMENTS, WHICH CORRESPONDS TO THE RESULTING NOISE LEVEL, WAS CALCULATED FOR EACH TEST POINT. FOR STRAIN LEVELS NEAR 1 $n\varepsilon$, THE IMPRECISION DUE TO NOISE IS LARGE, BUT DIMINISHES RAPIDLY AT HIGHER STRAINS. FOR MOST TEST CASES. THE PRECISION LIMIT FROM THE NOISE IS LESS THAN $\pm 0.01 \times 10^{-3}$ IN ζ. TO FURTHER BOUND THE PRECISION, THE MINIMUM, MAXIMUM, AND AVERAGE OF THE SIX OR MORE SINE SWEEPS ARE SHOWN ON THE DATA FIGURES. THE DIFFERENCE BETWEEN THE MINIMUM AND MAXIMUM WAS GENERALLY LESS THAN 0.04×10^{-3} IN ζ. FROM THIS IT CAN BE INFERRED THAT THE PRECISION OF THE EXPERIMENT WAS ON THE ORDER OF $\pm 0.02 \times 10^{-3}$ IN ζ. TO REDUCE THE EFFECTS OF AIR, MEASUREMENTS AT LARGE STRAIN LEVELS, FROM 10 $\mu\varepsilon$ TO 1000 $\mu\varepsilon$, WERE PERFORMED IN A VACUUM OF 10^{-2} TORR. MEASUREMENTS AT SMALL STRAIN LEVELS, FROM 1 $n\varepsilon$ TO 10 $\mu\varepsilon$, WERE PERFORMED IN AIR AND WERE CORRECTED USING TWO MODELS OF AERODYNAMIC DAMPING. ONE MODEL WAS BASED ON THE FRICTION IN A VISCOUS BOUNDARY LAYER AROUND THE OSCILLATING SPECIMENS WHICH WAS INDEPENDENT OF AMPLITUDE AND ANOTHER MODEL WAS BASED ON THE DAMPING DUE TO QUASI-STEADY DRAG WHICH WAS DEPENDENT ON AMPLITUDE. RESULTS FROM TESTS PERFORMED IN VACUUM AND AIR SHOWED THAT THE MODEL BASED ON VISCOUS EFFECTS WAS THE MORE APPLICABLE MODEL UP TO THE MAXIMUM STRAIN LEVELS REACHED IN THIS STUDY. WHEN NOT TAKEN IN VACUUM, THE RESULTS IN THE DATA FIGURES ARE CORRECTED FOR AIR USING THIS UNSTEADY VISCOUS MODEL. THESE CORRECTIONS ARE ON THE ORDER OF 0.1×10^{-3} TO 0.01×10^{-3} IN ζ. THIS CORRECTION IS THOUGHT TO REMOVE MOST OF THE INACCURACY DUE TO AIR DRAG. THE TRANSMISSION LOSSES WERE MINIMIZED BY SUSPENDING THE TEST SPECIMENS AT THEIR NODES AND WERE ESTIMATED TO BE 0.2×10^{-3} IN ζ. THUS, THE EXPERIMENT IS EXPECTED TO HAVE AN ACCURACY OF 0.2×10^{-3} AND A PRECISION OF 0.02×10^{-3} IN ζ.

Precision and Accuracy ofResults

- Limitations on accuracy of results imposed by aerodynamic damping and suspension
  - Aerodynamic damping dominated by friction in the boundary layer - independent of amplitude
    - Results correlated by comparing results of model with tests in vacuum and air
    - Air damping dependent on geometry, but is small $\zeta < 1e\text{-}5$
    - Corrections made to results obtained in air
  - Suspension effects found to be significant
    - 0.9 m uniply bar suspended and resuspended
    - Around $\zeta = 4e\text{-}4$
- Limits on precision from electrical and mechanical noise
  - Effects small

THE FIRST TESTS WERE CONDUCTED ON BARS OF ALUMINUM IN ORDER TO DETERMINE THE PURE MATERIAL DAMPING IN A SPECIMEN WITH A SIMPLE GEOMETRY MADE OF A COMMON AEROSPACE MATERIAL WHICH HAS A WELL DOCUMENTED AND PREDICTABLE LEVEL OF DAMPING. RECTANGULAR BARS OF 6061-T6 ALUMINUM WERE CONSTRUCTED WITH A CROSS-SECTION 26 MM WIDE AND 3.2 MM THICK. THE ZENER THERMOELASTIC MODEL CALLS FOR A MAXIMUM DAMPING AT THE RELAXATION FREQUENCY, ω_r, GIVEN BY,

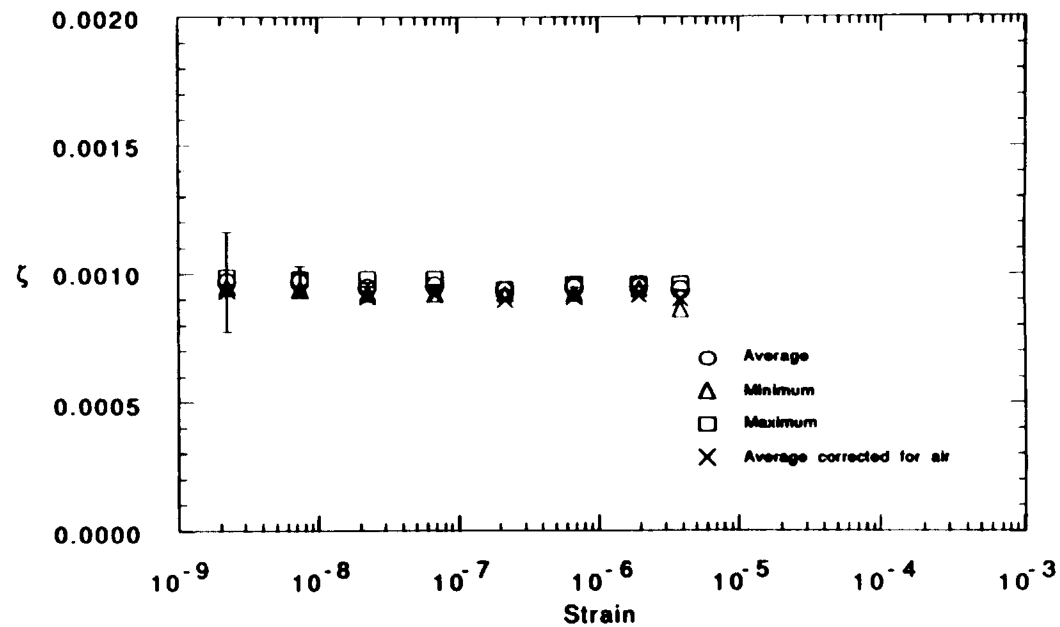
$$\omega_r = \frac{h^2 C \rho}{\pi^2 k}$$

WHERE H IS THE THICKNESS OF THE SPECIMEN, C IS THE SPECIFIC HEAT PER MASS, ρ, THE MATERIAL DENSITY, AND K THE THERMAL CONDUCTIVITY. FOR THE MATERIAL PROPERTIES AND DIMENSIONS OF THE ALUMINUM SPECIMENS, $\omega_r = 9.91$ Hz. THE BARS WERE MANUFACTURED WITH LENGTHS OF 0.5 M, 0.9 M, 1.3 M, AND 1.7 M TO GIVE A WIDE RANGE OF TEST FREQUENCIES WHICH BRACKETS THE ZENER RELAXATION FREQUENCY.

A TOTAL OF SEVEN MODES OF THE RECTANGULAR BARS WERE TESTED OVER THE LOWER STRAIN RANGE IN AIR FROM 1 nε TO 10 με. SHOWN IN THE ATTACHED VIEWGRAPH IS A TYPICAL PLOT OF DAMPING RATIO OBTAINED OVER THE SMALL STRAIN RANGE IN AIR AS A FUNCTION OF STRAIN FOR THE SECOND MODE OF THE 1.3 M ALUMINUM BAR AT 25.8 Hz. AS INDICATED IN THE ATTACHED VIEWGRAPH, DAMPING SHOWS LITTLE VARIATION WITH STRAIN AMPLITUDE BELOW 10 με, WITHIN THE BOUNDS OF THE PRECISION AND ACCURACY OF THE MEASUREMENTS.

ALUMINUM BAR-LOW STRAIN

- 3.2 mm thick bars tested in air for 6, 9, 17, 19, 25, 50, 173 Hz
- Results show low scatter even at low amplitude where noise indicates scatter possible
- Damping insensitive from 1 $\eta\varepsilon$ to 10 $\mu\varepsilon$

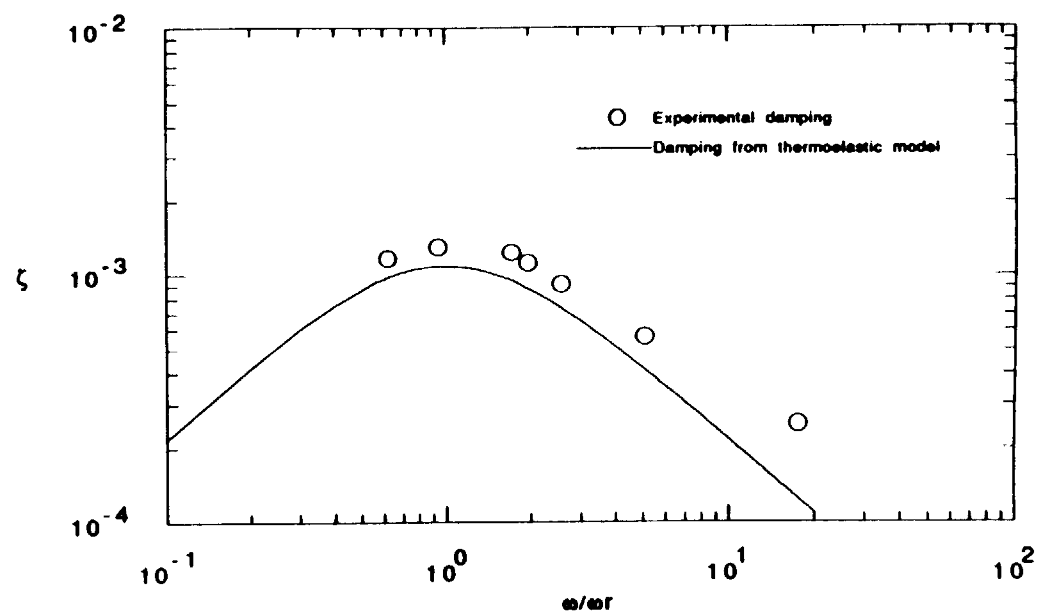


Damping in Second Mode of 1.3 m Al Bar in Air, f=25.8 Hz

THE RESULTS FROM THE ALUMINUM BARS AT SMALL STRAINS CAN BE COMPARED WITH PREDICTIONS FROM THE ZENER MODEL BASED ON THE THERMOELASTIC RELAXATION. THOSE PREDICTIONS ARE PLOTTED AGAINST FREQUENCY, NON-DIMENSIONALIZED USING THE RELAXATION FREQUENCY, ALONG WITH THE EXPERIMENTAL RESULTS IN THE ATTACHED VIEWGRAPH. AS SHOWN, THE EXPERIMENTAL RESULTS SHOW GOOD CORRELATION WITH THE THEORETICAL RESULTS. THE MEASURED DATA ARE CONSISTENTLY 0.1 TO 0.2 X 10^{-3} IN ζ ABOVE THE PREDICTED RESULT, WHICH IS CONSISTENT WITH THE EXPECTED ACCURACY OF THE EXPERIMENTAL PROCEDURE.

ALUMINUM BAR-ZENER CORRELATION

- Correlation with Zener thermomechanical model of transverse thermal flow in bar
- Low strain values of damping indicate excellent correlation
- Accuracy established to 1.2×10^{-4} in ξ

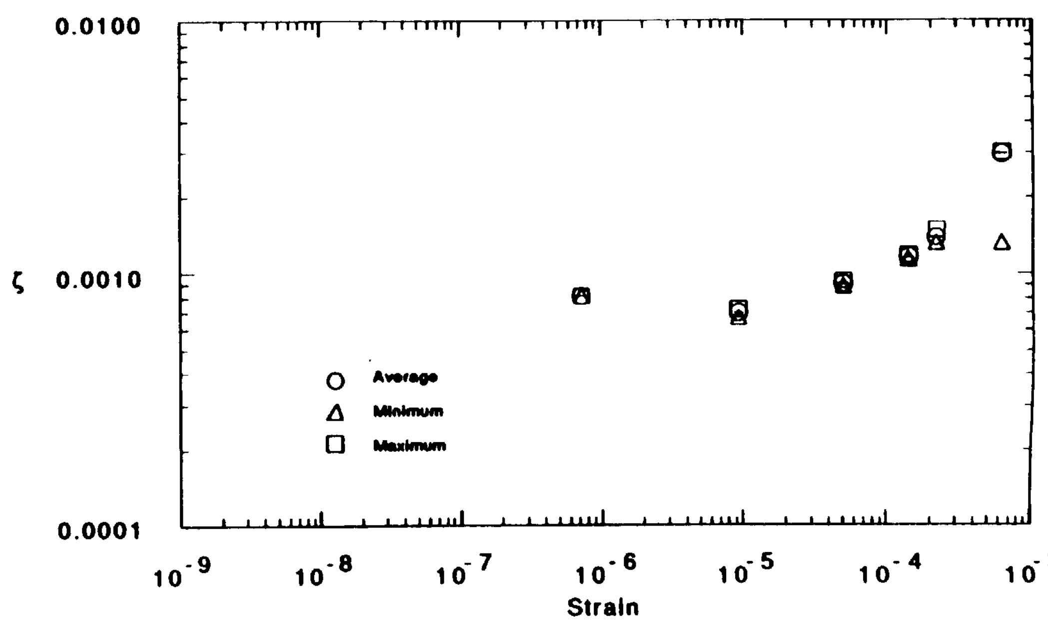


Theoretical and Experimental Damping. Ratios Plotted vs. Non-Dimensional Frequency

TESTS OVER THE RANGE OF LARGER STRAIN (1 TO 1000 $\mu\varepsilon$), WHICH WERE PERFORMED IN VACUUM, SHOW AN INCREASE IN DAMPING WITH STRAIN AMPLITUDE. AS SHOWN IN THE ATTACHED FIGURE, FOR THE TEST OF THE FIRST MODE OF THE 0.5 M ALUMINUM BAR, THE DAMPING SHOWS A DEFINITE INCREASE WITH INCREASING AMPLITUDE. THIS EFFECT, WHICH IS EVIDENT AT STRAINS ABOVE 10 $\mu\varepsilon$, CAN BE CORRELATED WITH A MODEL DEVELOPED BY BOSER FOR DAMPING AT LARGE STRAINS. THIS MODEL ASSUMESTHAT DAMPING AT LARGE STRAIN LEVELS IS DOMINATED BY FRICTION DUE TO THE MOTION OF DISLOCATIONS OCCURRING DURING PLASTIC DEFORMATION. THE FRICTION IS RELATED TO THE AMOUNT AND CONCENTRATION OF IMPURITIES IN THE MATERIAL, THE LATTICE STRUCTURE OF THE MATERIAL, AND THE STRAIN LEVELS IN THE STRUCTURE. APPLYING THIS MODEL TO THE DATA OBTAINED FROM THE DAMPING IN THE FIRST MODE OF THE 0.5 M ALUMINUM BAR TO OBTAIN A DISLOCATION DENSITY YIELDS A DENSITY OF 1.74×10^{6} CM^{-2}. THIS NUMBER IS WITHIN THE RANGE 10^{5} TO 10^{7}, WHICH IS TYPICAL OF METALS.

ALUMINUM BAR-HIGH STRAIN

- Tested in a vacuum
- Strain dependence becomes visible at 10 με
- Dependence approximately correlates with dislocation motion model of Boser
- Indicates nanostrain simulations should occur at strains less than 10 με

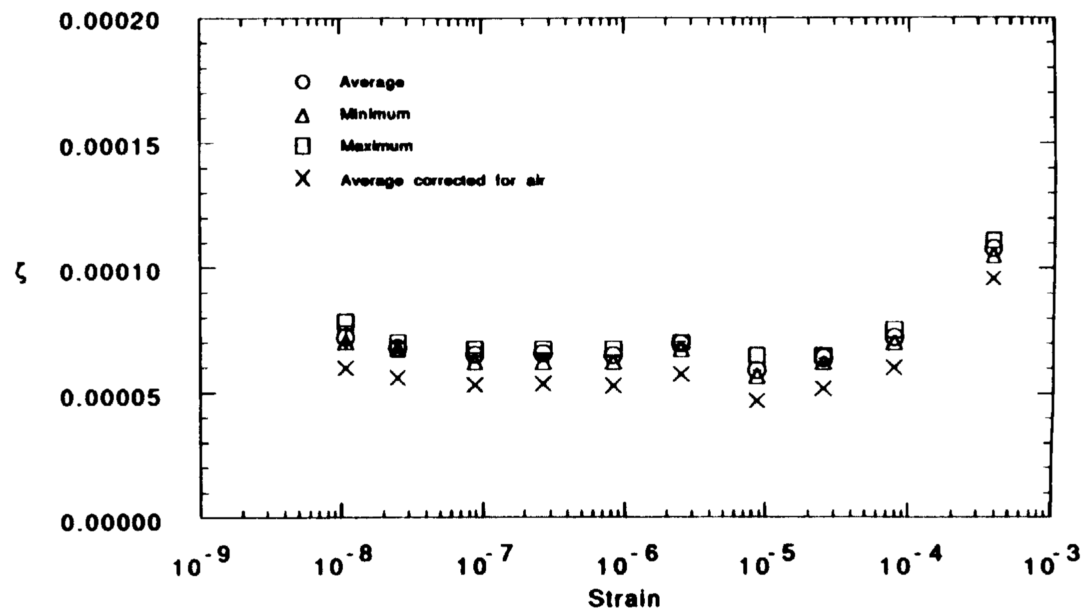


Damping in First Mode of 0.5 m Al Bar in Vacuum, f=58.1 Hz

TO OBTAIN VALUES OF MATERIAL DAMPING OF THE SAME MATERIAL IN A MORE TYPICAL GEOMETRY, TUBES WERE FABRICATED OF 6061-T6 ALUMINUM WITH A 26 MM OUTER DIAMETER AND A 1.7 MM WALL THICKNESS. THESE DIMENSIONS WERE CHOSEN TO GIVE THE SAME WIDTH AND TOTAL WALL THICKNESS AS THE BARS. THE TUBE SPECIMENS WERE CUT FROM THE SAME PIECE OF TUBING STOCK INTO 0.9 M AND 0.7 M LENGTHS. RESULTS FROM THE TESTS OF THE ALUMINUM TUBES SHOW LOWER AVERAGE VALUES OF DAMPING THAN THOSE OF THE BARS. AT SMALL STRAIN LEVELS, FROM 10 $n\varepsilon$ TO 100 $\mu\varepsilon$, DAMPING IS LARGELY INDEPENDENT OF STRAIN AMPLITUDE. THIS STRAIN INDEPENDENCE IS ILLUSTRATED IN THE ATTACHED VIEWGRAPH BY THE RESULTS FROM THE TESTS OF THE FIRST MODE OF THE 0.9 M ALUMINUM TUBE. AS SHOWN IN THE FIGURE IN THE VIEWGRAPH, FOR THE FIRST MODE OF THE 0.9 M ALUMINUM TUBE, DAMPING IS INDEPENDENT OF STRAIN UP TO A LEVEL OF 100 $\mu\varepsilon$. DUE TO THE MORE COMPLICATED GEOMETRY OF THE TUBE, THE THERMOELASTIC RELAXATION OF THE ZENER MODEL IS NOT AS PREVALENT, RESULTING IN SIGNIFICANTLY LOWER DAMPING. RESULTS FROM THE FIRST MODE OF THE 0.7 M ALUMINUM TUBE TESTED IN VACUUM OVER THE LARGER STRAIN LEVEL AGAIN SHOWS AN INCREASE IN DAMPING AT A STRAIN OF 10 $\mu\varepsilon$, YIELDING A DISLOCATION DENSITY OF 2.94×10^5 CM^{-2} WHICH AGAIN LIES IN THE TYPICAL RANGE FOR METALS.

ALUMINUM TUBES

- 25.4 mm OD, 1.6 mm wall tubes tested in air for 185, 308, 510, 830 Hz
- Measured values are below estimated accuracy but within precision of experiment
- Extremely low values are actually upper bound on possible aluminum tube damping in flexure
- No significant frequency dependence measured

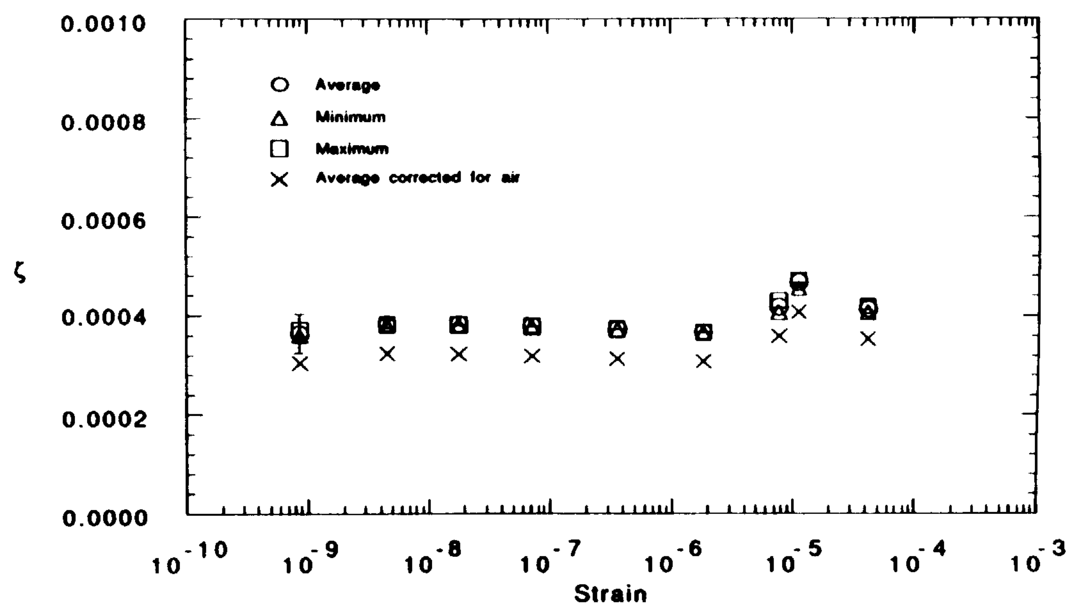


Damping in First Mode of 0.9 m Aluminum Tube in Air, f=185.1 Hz

ALTHOUGH ALUMINUM IS A COMMON MATERIAL WITH WELL DOCUMENTED DAMPING, ITS HIGH COEFFICIENT OF THERMAL EXPANSION RULES OUT ITS USE AS THE PRIMARY STRUCTURE OF A PRECISION SPACECRAFT. AS4/3501-6 GRAPHITE/EPOXY, A COMPOSITE MATERIAL COMMONLY USED IN THE AEROSPACE INDUSTRY, IS MORE TYPICAL OF MATERIALS TO BE USED IN THE CONSTRUCTION OF SPACE BASED ASTRONOMICAL STRUCTURES AND WAS THE SECOND MATERIAL STUDIED. THREE DIFFERENT TYPES OF SPECIMENS WERE CONSTRUCTED FROM GRAPHITE/EPOXY. TO TEST THE GRAPHITE/EPOXY MATERIAL IN A LAYUP WHICH MIGHT BE USED IN ASTRONOMICAL DEVICES, ONE SET OF THE 0.9 M AND 0.7 M LONG RECTANGULAR BARS WAS CONSTRUCTED WITH A $[\pm 15]_{6S}$ LAYUP WHICH GAVE AN EFFECTIVELY ZERO COEFFICIENT OF THERMAL EXPANSION. SIMILARLY, THE TUBES, 0.9 M AND 0.68 M LONG, WERE CONSTRUCTED WITH A $[\pm 15]_{3S}$ LAYUP. IN ORDER TO DETERMINE THE EFFECT OF LAYUP ON DAMPING, THE SECOND SET OF BARS, 0.9 M AND 0.7 M LONG, WERE CONSTRUCTED WITH A $[0]_{24}$, OR UNIPLY LAYUP. THE GRAPHITE/EPOXY BARS AND TUBE WERE BUILT WITH THE SAME CROSS SECTION AS THE ALUMINUM BARS AND TUBES. A TOTAL OF FOUR MODES OF THE TWO BARS WITH THE UNIDIRECTIONAL LAYUP WERE TESTED OVER THE SMALL STRAIN RANGE IN AIR. FOR ALL FOUR MODES THE DAMPING IS INDEPENDENT OF STRAIN AMPLITUDE. THIS BEHAVIOR IS ILLUSTRATED IN ATTACHED VIEWGRAPH IN THE PLOT OF DAMPING VS. STRAIN FOR THE SECOND MODE OF THE 0.9 M LONG UNIPLY BAR. THE RESULTS FROM THE TESTS OF THE UNIPLY BARS CAN BE COMPARED WITH THE MODEL OF DAMPING OF UNIPLY STRUCTURES DEVELOPED BY HASHIN AND ADAMS AND BACON. THIS MODEL PREDICTS THAT DAMPING OF UNIPLY LAMINATES ARE INDEPENDENT OF AMPLITUDE. USING THE PROPERTIES OF THE AS4/3501-6 MATERIAL, THESE MODELS PRODUCE AN ESTIMATED VALUE FOR ζ OF 0.19×10^{-3}. THIS VALUE, COMPARES WELL WITH THE EXPERIMENTAL RESULTS, SLIGHTLY UNDERPREDICTING THE ACTUAL DAMPING.

UNIDIRECTIONAL G/E BARS $[0]_{24}$

- 3.2 mm thick bars tested in air for 31, 57, 84, 157 Hz
- No damping dependence on strain below 10 $\mu\varepsilon$
- No dependence on frequency
- Low strain damping agrees moderately well with prediction of $\zeta=2x10^{-4}$ of Adams and Bacon



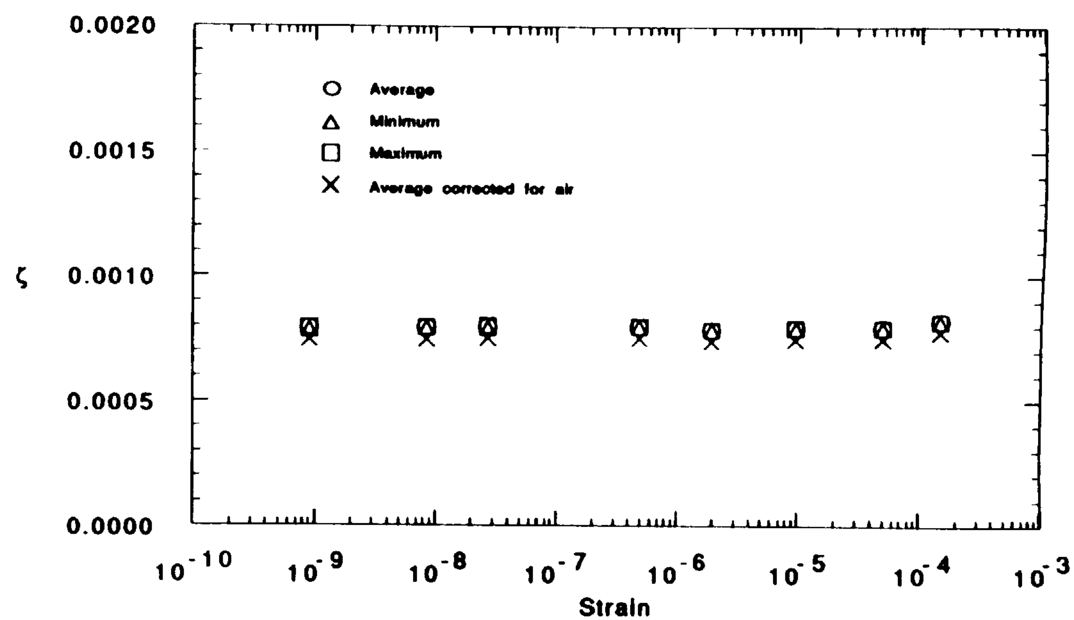
Damping in Second Mode of 0.9 m Uniply Bar in Air, f=83.5 Hz

A TOTAL OF FIVE MODES OF THE TWO BARS WITH THE $[\pm 15]_{6s}$ LAYUP WERE TESTED OVER THE SMALL STRAIN RANGE IN AIR. ALL FIVE MODES SHOWED LITTLE VARIATION IN DAMPING WITH AMPLITUDE. THIS BEHAVIOR IS ILLUSTRATED IN THE ATTACHED VIEWGRAPH WHICH SHOWS A PLOT OF DAMPING VS. STRAIN OVER A RANGE OF STRAIN FROM 0.899 $n\varepsilon$ TO 148 $\mu\varepsilon$ FOR THE SECOND MODE OF THE 0.7 M GR/EP BEAM WITH THE $[\pm 15]_{6s}$ LAYUP. WHEN COMBINED WITH THE RESULTS FROM TESTS CONDUCTED IN AIR AT HIGHER STRAINS, THE RESULTS SHOW LITTLE VARIATION IN STRAIN, OVER A RANGE OF STRAIN FROM 1 $\mu\varepsilon$ TO 1000 $\mu\varepsilon$.

THESE DAMPING RATIOS CAN BE COMPARED WITH THEORY DEVELOPED BY NI AND ADAMS FOR LAMINATES WITH SYMMETRIC LAYUPS. THIS MODEL, WHICH PREDICTS THAT DAMPING IS INDEPENDENT OF FREQUENCY AND DEPENDS ONLY ON THE MATERIAL AND GEOMETRIC PROPERTIES, GIVES AN ANALYTICAL VALUE OF ζ OF 0.84×10^{-3}. THISVALUE COMPARES WELL WITH THE EXPERIMENTAL RESULTS, SLIGHTLY OVERPREDICTING THE DAMPING FOR MOST CASES.

ZERO CTE G/E BARS $[\pm 15]_{6S}$

- 3.2 mm thick bars tested in air for 33, 53, 90, 145, 287 Hz
- No damping dependence on strain below 100 $\mu\varepsilon$
- Weak increase in damping with frequency above 150 Hz
- Low strain damping agrees with prediction of $\zeta = 8 \times 10^{-4}$ of Ni and Adams model



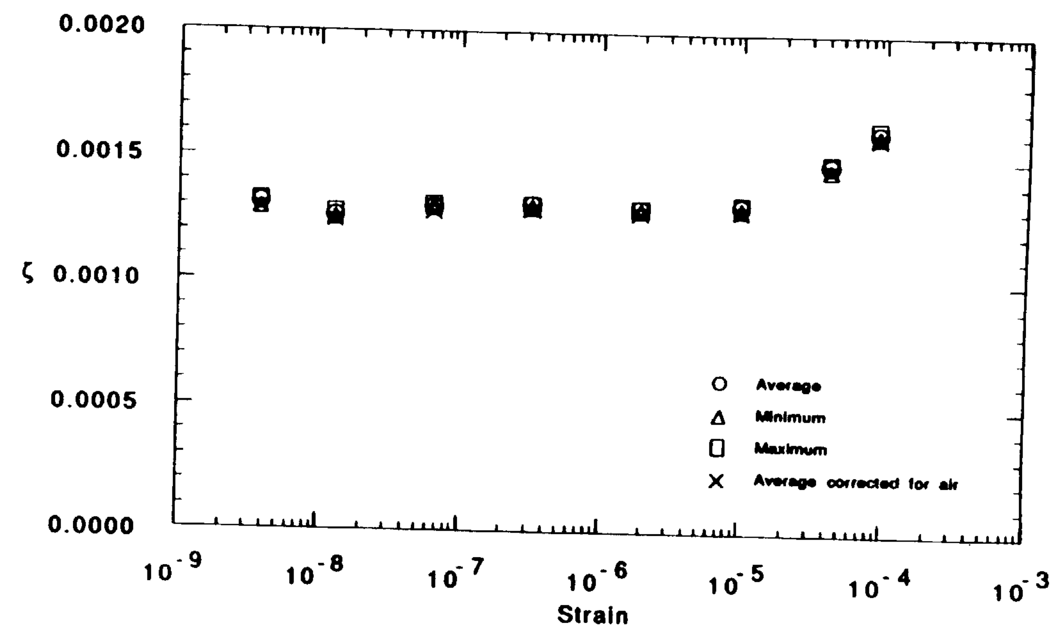
Damping in Second Mode of 0.7 m 0 CTE Gr/Ep Bar in Air, f=145.0 Hz

A TOTAL OF TWO MODES OF THE GRAPHITE/EPOXY TUBES WERE TESTED IN AIR AND ONE IN VACUUM. DAMPING SHOWS LITTLE VARIATION WITH AMPLITUDE IN THE SMALL STRAIN RANGE AS ILLUSTRATED IN THE ATTACHED VIEWGRAPH IN THE PLOT OF DAMPING VS. STRAIN IN THE FIRST MODE OF THE 0.7 M LONG TUBE. NO MODELS OF DAMPING OF LAMINATED TUBES COULD BE FOUND TO CORRELATE THE EXPERIMENTAL RESULTS.

UNLIKE THE ALUMINUM SPECIMENS, THE DAMPING OF THE GRAPHITE/EPOXY TUBES IS GENERALLY HIGHER THAN THAT OF THE BAR SPECIMENS WITH A SIMILAR LAYUP. IT IS PROBABLE THAT IN THE GRAPHITE/EPOXY TUBES AND BARS, A SIMILAR LOSS MECHANISM DOMINATES THE DAMPING, WHEREAS IN THE METAL SPECIMENS, DIFFERENT MECHANISMS DOMINATE THE DAMPING IN THE TUBES AND BARS.

ZERO CTE G/E TUBES $[\pm 15]_{3S}$

- 25 mm OD, 1.7 mm wall tubes tested in air at 275 and 503 Hz
- No damping dependence below 10 $\mu\varepsilon$
- Low strain value was somewhat above prediction of $\zeta = 8 \times 10^{-4}$ of Ni and Adams
- Second tube at 275 Hz gave about twice this damping level, but laminate cracking is suspected



Damping in First Mode of 0.7 m $[\pm 15]_{3s}$ Gr/Ep Tube in Air, f=503.0 Hz

To obtain values of structural damping, tests were performed on a model of an interferometric truss, part of a Controlled Structures Testbed developed at M.I.T. in the Space Engineering Research Center. The essential form of the testbed was a tetrahedral truss, 3.5 m on a side. The six legs of the truss were identical, consisting of 14 bays of truss, each 0.25 m long as shown in Figure 10. Because of the geometry of the truss layup, two different strut lengths were required: 0.25 m and 0.16 m. Each strut had an outer diameter of 9.5 mm or 3/8 inch and a wall thickness of 1.5 mm. Each node had a 30 mm diameter and had 18 10/32 threaded holes. Both the nodes and the struts were constructed of 6061-T6 aluminum.

Due to the symmetric geometry of the tetrahedral truss, the lowest global modes were clustered in two frequency ranges. The first bending modes, dominated by the first bending modes of the legs of the truss were clustered between 38 and 58 Hz and the second bending modes, dominated by second bending modes of the legs, were clustered between 94 and 195 Hz. The first bending mode of the longest struts was around 370 Hz.

To sense vibrations, the strut instrumented with the piezoceramic strain sensors was used to replace one of the normal members. The strut was placed in a bay to best sense as many modes as possible. To actuate vibrations, the piezoceramic proof mass actuator or D.C. motor driven proof mass actuator was attached to the node next to the instrumented strut.

Interferometer Testbed

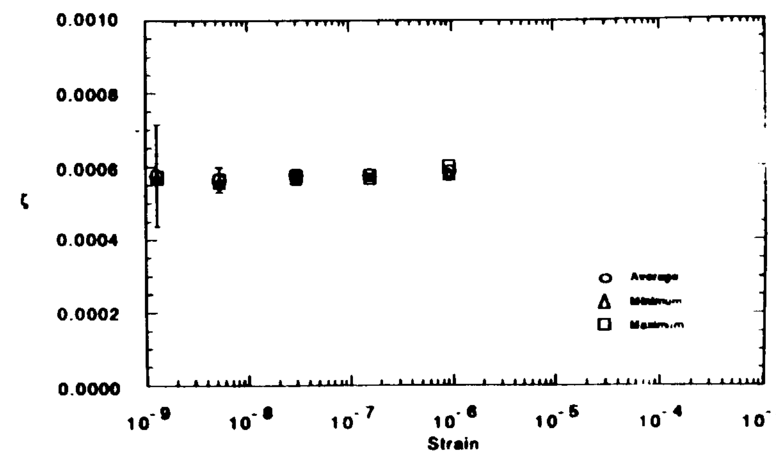
- Designed to model the behavior of a space based interferometer
- Designed to have tight, linear joints
- Essential form tetrahedral truss with six legs
  - 3.5 m on a side
  - 14 bays in a leg
  - Constructed with aluminum struts and joints
- Control bandwidth between 38 Hz and 195 Hz

LIKE THE MATERIAL TEST RESULTS, THE STRUCTURAL DAMPING SHOWS LITTLE VARIATION WITH STRAIN OVER THE SMALL STRAIN RANGE PERFORMED USING THE PIEZOCERAMIC ACTUATOR. THIS BEHAVIOR IS ILLUSTRATED IN THE ATTACHED VIEWGRAPH IN THE PLOT OF DAMPING VS. STRAIN OBTAINED FROM THE TESTS OF THE MODE OF THE TESTBED AT 44.1 HZ. AS SHOWN IN THE VIEWGRAPH, AT LARGER STRAIN LEVELS, USING THE PROOF MASS ACTUATOR FOR THE SAME MODE, THE DAMPING INCREASES WITH AMPLITUDE. IN ADDITION, THE MASS OF THE D.C. MOTOR CAUSES THE FREQUENCY TO DECREASE SLIGHTLY FROM 44.1 HZ TO 43.9 HZ.

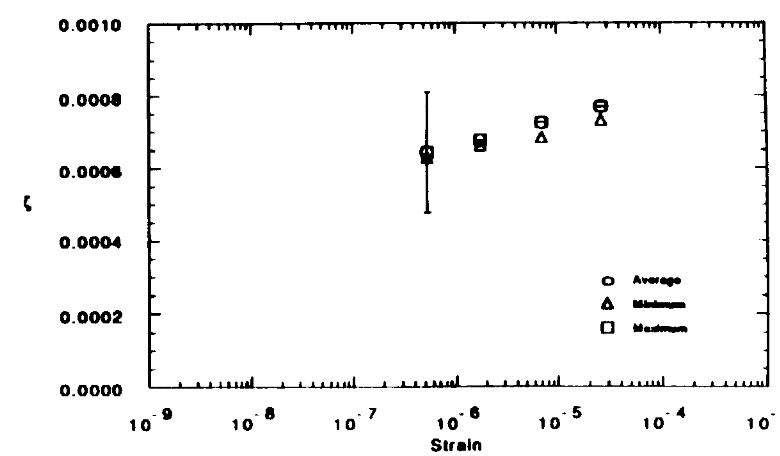
IN SUMMARY, THE DAMPING IN THE TETRAHEDRAL TESTBED, LIKE THE DAMPING IN THE MATERIAL DAMPING SPECIMENS, IS INDEPENDENT OF STRAIN BELOW A CERTAIN LEVEL. HOWEVER, THIS LIMIT UNDER WHICH THE DAMPING IS INDEPENDENT OF STRAIN AMPLITUDE IS SLIGHTLY LOWER THAN THE LIMIT FOR THE MATERIAL DAMPING SPECIMENS: 1 $\mu\varepsilon$ AS COMPARED TO 10 $\mu\varepsilon$. AT STRAIN LEVELS HIGHER THAN 1 $\mu\varepsilon$, DAMPING INCREASES WITH STRAIN.

INTERFEROMETER TESTBED DAMPING

- Tested in air at 40, 44, 55 Hz
- No dependence on strain below 10 με
- No dependence on mode tested with "family" of 12 first modes
- Since damping is 6 to 8 times that measured for an aluminum tube, damping originates in connections
- Remarkably similar to damping measured by Fanson at VPL for similar truss construction



Damping in Mode #1 of Testbed, f=44.1 Hz, Small Strain Range



Damping in Mode #1 in Testbed, f=43.9 Hz, Large Strain Range

RESULTS FROM BOTH TESTS PERFORMED TO CHARACTERIZE THE BEHAVIOR OF MATERIAL AND STRUCTURAL DAMPING, SHOW THAT THE BEHAVIOR OF DAMPING CAN BE DIVIDED INTO TWO DISTICT REGIONS. THE BOUNDS ON THESE REGIONS DEPEND ON THE GEOMETRY OF THE TEST SPECIMENS. IN THE REGION OF SMALL STRAIN, THE DAMPING IS INDEPENDENT OF STRAIN AND IS CONSTANT DOWN TO 1 $n\varepsilon$. IN THE REGION OF LARGE STRAIN, DAMPING INCREASES WITH STRAIN. THE RATE OF DAMPING INCREASE WITH STRAIN DEPENDS ON BOTH THE MATERIAL AND GEOMETRY OF THE TEST SPECIMENS.

DAMPING OF THE ALUMINUM BARS AND TUBES SHOWS LITTLE VARIATION WITH STRAIN LEVEL IN THE SMALL STRAIN REGION, FROM 1 $n\varepsilon$ TO ABOUT 10 $\mu\varepsilon$. THIS ASYMPTOTIC LOWER LIMIT ON DAMPING FOR SMALL STRAIN CAN BE PREDICTED FOR TRANSVERSE VIBRATION OF THE ALUMINUM BARS WITH THE THERMOELASTIC MODEL DEVELOPED BY ZENER. FOR TRANSVERSE VIBRATION OF THE ALUMINUM TUBES, THE DAMPING IS MORE THAN AN ORDER OF MAGNITUDE LOWER AND IS NOT CORRELATED WITH A THERMOELASTIC MODEL. THIS IMPLIES THAT THE DAMPING OF THE ALUMINUM TUBES HAS A DIFFERENT ORIGIN THAN THAT OF THE BARS. AT LARGER STRAIN LEVELS, ABOVE 10 $\mu\varepsilon$, THE DAMPING OF BOTH THE BARS AND TUBES CAN BE MODELED USING THE DISLOCATION MODEL DEVELOPED BY BOSER.

THE RESULTS FROM THE MATERIAL DAMPING TESTS OF THE GRAPHITE/EPOXY BARS AND TUBES SHOW THAT THE DAMPING OF GRAPHITE/EPOXY LAMINATES ALSO HAS AN ASYMPTOTIC LOWER LIMIT IN THE SMALL STRAIN REGION, BELOW 10 $\mu\varepsilon$. FOR LAMINATES WITH UNIPLY LAYUPS, AND SIMPLE, RECTANGULAR

CONCLUSIONS

- For the materials tested, damping
  - was independent of strain from 1 $\eta\varepsilon$ to 10 $\mu\varepsilon$
  - rose in almost all cases above 10-100 $\mu\varepsilon$
  - followed predicted frequency trends
  - agreed reasonably well with predicted magnitude
  - was measured with an accuracy of $1\text{-}2\times10^{-4}$ and precision of 2×10^{-5}
- For the precision interferometer truss tested, damping
  - was independent of strain from 1 $\eta\varepsilon$ to 10 $\mu\varepsilon$
  - rose gradually above 10 $\mu\varepsilon$
  - is attributed to joints

GEOMETRIES, THIS LIMIT CAN PREDICTED WITH MODELS DEVELOPED BY ADAMS AND BACON. THESE MODELS RELATE THE DAMPING OF A UNIPLY LAMINATE TO THE PROPERTIES OF THE MATRIX. FOR MORE COMPLICATED LAMINATES, THIS LIMIT CAN BE PREDICTED WITH A MODEL DEVELOPED BY NI AND ADAMS. A CHANGE IN GEOMETRY TO A CYLINDRICAL CROSS SECTION APPEARS TO HAVE ONLY A SMALL EFFECT ON DAMPING. THIS IMPLIES THAT, UNLIKE THE ALUMINUM SPECIMENS, THE SAME LOSS MECHANISM IS PRESENT IN BOTH THE GRAPHITE/EPOXY BARS AND TUBES. THIS DAMPING SHOWS A WEAKER DEPENDENCE ON STRAIN THAN THE DAMPING OF THE ALUMINUM SPECIMENS IN THE LARGE STRAIN REGION, ABOVE 10 $\mu\varepsilon$.

STRUCTURAL DAMPING, MEASURED FROM THE PRECISION INTERFEROMETER TRUSS, ALSO SHOWS AN ASYMPTOTIC LOWER LIMIT IN THE SMALL STRAIN REGION, MEASURED USING THE PIEZOELECTRIC ACTUATOR, AT LEAST TO 1 $n\varepsilon$. IN THE LARGE STRAIN REGION, ABOVE 1 $\mu\varepsilon$, THE DAMPING OBTAINED USING THE PROOF MASS ACTUATOR, APPEARS TO DEPEND ON STRAIN LEVEL. THE LOW STRAIN DAMPING RATIOS OBTAINED FROM THE TESTS OF THREE MODES OF THE TESTBED ARE AN ORDER OF MAGNITUDE GREATER THAN THE DAMPING RATIOS OF THE ALUMINUM TUBE DAMPING SPECIMENS. SINCE THE TESTBED IS CONSTRUCTED OF TUBES MADE FROM THE SAME 6061-T6 ALUMINUM AS THE ALUMINUM TUBE DAMPING SPECIMENS, THIS RESULT IMPLIES THAT THE PRIMARY SOURCE OF THE STRUCTURAL DAMPING OF THE TESTBED IS IN THE JOINTS AND ATTACHMENTS.

THE VIRTUES OF PASSIVE DAMPING IN CONTROLLED STRUCTURES

A. von Flotow
D. Vos
L. Sievers
K. Scribner
J. Garcia

Fall 1990

THIS SLIDE PRESENTS A PARODY OF A CLASSIC PROBLEM STATEMENT OF CONTROL OF LARGE SPACE STRUCTURES. THE SLIDE IS MEANT TO MAKE THE POINT THAT THIS PROBLEM STATEMENT IS SUSPECT.

The Virtues of Passive Damping

" the control bandwidth must include many

closely spaced

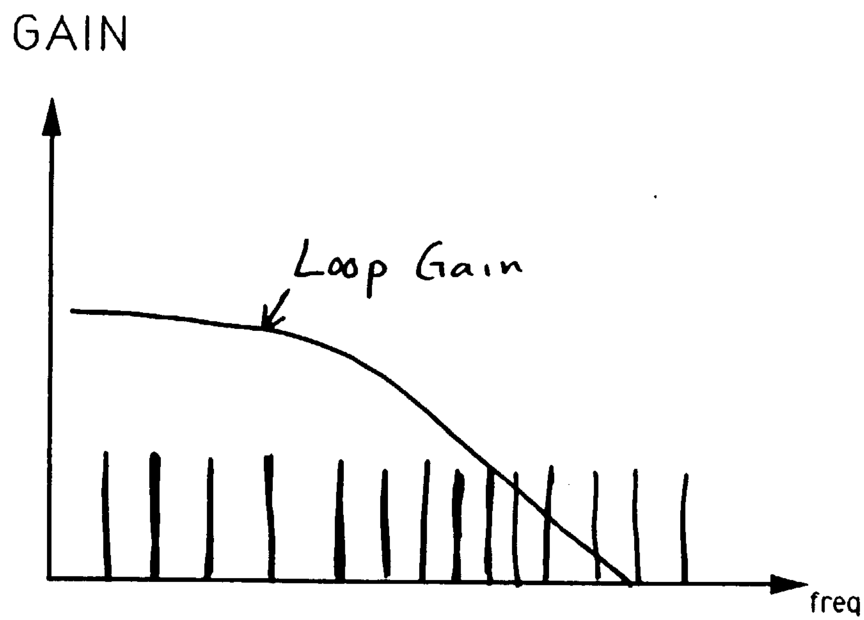
poorly modeled

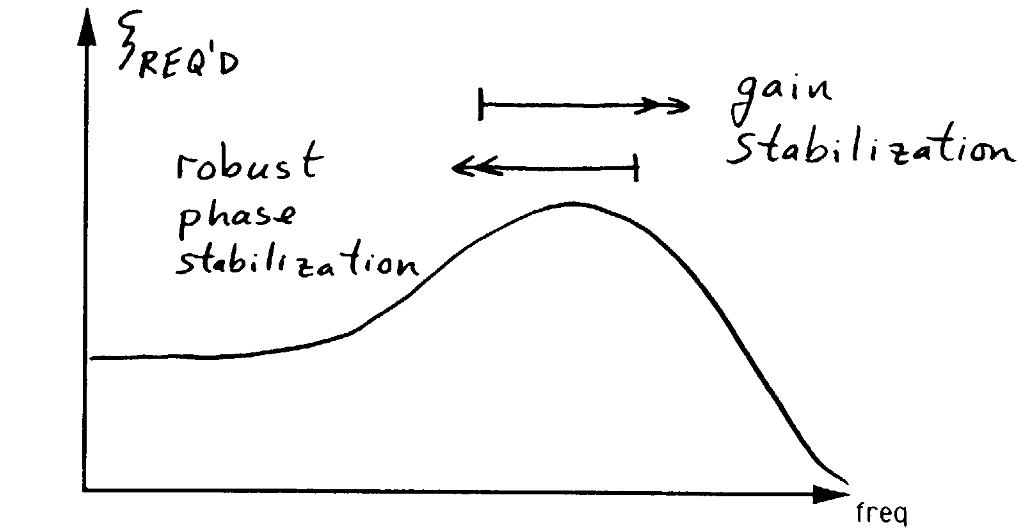
lightly damped

resonant modes of the flexible structure"

THIS SLIDE PRESENTS EMPIRICALLY THE CLAIM THAT A MINIMUM LEVEL OF PASSIVE DAMPING IS REQUIRED TO ACHIEVE ROBUST CONTROL OF STRUCTURAL DYNAMICS.

The Virtues of Passive Damping





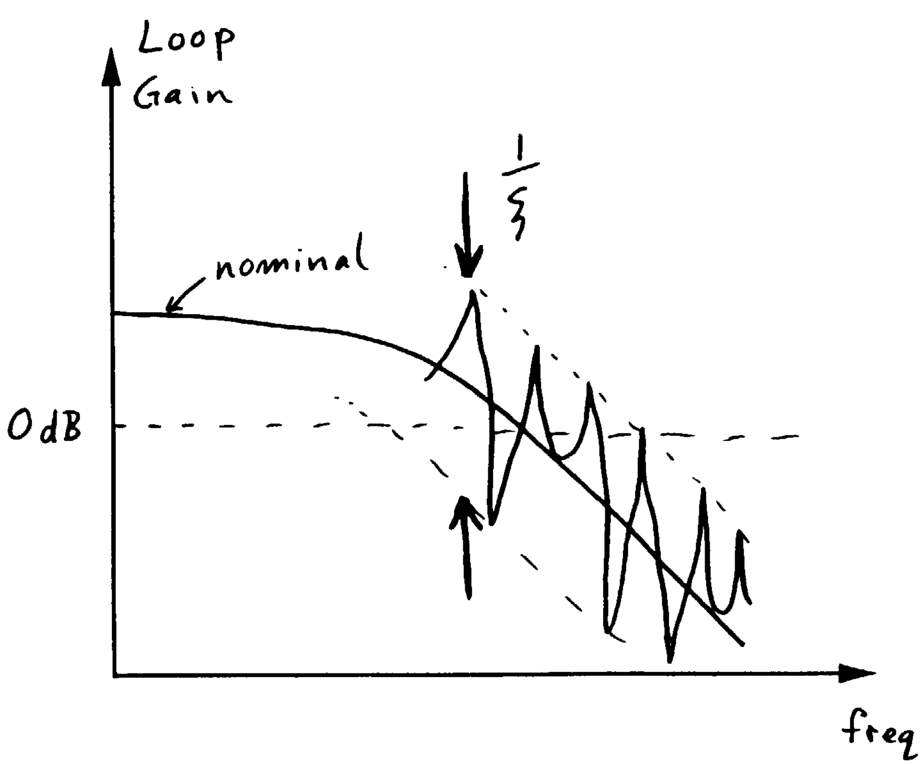
- GAIN STABILIZATION OUTSIDE THE BANDWIDTH

- PHASE STABILIZATION WITHIN THE BANDWIDTH

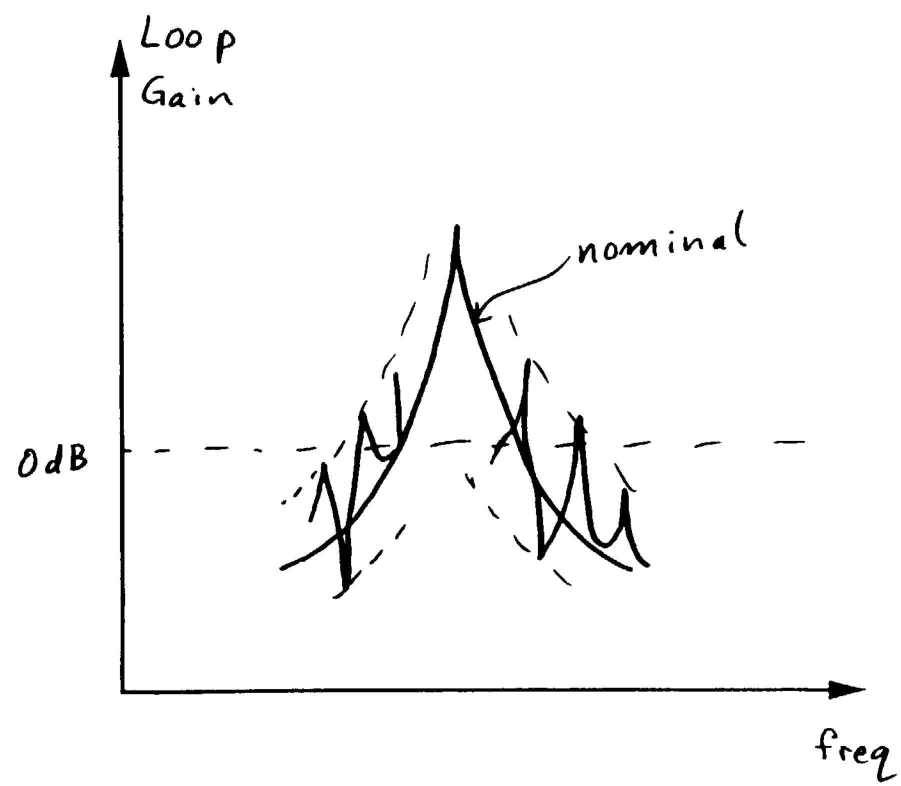
THIS SLIDE EXPLAINS THE VIRTUES OF PASSIVE DAMPING IN TERMS OF GAIN STABILIZATION.

Gain Stabilization Beyond Bandwidth

Broadband

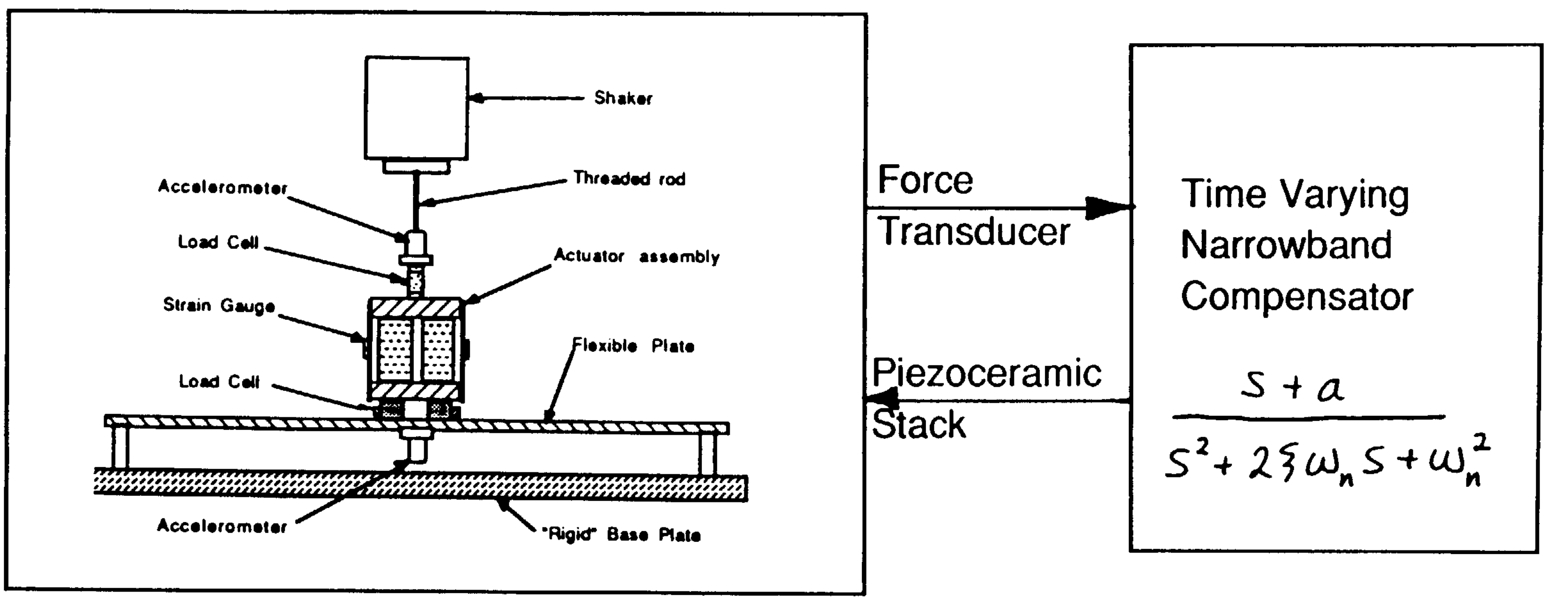


Narrowband



Gain Stabilization Beyond Bandwidth

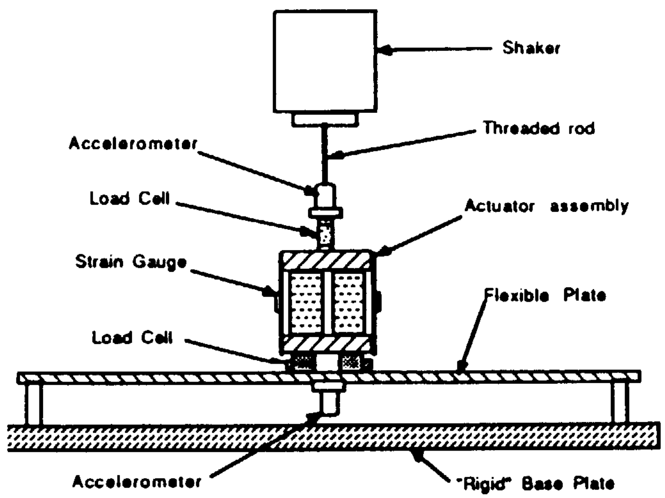
Narrowband Vibration Isolation



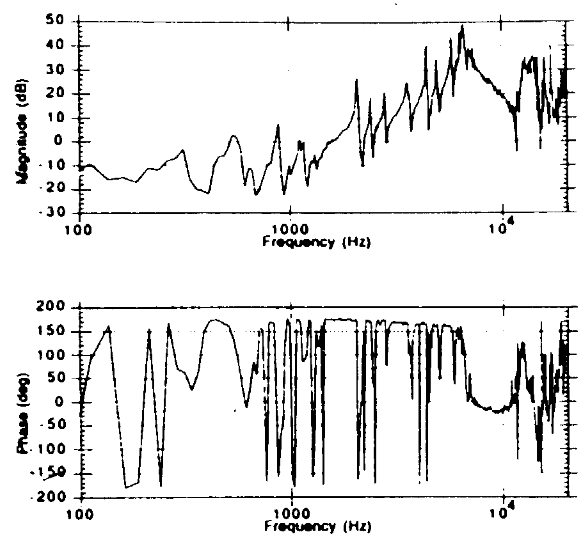
$\omega_n = \omega_{dist}$ (time-varying)

$\zeta = \zeta_{plant}$ (best)

Narrowband Vibration Isolation

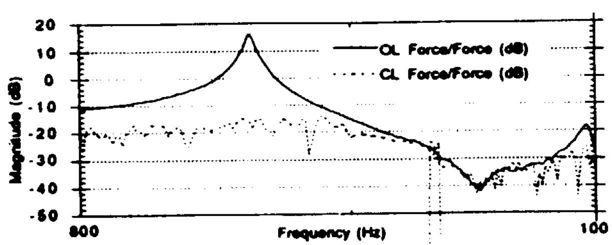


Apparatus



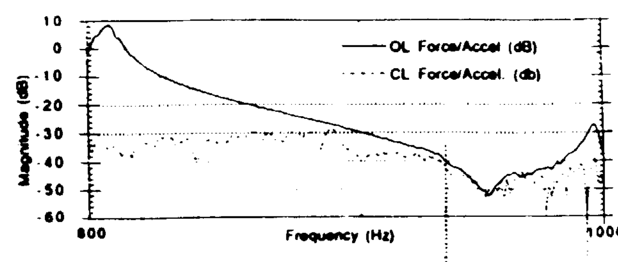
Measured Plant Transfer Function

Active Isolation Performance:
(Time Varying Sinusoidal Disturbance)



Light Machine

⟼ swept in 1 second ⟶



Massive Machine

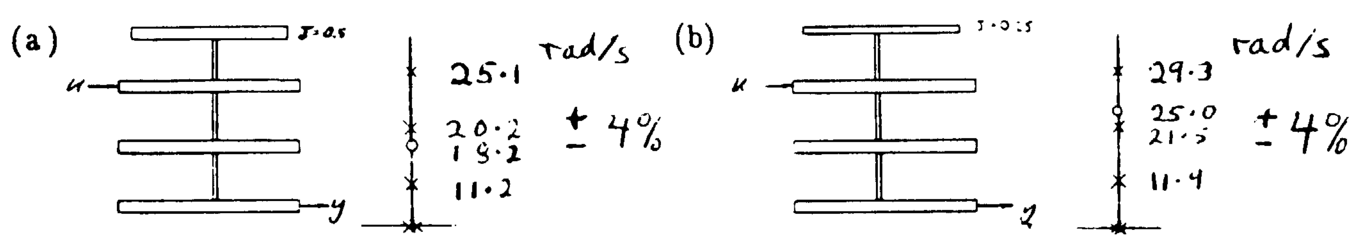
⟼ swept in 1 second ⟶

THIS SLIDE ARGUES THAT STRUCTURAL DYNAMIC UNCERTAINTY WILL LEAD TO UNCERTAINTY IN PLANT PHASE OF MANY RADIANS, PARTICULARLY IN NON-COLLOCATED TRANSFER FUNCTIONS.

Phase Stabilisation Within Bandwidth

In lightly damped plants the phase uncertainty can be large and of unknown sign:

Rosenthal PhD (4 Disks, 4 Modes, ζ=0.004)



4–Disk example by Rosenthal.

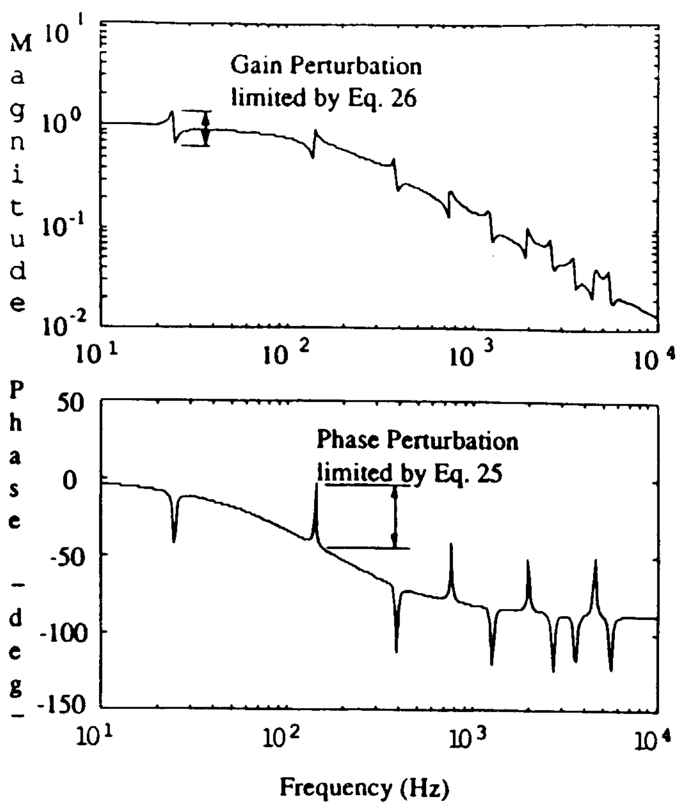
- phase uncertaintly near second mode of +/- 180 degrees if $\zeta=0$
- highest bandwidth achieved approximately 6 rad/s

THIS SLIDE QUANTIFIES PHASE UNCERTAINTY DUE TO APPROXIMATE POLE-ZERO CANCELLATION.

Phase Stabilization Within Bandwidth

- High Bandwidth Control Implies Approximate Plant Inversion

- Gain and Phase Excursions due to near pole-zero cancellation:



Multiplicative gain excursions:

$$\delta G \doteq \sqrt{1 + \left(\frac{1}{\xi} \frac{p-z}{p} \right)^2}$$

Phase Excursion:

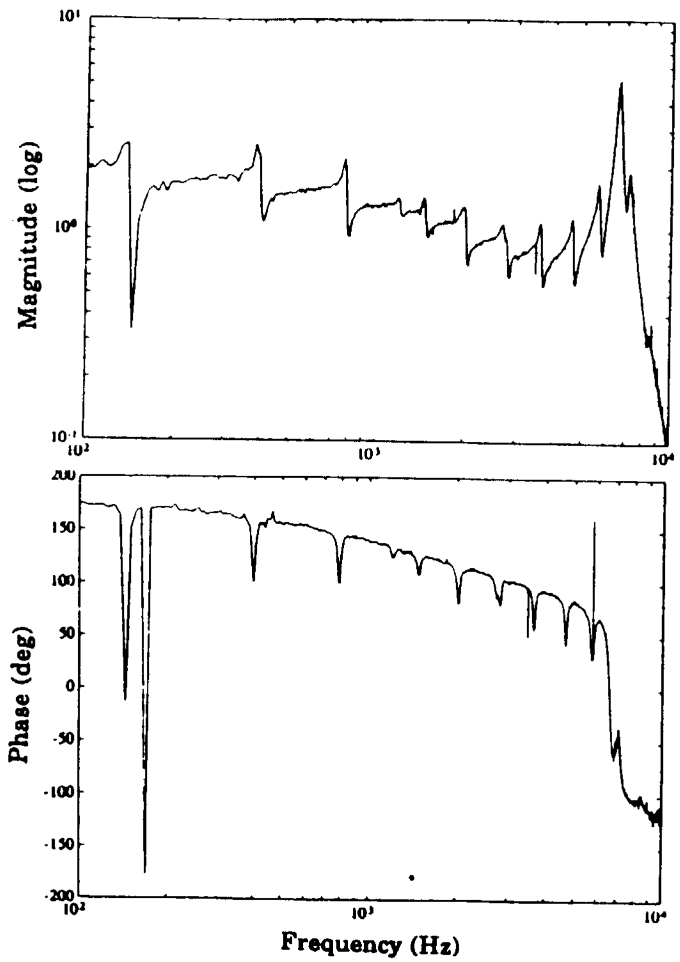
$$\delta \phi \doteq 2 \tan^{-1} \left(\frac{1}{2\xi} \frac{p-z}{p} \right)$$

$$\doteq \frac{1}{\xi} \frac{p-z}{p}$$

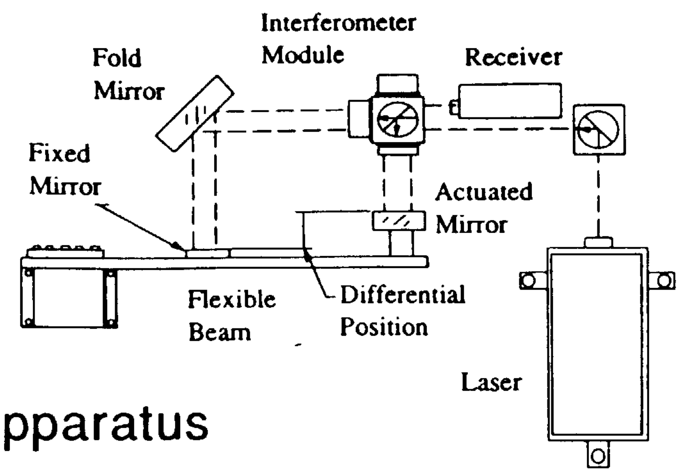
THIS SLIDE EXPLAINS A SIMPLE MIRROR-POSITIONING CONTROL EXPERIMENT ENABLED BY PASSIVE DAMPING AND APPROXIMATE POLE-ZERO CANCELLATION.

Phase Stabilization Within Bandwidth

Positioning of Small Mirrors on a Flexible Structure



Plant Transfer Function



Apparatus

Gain Excursion (multiplicative):

$$\delta G = \sqrt{1 + \frac{1}{4\xi_i^2}\left(\frac{m_m}{\bar{m}_i} - \frac{m_m}{\hat{m}_i}\right)^2}$$

Phase Excursion:

$$\delta\phi = -2\tan^{-1}\left[\frac{1}{4\xi_i}\left(\frac{m_m}{\bar{m}_i} - \frac{m_m}{\hat{m}_i}\right)\right]$$

m_m=mirror mass

$\bar{m}_i, \hat{m}_i$ = effective modal mass

THIS SLIDE SUMMARIZES THE LEVEL OF REQUIRED PASSIVE DAMPING IN THE BANDWIDTH.

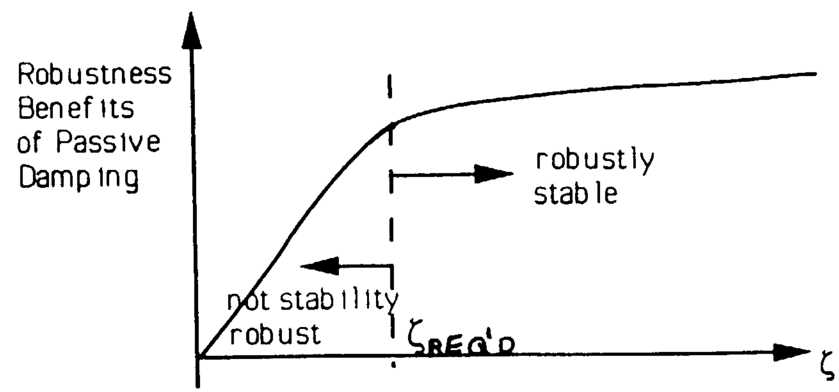
Phase Stabilization Within Bandwidth

How much ζ is sufficient for robust plant inversion????

$\zeta_{REQ'D} = (1/\delta\phi) * (\delta\omega/\omega)$

$\delta\phi$ is nominal phase margin

$\delta\omega/\omega$ is relative uncertainty in pole, zero location

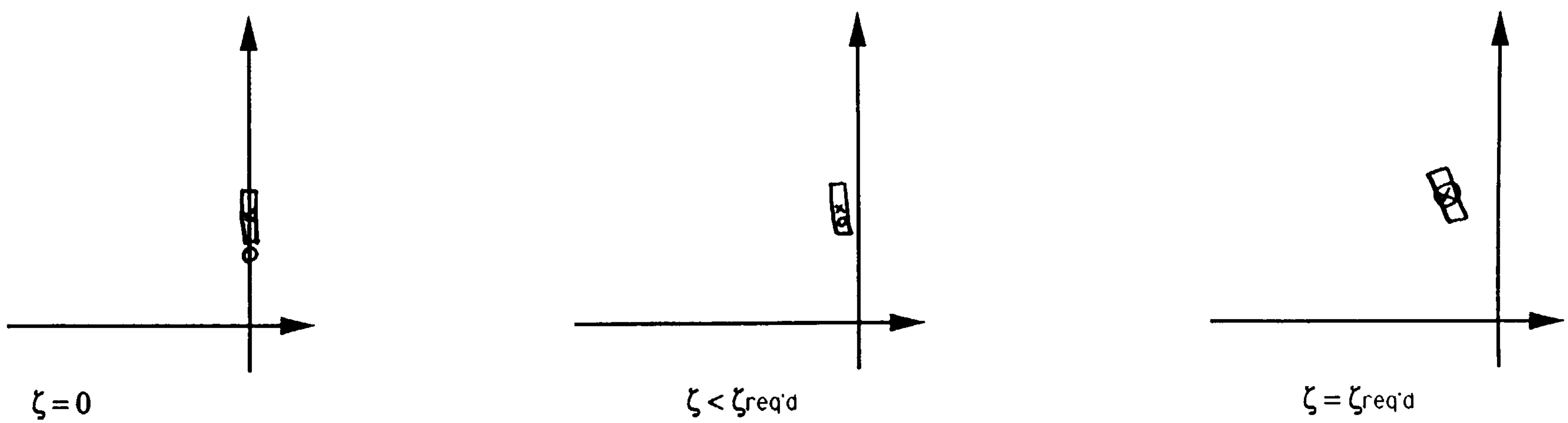


$\zeta_{REQ'D}$:
- 1 to 4 percent for well identified laboratory structures
- much greater for uncertain structures

THIS SLIDE MAKES SOME HEURISTIC EXTENSIONS TO MIMO PLANTS.

Phase Stabilization Within Bandwidth

If $\zeta < \zeta_{REQ'D}$, can sacrifice performance for stability



For $\zeta < \zeta_{REQ'D}$ will have oscillatory pole within bandwidth

The Virtues of Passive Damping

Inferences for MIMO

$$\vec{Y}(s) = \left[H_{ij}(s) \right] \vec{U}(s)$$

- Each Hij(s) is subject to large phase uncertainty near uncertain poles, zeros
- "Directionality" of plant is highly uncertain at <u>each</u> uncertain zero of <u>each</u> Hij(s)
- MIMO is likely to be "worse" than SISO

The Virtues of Passive Damping

summary

- Passive damping limits the effect (in gain and phase) of mis-modeled structural dynamics

- A critical level of passive damping is needed in controlled structures (in the bandwidth):

    - 1 to 4 percent for well identified laboratory structures

    - much more for uncertain structures

- Adaptive control (not LTI) may void these conclusions

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Research
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A Systems Engineering Approach to Disturbance Minimization for Spacecraft Utilizing Controlled Structures Technology

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PRESENTATION SUMMARIZES THESIS WRITTEN BY CHRIS EYERMAN. THE THESIS INVESTIGATES THE SYSTEMS RESPONSIBILITIES IN THE DESIGN OF A CST SPACECRAFT TO MINIMIZE DISTURBANCES, AND PROVIDES A BASIS FOR FURTHER SYSTEM STUDIES IN THIS AREA.

SHOWN ARE THE MAJOR SECTIONS OF THE THESIS. THE FIRST THREE SECTIONS IDENTIFY A PROCESS BY WHICH A SYSTEMS ENGINEER CAN APPROACH THE DISTURBANCE MINIMIZATION PROBLEM, AND THE LAST TWO SECTIONS APPLY THIS FRAMEWORK TO THE CONCEPTUAL DESIGN OF AN OPTICAL INTERFEROMETER SPACECRAFT AND SOME OF ITS SUBSYSTEMS.

Overview

- Spacecraft Disturbances
- CST Tools
- System Approach to Disturbance Minimization
- Space-Based Interferometer: OPTICS
- Subsystem Design:

  Power Subsystem
  Attitude Control Subsystem
  ...

THE FIRST SECTION OF THE THESIS IDENTIFIES THE DISTURBANCES RELEVANT TO THIS CLASS OF SPACECRAFT. THE DISTURBANCES ARE CHARACTERIZED ACCORDING TO MAGNITUDE, FREQUENCY, AND OPERATIONAL CHARACTERISTICS. A SIMPLE NUMERICAL MODEL FOR EACH IS PROVIDED.

THE DISTURBANCES ARE CATEGORIZED INTO FREQUENCY GROUPINGS RELATIVE TO THE STRUCTURAL MODES OF THE SPACECRAFT BUS AS SHOWN:

I: EXPONENTIAL (VERY LOW FREQUENCY) DUE MOSTLY TO THERMAL AND ENVIRONMENTAL DISTURBANCES.

II: LOW FREQUENCY IS JUST BELOW THE PRIMARY MODES OF THE STRUCTURE.

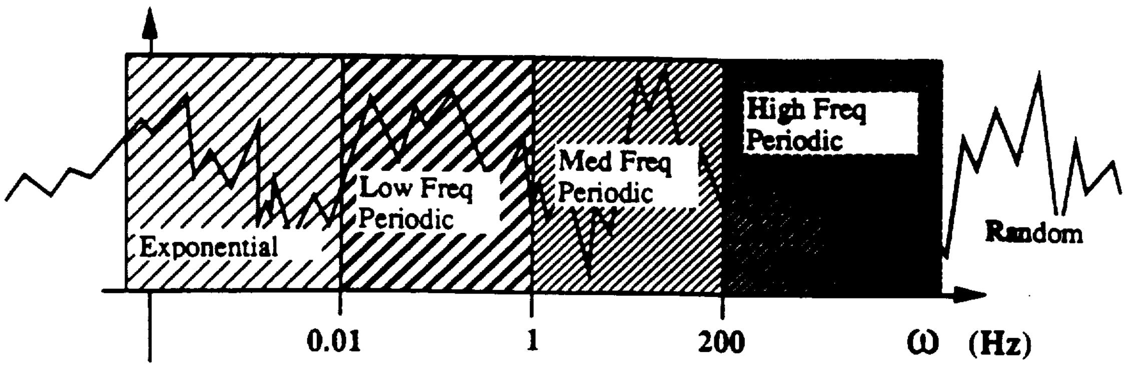
III: MEDIUM FREQUENCY COVERS THE RANGE OF THE PRIMARY STRUCTURAL MODES.

IV: HIGH FREQUENCY IS FAR BEYOND THE PRIMARY MODES AND APPEARS AS GENERAL BACKGROUND NOISE.

V: RANDOM IS MOSTLY BROADBAND OR NON-PERIODIC.

DISTURBANCES SHOULD BE WELL UNDERSTOOD BEFORE DESIGNING THE SYSTEM SINCE THEY USUALLY DETERMINE WHICH CST TOOLS ARE MOST APPLICABLE TO THE PROBLEM.

Spacecraft Disturbances



I. Exponential (E): $10^{-6} \Rightarrow 10^{-2}$ Hz
   A. Environmental disturbances (GGT, Aero, Rad'n, Mag'nt)
   B. Gross thermal-mechanical deformations

II. Low Frequency Periodic (L): $\sim 10^{-2} \Rightarrow 1$ Hz
   A. Flexi-body interaction torques (damping), and momentum exchange with flexible appendages and movable elements; vehicle fundamental modes
   B. Internal thermal-mechanical fluctuations: active cycling of electric heaters; cyclic operation of dissipating devices
   C. Fliud slosh: propellant, cryogenics
   D. Low frequency electromechanical

III. Med. Frequency Periodic (M): ~1 => 200 Hz
   A. Electromechanical devices: RWA/CMG's, data recording devices, servomechanisms, pumps, displacement actuators, active mounts,etc.
   B. 'Stiffer' flexi-body interactions
   C. Higher harmonics of fluid slosh

IV. High Frequency Periodic (H): ~200 => + Hz
   A. Higher harmonics of electromechanical devices
   B. Electrical noise: AC current

V. Random (R):
   A. Fluid turbulence: broadband
   B. Friction
   C. Sensor / electrical noise (photon, thermal, shot): broadband
   D. Micrometeoroid impact: very low freq
   E. Non-periodic gimballing (thrusters, sensors, antennae): low freq
   F. Thruster firing - transients: low-high freq

THESE ARE THE TOOLS AVAILABLE TO THE DESIGN ENGINEER IN ATTACKING THE DISTURBANCE MINIMIZATION PROBLEM. THE FIRST FOUR CATEGORIES ARE TOOLS THAT ARE AVAILABLE TO ANY CST PROBLEM. THE SPECIFIC APPLICATION OF A SPACE-BASED INTERFEROMETER ADDS THE LAST TO CATEGORIES TO THE CST TOOLBOX.

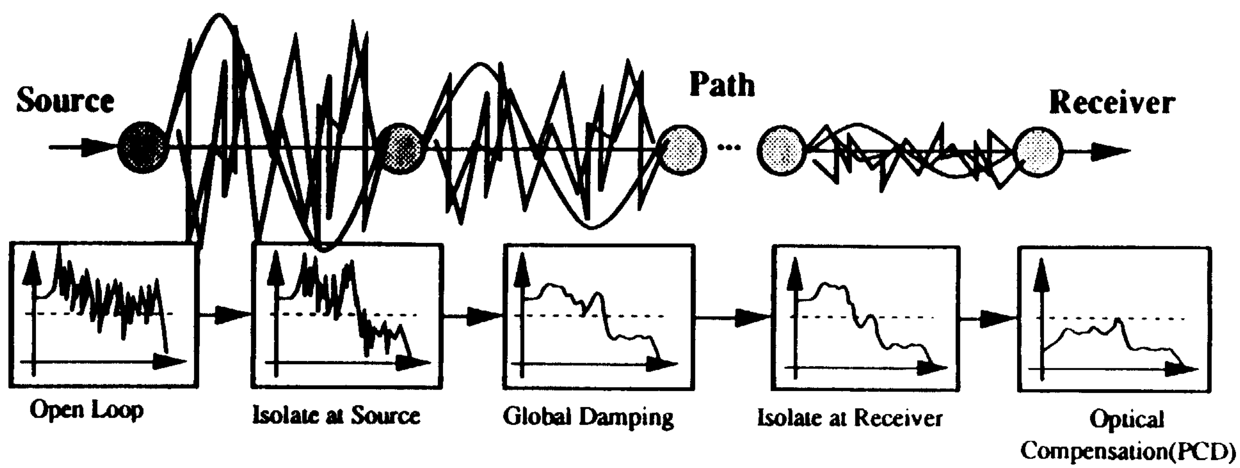
CST Tools

- Passive Structural Techniques:
  - Structural Design
  - Thermal Control Techniques
  - ``Zero'' CTE Structures
  - Structural Tailoring (Jacques)
- Passive Damping:
  - Material Damping
  - Structural Damping: bonded/mechanical interfaces
  - Space Viscoelastics
  - Friction Dampers
  - Vibration Absorbers: resonant / viscous dampers
  - Shunted Piezoelectrics (Hagood)
- Vibration Isolation:
  - Passive Isolation: soft/tuned mounts
  - Active Isolation (Schribner, Stampleman,...)
- Active Structural Control: (Control Weenies,...)
  - Active Damping
  - Shape Control
- Optical Compensation Techniques:
  - Active Optical Elements, Delay Lines,...
  - Image Reconstruction Techniques,...
- Attitude Control System

THIS SECTION OUTLINES A FRAMEWORK FOR APPLYING THE CST TOOLS TO ACHIEVE THE DISTURBANCE MINIMIZATION GOALS GIVEN THE RELEVANT DISTURBANCE SOURCES.

THE INPUT-OUTPUT RELATIONSHIP OF DISTURBANCE SOURCE TO OPTICAL COMPONENT IMMEDIATELY SUGGESTS THE APPROACH SHOWN. THE PROBLEM IS FIRST ATTACKED AT THE SOURCE THROUGH SELECTION OF INHERENTLY QUIET COMPONENTS AND ISOLATION. NEXT, THE PATH FROM SOURCE TO RECEIVER CAN BE ATTACKED WITH BOTH PASSIVE AND ACTIVE TECHNIQUES, AND, FINALLY, THE RECEIVER CAN BE ISOLATED OR LOCALLY DAMPED. FOR THE INTERFEROMETER, THE LAST RESORT IS SOME SORT OF PATH-LENGTH OPTICAL COMPENSATION DEVICE.

System Approach to Disturbance Minimization



Source	Path	Receiver
Quiet the Source	Passive Structural Techniques	Vibration Isolation
Local Damping	Global Thermal Design	Local Damping
Vibration Isolation	Global Damping	Disturbance Compensation:
Command Shaping	Structural Control	Optical Compensation
	Attitude Control	Quasi-static Correction

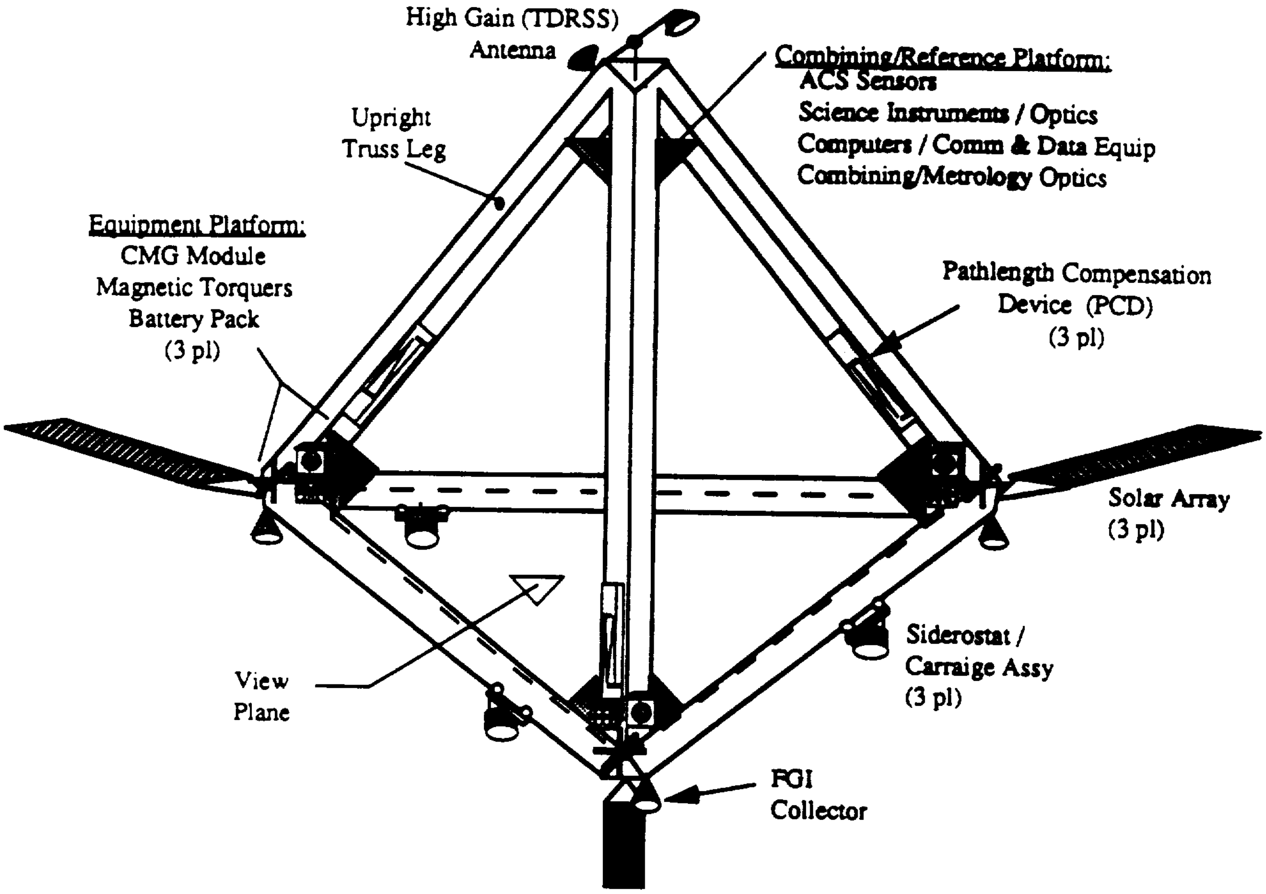
THE SECOND HALF OF THE THESIS DEMONSTRATES THE APPLICATION OF THE PRECEDING FRAMEWORK TO THE CONCEPTUAL DESIGN OF AN INTERFEROMETER SPACECRAFT BASED ON A 35 METER TETRAHEDRAL TRUSS STRUCTURE. A NUMERICAL MODEL OF THE STRUCTURE AND ITS RESPONSE TO ARBITRARY DISTURBANCES WAS USED IN THE DESIGN. ALTHOUGH THE MODEL WAS RELATIVELY SIMPLE, AND CERTAINLY NOT COMPLETE, IT IS ADEQUATE FOR A FIRST ORDER ANALYSIS. THE PICTURE SHOWS THE RESULT OF THIS DESIGN.

TWO IMPORTANT NOTES:

THE SPACECRAFT IS AS INERTIALLY SYMMETRIC AS POSSIBLE SO AS TO MINIMIZE THE EFFECTS OF MOST ENVIRONMENTAL DISTURBANCES.

HIGHLY DISTRIBUTED DISTURBANCE SOURCES AND OPTICAL COMPONENT RECEIVERS LEADS TO A VERY HIGHLY MIMO SYSTEM.

Space-Based Interferometer: OPTICS



- Illustrates Need for <u>Total</u> System Approach:

  Distributed Disturbances and Sensitive Elements (Optics)

  ACS; Science / Guidance Interferometer

  Internal / External Laser Metrology

  CST Tools; Optical Compens.; Image Reconstruction;...

  •••

- Numerical Model

THIS SUMMARIZES THE TWO MAJOR DESIGN OPTIONS FOR THE POWER SUBSYSTEM: PHOTOVOLTAIC AND RTG. AFTER COMPARING THE IMPORTANT PARAMETERS OF SUBSYSTEM MASS, PATH-LENGTH JITTER, AND CST COMPLEXITY, IT IS FAIRLY OBVIOUS THAT THE PHOTOVOLTAIC OPTION IS SUPERIOR.

Power Subsystem Design Summary

Parameter	*Photovoltaic System*	*RTG System*
1.Primary Elements (Qty):	Solar Arrays (3), Batteries (3)	RTG Generators (12), Radiators (3)
2.Total Subsystem Mass:	528 kg	900 kg
3.Total Array/Radiator Area:	116 m^2	70 m^2
4.Array/Radiator Bending Fundamental Frequency:	0.1 - 1.0 Hz	> 10 Hz
5.Total System Thermal Power:	8.3 kW_{th}	41 kW_{th}
6.Net Conversion Efficiency(η_{NET}):	0.38	0.11
7.Disturbance Types:	L,M,H - Transient Env't - Steady-State	R - Steady-State Env't - Steady State
8.On During Science:	NO	YES
9.Max Pathlength Jitter: (nanometers) Peak: RMS: Settling Time:	43 n-m NA 10 nm, 10 sec, 10% damp.	700-70,000 n-m 14-1440 n-m NA
10.Recommended CST Tools:	PCD Control: 1 Hz Array Damping Ratio: 10% max	PCD Control: 10 Hz Structural Damping Ratio: 1% Isolation: 4 Hz
11.Mass of CST System:	md	Md+Mi
12.CST Complexity:	Low	Low-Med

CHARTS SHOW FIRST-ORDER UTILITY OF GENERAL CST TECHNIQUES TO MINIMIZE MAXIMUM RTG DISTURBANCES (BAND-LIMITED WHITE NOISE DUE TO COOLANT FLOW). DAMPING, OPTICAL COMPENSATION, AND ISOLATION MAY REPRESENT ANY OF THE AVAILABLE CST TOOLS OF THESE TYPES (PASSIVE OR ACTIVE).

CHARTS SHOW THAT SINGLE TOOLS ALONE ARE NOT ADEQUATE TO ACHIEVE THE PERFORMANCE SPECIFICATION OF 10 NM RMS.

Power Subsystem Design

- CST Tool Implementation: (RTG)

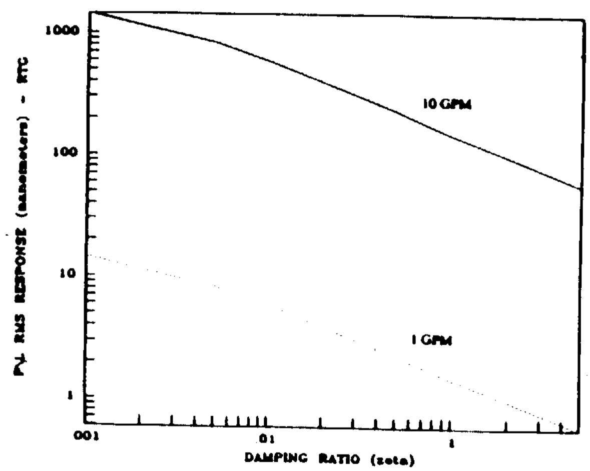


Figure 5.16: Pathlength RMS Response vs. Structural Damping Ratio: RTG Flow Noise at 1 and 10 GPM.

Damping Augmentation

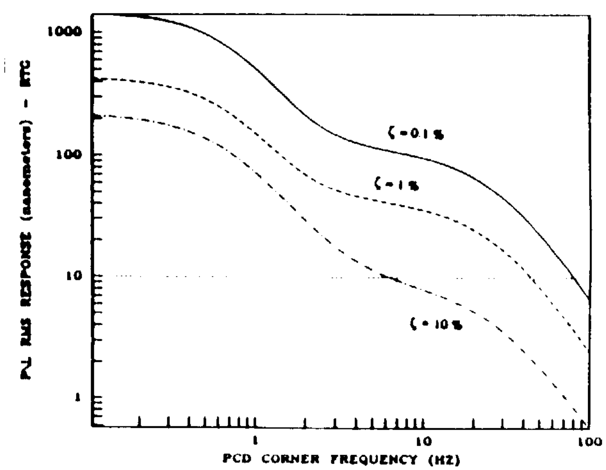


Figure 5.13: Pathlength RMS Response versus PCD Corner Frequency for Structural Damping Ratios of: ζ =0.1, 1 and 10%.

Optical Compensation (PCD)

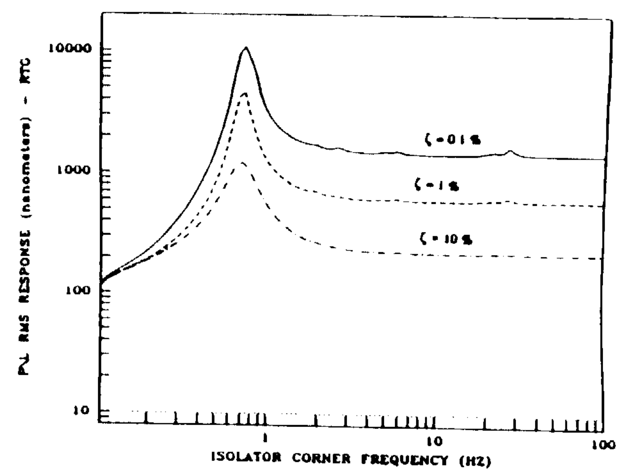


Figure 5.17: Pathlength RMS Response vs. Isolator Corner Frequency for Structural Damping Ratios of: ζ =0.1, 1 and 10%, and Maximum RTG Flow Noise.

Vibration Isolation

THESE CHARTS SHOW THE EFFECTIVENESS AND COMPLEMENTARY NATURE OF VARIOUS COMBINATIONS OF CST TOOLS. THEY THAT ONE WAY OF ACHIEVING THE PERFORMANCE IS TO USE 1% STRUCTURAL DAMPING, 10 HZ OPTICAL COMPENSATION, AND 4 HZ ISOLATORS. NOTE THE FOLLOWING:

OPTICAL COMPENSATION IS MORE EFFECTIVE AT LOWER FREQUENCIES.

ISOLATION IS MORE EFFECTIVE AT HIGHER FREQUENCIES.

DAMPING GENERALLY ATTENUATES OVER ALL FREQUENCIES.

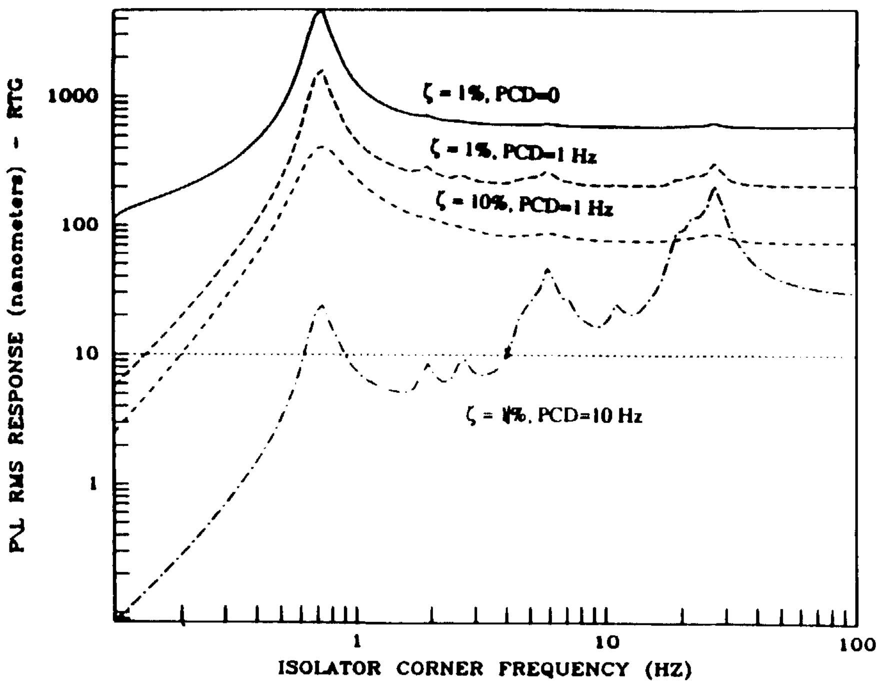


Figure 5.18: Pathlength RMS Response vs. Isolator Corner Frequency for Various Levels of PCD Control and Damping, and Maximum RTG Flow Noise.

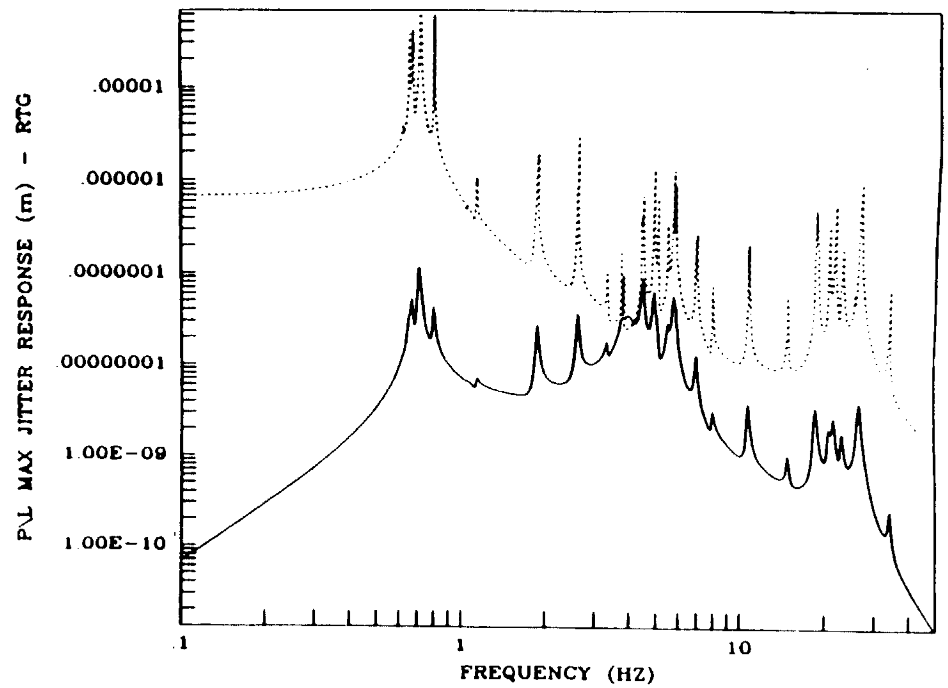


Figure 5.20: Pathlength Maximum Jitter for: 10 Hz PCD, 4 Hz Isolation and ζ=1%.

THIS SUMMARIZES THE TWO MAJOR ATTITUDE CONTROL SUBSYSTEM DESIGN OPTIONS: A REACTION WHEEL SYSTEM, AND A CONTROL MOMENT GYRO SYSTEM. BOTH THE IMPROVED-RWA AND THE ADVANCED CMG CONSIDERED HERE ARE EXTRAPOLATIONS BASED ON CURRENT SYSTEMS. THE PRIMARY DIFFERENTIATOR HERE IS THE TOTAL SUBSYSTEM MASS, WHICH STRONGLY FAVORS THE CMG BASED SYSTEM.

C-2

Attitude Control Subsystem Design

Parameter	*Improved-RWA System*	*Advanced CMG System*
1.Primary Elements (Qty):	RWA's (30), Magnetic Torquers (12), Sensor Complement	CMG's (3), Magnetic Torquers (12), Sensor Complement
2.Total Subsystem Mass:	3016 kg	1321 kg
3. Net Actuator Capacity: (per axis)	20 Nm / 5000 Nms	4150 Nm / 2300 Nms
4.Total (Avg) System Power:	1790 W_E	635 W_E
5.Disturbance Types:	L,M,H,R - Steady State/Swept	M,H,R - Steady State
6.On During Science:	YES	YES
7.Pathlength Response: (nanometers) Max Jitter: RMS:	8 micron 803 n-m	10 micron ?? n-m
8.Recommended CST Tools:	PCD Control: 1 Hz Structural Damping Ratio: 1% Isolation: 3 Hz	PCD Control: 1 Hz Structural Damping Ratio: 1% Isolation: 3 Hz
9.Mass of CST System:	M_d + (5) M_i	$M_d + M_i$
10.CST Complexity:	Low-Med	Low-Med
11.MISC Constraints:	Constrains Maximum Siderostat Velocity to 0.1 m/s: 50% Science Time	100% Science Time

SINGLE CST TOOLS ARE NOT ADEQUATE FOR MEETING PERFORMANCE SPECIFICATIONS. COMBINATIONS OF TOOLS WILL HAVE TO BE USED.

PASSIVE GLOBAL DAMPING IS ESSENTIAL FOR ACHIEVING DESIRED PERFORMANCE.

BETTER NUMERICAL MODELS ARE REQUIRED FOR FURTHER INVESTIGATION.

A DETAILED DESIGN AND PERFORMANCE SPECIFICATIONS ARE NEEDED FOR AN OPTICAL PATH-LENGTH COMPENSATOR DEVICE.

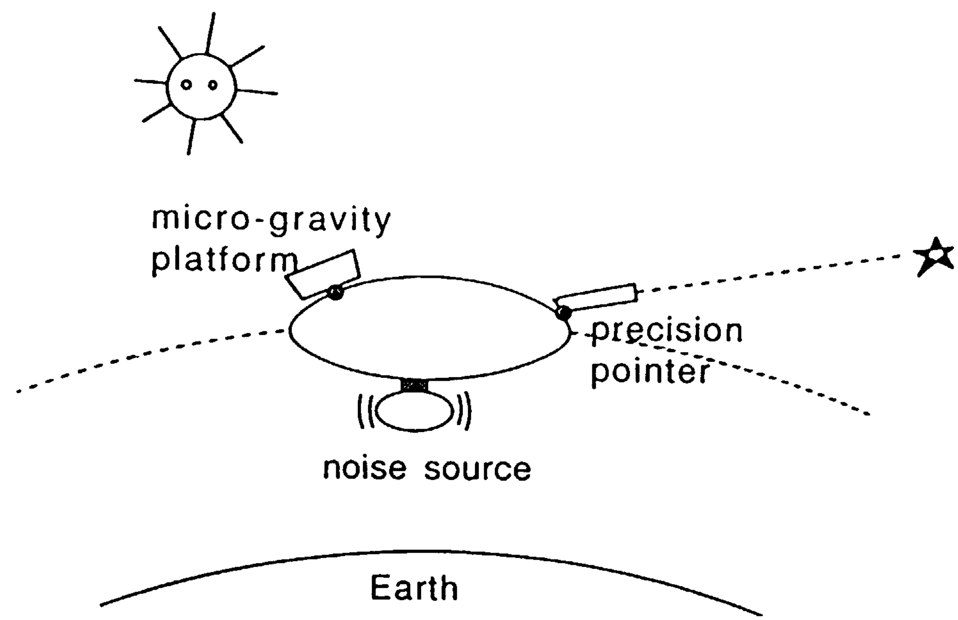
THE PROCEDURE FOR OBTAINING DIFFERENT BASELINES DURING DATA ACQUISITION NEEDS TO BE REEXAMINED IN THE CONTEXT OF DISTURBANCE MINIMIZATION.

Conclusions

- Need to use CST techniques in combination
- Passive global damping is essential
- Need better math models:
  - Couple solar array to spacecraft structural modes
- Need detailed design of optical path-length compensator
- Review procedure for science data acquisition as means of minimizing disturbances

ISOLATION

A. von Flotow
G. Blackwood
D. Stampleman
J. Garcia
L. Sievers



Source or destination is small

Sources: machinary, mirrors

Destinations: mirrors, payloads, laboratories

A LIST OF PROJECTS DISCUSSED IN THIS PRESENTATION.

ISOLATION PROJECTS

Micro-G Facility Aboard Milli-G Spacecraft

Machinary Mounts, Periodic Disturbances

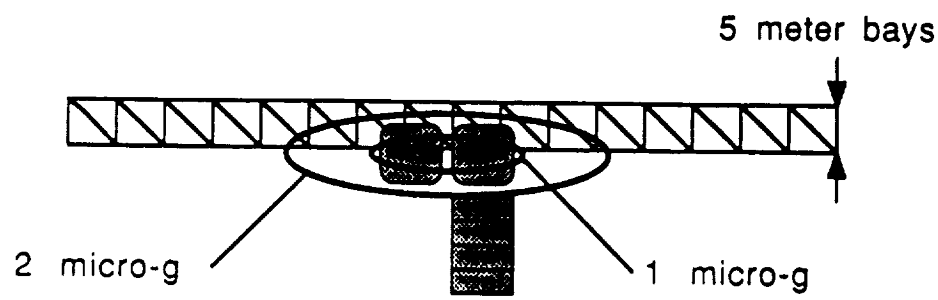
Small Mirrors, Broadband Disturbances

ORBITAL MICRO-GEE DISTURBANCES.

Disturbances

ORBITAL - "Static" 10^{-4} Hz formation flying

- Gravity Gradient 0.5 μg / m offset from the platform c.g. in the direction of the orbital radius in LEO



Space Station Low Frequency Acceleration gradient

- Aerodynamic The major disturbance torque in LEO

- 1 μg is approximately a 3 mm sinusoid at 0.01 Hz

SPECIFICATION FOR MICRO-GEE MATERIALS PROCESSING.

Isolation of a Space Station Laboratory Facility

Acceleration Environment and Specification:

ref: Jones, Owens, and Owen, " A Microgravity Isolation Mount," Acta Astronautica, Vol. 15, No. 6/7 1987

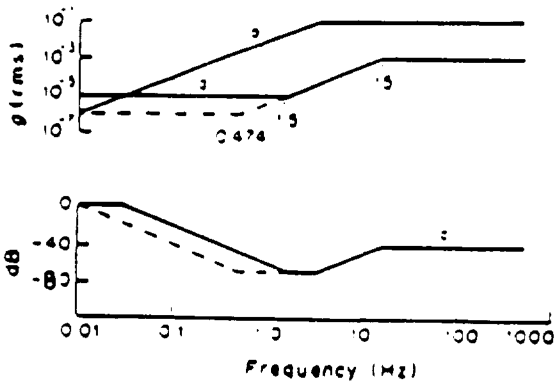


Fig. 2 Sinusoidal specification.

a = desired
b = anticipated
c = required isolation

- Mount travel of less than 1 cm. must be accommodated

- Prevent 6 axis payload drift (bias force ~ 1 mN)

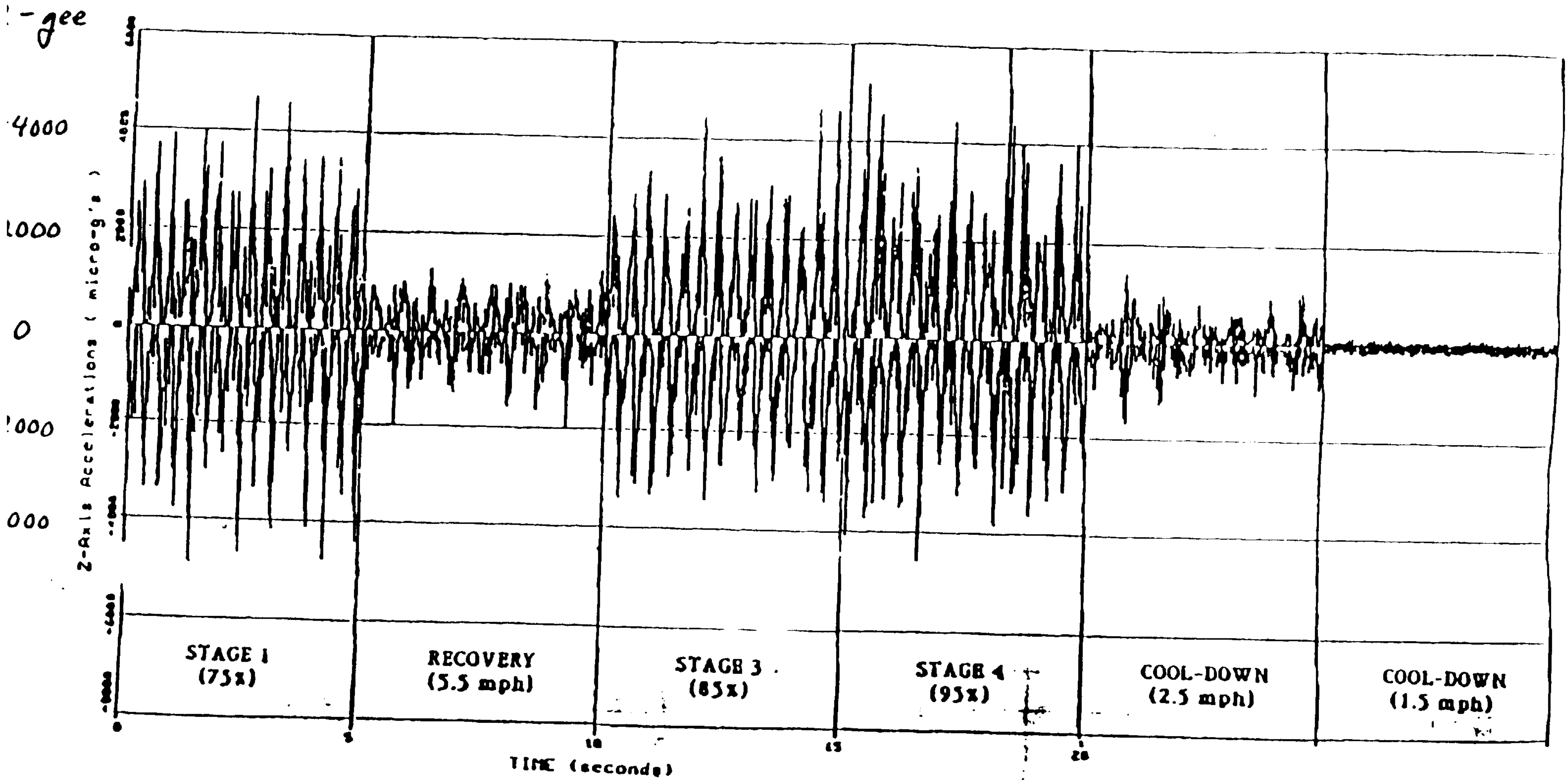
- Accommodate umbilical cord (uncertain dynamics) or non-contact substitutes which allow
  1) Data Transfer,
  2) Payload Cooling, and
  3) Power Flow.

- Passively stable, graceful failure

MEASURED MIDDECK ACCELERATION. HUMAN ON A TREADMILL.

HISA ACCELEROMETER DATA

DSO 476 PROTOCOL B (PLT)



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MEASURED MIDDECK ACCELERATIONS.

4

ATTACHMENT 2.

HISA ACCELEROMETER DATA

NCC ENGINE BURN

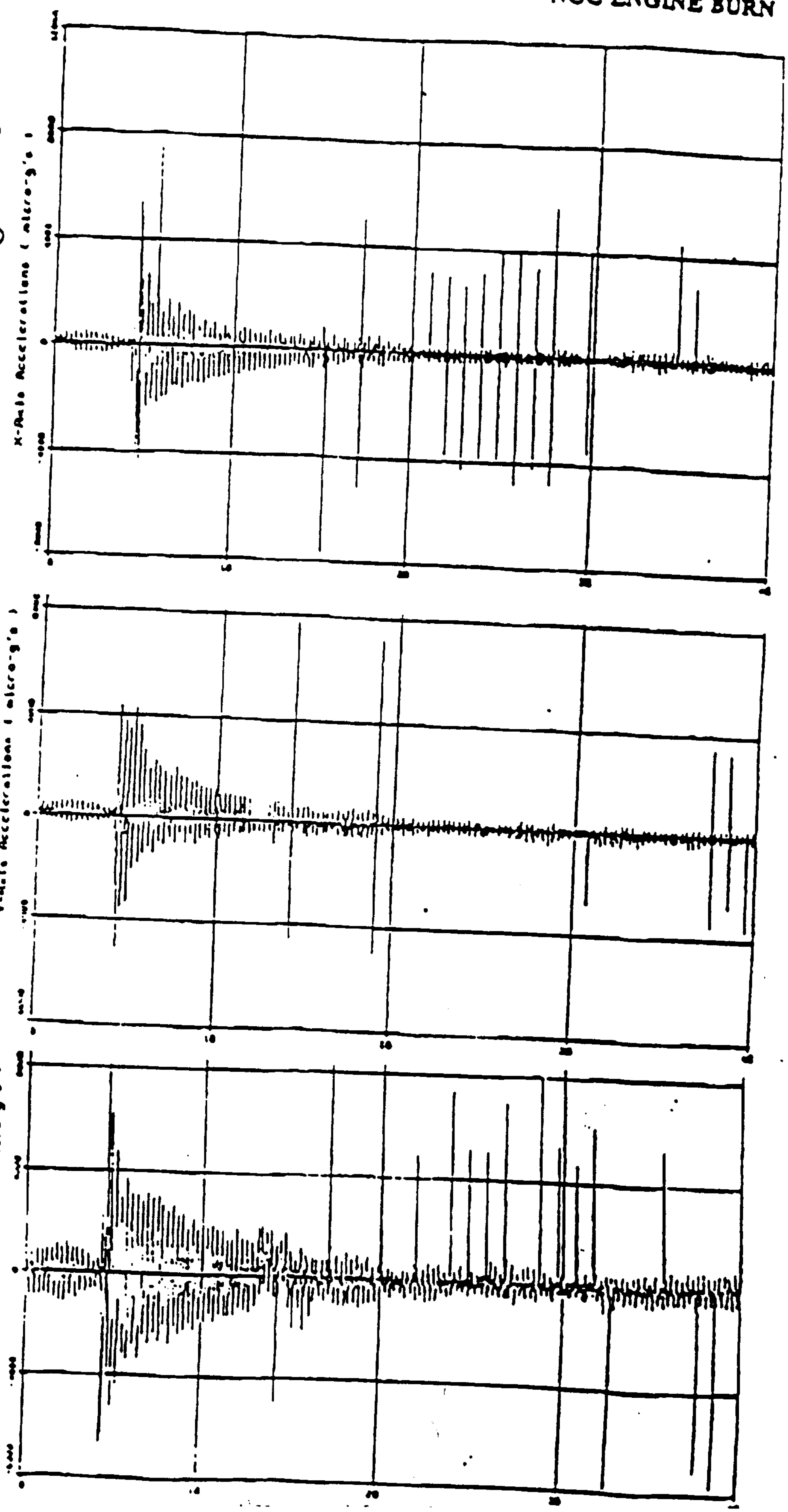
800

4000

0

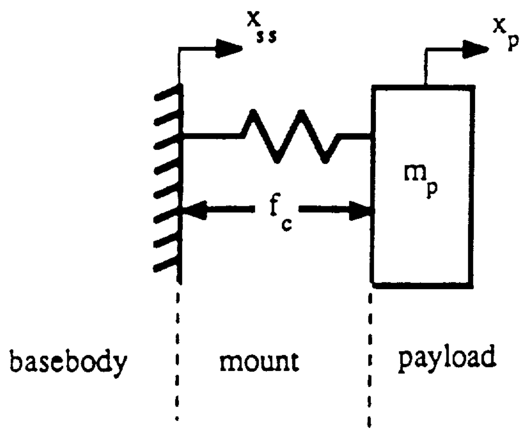
μ-gee

-4000

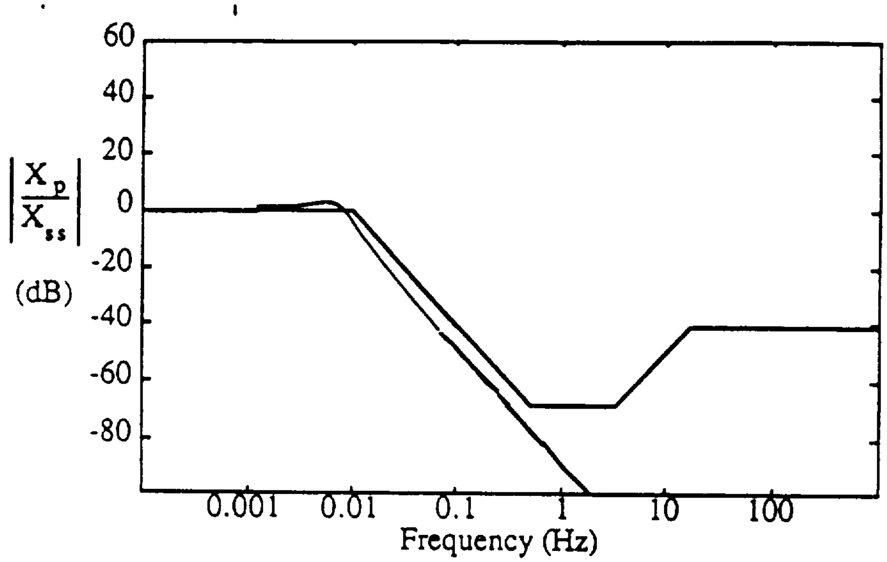


HEURISTIC DISCUSSION OF THE MICRO-GEE ISOLATION PROBLEM.

Why Not Passive Isolation?



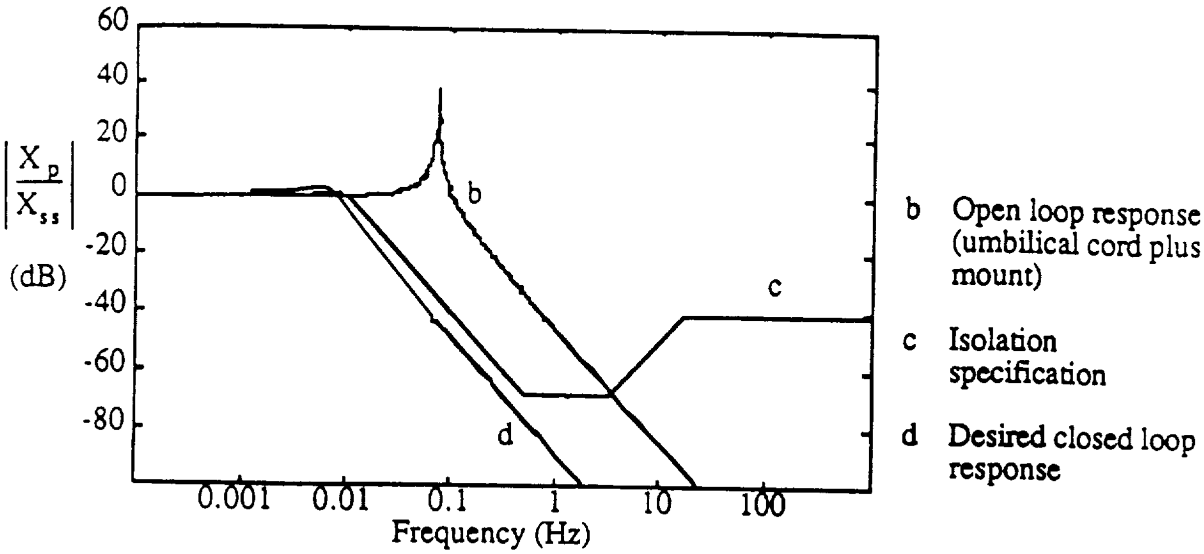
m_p	Payload mass
k_m	Mount stiffness
x_p	Payload displacment
x_{ss}	Basebody (space station) disturbance displacement
f_c	Control force (from actuator)



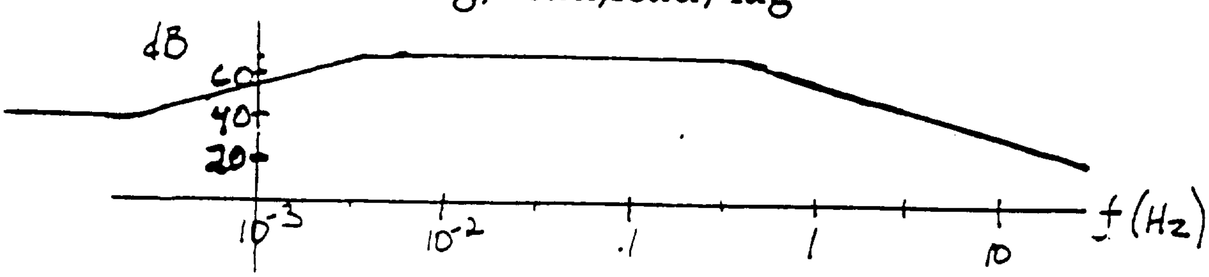
- If m_p = 100 kg., then $k_m \leq 0.4$ N/m and static deflection = 0.75 mm. ($k_{telephone} \approx 20$ N/m)
- Conclusion:
  - Active isolation needed only if a stiff umbilical cord is needed
  - Otherwise use soft passive springs
- Control/Isolation Goal is to lower the resonant frequency, f_r, below 0.01 Hz

HEURISTIC DISCUSSION OF MICRO-GEE ISOLATION ENHANCEMENT THROUGH FEEDBACK.,

Control Problem (Each axis)



- Km ≈ 40 N/m to bury the uncertain umbilical cord dynamics
- Axes decoupled by using modal analysis
- At least a factor of 10 active enhancement of isolation is necessary between 0.01 Hz and 10 Hz
- Compensator (each axis) from accelerometer to control force is lag/lead,lead/lag

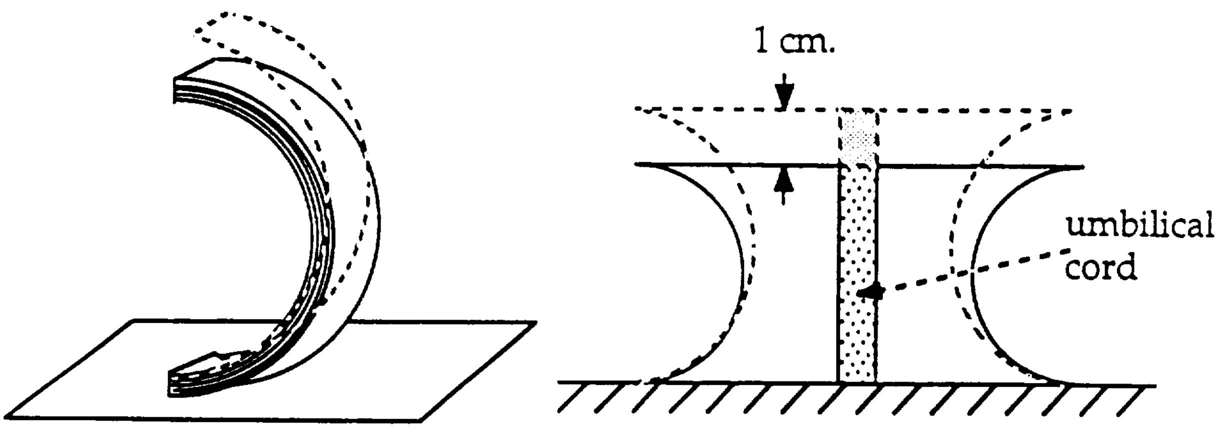


- Mount is essentially passive outside the active range of 0.01 Hz ≤ f ≤ 1 Hz

DESCRIPTION OF A MICRO-GEE ISOLATION ACTUATOR.

The PVDF Actuator

(SIZED TO OVERCOME A 40 N/M MOUNT STIFFNESS WHICH INCLUDES BOTH THE ACTUATOR AND UMBILICAL CORD)



- Layered PVDF piezoelectric film used in a "bimorph" format

- Arcs can be used in various configurations as semi-active mounts

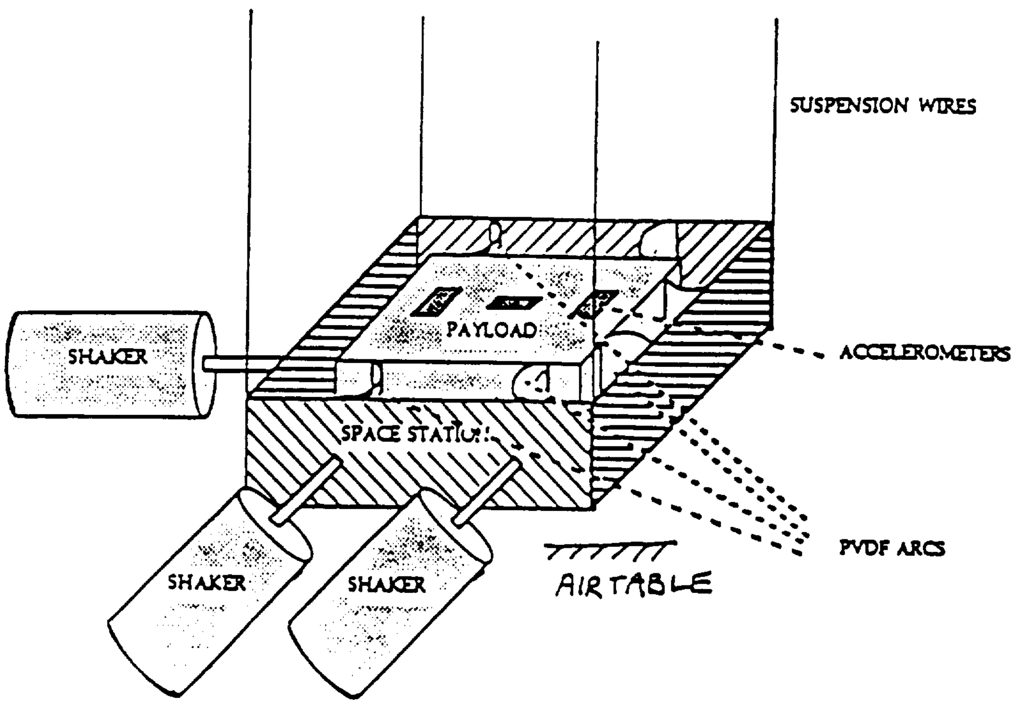
Each Arc:
- internal modes > 40 Hz
- can overpower itself for 1 cm. travel
- workable example:

width	=	0.05 m
radius	=	0.04 m
layer thickness	=	28 μm
number of layers	=	6
force @ 1 cm. travel	=	0.12 N
piezoelectric induced force	=	0.13 N

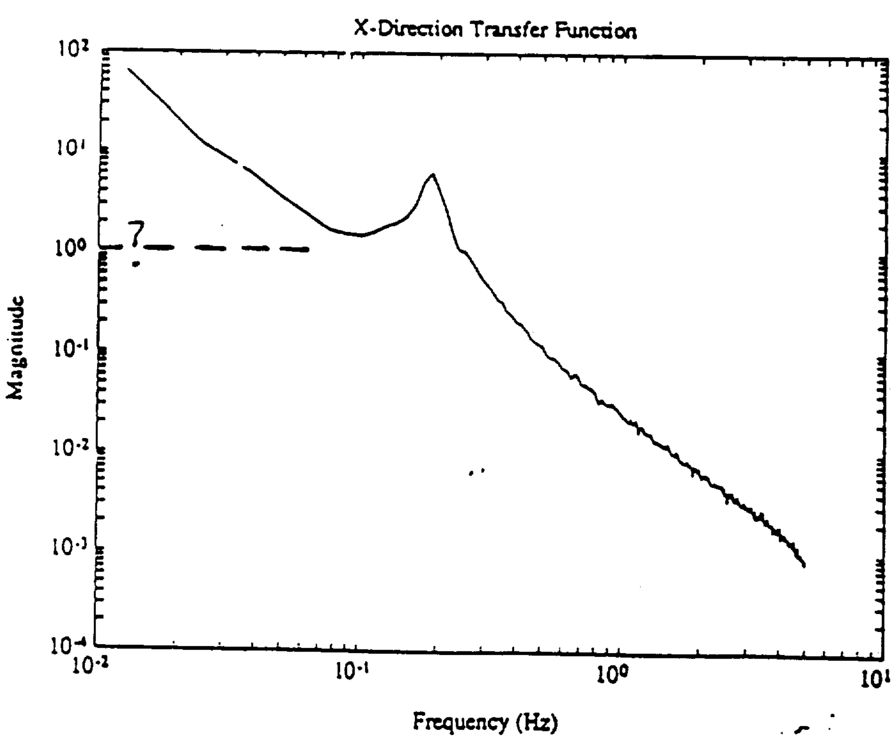
PRELIMINARY PERFORMANCE MEASUREMENTS ON A 3-AXIS MICRO-GEE LABORATORY ISOLATION DEMONSTRATION.

Accomplished: (1990)

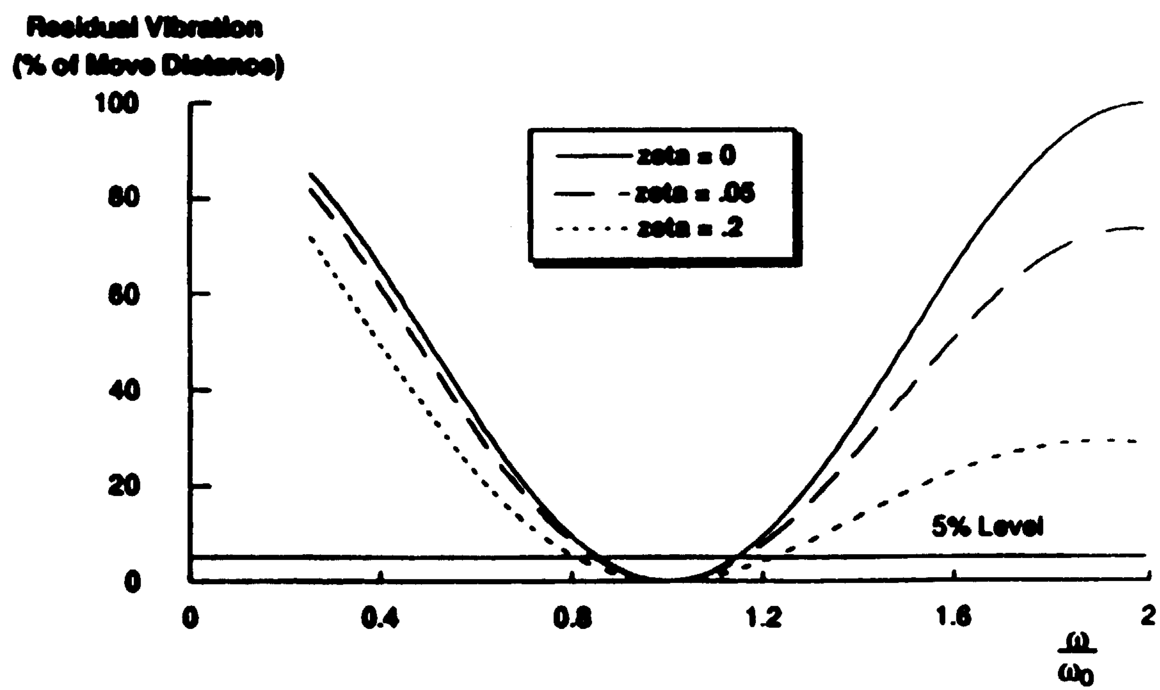
3-axis experiment operating:



Preliminary Data : (Open Loop)

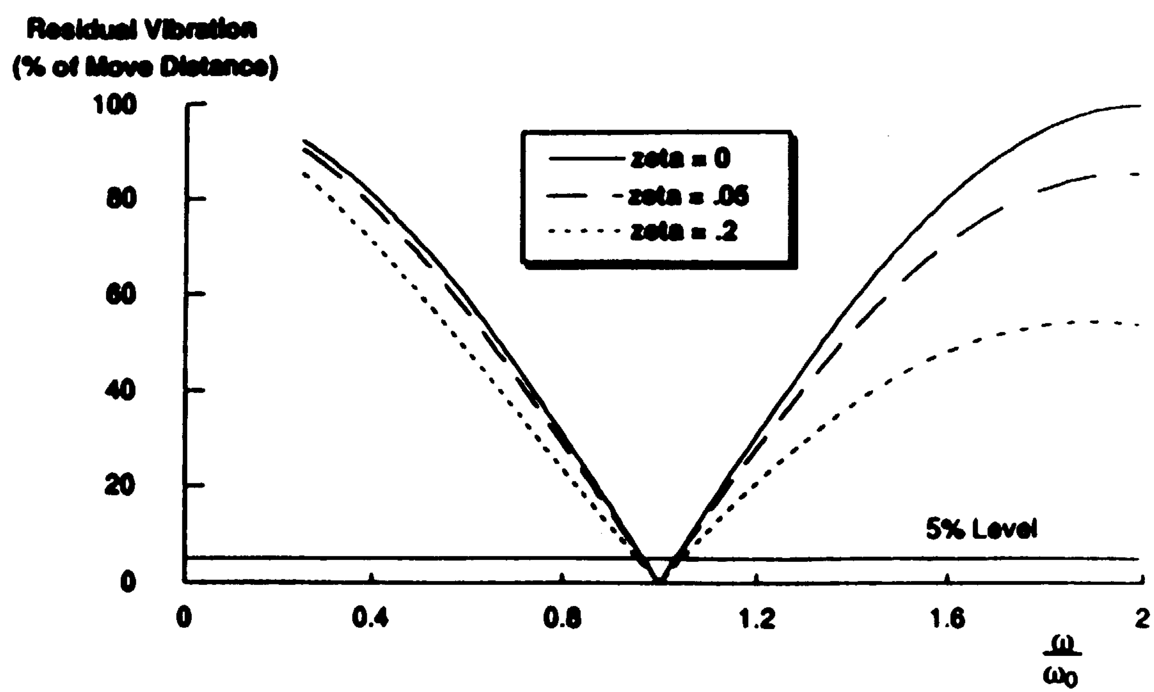


SHAPER INSENSITIVITY
THREE IMPULSE SEQUENCE



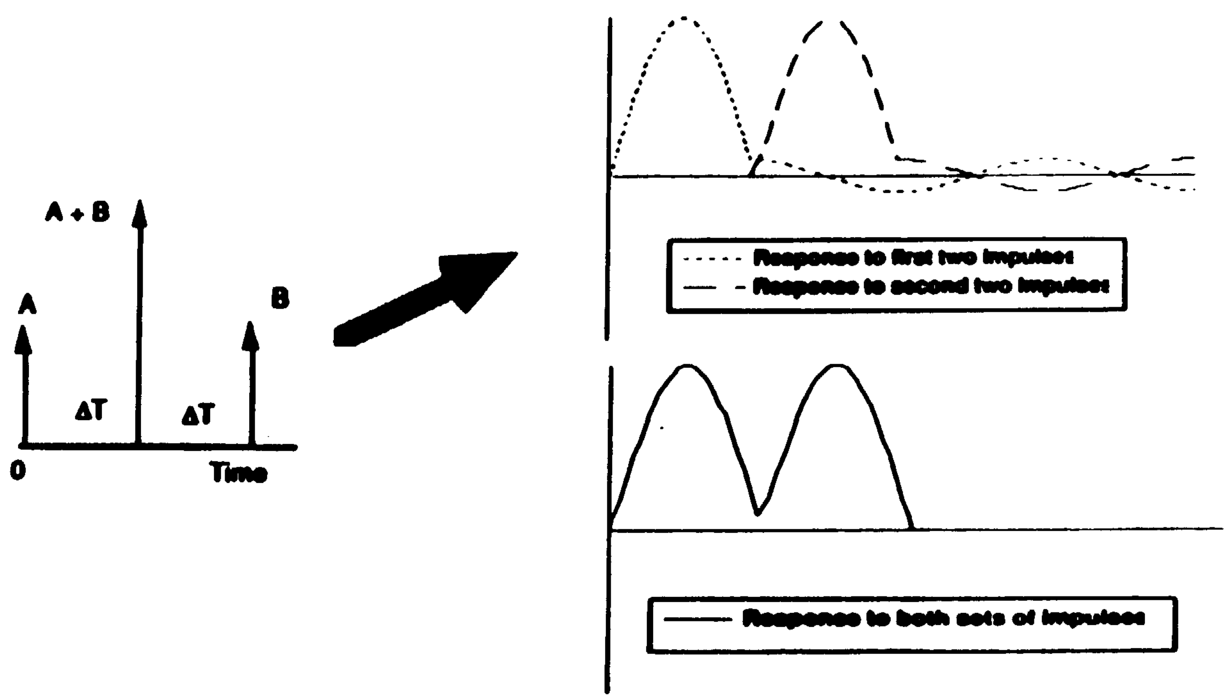
THIS GRAPH SHOWS THE IMPROVED INSENSITIVITY OF THE THREE-IMPULSE SEQUENCE. AS THE NATURAL FREQUENCY RATIO DRIFTS AWAY FROM UNITY, THE RESIDUAL VIBRATION LEVEL CLIMBS GRADUALLY, REMAINING BELOW 5% FOR SIGNIFICANT FREQUENCY ESTIMATION ERRORS. NOTICE THAT THE VIBRATION LEVEL IS EVEN LESS SENSITIVE TO DAMPING RATIO ESTIMATE ERRORS.

SHAPER INSENSITIVITY
TWO IMPULSE SEQUENCE



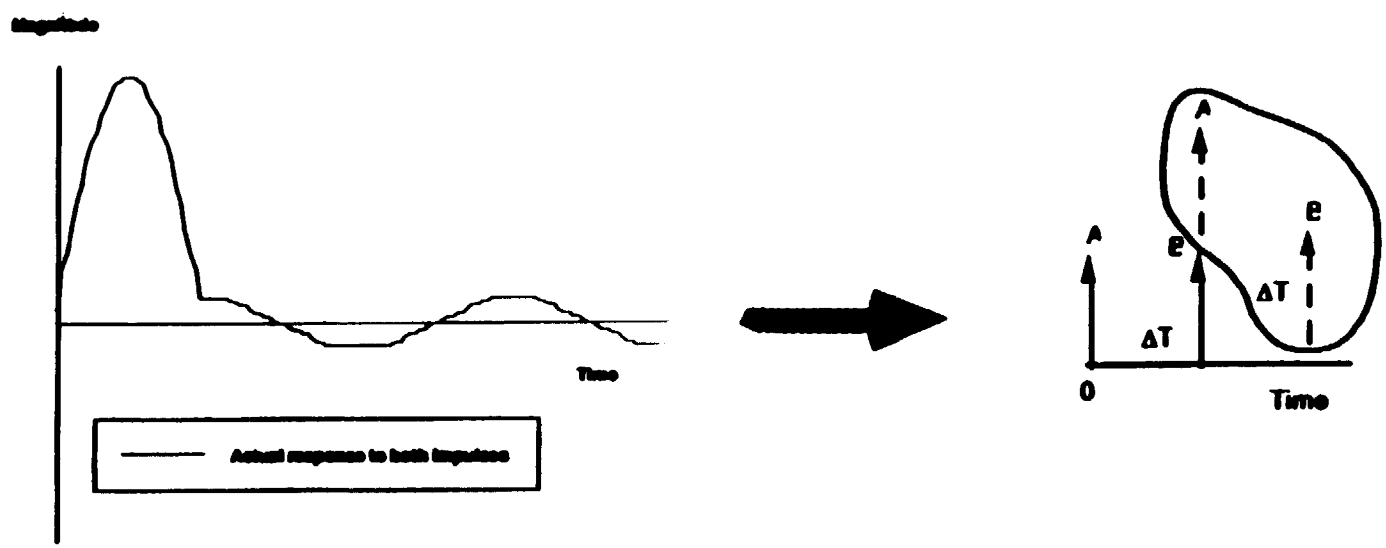
THIS GRAPH QUANTIFIES THE TWO-IMPULSE SHAPER'S INSENSITIVITY TO SYSTEM PARAMETER ESTIMATES. THE X-AXIS IS A RATIO OF ESTIMATED NATURAL FREQUENCY TO ACTUAL NATURAL FREQUENCY. THE Y-AXIS REPRESENTS THE MAGNITUDE OF THE SYSTEM RESIDUAL VIBRATION, CALIBRATED AS A PERCENTAGE OF THE ORIGINAL COMMANDED MOVE DISTANCE. IF THE NATURAL FREQUENCY RATIO IS UNITY, ESSENTIALLY MEANING THAT OUR PARAMETER GUESS IS PERFECT, THE VIBRATION LEVEL IS ZERO. AS THE PARAMETER GUESS BECOMES LESS ACCURATE, HOWEVER, THE VIBRATION LEVEL CLIMBS STEEPLY.

IMPULSE SEQUENCE DEVELOPMENT RESPONSE TO THREE IMPULSES



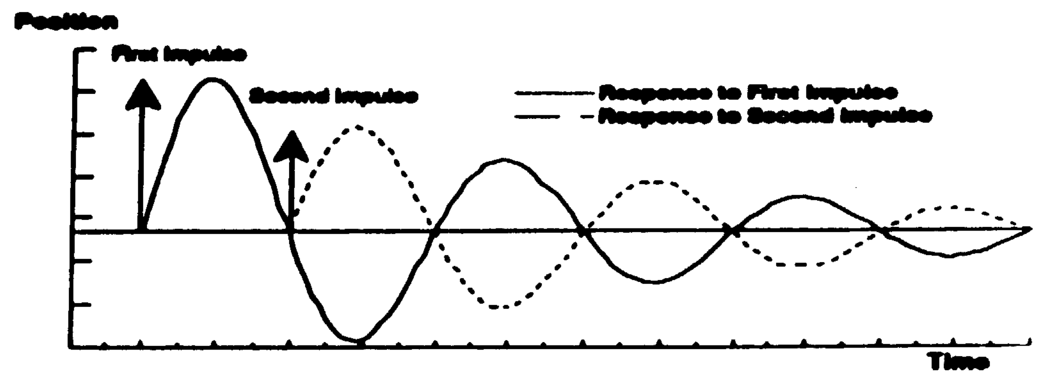
STACKING THE DUPLICATE TWO-IMPULSE SHAPER ONTO THE ORIGINAL TWO-IMPULSE SHAPER YIELDS THE THREE-IMPULSE SEQUENCE SHOWN HERE. THIS SEQUENCE WILL CANCEL THE SYSTEM'S RESIDUAL VIBRATION EVEN IN THE PRESENCE OF ERRORS IN THE NATURAL FREQUENCY AND DAMPING RATIO ESTIMATES.

IMPULSE SEQUENCE DEVELOPMENT
ADDING A THIRD IMPULSE

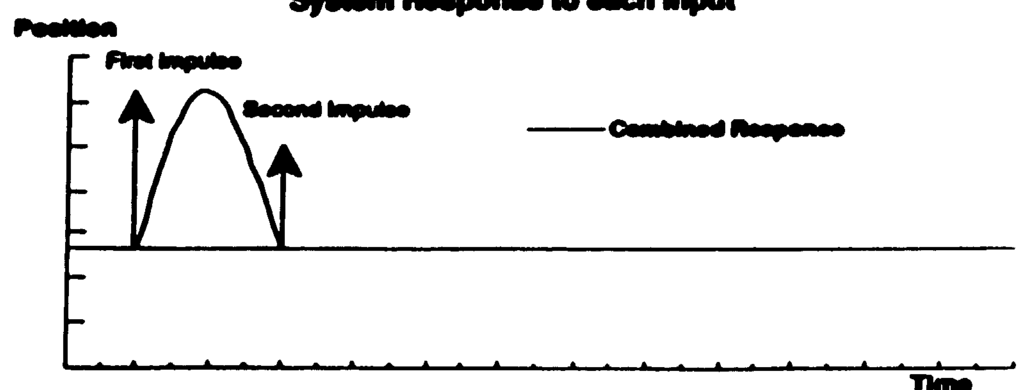


POSITIONING THE SECOND IMPULSE PROPERLY DEPENDS ON A PRECISE KNOWLEDGE OF THE SYSTEM PARAMETERS, NAMELY THE NATURAL FREQUENCY AND DAMPING RATIO OF THE SYSTEM. IF THESE PARAMETERS ARE ESTIMATED POORLY, THE SECOND IMPULSE WILL BE PLACED INCORRECTLY, AND RESIDUAL VIBRATION WILL RESULT. THE KEY POINT ABOUT THE RESIDUAL VIBRATION IS THAT IT IS REGULAR, SINUSOIDAL, AND THUS PREDICTABLE, AND PRESUMABLY REPRODUCIBLE. IF THE SHAPER CAUSED THIS RESIDUAL VIBRATION IN THE FIRST PLACE, A DUPLICATE SHAPER, DISPLACED IN TIME, COULD PRODUCE IDENTICAL RESIDUAL VIBRATION 180° OUT OF PHASE WITH THE INITIAL ERROR.

IMPULSE SEQUENCE DEVELOPMENT
RESPONSE TO TWO IMPULSES



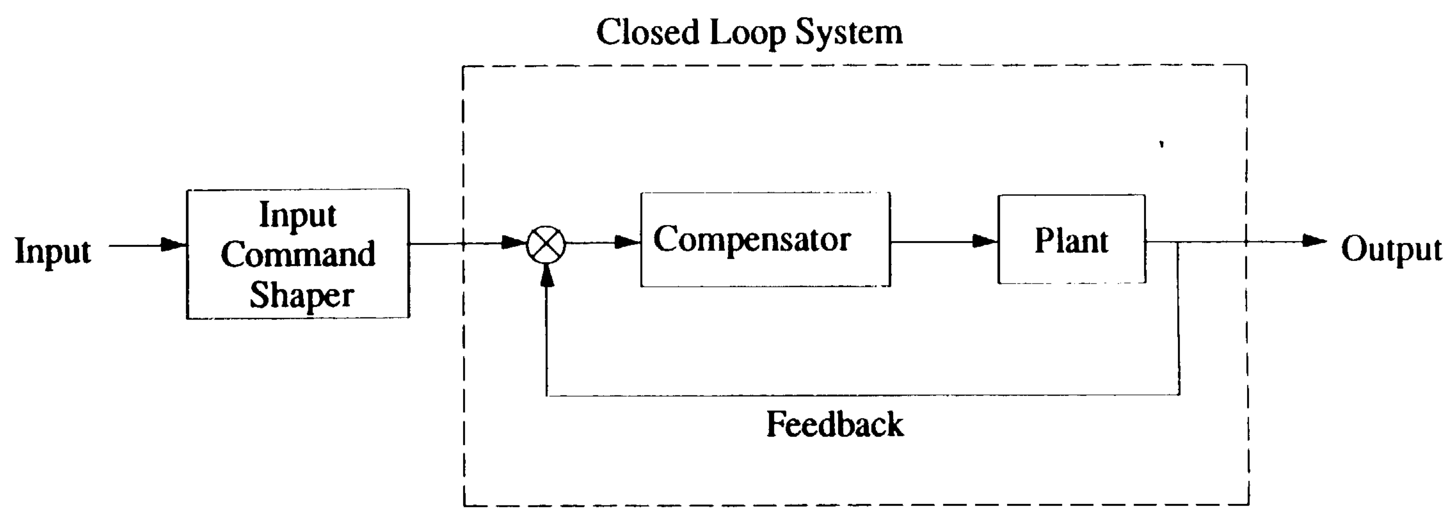
System Response to each Input



System Response to both Inputs

A STANDARD 2ND ORDER SYSTEM WILL VIBRATE AFTER EXPERIENCING AN IMPULSE INPUT. A SECOND IMPULSE, OF A CERTAIN MAGNITUDE AND POSITIONED AT THE PROPER TIME, WILL SET UP VIBRATIONS THAT CANCEL THE MOTION CAUSED BY THE FIRST IMPULSE.

SHAPER POSITION IN CONTROL SYSTEM



THE INPUT COMMAND SHAPER RESIDES COMPLETELY OUTSIDE A GIVEN CLOSED-LOOP CONTROL SYSTEM, WHERE IT CAN ACT ALONE OR AUGMENT ANY EXISTING VIBRATION REDUCTION SCHEMES. THE SHAPER ACTS AS A BLACK BOX, MODIFYING INPUTS TO YIELD VIBRATION-FREE OUTPUTS, BUT WHAT'S INSIDE THE BOX? THE SHAPER IS ACTUALLY A SIMPLE IMPULSE TRAIN WHICH, WHEN CONVOLVED WITH THE SYSTEM INPUTS, WORKS TO SUPPRESS THE OUTPUT VIBRATION. NOW WE'LL LOOK AT HOW WE CONSTRUCT THE SHAPER'S IMPULSE SEQUENCE.

OUTLINE

- Pre-Shaper Development
- Early Experiments & Results
- Applications to MACE
- Future Work

Presentation Topics: Pre-Shaper Development, Early Experiments and Results, Applications to MACE, and Future Work.

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USING INPUT COMMAND PRE-SHAPING TO SUPPRESS MULTIPLE MODE VIBRATION

James Hyde
Ken Chang
Whit Rappole
Prof. Warren Seering

MIT Mechanical Engineering Department

January 15, 1991

BROADBAND MIMO ISOLATION: FLEXIBLE STRUCTURE, UNCERTAIN DYNAMICS

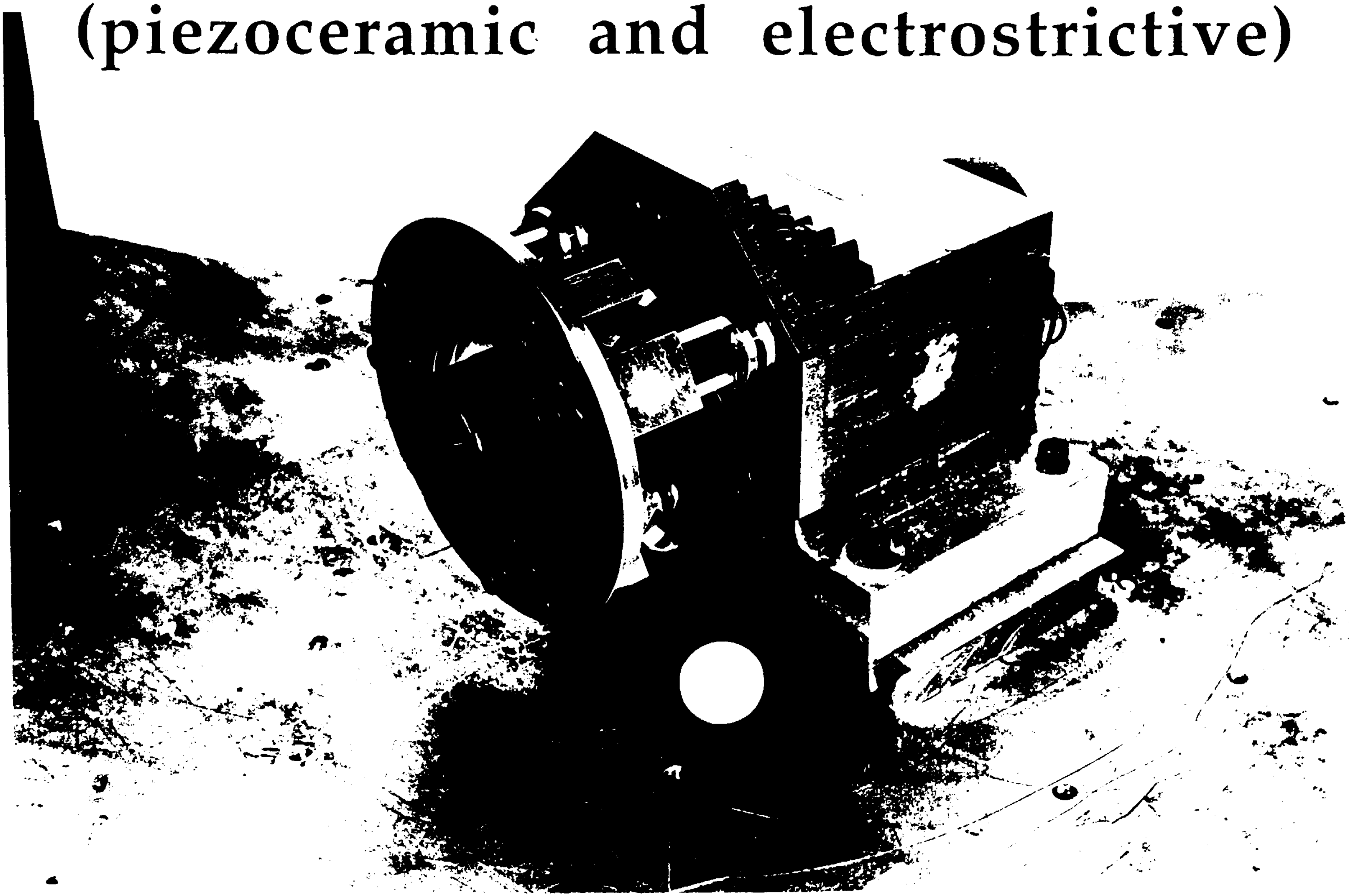
Design/Redesign Options:

- add passive damping to global modes
- reactuate moving mass of active optics
- reduce apparant modal mass through local stiffness modifications (passive, possibly active)

Research Issues:

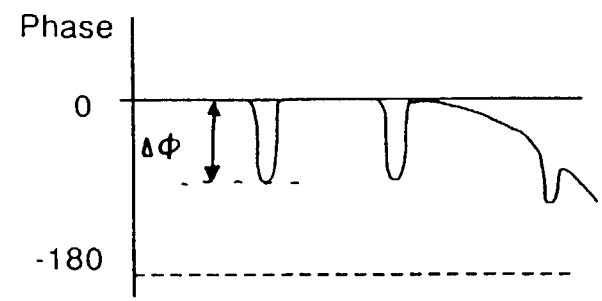
- extend SISO analysis to MIMO case: MIMO error bounds on magnitude and phase excursions
- noncollocation
- multiple modes
- control 3 pathlengths (on Interferometer Tesbed) to 50 nm rms using two active optical stages (6 dof total) in presence of disturbances
- design low-order controllers that
  - ignore detailed plant dynamics
  - incorporate some plant knowledge

Active Mirror Mounts
(piezoceramic and electrostrictive)



SISO ANALYSIS APPLIED TO TESTBED FE MODEL

naked truss damping:	.06%	(meas.)
instrumented truss:	.6 %	(est.)
damped, active + passive:	6%	(est.)
moving mirror mass:	1.5 kg	



$\Delta\Phi$ in degrees (single mode)

Direction	x	y	z
z = .06%	163	84	167
z = .6%	68	10	84
z = 6%	8	1	10
m = .15, z = .06%	8	1	10

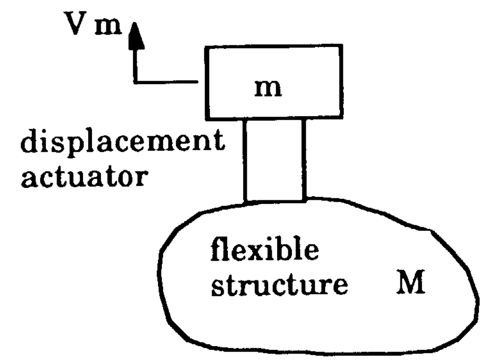
SUMMARY OF THE ANTICIPATED LEVEL OF STRUCTURAL DYNAMIC CONTRIBUTIONS TO THE INTERFEROMETER PLANT TRANSFER FUNCTIONS, FROM COMMANDED TO ACHIEVED PATH LENGTH CORRECTIONS.

POSITIONING "LIGHT" MIRRORS ON A FLEXIBLE STRUCTURE: IMPEDANCES

Mobilities of moving mass (Y_m) and structure (Y_s) at r^{th} mode:

$$Y_m = \frac{1}{j\omega m}$$

$$Y_s = \frac{\psi_r^2}{\omega_r \eta M}$$

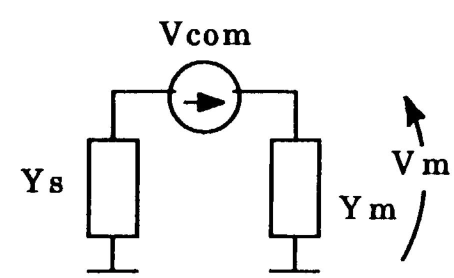


Prescribe v_{com} into actuator:

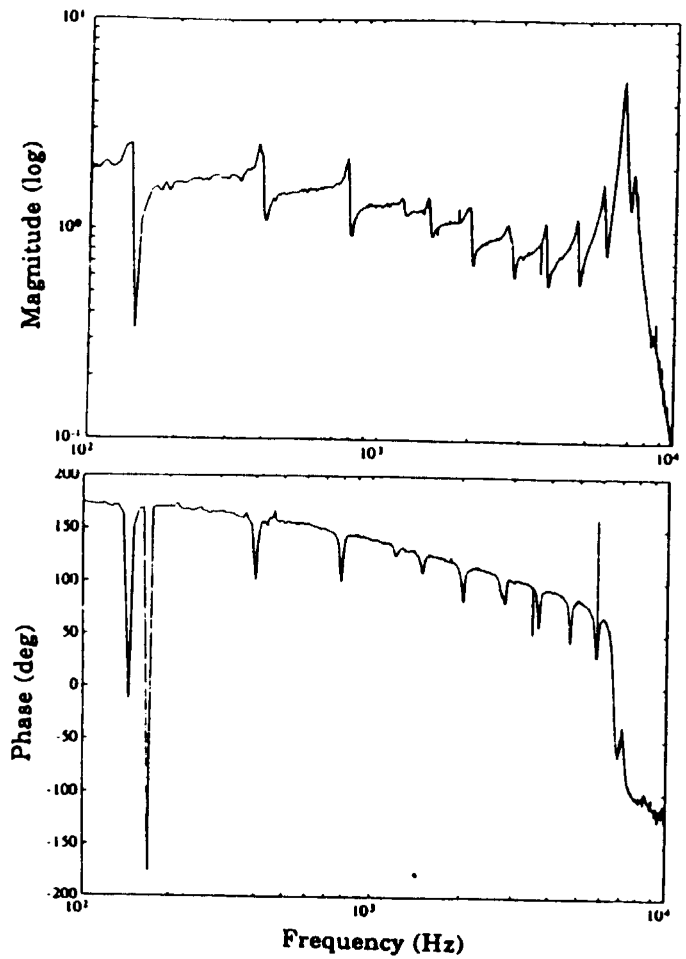
$$\frac{v_m}{v_{com}} = \frac{\frac{1}{j\omega m}}{\frac{1}{j\omega m} + \frac{\psi_r^2}{\omega_r \eta M}}$$

For output position control, want $Y_m >> Y_s$

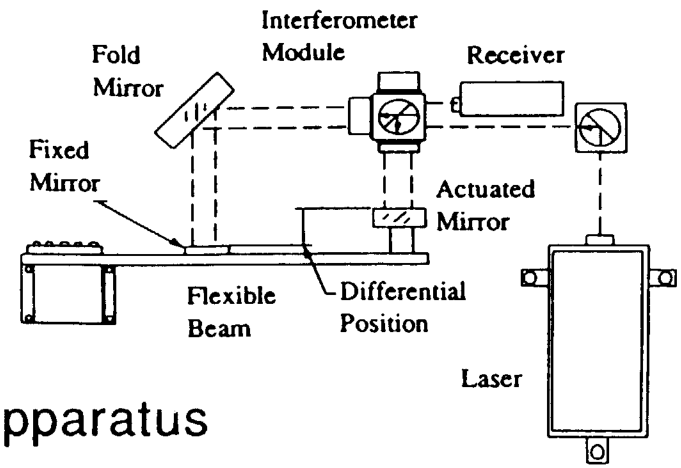
$$\Rightarrow \frac{m\psi_r^2}{\eta M} << 1$$



Positioning of Small Mirrors on a Flexible Structure



Plant Transfer Function



Apparatus

Gain Excursion (multiplicative):

$$\delta G = \sqrt{1 + \frac{1}{4\xi_i^2}\left(\frac{m_m}{\bar{m}_i} - \frac{m_m}{\hat{m}_i}\right)^2}$$

Phase Excursion:

$$\delta\phi = -2\tan^{-1}\left[\frac{1}{4\xi_i}\left(\frac{m_m}{\bar{m}_i} - \frac{m_m}{\hat{m}_i}\right)\right]$$

m_m=mirror mass

$\bar{m}_i, \hat{m}_i$ = effective modal mass

A PRELIMINARY EXPERIMENT IN HIGH BANDWIDTH POSITIONING OF SMALL MIRRORS ON A FLEXIBLE STRUCTURE, REPRESENTATIVE OF THE ANTICIPATED PROBLEMS IN THE LARGE INTERFEROMETER.

INTERFEROMETER PERFORMANCE METRIC

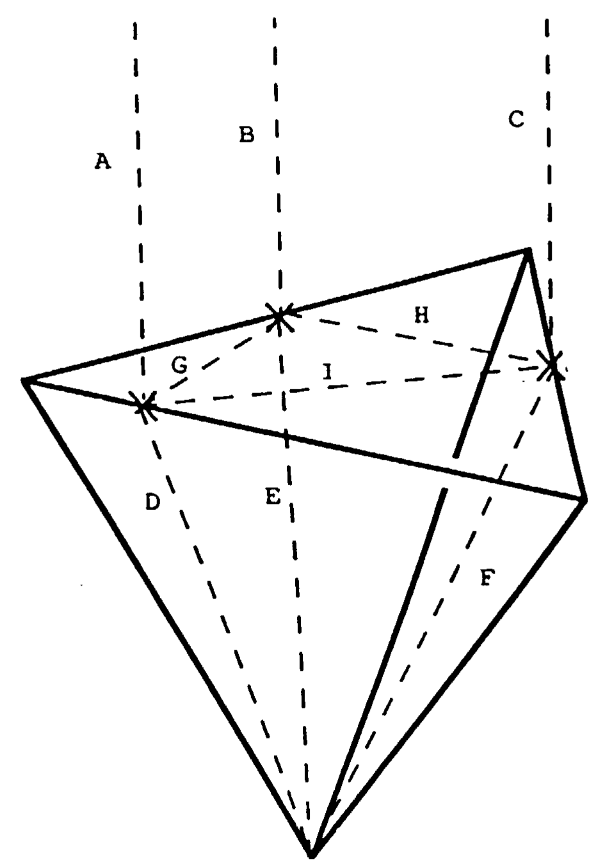
- require *total differential pathlength* stability of 50 nm rms above 0.1 Hz for 10 second duration.
- some elements of performance metric are not measurable for all typical mission scenarios, ie, the external differential path.

Testbed performance metric:

$$J = \left\langle \left[G + D - (H + F) \right]^2 \right\rangle^{1/2}$$

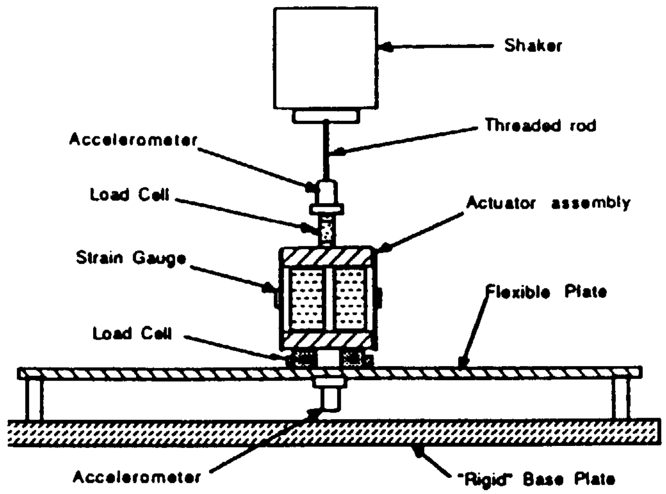
Science performance metric:

$$J' = \left\langle \left[A + D - (C + F) \right]^2 \right\rangle^{1/2}$$

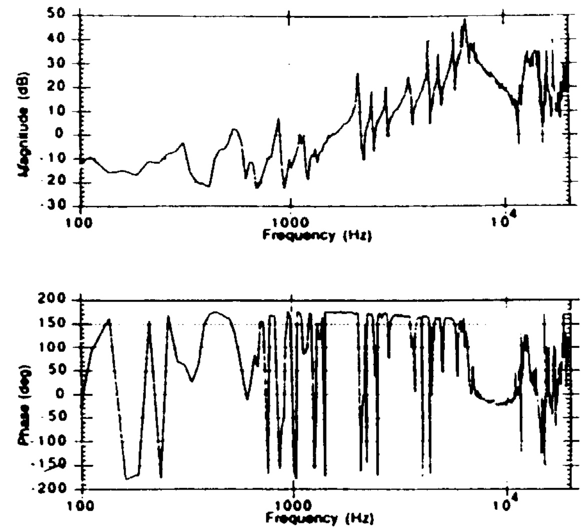


A SUMMARY OF THE PERFORMANCE METRIC FOR REAL-TIME INTERFEROMETRIC IMAGING.

Narrowband Vibration Isolation

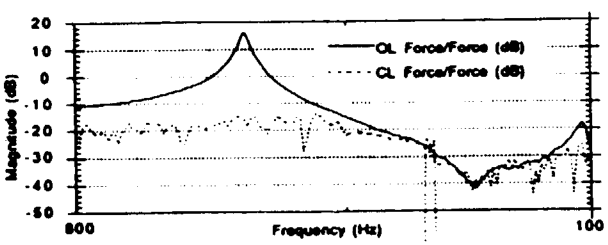


Apparatus



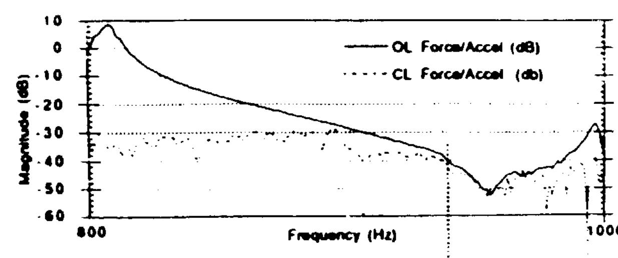
Measured Plant Transfer Function

Active Isolation Performance: (Time Varying Sinusoidal Disturbance)



Light Machine

⟼ swept in 1 second ⟶|

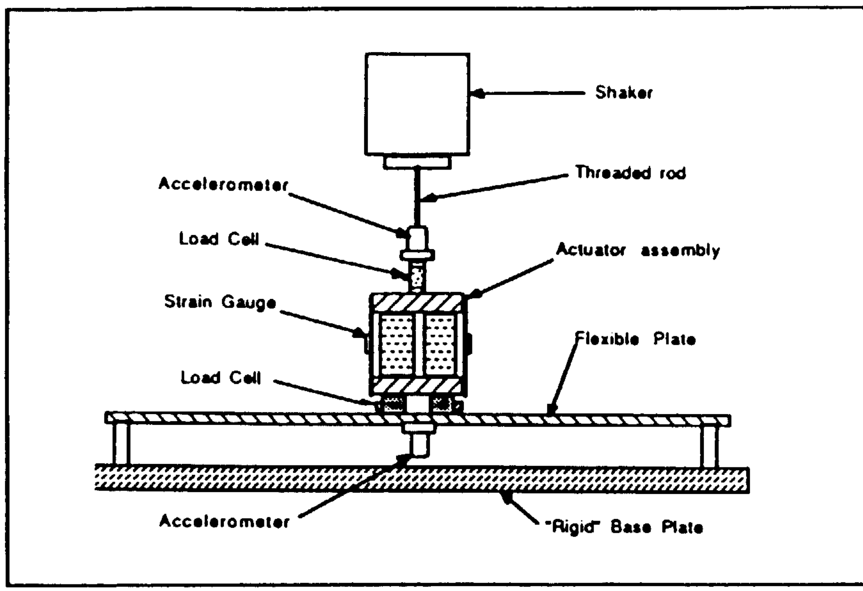


Massive Machine

⟼ swept in 1 second ⟶|

LABORATORY ACHIEVED ISOLATION OF A TIME-VARYING SINUSOIDAL DISTURBANCE.

Narrowband Vibration Isolation



Force Transducer

Piezoceramic Stack

Time Varying Narrowband Compensator

$$\frac{s + a}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n = \omega_{dist}$ (time-varying)

$\zeta = \zeta_{plant}$ (best)

A LABORATORY DEMONSTRATION OF ISOLATION OF A TIME-VARYING SINUSOIDAL DISTURBANCE. THIS IS THOUGHT TO BE REPRESENTATIVE OF THE HARMONIC DISTURBANCE FROM A ROTATING MACHINE.

Summary

- Microgravity isolation is easily achievable
- Active isolation only required by stiff umbilical cord
- This mount is passive with active enhancement

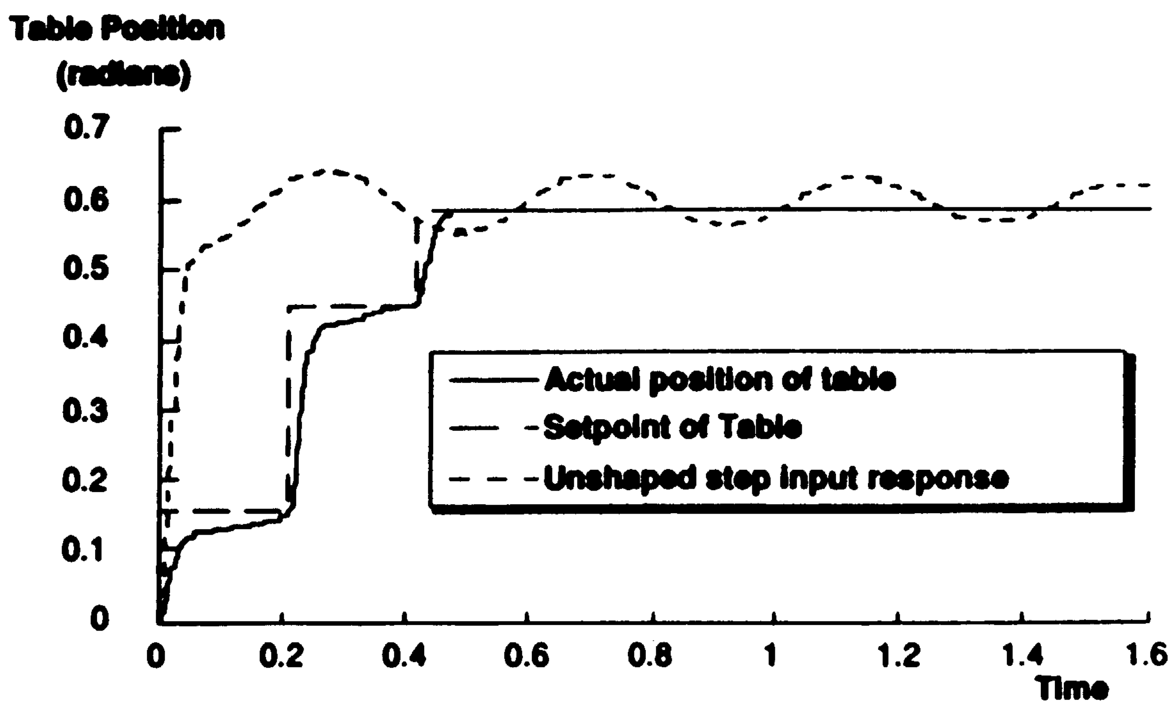
Future Work

- Six axes ground demonstration too difficult
- Perform 3 sequential 3-axis demonstrations with 6-axis hardware
- Fly with a microgravity customer

EARLY EXPERIMENTS AND RESULTS

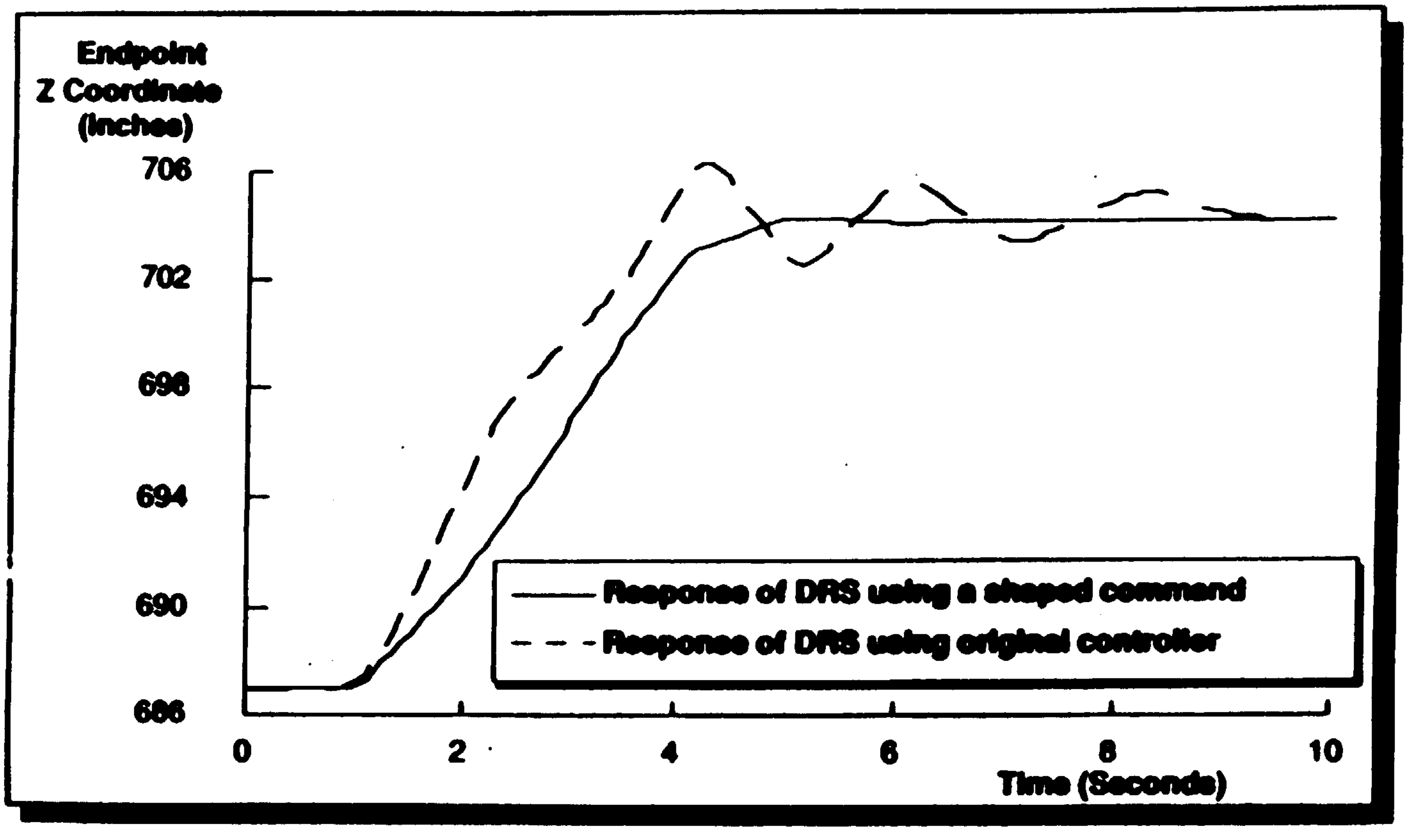
THIS GRAPH DEPICTS SOME EARLY VIBRATION SUPPRESSION WORK CONDUCTED ON A ROTARY TABLE FITTED WITH A LONG, FLEXIBLE BEAM. NOTE HOW THE STEP INPUT IS "STAIRCASED" BY CONVOLVING IT WITH THE INPUT SHAPER. WE AREN'T ACTUALLY SENDING IMPULSE COMMANDS TO THE SYSTEM, WE'RE CONVOLVING ANY GIVEN INPUT WITH THE IMPULSE SEQUENCE, AND THEN TRANSMITTING A *SHAPED* INPUT. THIS GRAPH ALSO POINTS OUT THE PENALTY FOR EMPLOYING THE SHAPING TECHNIQUE: A DELAY IS INCURRED IN THE SYSTEM RISE TIME. ALSO EVIDENT, HOWEVER, IS THE SAVINGS IN SETTLING TIME AFFORDED BY THE SHAPER, AND IN MANY CASES, THE RISE TIME DELAY IS ACCEPTABLE GIVEN THE NET RESPONSE TIME SAVINGS.

STEP INPUT CONVOLUTION



PERSONNEL AT THE DRAPER LABORATORIES HAVE DEVELOPED A COMPUTER SIMULATION OF THE SPACE SHUTTLE REMOTE MANIPULATOR SYSTEM. EMPLOYING THE COMMAND SHAPING TECHNIQUE DRAMATICALLY REDUCED THE RESIDUAL VIBRATION EXPERIENCED DURING A STANDARD MANEUVER.

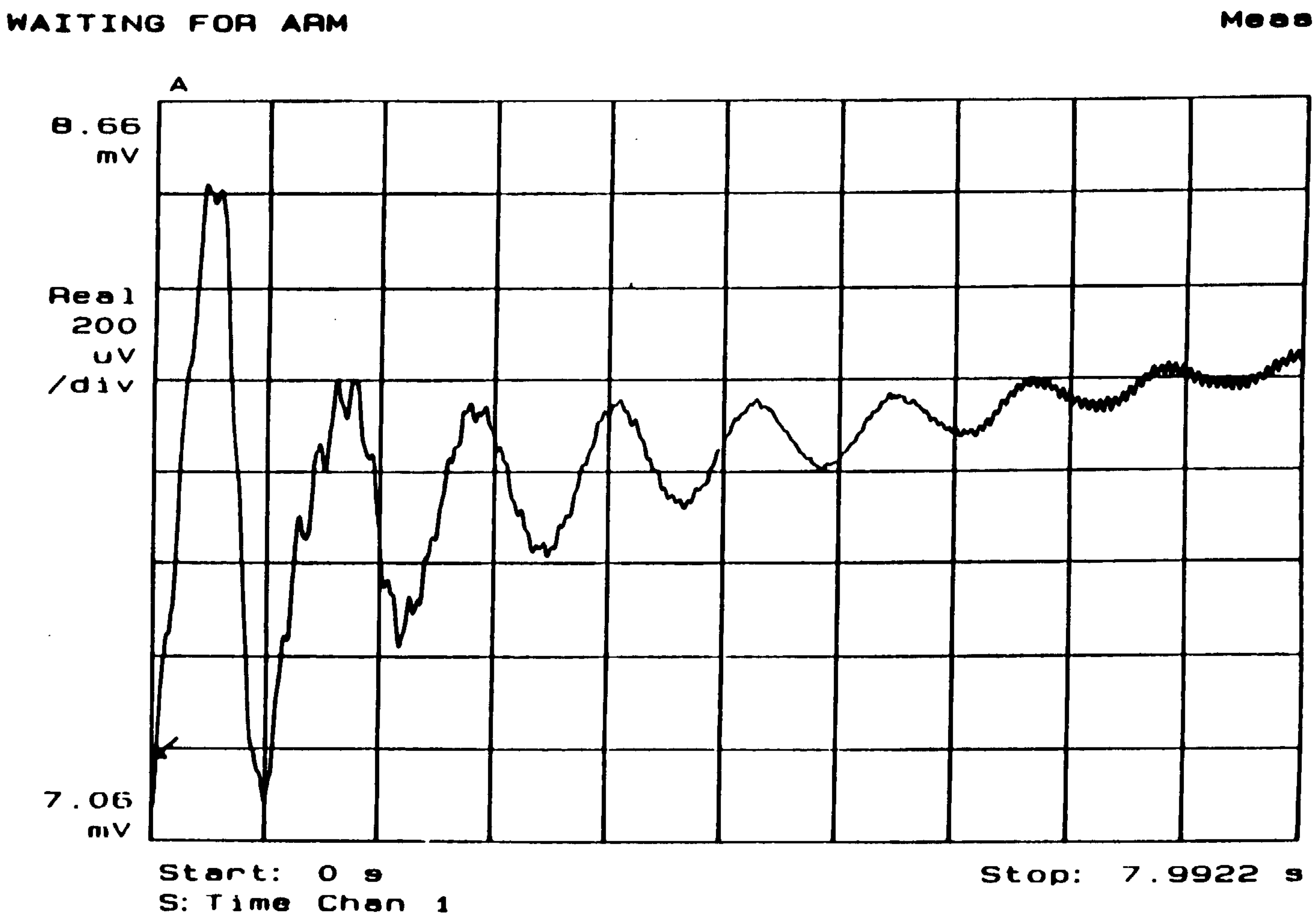
DRAPER RMS SIMULATION RESULTS



ORIGINAL PAGE IS
OF POOR QUALITY

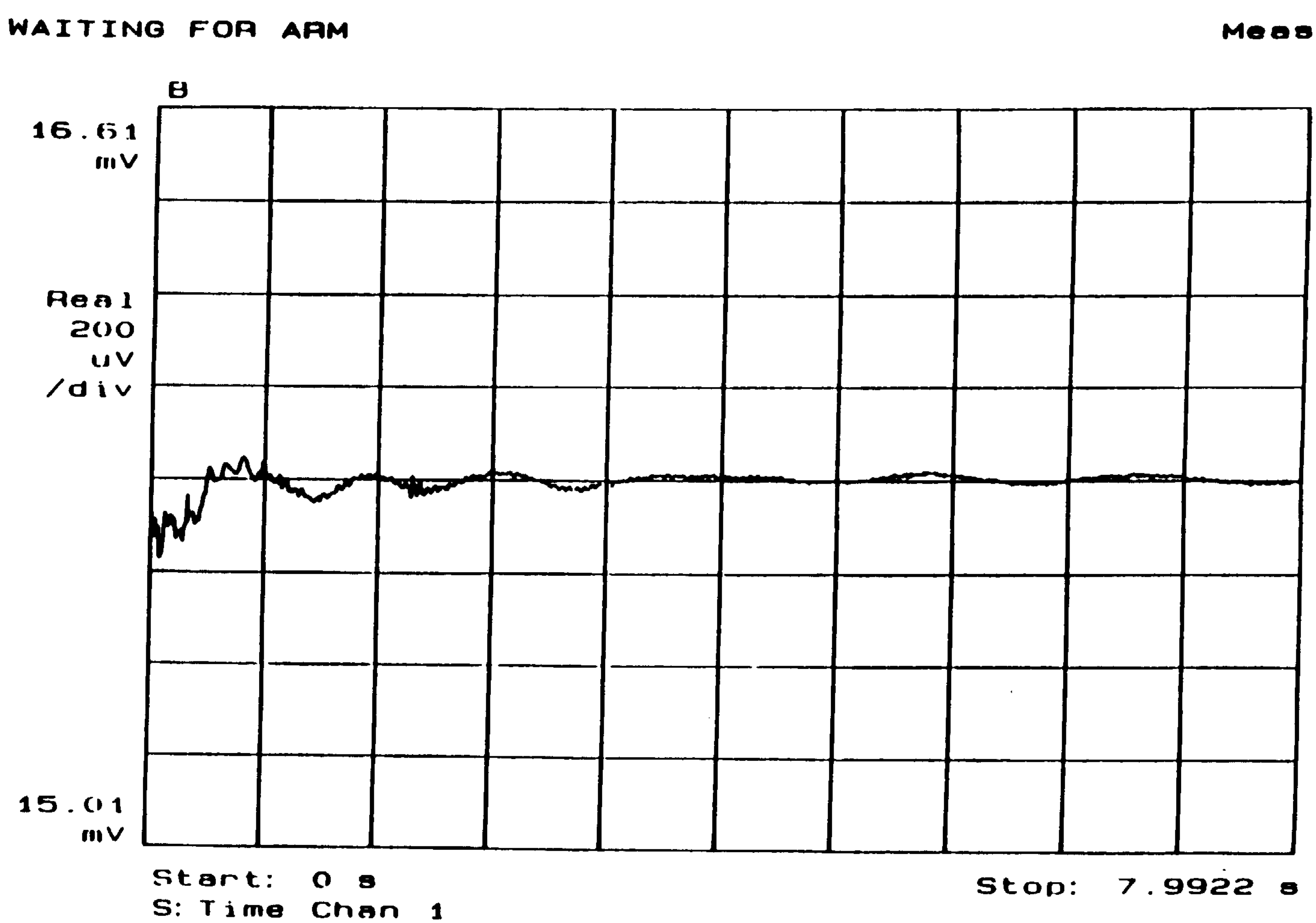
FURTHER TESTS HAVE BEEN CONDUCTED ON A FULL-SCALE REPLICA OF THE SPACE SHUTTLE ARM AT JOHNSON SPACE CENTER'S MANIPULATOR DEVELOPMENT FACILITY. THE ARM'S FLEXIBILITY IS EVIDENT FROM THIS GRAPH.

MDF RESULTS - UNSHAPED INPUT



THE SAME MANEUVER, UTILIZING A SHAPED COMMAND, RESULTS IN ABOUT 90% LESS RESIDUAL VIBRATION.

MDF RESULTS - SHAPED INPUT



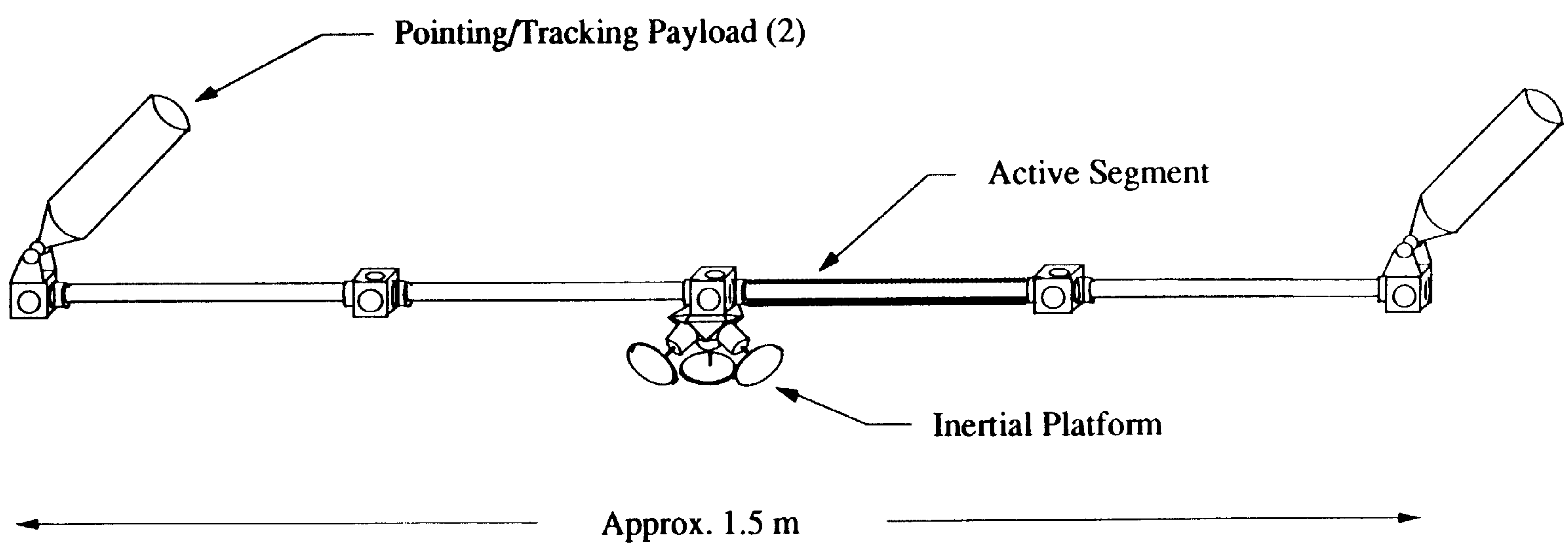
THE PREVIOUS EXAMPLES DEMONSTRATED THE EFFECTIVENESS OF "SINGLE MODE SHAPERS," INCAPABLE OF SUPPRESSING VIBRATION AT MORE THAN ONE MODE. OUR MOST RECENT WORK HAS FOCUSED ON DEVELOPING "MULTIPLE MODE SHAPERS" THAT RETAIN ALL THE PARAMETER INSENSITIVITY OF THE EARLIER SEQUENCES. THROUGH COLLABORATION WITH SERC, WE ARE APPLYING OUR MULTIPLE MODE SEQUENCES TO ONE OF THEIR EXPERIMENTS: THE MACE SYSTEM.

APPLICATION TO THE MID-DECK ACTIVE CONTROL EXPERIMENT

- MACE -

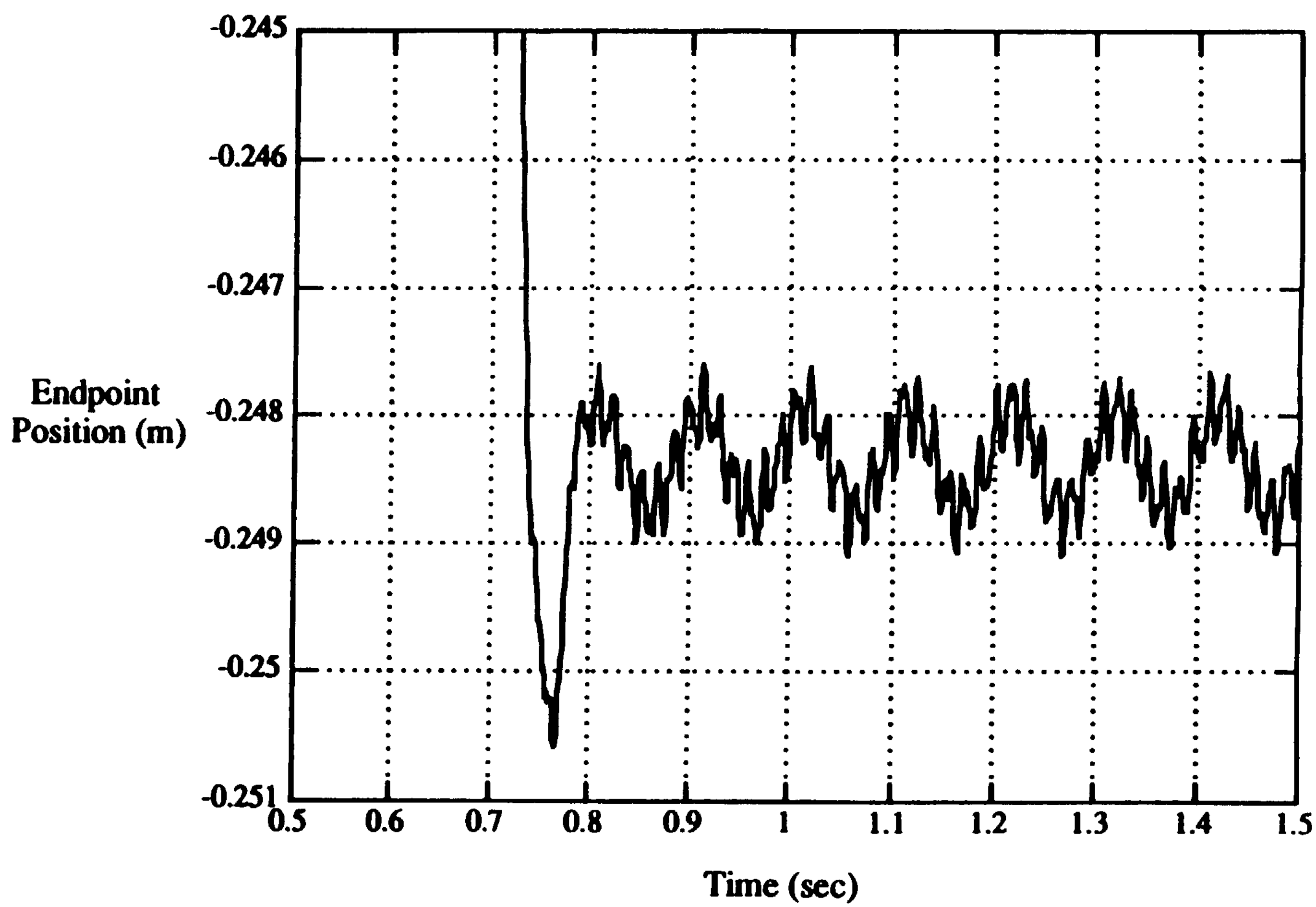
MACE IS A JOINT MIT/NASA PROJECT DESIGNED TO STUDY METHODS FOR CONTROLLING FLEXIBLE SYSTEMS IN MICRO OR ZERO GRAVITY FIELDS. MACE IS A FLEXIBLE STRUCTURE WITH TWO MULTI-AXIS POINTING PAYLOADS RESIDING ON EITHER END OF A TUBULAR BUS. THE SYSTEM INCORPORATES ATTITUDE CONTROL THROUGH A SET OF THREE-AXIS TORQUE WHEELS, AND UTILIZES INERTIAL POSITION SENSING INFORMATION GAINED FROM GYROSCOPE PACKAGES MOUNTED AT THE CENTER OF THE BUS AND INSIDE EACH PAYLOAD.

MACE SYSTEM SCHEMATIC



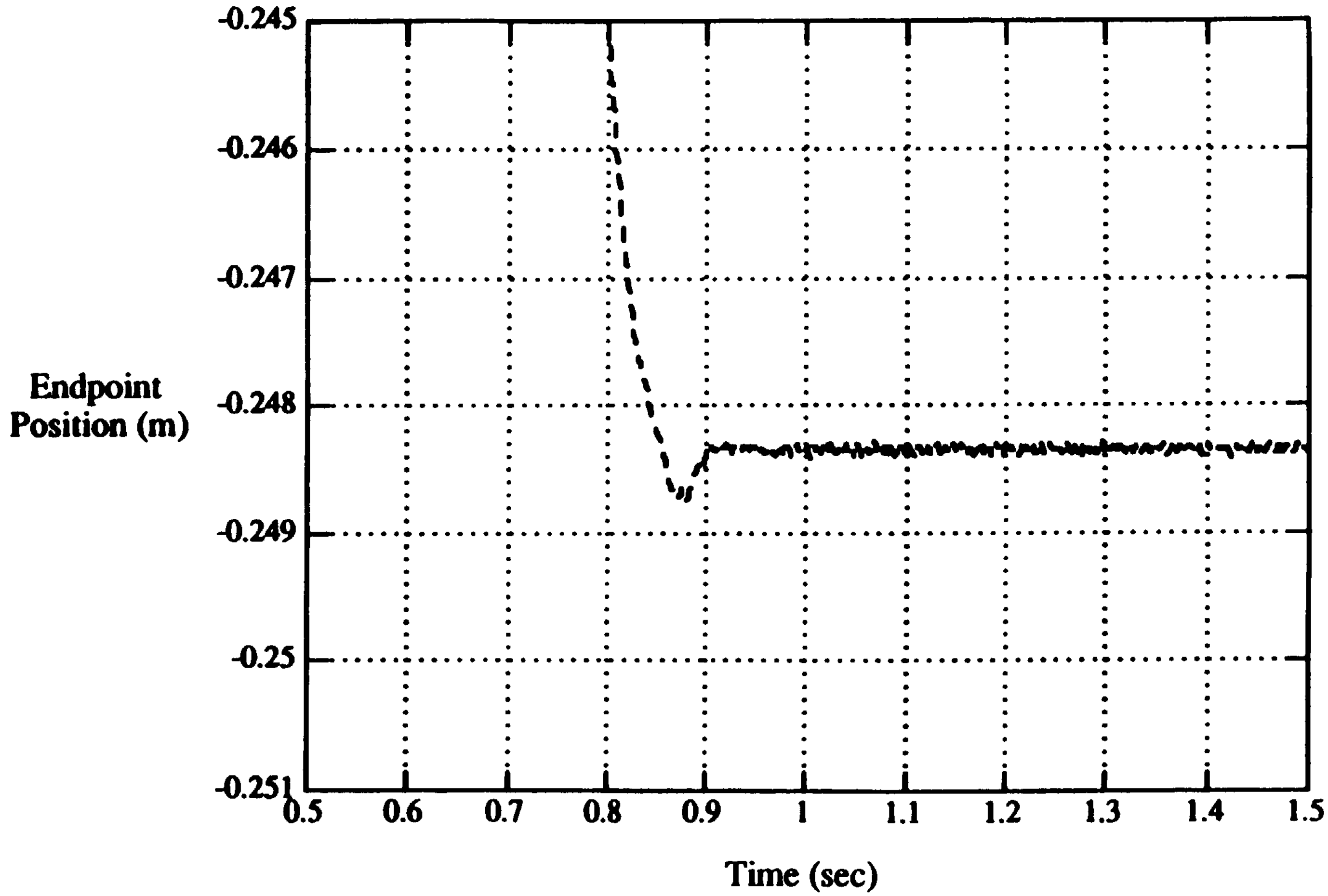
SERC PERSONNEL HAVE DEVELOPED SEVERAL COMPUTER MODELS OF MACE. USING A FINITE ELEMENT MATLAB MODEL, WE EXAMINED THE BUS RESPONSE WHEN ONE OF THE PAYLOADS WAS ROTATED SEVERAL DEGREES. THE MOTION OF THE FINITE ELEMENT ON THE OPPOSITE END OF THE BUS IS SHOWN IN THIS GRAPH. THE MULTIPLE MODE VIBRATION IS CLEARLY VISIBLE.

RESPONSE TO UNSHAPED INPUT



WHEN THE PAYLOAD IS SLEWED USING COMMANDS THAT WERE SHAPED BY OUR MULTIPLE MODE SEQUENCE, THE RESIDUAL VIBRATION IS DRAMATICALLY DECREASED. THE FINITE ELEMENT MODEL HAD A SYSTEM OF EIGHT MODES OF VIBRATION, AND OUR SHAPER WAS DEVELOPED TO SUPPRESS ONLY THREE OF THOSE MODES, BUT THE GRAPH CLEARLY SHOWS THAT THIS WAS SUFFICIENT TO SUPPRESS THE MAJORITY OF THE SYSTEM VIBRATION.

RESPONSE TO SHAPED INPUT



PUSH MULTIPLE MODE TECHNIQUE TO CAPTURE MORE MODES; EMPLOY NON-LINEAR DISCOS MODELS OF MACE; TEST PRE-SHAPERS ON MACE GROUND TEST ARTICLE. THE BIG IMMEDIATE PUSH HERE IS TO EXAMINE THE EFFECTS OF SYSTEM NON-LINEARITIES ON THE SHAPER'S SUCCESS IN MINIMIZING RESIDUAL VIBRATION.

FUTURE WORK

- Push Multiple Mode Technique to Capture More Modes
- Employ Non-Linear DISCOS Models of MACE
- Test Pre-Shapers on MACE Ground Test Article

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DEVELOPMENT OF AREA AVERAGING SENSORS FOR ACTIVE STRUCTURAL CONTROL

Dr. David W. Miller

January 1991

IN THE FIELD OF CONTROLLED STRUCTURES, STRUCTURAL SYSTEMS CAN BE DESCRIBED BY PARTIAL DIFFERENTIAL EQUATIONS INDICATING THAT THE STRUCTURE IS NOT AN INTERCONNECTION OF LUMPED BODIES BUT RATHER A DISTRIBUTED MEDIUM. THIS DISTRIBUTED MEDIUM OPENS THE OPPORTUNITY TO INTRODUCE A NEW CLASS OF SENSORS TERMED AREA-AVERAGING SENSORS WHICH ARE CONTINUOUSLY DISTRIBUTED ALONG THE STRUCTURE TO GENERATE A SIGNAL WHICH IS PROPORTIONAL TO THE WEIGHTED AVERAGE OF SOME MOTION VARIABLE DISTRIBUTED ALONG THE STRUCTURE. THE WORK PRESENTED IN THESE VIEWGRAPHS DISCUSS FOUR MOTIVATIONS FOR USING THIS TYPE OF SENSOR.

RESEARCH DEVELOPMENT

- Modal sensors.

  Distribute PVDF in the shape of a particular mode to realize a sensor which is only sensitive to that mode.

- Sensors which exhibit rolloff without phase lag.

  By filtering wave numbers, rather than frequencies, sensor rolloff dynamics are not restricted by causality constraints.

- Implementation of full state feedback for infinite order structural systems.

  LQR solutions for P.D.E. systems are spatial convolution kernels. These kernels can be inferred from numerical analysis and realized using an area-averaging sensor.

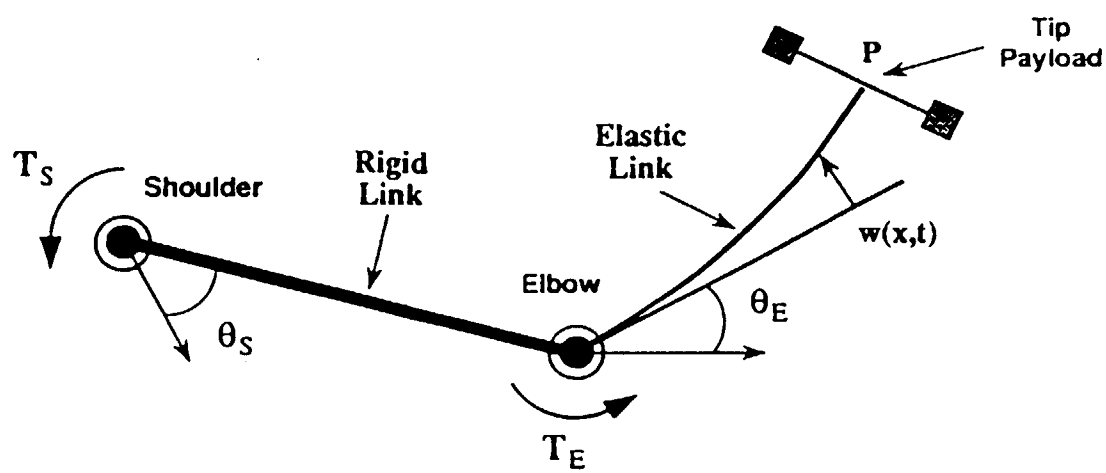
- Realization of noncausal compensators through distributed sensing.

  Temporal compensators, such as some matched terminations, cannot be implemented due to causality constraints.

PIEZO-POLYMER (PVDF) WHICH IS SENSITIVE TO STRAIN IS PARTICULARLY SUITED FOR USE AS AN AREA-AVERAGING SENSOR. IT CAN BE ACQUIRED IN SHEETS, BONDED TO THE SURFACE OF THE STRUCTURE AND THE WIDTH OF ITS SURFACE ELECTRODE CAN BE VARIED IN A DESIRED SHAPE OF SPATIAL WEIGHTING PATTERN. ONE DESIRABLE PATTERN IS THAT OF A STRAIN MODE SHAPE IN A STRUCTURE. M.I.T. SERC HAS DEVELOPED STRAIN MODE SENSOR FOR THE FIRST FOUR MODES OF A FLEXIBLE LINK IN A ROBOTIC MANIPULATOR LOCATED AT MARTIN MARRIETTA. THESE STRAIN MODE SENSORS ACT AS NARROW BAND SENSORS WITHOUT THE ASSOCIATED PHASE LAG ASSOCIATED WITH EQUIVALENT TEMPORAL SENSORS.

MANUFACTURE

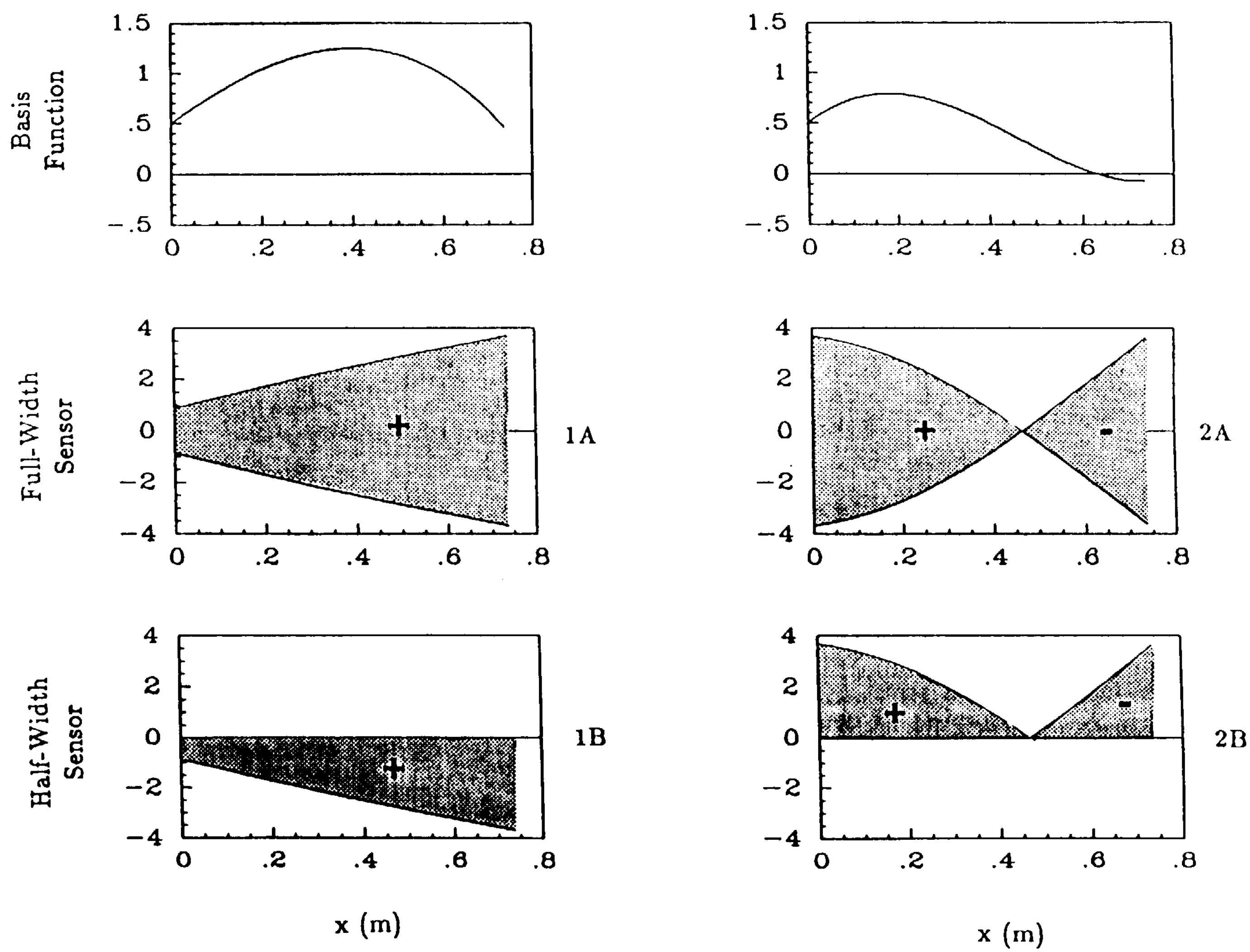
SENSORS DESIGNED FOR PLANAR RIGID-FLEXIBLE MANIPULATOR:



PVDF SENSORS SHAPED TO DETECT THE AMPLITUDES OF THE FIRST TWO NATURAL MODE SHAPES OF OUTER LINK WHEN ELBOW JOINT AXIS FIXED

THE WIDTH OF THE ELECTRODE IS VARIED PROPORTIONAL TO THE MAGNITUDE OF THE STRAIN MODE SHAPE. NEGATIVE VALUES IN THE MODE SHAPE ARE GENERATED BY EITHER FLIPPING THE PVDF IN THAT REGION, TO REVERSE THE POLARIZATION, OR WIRING TOGETHER THE SURFACES OF THE VARIOUS SEGMENTS TO APPROPRIATELY.

Eigenfunctions of beam pinned at one end with root inertia and tip mass and inertia.



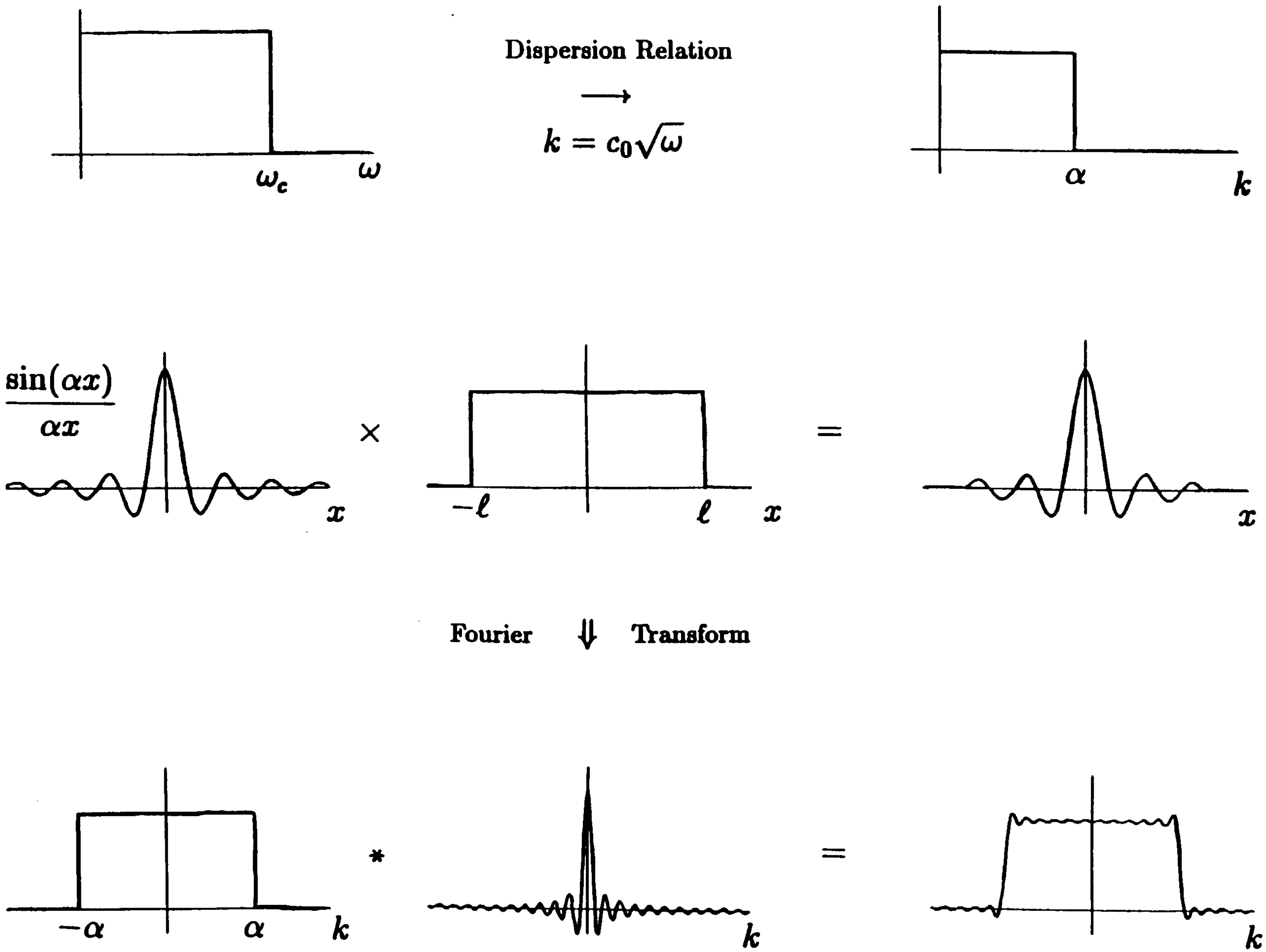
A SECOND MOTIVATION FOR THE USE OF AREA-AVERAGING SENSORS IS TO DEVELOP OTHER TYPES OF SENSORY DYNAMICS WHICH ARE NOT ACHIEVABLE USING A TEMPORAL COMPENSATOR. ONE SUCH EXAMPLE IS TO DEVELOP A SENSOR WHICH EXHIBITS MAGNITUDE ROLLOFF WITH FREQUENCY (A LOW PASS FILTER) WITHOUT THE ASSOCIATED PHASE LAG WHICH EXHAUSTS PHASE MARGIN IN A REALTIME CONTROL APPLICATION.

Motivation

- Want to roll off controller gain.
- Gain and phase are related in conventional designs.
- Sensor lag can exhaust phase margin.
- So, design sensor with rolloff but without phase lag.
- Can be achieved using past *and* future information.
- Future response at given location will be caused by energy which is presently distributed elsewhere in structure.
- Suggests use of spatially distributed structural measurements.

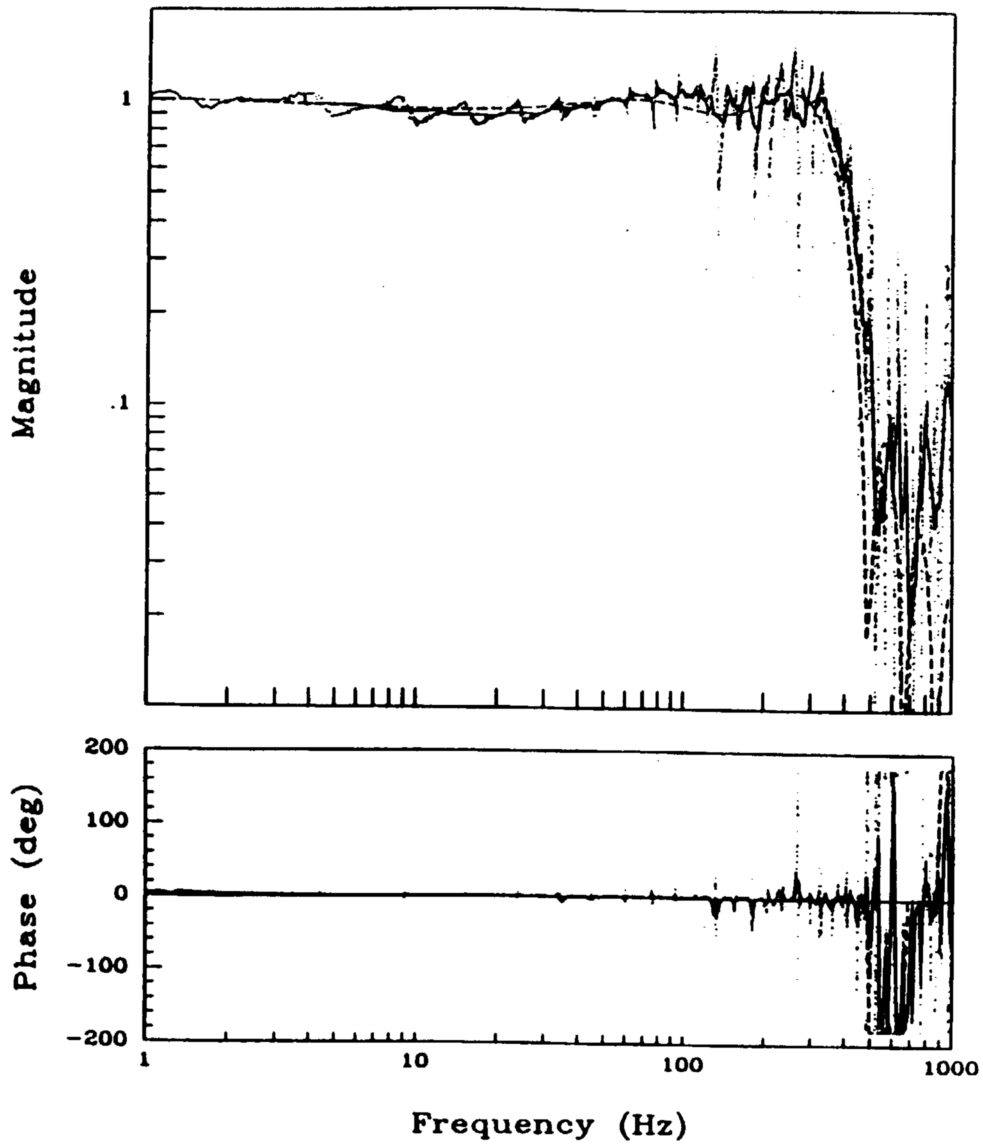
EXPONENTIAL ELECTRODE SHAPES CAN BE USED TO GENERATE FINITE ROLLOFF SLOPES. MORE IMPORTANTLY, HOWEVER, VERY SHARP CUTOFF CAN BE ACHIEVED BY USING AN AREA-AVERAGING SENSOR WHOSE ELECTRODE IS IN THE SHAPE OF A SINC FUNCTION. A LIMITATION TO THIS SENSOR IS THE FACT THAT THE SINC FUNCTION SHAPE IS INFINITE IN EXTENT AND MUST BE TRUNCATION, OR WINDOWED, IN ACTUAL APPLICATION. THIS WINDOWING CAUSES SOME DEGRADATION IN PERFORMANCE. DIFFERENT WINDOWING SCHEMES CAN BE EMPLOYED WHICH, IN CONJUNCTION WITH THE SENSOR SHAPE, ELIMINATES ALL PHASE VARIATIONS.

Approach — Sinc Function Sensor



A SINC FUNCTION SENSOR WHICH WAS SIMPLY TRUNCATED (BOXCAR WINDOW), WAS IMPLEMENTED AND THE TRANSFER FUNCTION FROM POINT STRAIN AT THE CENTER OF THE SENSOR TO THE OUTPUT OF THE SENSOR WAS CALCULATED AND MEASURED. THE TWO ARE IN GOOD AGREEMENT. THE 180 DEGREE PHASE REVERSALS ARE DUE TO THE BOXCAR WINDOW, A BARTLETT WINDOW (TRIANGULAR) ELIMINATES THESE PHASE REVERSALS AND THE TRANSFER FUNCTION REMAINS AT ZERO PHASE FOR ALL FREQUENCIES.

Experimental Results — Sinc Function Sensor

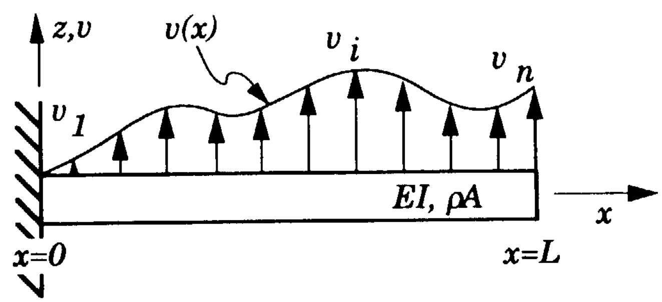


$$\frac{\bar{v}''}{v_c''}$$

From strain gauge signal to sinc function sensor signal (5 scale lengths).

A THIRD APPLICATION OF AN AREA-AVERAGING SENSOR IS AS THE EXACT FEEDBACK SOLUTION TO THE INFINITE DIMENSIONAL LINEAR QUADRATIC REGULATOR PROBLEM. SINCE STRUCTURES CAN BE MODELLED USING PARTIAL DIFFERENTIAL EQUATIONS, THE LQR PROBLEM IS NO LONGER A MATRIX PROBLEM BUT A FUNCTIONAL PROBLEM. THE FEEDBACK IS NO LONGER IN TERMS OF DISTRIBUTED POINT MEASUREMENTS BUT THE CONVOLUTION OF CONTINUOUSLY DISTRIBUTED STATE FUNCTIONS.

MOTIVATION



Structures are infinite order.

Exact model from PDE's subjected to BC's.

Exact LQR solution to exact model difficult to impossible to find.

In practice, use discrete models to find approximate LQR solution.

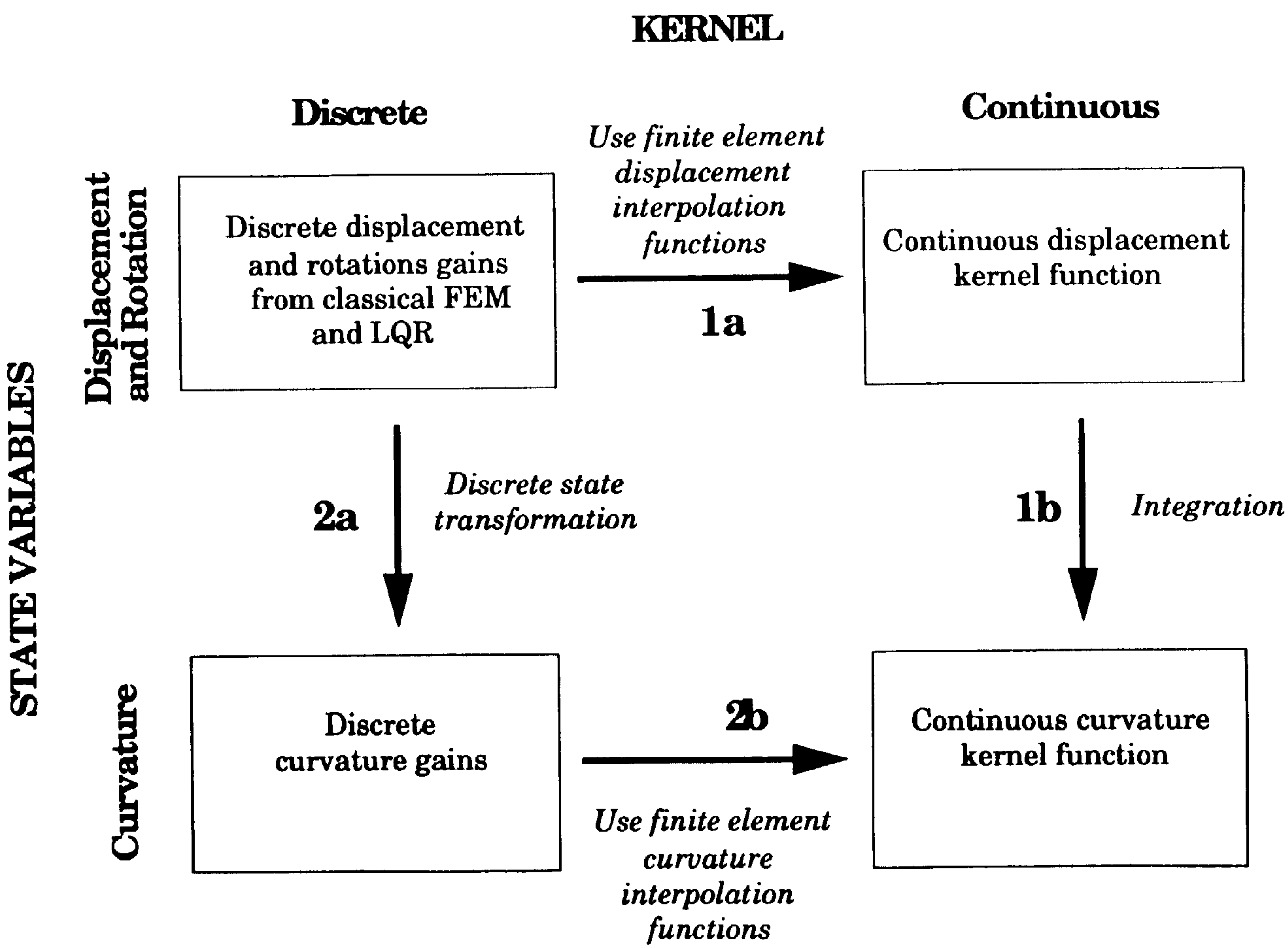
Can exact LQR solution be inferred from approximate LQR solution?

Can such an exact solution be implemented?

Are there alternative state functions which simplify implementation?

THE EXACT, INFINITE ORDER FEEDBACK KERNEL IS DIFFICULT TO IMPOSSIBLE TO FIND FOR COMPLEX STRUCTURES. HOWEVER, IT CAN BE INFERRED FROM THE SOLUTIONS OF FINITE DIMENSIONAL PROBLEMS. PATHS 1 AND 2 DEMONSTRATE TWO APPROACHES TO APPROXIMATING THESE FEEDBACK CONVOLUTION KERNELS.

APPROACHES TO FINDING THE FEEDBACK KERNEL



THE EXACT DISPLACEMENT AND VELOCITY FEEDBACK KERNELS CAN BE TRANSFORMED INTO STRAIN AND STRAIN RATE KERNELS AUGMENTED BY POINT INERTIAL MEASUREMENTS TO RETAIN RIGID BODY INFORMATION. THESE INERTIAL MEASUREMENTS CAN BE MOVED TO ANY DESIRED LOCATION ON THE STRUCTURE. NOW THAT THE DISTRIBUTED PORTION OF THE SENSOR IS EXPRESSED IN TERMS OF CURVATURE STRAIN, IT CAN BE IMPLEMENTED USING PVDF.

EQUIVALENT FEEDBACK USING ALTERNATIVE STATE FUNCTIONS

- Distributed displacement kernel can be expressed in terms of point inertial gains (rigid body control) and distributed curvature kernel.

$$
\begin{aligned}
u_D(t) &= \int_0^L \kappa_D(w) v(w,t)\,dw \\
&= v(0,t)\int_0^L \kappa_D(w)\,dw + \frac{\partial v(L,t)}{\partial x}\int_0^L\int_w^L \kappa_D(\gamma)\,d\gamma\,dw - \\
&\int_0^L\int_0^w\int_\gamma^L \kappa_D(\tau)\,d\tau\,d\gamma\,\frac{\partial^2 v(w)}{\partial w^2}\,dw
\end{aligned}
$$

- Point measurements can be relocated within the structure (locations are not unique).

$$\frac{\partial v(L,t)}{\partial x} = \frac{\partial v(0,t)}{\partial x} + \int_0^L \frac{\partial^2 v(w,t)}{\partial w^2}\,dw$$

- This results in a change of integration limits to find the curvature kernel.

A FINAL WAY IN WHICH AREA-AVERAGING SENSORS ARE BEING USED IN THE RESEARCH AT M.I.T. SERC IS AS NONCAUSAL FEEDBACK COMPENSATORS. HERE THE TERM NONCAUSAL IS NOT BEING USED CORRECTLY. MORE TO THE POINT, TEMPORAL COMPENSATORS WHICH ARE NONCAUSAL CAN BE IMPLEMENTED IN REALTIME BY USING AN AREA-AVERAGING SENSOR TO REALIZE THE COMPENSATOR SPATIALLY RATHER THAN TEMPORALLY. ONE APPLICATION IS AS A WAVE MODE SENSOR WHICH MEASURES NOT ONLY THE MAGNITUDE OF THE RESPONSE BUT ALSO THE DIRECTION IN WHICH THE ENERGY CAUSING THE RESPONSE IS PROPAGATING. A SECOND APPLICATION IS AS A MATCHED TERMINATION WHICH ABSORBS ALL VIBRATIONAL ENERGY WHICH ARRIVES AT A PARTICULAR LOCATION.

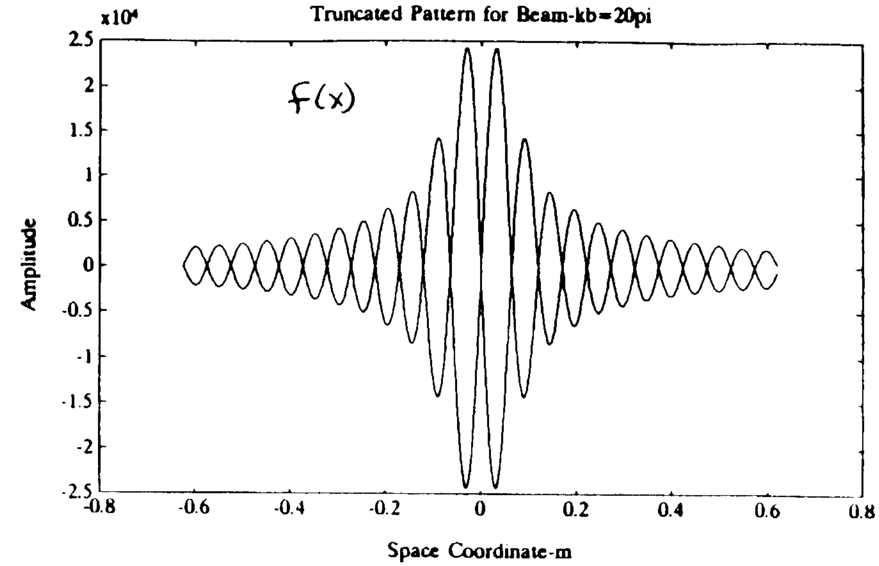
MOTIVATION: Why Sense Waves?

- Compute Experimental Wave Response of a Structure
- Determine How Power Flows in a Structure-(Magnitude and Direction)
- Compute Experimental Scattering Coefficients for various Structural Discontinuities
- Facilitate Passive Control Methods
- Implement Causal and Noncausal Collocated Wave Compensator Designs for Active Isolation and Damping.
- Learn the Wave Model

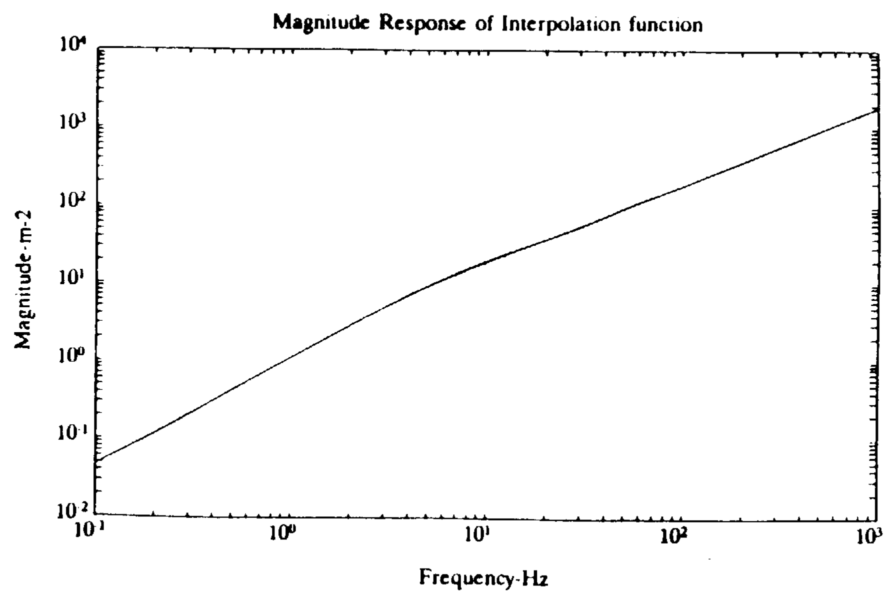
THE SPATIAL PATTERN SHOWN ON THE VIEWGRAPH IS AN EXAMPLE OF A DISTRIBUTED SENSOR PATTERN WHICH CAN DIFFERENTIATE RESPONSE BASED UPON DIRECTION OF PROPAGATION.

PROPERTIES OF WEIGHTING PATTERN

•Shape:

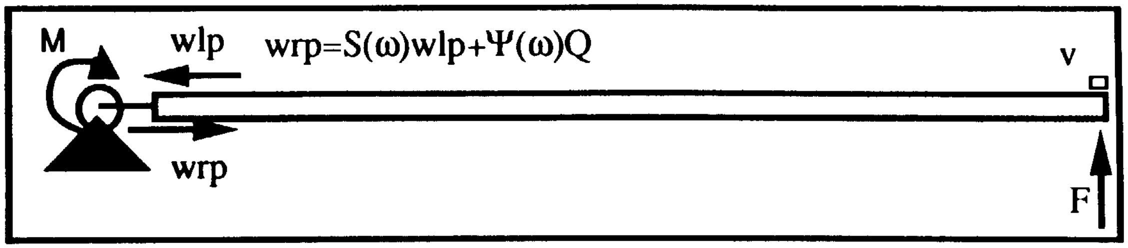


•Transform Charactertiestics:



THE MATCHED TERMINATION FOR THE PINNED END OF A BERNOULLI-EULER BEAM USING A MOMENT ACTUATOR IS THE FEEDBACK OF COLLOCATED ROTATION THROUGH A COMPENSATOR OF THE FORM $(-s)^{1/2}$. THIS COMPENSATOR IS NONCAUSAL. IF THE COLLOCATION RESTRICTION IS REMOVED AND A DISTRIBUTED SENSOR IS USED, IT IS POSSIBLE TO REALIZE THE SAME PERFORMANCE IN REALTIME.

APPLICATION TO NONCAUSAL WAVE CONTROL
(A Pinned-Free Example)



•Non-Realizable Form of Noncausal Control at Pinned End with Moment Actuation-(Near Field components assumed to be negligible)

$$M = \sqrt{2}\, EI\beta \sqrt{-s}\, [1 \;\; 0] \begin{Bmatrix} v' \\ -EIv'' \end{Bmatrix}; \quad \beta = \left(\frac{\rho A}{EI}\right)^{1/4}$$

$$= EI\beta^2 \omega (1 + i)(w_{rp}(0, \omega) - w_{lp}(0, \omega))$$

•Realizable Form of compensation using a convolving sensor

$$M = \frac{-\sqrt{2}\, EI}{\sqrt{s}\, \beta} \int_0^d (e^{-ax'} + e^{-bx'} - (a + b)\delta(x'))\beta^2 \omega \left(w_{rp}(0, \omega) e^{-ikx'} + w_{lp}(0, \omega) e^{ikx'}\right) dx'$$

$$\approx EI\beta^2 \omega (1 + i)(w_{rp}(0, \omega) - w_{lp}(0, \omega)); \quad k < 0.1\sqrt{ab}$$

CONCLUSIONS

Spatial properties of structures can be exploited to provide:

- Performance which is not achievable through temporal compensation,
- Reduced sensitivity to short wavelength, poorly modelled phenomena,
- Direct measurement of propagating energy in structures,
- The natural feedback architecture for structural control.

Spatially distributed strain measurements are typically used because:

- PVDF facilitates implementation,
- Distributed displacement and rotation measurements can be readily converted to distributed strain measurements augmented by point inertial measurements,
- Strain measurements are directly related to internal structural deformations.

Area-averaging sensors are applicable to:

- Control of flexible structures,
- Control of aeroelastic structures,
- Energy transmission control in isolation systems.

MIT Space Engineering Research Center

THE ZEROES OF CONTROLLED STRUCTURES:SENSOR AND ACTUATOR ATTRIBUTES AND STRUCTURAL MODELLING

Farla M. Fleming
Edward F. Crawley

C-3

THE ZEROES OF CONTROLLED STRUCTURES TRANSFER FUNCTIONS ARE AS IMPORTANT AS THE POLES FOR CONTROLLED STRUCTURE DESIGN. THE STUDY OF ZEROES HOWEVER HAS NOT BEEN EXTENSIVE. THERE ARE MANY REASONS WHY ZEROES ARE MORE DESIRABLE FOR THE PURPOSES OF ANALYSIS AND DESIGN. RESIDUES WILL BE LATER DEFINED.

Motivation

- Zeroes, like residues, effectively encapsulate information regarding the input to output characteristics of a system.
- Zeroes may be found directly from experimental data, or easily estimated from transfer functions.
- Zeroes allow for a direct assessment in the frequency domain of the impact on performance of various design and modelling issues.
- Zeroes can be calculated exactly while an infinity of the residues can not.
- Zeroes are a reflection of the actuator and sensor attributes as well as the structure, just as the poles are a direct reflection of the nature of the structure.

FOR SINGLE INPUT SINGLE OUTPUT SYSTEMS, THE SPACING BETWEEN POLES AND ZEROES DETERMINES THE EFFECTIVENESS OF FEEDBACK TO DAMP OR STIFFEN A SYSTEM. THROUGH STUDY OF THE ZEROES, THE EFFECT OF SENSOR AND ACTUATOR TYPE, IMPEDANCE AND LOCATION ON THE ACHIEVABLE PERFORMANCE OF A CONTROL SYSTEM CAN BE UNDERSTOOD.

JUST AS A MODEL HAS AN EFFECT ON THE PREDICTION OF THE POLES OF A SYSTEM, THE EFFECT ON THE ZEROES OF STRUCTURAL MODELLING WILL BE STUDIED AS WELL IN ORDER TO PROPOSE TECHNIQUES FOR IMPROVING ZERO PREDICTION ACCURACY. STRUCTURAL MODELLING CAN BE IMPROVED FOR THE PURPOSE OF ACCURATELY PREDICTING THE ZEROES, AS OPPOSED TO THE STANDARD PERSPECTIVE OF CAPTURING THE STRAIN ENERGY OR MOMENTUM OF THE SYSTEM AS IN THE CASE OF POLES BUT .

Objective

The objective of this work is to demonstrate how the zero frequencies are effected by the design decisions of sensor and actuator, namely their type, their impedance relative to the structure, and their location and to develop methods for improving structural modelling and to increase the accuracy of zero predictions.

TO STUDY THE ZEROES OF SINGLE INPUT SINGLE OUTPUT SYSTEMS A VARIETY OF DEFINITIONS CAN BE USED TO DESCRIBE THEM, THREE OF WHICH ARE DEFINITIONS IN THE FREQUENCY DOMAIN:

- -TRANSFER FUNCTION DEFINITION
- -RESIDUE EXPANSION DEFINITION
- -ROOT LOCUS DEFINITION

AND ONE DEFINITION OF A ZERO IN THE TIME DOMAIN:

- -TRANSMISSION BLOCKING DEFINITION

THE ZERO DEFINITIONS ASSUME A SINGLE INPUT SINGLE OUTPUT (SISO) SYSTEM DESCRIPTION OF THE FORM:

$$\dot{x} = Ax + bu$$

$$y = cx$$

ALL OF THESE DEFINITIONS YIELD NUMERICALLY EQUIVALENT ZEROES, AND HENCE THE DEFINITIONS MAY BE USED AS NEEDED FOR ANALYSIS OR DESIGN. THE SISO DEFINITIONS GIVEN HERE MAT BE EASILY EXTENDED TO THE MULTI INPUT MULTI OUTPUT (MIMO) CASE.

Definitions of Zeroes

There are **two** types of zeroes for the SISO case: transmission zeroes and their limiting case of pole-cancellation zeroes.

Transfer Function Definition

$$\frac{y(s)}{u(s)}=c(sI-A)^{-1}b=\frac{\psi(s)}{\phi(s)}=\prod_i\frac{(s-z_i)}{(s-p_i)}$$

- Zeroes are the roots of $\psi(s)$. Observability and controllability tests must be calculated separately. Pole-cancellation zeroes may be "lost".

Residue Expansion Definition

$$\frac{y(s)}{u(s)}=c(sI-A)^{-1}b=\frac{\psi(s)}{\phi(s)}=\sum_i\frac{cv_iw_i^Tb}{(s-p_i)}$$

- Zeroes are the frequencies at which the summation is zero or when $cv_iw_i^Tb$, the modal residue, is zero. Observability and controllability are available directly.

THE DEFINITIONS MAY BE EVALUATED AS WELL FOR THEIR ABILITY TO PROVIDE DIFFERENT INFORMATION. FOR EXAMPLE, CERTAIN DEFINITIONS PROVIDE MODAL CONTROLLABILITY AND OBSERVABILITY DIRECTLY, NAMELY THE RESIDUE EXPANSION DEFINITION AND THE TRANSMISSION BLOCKING DEFINITION. MODAL OBSERVABILITY AND CONTROLLABILITY MUST BE CALCULATED SEPARATELY FROM THE TRANSFER FUNCTION DEFINITION AND ROOT LOCUS DEFINITION. MODAL CONTROLLABILITY AND OBSERVABILITY ARE KEY PROPERTIES FOR ASSESSING THE ACHIEVABLE PERFORMANCE OF A SYSTEM ON A MODE BY MODE BASIS.

Definitions of Zeroes, Cont'd

Transmission Blocking Definition

A plant has a zero at a frequency z_k if there exists a vector of initial states ξ_k, and a force u_k, not both zero so that the scalar output $y(t)$, is zero for all $t \geq 0$. This translates to a generalized eigenvalue problem

$$\begin{bmatrix} z_k I - A & -b \\ -c & 0 \end{bmatrix} \begin{bmatrix} \xi_k \\ u_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- Observability and controllability are available from the zero direction (eigenvector of the generalized eigenvalue problem). This definition gives an eigenvalue formulation of the problem.

Root Locus Definition

The finite zeroes of $g(s)=c(sI-A)^{-1}b$ are the finite zeroes of the return difference $1+1\backslash\rho g(s)$ in the limit as $\rho \to 0$.

- Observability and controllability must be calculated separately. This definition yields the result that the zeroes are the poles at high gain of the closed loop.

AS MAY BE SEEN IN THE ZERO DEFINITIONS, THE ZEROES ARE A FUNCTION NOT ONLY OF THE POLES OF A SYSTEM BUT OF THE ACTUATION AND SENSING MECHANISMS AS WELL. THIS CAN BE SEEN BY THE ROLE PLAYED BY THE SYSTEM MATRIX A, AS WELL AS THE CONTROL INFLUENCE MATRIX B AND MEASUREMENT MATRIX C.

IN FACT, THE ACTUATION AND SENSING MECHANISMS MAY BE DEFINED BY THREE ATTRIBUTES OF AN ACTUATOR OR SENSOR.

Sensor and Actuator Attributes

Zeroes may be considered a combined measure of modal controllability and observability. Their locations are reflections of the following attributes of the sensor/actuator pair.

- TYPE

  Sensor and actuator of the same type are referred to as DUAL.

- IMPEDANCE relative to the structure

  Sensor and actuator of the same impedance are referred to as COMPLEMENTARY.

- LOCATION

  Sensor and actuator at the same location are referred to as COLLOCATED.

THE TYPE OF AN ACTUATOR OR SENSOR REFERS TO THE AXIS ABOUT OR ALONG WHICH THE SENSOR OR ACTUATOR ACTS, ITS DISTRIBUTED OR LOCALIZED NATURE, AND ITS REACTION CHARACTERISTICS, RELATIVE OR INERTIALLY REACTING.

THE IMPEDANCE OF AN ACTUATOR OR SENSOR RELATIVE TO A STRUCTURE REFERS TO WHETHER AN ACTUATOR OR SENSOR IS STIFF OR FLEXIBLE RELATIVE TO THE STRUCTURE. A DISPLACEMENT AND A FORCE ACTUATOR ARE AT OPPOSITE ENDS OF THE IMPEDANCE SPECTRUM.

THE LOCATION OF AN ACTUATOR OR SENSOR IS THE PLACEMENT OF THE ACTUATOR OR SENSOR ON THE STRUCTURE.

NOTE THAT THE ZEROES ARE NOT A FUNCTION ONLY OF THE LOCATION OF THE SENSOR AND ACTUATOR.

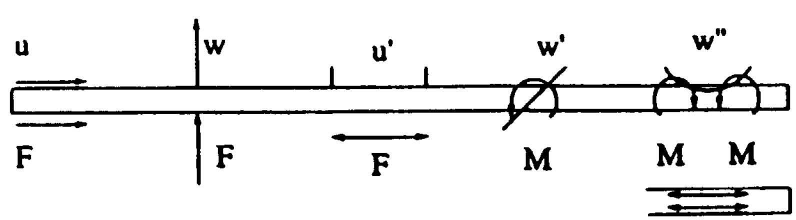
Type of Actuator and Sensor Pairs

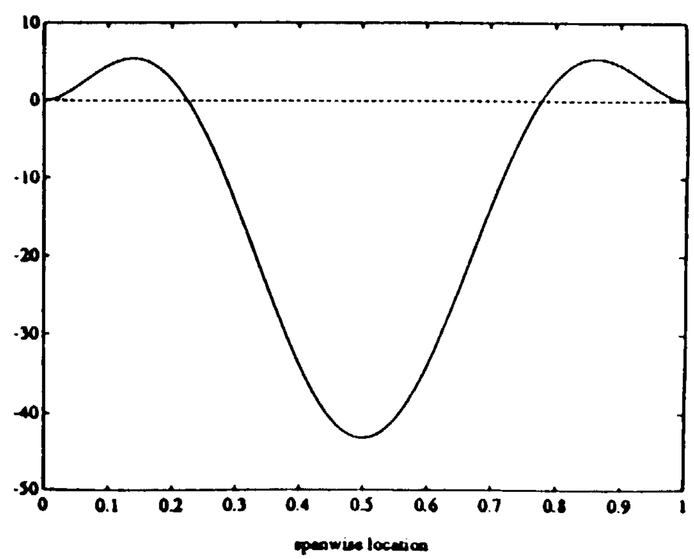
Type may be characterized by the sensor or actuator's

- direction (transverse, axial...)
- spatial distribution (point or distributed)
- reaction characteristics (requiring an inertial reference, or reacting relative to another portion of the structure)

- When the sensor and actuator pair are collocated and dual (i.e. the same type) and complementary extremes, positive residues result for all modes.

- Alternating pole-zero patterns arise when the modal residues are all consistently of the same sign.





GEVARTER SHOWED THAT FOR A SINGLE INPUT SINGLE OUTPUT SYSTEM, A SENSOR AND ACTUATOR PAIR WHICH PRODUCES RESIDUES ALL OF THE SAME SIGN HAS THE POLES AND ZEROES ALTERNATE UP THE IMAGINARY AXIS. SUCH A CONFIGURATION IS DESIRABLE FOR ROBUSTNESS PROPERTIES OF THE SYSTEM.

CLEARLY SENSORS AND ACTUATORS WHICH ARE DUAL, COLLOCATED AND COMPLEMENTARY EXTREMES WILL HAVE ALL POSITIVE RESIDUES. COMPLEMENTARY EXTREMES ARE SENSOR/ACTUATOR PAIRS WHICH ARE PURE FORCE/DISPLACEMENT OR DISPLACEMENT/FORCE PAIRS AS OPPOSED TO SOME INTERMEDIATE MIX. EXAMPLES OF SUCH SENSOR/ACTUATOR PAIRS ARE SHOWN IN THE FIGURE TO THE LEFT.

IN FACT, SENSOR/ACTUATOR PAIRS WHICH ARE NON-DUAL, COLLOCATED AND COMPLEMENTARY EXTREMES WILL PRODUCE RESIDUES ALL OF THE SAME SIGN AND HENCE PRODUCE ALTERNATING POLE-ZERO PATTERNS. THE COMBINATION OF DUALITY, COLLOCATION AND COMPLEMENTARY EXTREMES IS A SUFFICIENT BUT NOT NECESSARY CONDITION FOR ALTERNATING PATTERNS.

THE FIGURE TO THE RIGHT DISPLAYS THE FUNCTION $\Phi(X)\Phi''(X)$ FOR THE FIRST MODE OF A FREE-FREE BERNOULLI EULER BEAM WHERE $\Phi(X)$ IS THE TRANSVERSE DISPLACEMENT MODE SHAPE. THIS FUNCTION $F(X)F''(X)$ CAN BE USED TO TEST STRUCTURES OR AREAS OF STRUCTURES WHICH WITH NON-DUAL BUT COLLOCATED AND COMPLEMENTARY EXTREME SENSOR/ACTUATOR PAIRS WILL PRODUCE ALTERNATING POLE-ZERO PATTERNS. IN THIS PARTICULAR CASE, THE PRODUCT $\Phi(X)\Phi''(X)$ SHOWS THE REGIONS ALONG THE STRUCTURE GOVERNED BY SPATIALLY EXPONENTIAL BEHAVIOR, WHERE $\Phi(X)\Phi''(X) > 0$, AND THOSE REGIONS GOVERNED BY SPATIALLY SINUSOIDAL BEHAVIOR WHERE $\Phi(X)\Phi''(X) < 0$.

FOR DUAL, COLLOCATED, AND COMPLEMENTARY EXTREME SENSOR AND ACTUATOR PAIRS, ALTERNATING POLE-ZERO PATTERNS RESULT IRRESPECTIVE OF THE STRUCTURE ON WHICH THE SENSOR/ACTUATOR PAIR ACT.

FOR THE NON-DUAL CASE, THIS IS NO LONGER TRUE. WHILE IT IS STILL POSSIBLE FOR CERTAIN SENSOR AND ACTUATOR COMBINATIONS, THE NATURE OF THE STRUCTURE IS CRITICAL AS WELL TO PRODUCE THE ALTERNATING POLE-ZERO PATTERNS.

THE NATURE OF THE STRUCTURE IS TAKEN TO MEAN WHETHER THE STRUCTURE MAY BE DESCRIBED BY SPATIALLY SINUSOIDAL OR SPATIALLY EXPONENTIAL BEHAVIOR. IN FACT, THESE CATEGORIES MAY BE BROADENED TO BE THOSE THAT ARE CHARACTERIZED BY $\Phi(X)\Phi''(X)>0$ OR $\Phi(X)\Phi''(X)<0$ AT A GIVEN LOCATION.

A PSEUDO-DUAL SENSOR AND ACTUATOR PAIR IS ONE WHERE A NON-DUAL, COLLOCATED AND COMPLEMENTARY EXTREME PAIR PRODUCES ALTERNATING POLE-ZERO PATTERNS.

Type of Actuator and Sensor Pair, Cont'd

There is a class of sensor and actuator pairs known as PSEUDO-DUAL which produce consistently positive or negative residues for certain classes of structures. Pseudo-duals can be used in place of duals to obtain desirable alternating or missing pole-zero patterns.

Pseudo-duals can be discussed with reference to two pure classes of structures, those governed completely by spatially sinusoidal functions, and those by spatially exponential functions.

TWO TABLES WERE CONSTRUCTED FOR A VARIETY OF SENSOR AND ACTUATOR COMBINATIONS. ONE TABLE CORRESPONDS TO THE CASE OF A STRUCTURE REPRESENTED BY SPATIAL SINUSOIDS EXCLUSIVELY, THE OTHER BY SPATIAL EXPONENTIALS.

CERTAIN SENSOR AND ACTUATOR PAIRS ARE THEN CLASSIFIED AS BEING DUAL OR PSEUDO-DUAL FOR THE SPECIFIC STRUCTURE, WHERE PSEUDO-DUAL DESCRIBES A NON-DUAL SENSOR AND ACTUATOR PAIR THAT STILL PRODUCES AN ALTERNATING POLE-ZERO PATTERN. A MINUS SIGN IS INCLUDED WHEN THE CHOICE OF SENSOR AND ACTUATOR PRODUCES RESIDUES ALL OF THE SAME SIGN AND HENCE ALTERNATING POLE-ZERO PATTERNS, BUT THE RESIDUES ARE CONSISTENTLY NEGATIVE.

THE SENSOR AND ACTUATOR CHOICES ARE THE SAME AS THOSE DEPICTED IN THE LEFT FIGURE TWO VIEWGRAPHS EARLIER.

Dual and pseudo-dual sensor/actuator pairs for structures represented by spatial sinusoids

	u, w	w', w'	w''
force	DUAL		(-) PSEUDO-DUAL
force couple/moment		DUAL	
moment couple	(-) PSEUDO-DUAL		DUAL

Dual and pseudo-dual sensor/actuator pairs for structures represented by spatial exponentials

	u,w	w', w'	w''
force	DUAL		PSEUDO-DUAL
force couple /moment		DUAL	
moment couple	PSEUDO-DUAL		DUAL

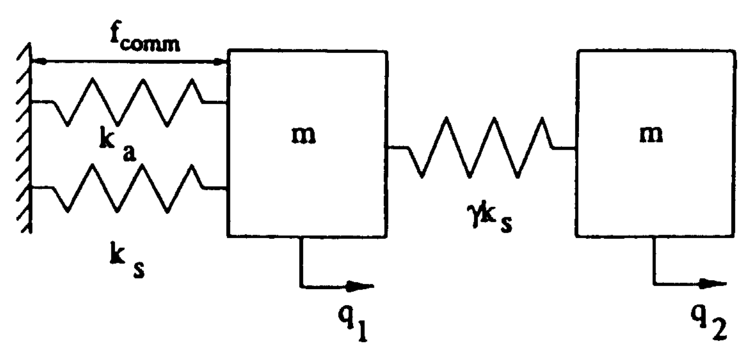
THE IMPORTANCE OF SENSOR AND ACTUATOR IMPEDANCE RELATIVE TO THE STRUCTURE HAS BEEN ACKNOWLEDGED BY FANSON ET.AL. AND BY CHEN ET.AL. AT JPL.

AN EXAMPLE OF THE EFFECT OF THE IMPEDANCE OF THE ACTUATOR RELATIVE TO THE STRUCTURE IS IN THE INSTANCE OF A PIEZOCERAMIC ACTUATOR BONDED TO THE SURFACE OF A PLATE-LIKE STRUCTURE. FOR A STIFF ACTUATOR, THE ACTUATOR HAS INFINITE AUTHORITY OVER THE STRUCTURE AND DEFLECTS IT AT WILL, ACTING AS A DISPLACEMENT SOURCE. CONVERSELY A VERY FLEXIBLE ACTUATOR RELATIVE TO THE STRUCTURE WILL NOT DEFLECT THE STRUCTURE AT ALL BUT INSTEAD APPLY A LOAD AT THE ACTUATOR LOCATION.

BY MEANS OF A SIMPLE MODEL, THE EFFECTS OF THE IMPEDANCE OF THE ACTUATOR RELATIVE TO THE STRUCTURE COULD BE FOUND. THE ACTUATOR STIFFNESS K_A COULD BE VARIED, AND CORRESPONDINGLY, THE ACTUATION MECHANISM COULD BE TRANSLATED FROM ONE END OF THE IMPEDANCE SPECTRUM TO THE OTHER.

Impedance of Actuator and Sensor Pairs

- The relative stiffness or mechanical impedance of the actuator to the structure determines whether the actuator acts as a generalized force actuator or a generalized displacement actuator along a certain axis.
- Similarly, the relative stiffness or mechanical impedance of the sensor to the structure determines whether the sensor acts as a generalized force sensor or a generalized displacement sensor along a certain axis.
- The pole-zero spacing can be altered by parametrically varying the impedance of the sensor and the actuator .
- This effect was studied by means of a simple model



NORMALIZING SO THAT $0<B<1$ TRAVERSES THE ACTUATION SPECTRUM FROM A PURE FORCE TO A PURE DISPLACEMENT, AND $0<D<1$ TRAVERSES THE SENSING SPECTRUM FROM A PURE DISPLACEMENT TO A PURE FORCE, A PARAMETRIC STUDY CAN BE PERFORMED.

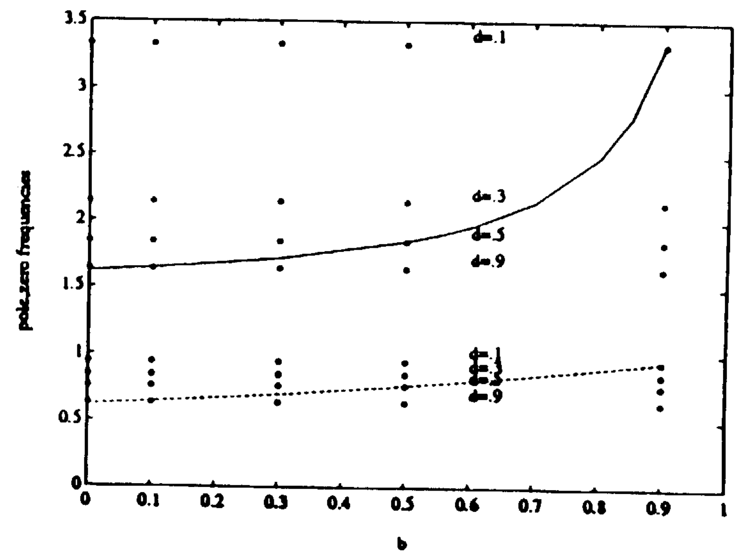
IN THE FIGURE, THE POLES OF THE SYSTEM ARE GIVEN BY THE DASHED AND SOLID LINES AND THE ZEROES BY THE CIRCLES AS A FUNCTION OF BOTH B AND D.

POLE-ZERO CANCELLATION OCCURS WHEN $B=1-D$.

THESE PRELIMINARY PREDICTIONS AGREE WELL WITH THE ACTUAL EXPERIMENTAL TRANSFER FUNCTIONS PRODUCED BY FANSON ET.AL. AND CHEN ET.AL.

Results of Parametric Impedance Study

- As b, the relative actuator stiffness rises, the pole migrates upwards.
- As d, the relative sensor stiffness rises, the zero migrates downwards.
- Force actuator and displacement sensor or vice versa give greatest spacing.
- Pole-zero cancellation can occur for incorrect actuator and sensor impedance choices.



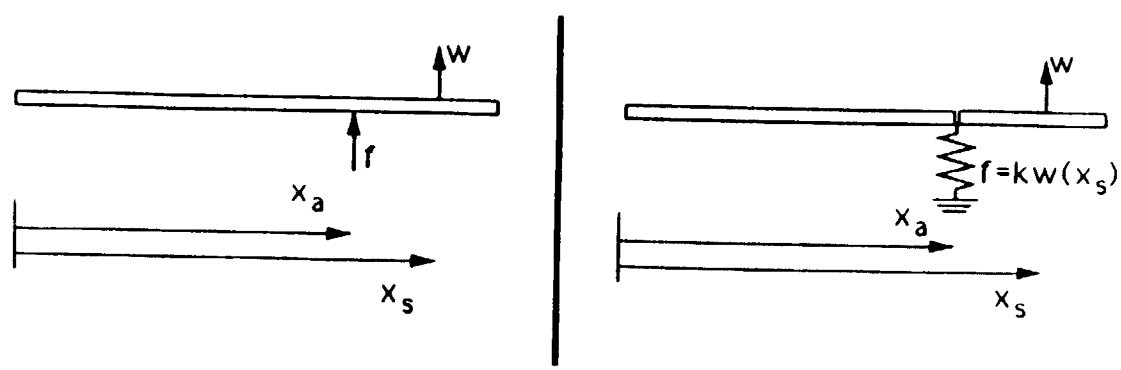
A PARAMETRIC STUDY OF THE SENSOR/ACTUATOR PAIR TO LOCATION WAS UNDERTAKEN, AND BY USING AN EXACT SOLUTION BASED ON THE PARTIAL DIFFERENTIAL EQUATION MODEL, NO SPURIOUS EFFECTS WERE PRESENT IN PREDICTING THE ZEROES AS A FUNCTION OF LOCATION.

THE SOLUTION METHOD TO FIND THE EXACT ZEROS WAS BASED ON THE FACT THAT THE ZEROES OF AN OPEN LOOP TRANSFER FUNCTION ARE EQUAL TO THE POLES OF THE CLOSED LOOP SYSTEM WITH INFINITE GAIN.

THE FEEDBACK ELEMENT WAS THEN PHYSICALLY REALIZED IN THE MODEL AND THE SOLUTION WAS FOUND AS THE GAIN ON THE FEEDBACK ELEMENT WAS INCREASED TO INFINITY.

Location of Actuator and Sensor Pairs

- A method was developed for exact solution of zeroes.
- Solution method relies on the definition that states that the zeroes of an open loop transfer function are equal to the poles of the closed loop system with infinite gain
- An internal boundary is used at the location of the point of application of the force actuator.
- Compatibility are enforced at the internal boundary.
- The feedback is incorporated into the equilibrium equations at the internal boundary.
- A generalized eigenvalue problem results which is larger than that of the original.



WHILE THE METHOD IS VERY GENERAL AND CAN BE USED AS WELL TO PROVE CONVERGENCE BEHAVIOR OF THE ZERO PREDICTIONS FROM APPROXIMATE METHODS, THE BEHAVIOR OF THE ZEROES WAS EXAMINED ON A VARIETY OF SIMPLE STRUCTURES, AND FOR A SINGLE ACTUATOR/SENSOR PAIR TYPE- A FORCE /DISPLACEMENT PAIR. FOR THE BEAM EXAMPLES, THE SENSOR AND ACTUATOR ACTED IN THE TRANSVERSE DIRECTION, FOR RODS, IN THE AXIAL DIRECTION.

AS AN EXAMPLE OF THE METHOD A BERNOULLI-EULER BEAM PROBLEM IS DISCUSSED.

Example

- For a Bernoulli-Euler beam with one force actuator, two separate solutions are assumed, one governing each region of the structure.

- For the simple case of constant gain feedback, a linear force actuator, the feedback mechanism is a force at x_a proportional to the displacement at the sensor x_s.

- The shear equilibrium at the internal boundary is given by

$$EI\frac{d^3 w_1}{dx^3}(x_a)-EI\frac{d^3 w_2}{dx^3}(x_a)-kw(x_s)=0$$

- The generalized eigenvalue problem that arises is of order twice as large as an ordinary structural eigenvalue problem. The limit is taken as the feedback gain is made infinitely large.

- The roots of the resulting transcendental equation yield the zeroes exactly as a function of the sensor and actuator location.

THE PARAMETRIC STUDY BEGINS FIRST WITH THE COLLOCATED CASE. HOW DO THE ZERO FREQUENCIES VARY AS THE COLLOCATED, DUAL COMPLEMENTARY EXTREME SENSOR/ACTUATOR PAIR IS MOVED ALONG THE LENGTH OF THE STRUCTURE?

A BERNOULLI-EULER BEAM WITH FREE-FREE BOUNDARY CONDITIONS WAS ADOPTED FOR THIS CASE. THE FREQUENCIES OF THE POLES AND ZEROES ARE PLOTTED AS Λ_I AS A FUNCTION OF THE SPANWISE LOCATION OF THE COLLOCATED SENSOR/ACTUATOR PAIR. Λ_I IS RELATED TO Ω_I BY THE RELATION

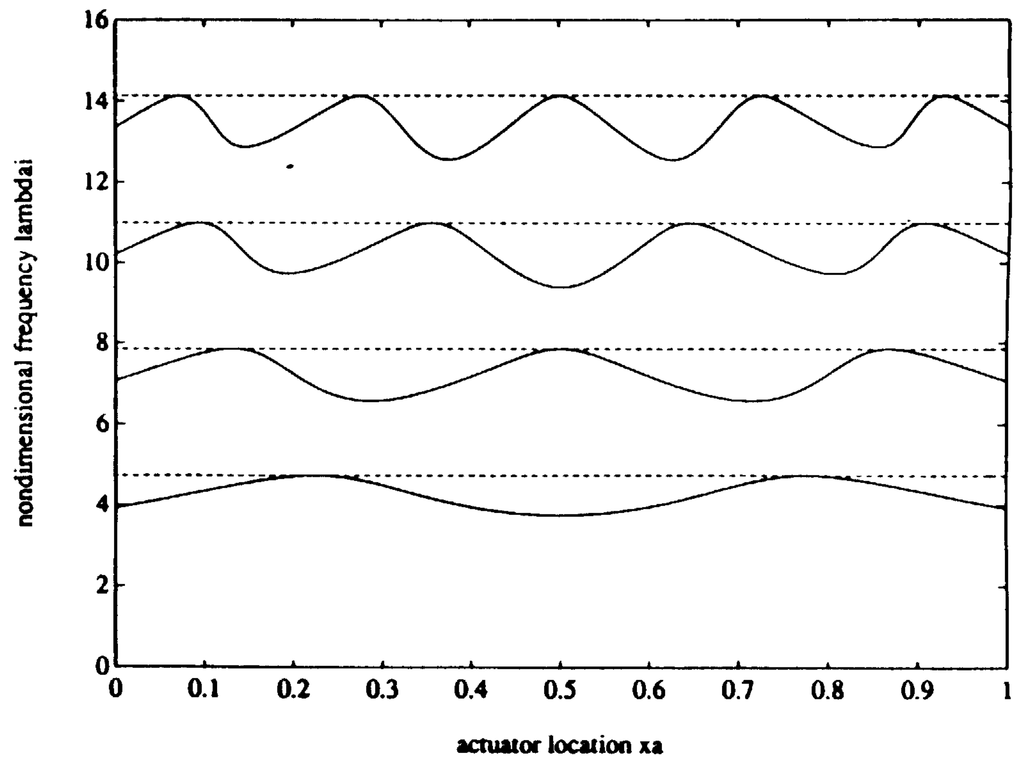
$$\omega_i = \lambda_i^2 \sqrt{\frac{EI}{mL^4}}$$

THE ZERO FREQUENCIES ARE GIVEN BY THE SOLID LINES AND THE POLE FREQUENCIES BY THE DASHED LINES. THE FIRST FOUR ZEROES AND POLES ARE SHOWN.

AT ANY SENSOR/ACTUATOR PAIR LOCATION, THE POLE AND ZERO FREQUENCIES ALTERNATE.

Free-Free Beam with Collocated Displacement Sensor and Force Actuator

- Plant symmetry ensures symmetry of the zero trajectory. Collocated and dual sensor and actuator pair ensures alternating pole-zero pattern.
- Slope of the zero trajectory is always zero at a pole-zero cancellation.
- Zeroes bounded from above and below by poles, although zero trajectory never reaches lower bound for unconstrained open loop plants.



WHILE THE DATA FROM THE ZERO TRAJECTORIES YIELDS INFORMATION ON THE SENSITIVITY OF THE ZERO FOR THAT PARTICULAR STRUCTURE AND SENSOR AND ACTUATOR CONFIGURATION OR MORE GENERAL RESULT FOR THE SENSITIVITY OF THE ZERO IS FOUND BY DOING A PERTURBATION ANALYSIS ON THE GENERALIZED EIGENVALUE PROBLEM OF THE TRANSMISSION BLOCKING DEFINITION OF A ZERO.

THE SUPERSCRIPT (0) CORRESPONDS TO THE ORIGINAL SYSTEM, (1) TO THE LINEAR CONTRIBUTION IN A TAYLOR SERIES EXPANSION OF THE VARIABLES ABOUT THE NOMINAL CONDITION TO PARAMETER VARIATIONS.

THE ZERO DIRECTIONS ARE VERY IMPORTANT IN THE SENSITIVITY ANALYSIS. FOR THE COLLOCATED CASE, THE RIGHT AND LEFT ZERO DIRECTIONS HAVE A SPECIAL FORM DUE TO THE SIMULTANEOUS LOSS OF CONTROLLABILITY, WTILDA^T=[$W_I{}^T$ 0], AND OBSERVABILITY, VTILDA=[V_I 0] WHERE W^T AND V ARE THE LEFT AND RIGHT EIGENVECTORS OF THE UNCONTROLLABLE AND UNOBSERVABLE MODE I.

IN THE NON-COLLOCATED CASE, A LOSS OF OBSERVABILITY OCCURS DISTINCTLY SEPARATE FROM A LOSS OF CONTROLLABILITY.

Sensitivity of the Zero Frequencies

- Using the transmission blocking definition of the zero, the first derivative of the zero with respect to a parameter variation may be found

$$z_k^{(1)} = \frac{[w_k^T]^{(0)} \mathcal{A}^{(1)} v_k^{(0)}}{[w_k^T]^{(0)} \mathcal{B}\, v_k^{(0)}}$$

where

$$\mathcal{A}^{(1)} = \begin{bmatrix} A^{(1)} & b^{(1)} \\ c^{(1)} & d^{(1)} \end{bmatrix} \qquad \mathcal{B} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$$

and w_k^T and v are the right and left zero direction vectors.

- A simultaneous loss of observability and controllability occurs for the collocated and dual case. The sensitivity of the zero at such a pole-zero cancellation to changes in the sensor or actuator attributes is zero.

- The sensitivity of the zero at a pole-zero cancellation is no longer zero when the sensor and actuator are non-collocated or non-dual.

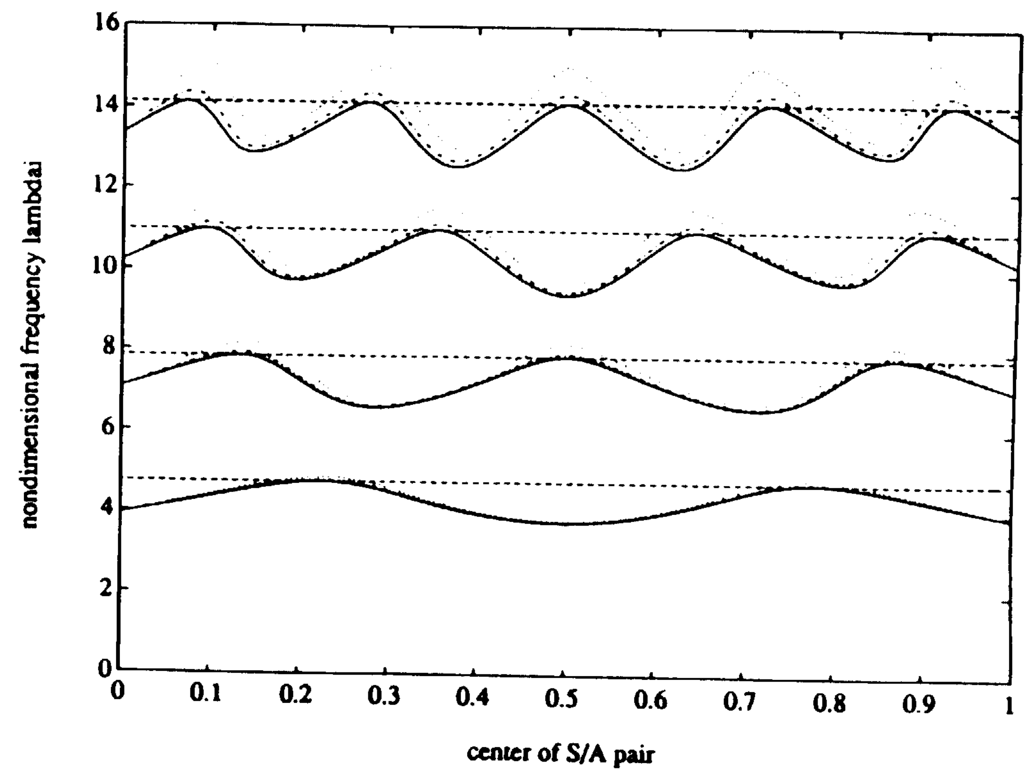
TO BEGIN TO UNDERSTAND THE CONSEQUENCES OF NON-COLLOCATED CONTROL SYSTEMS, A SECOND SET OF ZERO TRAJECTORIES ARE PLOTTED FOR NON-COLLOCATION DISTANCES SMALL RELATIVE TO THE WAVELENGTH OF THE MODES IN QUESTION. THE SEPARATION DISTANCE IS 5 OR 10% OF THE SPAN CORRESPONDING TO THE DASH-DOT AND DOT-DOT LINES. THE COLLOCATED CASE ZEROES ARE INCLUDED FOR REFERENCE AS THE SOLID LINES AND OF COURSE THE POLES ARE INCLUDED AS WELL.

THE NON-DIMENSIONALIZED ZERO FREQUENCIES ARE PLOTTED AS A FUNCTION OF THE CENTER OF THE SENSOR/ACTUATOR PAIR.

THESE ZEROES ARE STILL IMAGINARY.

Free-Free Beam with Slightly Non-Collocated Displacement Sensor and Force Actuator

- Two pole-zero cancellations occur in the vicinity of each node.
- Alternating pole-zero pattern destroyed.
- Slope of the żero trajectories in non-zero at pole-zero cancellations.

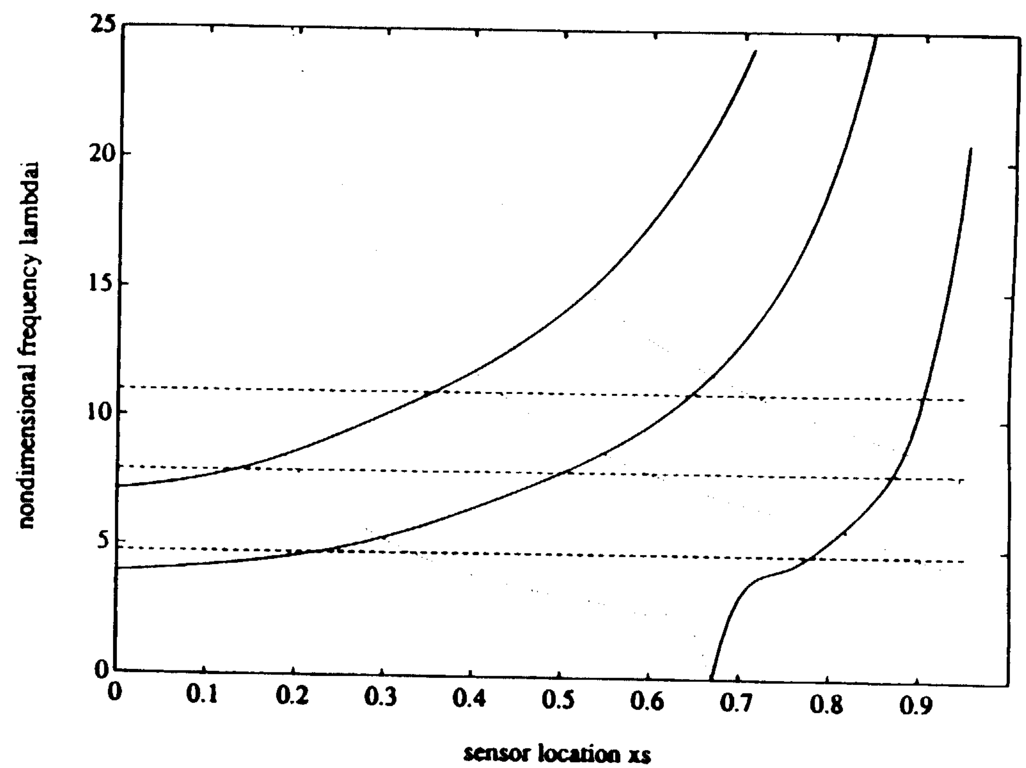


IN FACT, FOR LARGER NON-COLLOCATION DISTANCES, THE IMAGINARY ZEROES, ONCE MOVING TO HIGHER AND HIGHER FREQUENCY AS THE NON-COLLOCATION DISTANCE BECAME STRONGER, APPEAR IN SYMMETRIC PAIRS ON THE REAL AXIS AND MOVE IN CLOSER TO THE IMAGINARY AXIS AS THE NON-COLLOCATION BECOMES STRONGER.

THIS IS VISIBLE IN THE FIGURE WHERE THE ACTUATOR REMAINS FIXED AT $X=0$ AND THE SENSOR LOCATION IS VARIED, GENERATING A LOCUS OF ZEROES AS THE NON-COLLOCATION DISTANCE GROWS. THE SOLID LINES CORRESPOND TO THE IMAGINARY ZEROES AND THE DOT-DOT LINES TO REAL ZEROES. FOR THE REAL ZEROES, λI IS ACTUALLY A COMPLEX NUMBER AND THE FIGURE REPRESENTS THE MODULES OF λI.

Free-Free Beam with Non-Collocated Displacement Sensor and Force Actuator at one end.

- Imaginary zeroes begin at collocated location and migrate upwards.
- Real zeroes begin at infinity for the collocated case and become of lower frequency as the non-collocation distance grows.
- Lowest real zero reaches origin at center of percussion and becomes imaginary again.



Effect of Modelling on the Zero Frequencies

The accuracy of the zeroes is susceptible to the limitations of various modelling techniques which attempt to capture the structural behavior of an infinite order system with a finite number of states.

- Truncation effects (lack of completion of model)
    - Truncation of modes removes their residual from consideration and therefore causes inaccuracy in the zero.
    - Truncating modes below, within, and above the bandwidth have different effects on the zeroes.
- Spatial Discretization effects (lack of fidelity of model)
    - Discretization has two constraining effects: describing the continuum by a finite set of points or nodes, and restricting the motion between such points by assumed interpolation functions of a given order.

IN ADDITION TO THE BEHAVIOR OF THE ZEROES DUE TO PHYSICAL DESIGN DECISIONS SUCH AS SENSOR AND ACTUATOR ATTRIBUTES, THE NATURE OF THE STRUCTURAL MODEL ALSO AFFECTS THE PREDICTED FREQUENCY OF THE ZERO. WHILE THE EFFECTS OF SUCH MODELLING TECHNIQUES ARE WELL UNDERSTOOD FOR THE POLES OF A SYSTEM, THIS CANNOT BE SAID REGARDING ZEROES.

APPROXIMATE MODELLING TECHNIQUES MAY BE CATEGORIZED GENERALLY INTO TWO TYPES, EACH OF WHICH IS EXPLORED HERE VIA SIMPLE EXAMPLES. THE EXAMPLES CHOSEN HAVE EXACT SOLUTIONS THAT CAN BE COMPARED TO THE PREDICTIONS.

TRUNCATION EFFECTS ARE CRITICAL WHEN MODES ARE REMOVED FROM A MODEL, BE THEY ABOVE, BELOW, OR WITHIN THE BANDWIDTH. ONLY THOSE ABOVE THE BANDWIDTH ARE ADDRESSED HERE, ALTHOUGH ONE MUST BE AWARE OF THE EFFECTS ON ZEROES OF TRUNCATING A MODE WITHIN OR BELOW THE BANDWIDTH.

A VARIETY OF METHODS ENSURE THAT THE STATIC CONTRIBUTION OF THE TRUNCATED HIGHER MODES IS RETAINED IN THE MODEL. THIS HAS HISTORICALLY BEEN DONE NOT FOR THE PURPOSES OF IMPROVING ZERO PREDICTIONS, BUT FOR OVERALL IMPROVEMENT IN RESPONSE PREDICTIONS SUCH AS DISPLACEMENTS, ACCELERATIONS OR STRESSES.

Compensating for Modes Truncated Above the Bandwidth

- Modes above the bandwidth contribute essentially a static response in the bandwidth.
- The system response can be characterized by four terms:
    - static and dynamic response of the retained modes
    - static and dynamic response of the truncated modes
- Methods from a variety of disciplines which retain the static information from the truncated modes fall into two categories:
    - calculate the complete static response and correct with the dynamic response of the retained modes
        - Mode Acceleration Method
    - calculate the complete response of the retained modes and add the static response of the truncated modes
        - Residual Stiffness Method in Finite Elements, Residual Stiffness Method in Modal Identification, and Rayleigh Ritz Assumed Modes Method with a static mode shape assumed, static force feedthrough in state space model.

THE EFFECT OF INCLUDING THE STATIC RESPONSE OF THE HIGHER MODES TO BE TRUNCATED MAY BE SEEN FROM THESE SIMPLE MODEL RESULTS. A UNIFORM CLAMPED-FREE BEAM WITH TRANSVERSE FORCE ACTUATOR AND DISPLACEMENT SENSOR COLLOCATED AT THE TIP WAS MODELLED USING A RAYLEIGH RITZ ASSUMED MODES MODEL WHERE THE ASSUMED MODES WERE EXACT. THIS ENSURES THAT ONLY THE EFFECT OF TRUNCATION AND NOT DISCRETIZATION IS BEING STUDIED. THE RESULTS OF THE MODEL WITH AND WITHOUT THE EXACT STATIC MODE ARE SHOWN FOR THE FIRST AND SECOND ZEROES OF THESE MODELS AS A FUNCTION OF THE NUMBER OF DEGREES OF FREEDOM OR MODES INCLUDED IN THE MODEL. INCLUSION OF THE STATIC DEFLECTION SHAPE MEANT ONE LESS DYNAMIC MODE SHAPE INCLUDED IN THE N DEGREE OF FREEDOM MODEL.

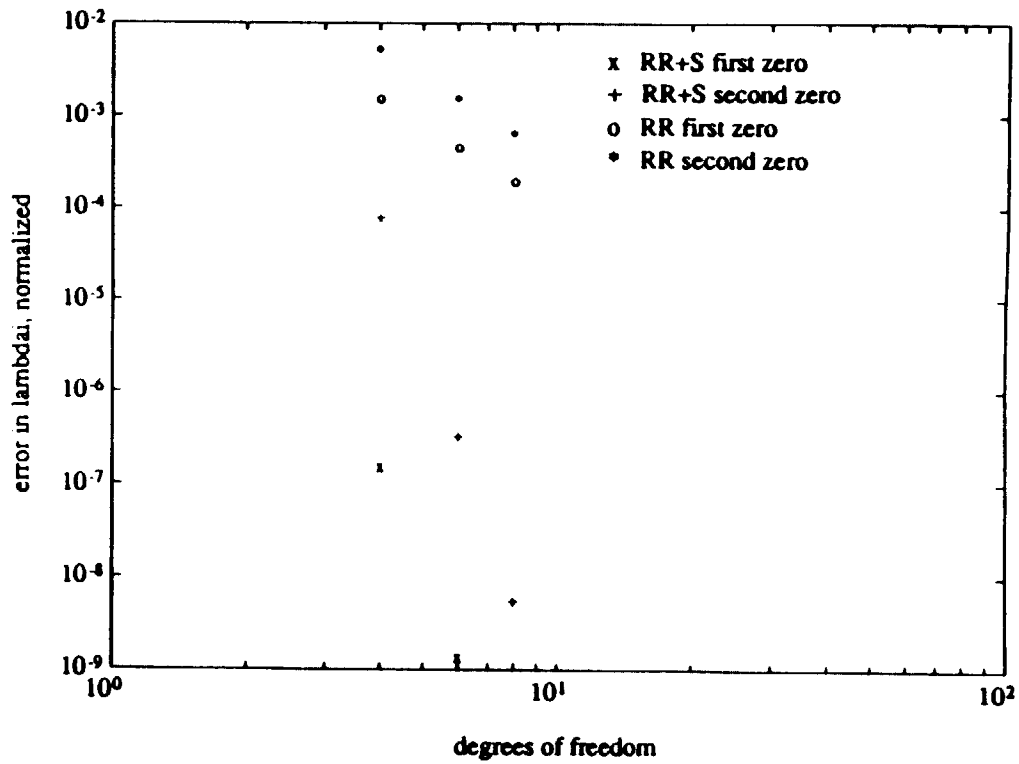
Value of Adding the Static Response

- Rayleigh Ritz model of clamped-free beam with collocated tip sensor and actuator.

- Exact mode shapes used without and with static "mode" shape used.

- Approximate improvement of four orders of magnitude in the zero accuracy prediction due to addition of static "mode".

- Convergence rate is also improved.



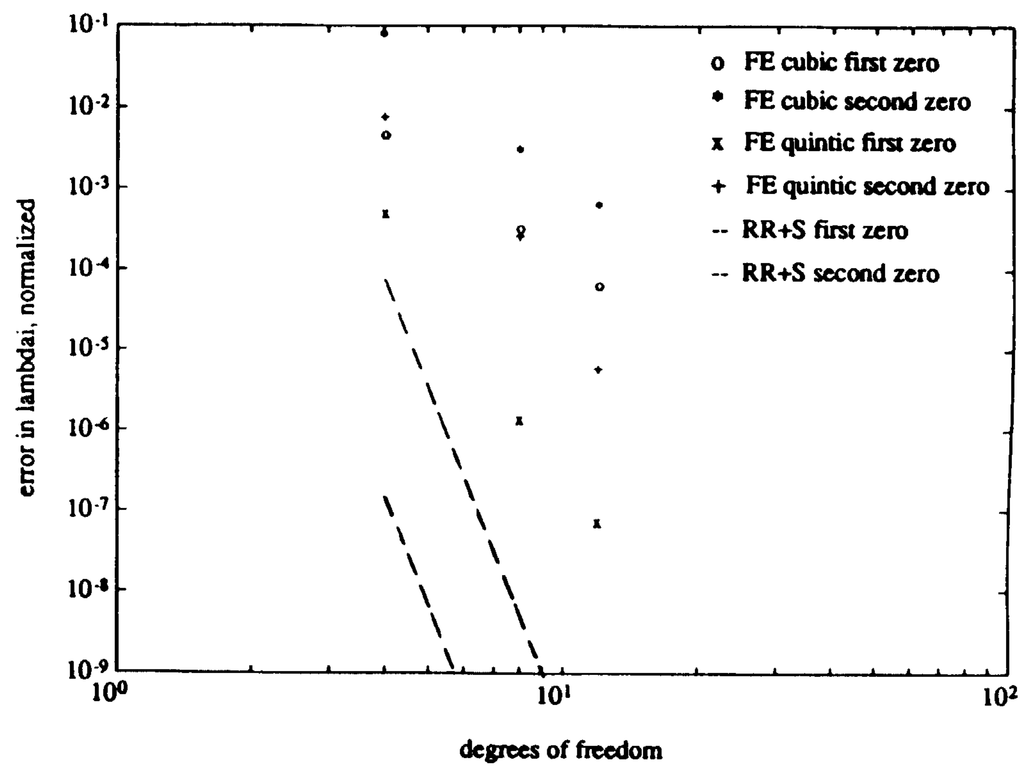
THE EFFECT OF DISCRETIZATION WAS STUDIED BY COMPARING THE ZERO PREDICTIONS USING TWO DIFFERENT INTERPOLATION FUNCTIONS, A CUBIC AND A QUINTIC BEAM ELEMENT FOR THE SAME STRUCTURE AND ACTUATOR/SENSOR MODEL AS THE PREVIOUS CASE. THE PREDICTIONS OF THE FIRST AND SECOND ZEROES FOR THE SAME NUMBER OF DEGREES OF FREEDOM ARE COMPARED. FOR REFERENCE, THE RAYLEIGH RITZ MODEL WHICH INCLUDES THE STATIC BEHAVIOR IS ALSO PRESENTED. COMPARING THE CUBIC AND QUINTIC RESULTS TO THOSE OF THE RAYLEIGH RITZ MODEL REMOVE THE TRUNCATION EFFECT. BOTH THE ABSOLUTE ERROR IN THE PREDICTION AND THE CONVERSION RATE ARE IMPROVED AS HIGHER ORDER ELEMENTS ARE USED.

The Effect of Discretization

- Clamped-free beam with collocated tip sensor and actuator. Rayleigh Ritz model with static "mode", cubic and quintic Finite Elements.

- Accuracy and convergence of quintic element is greater than cubic for equal number of dof's.

- Accuracy and convergence of Rayleigh Ritz model is greater than either Finite Element , revealing effect of discretization.

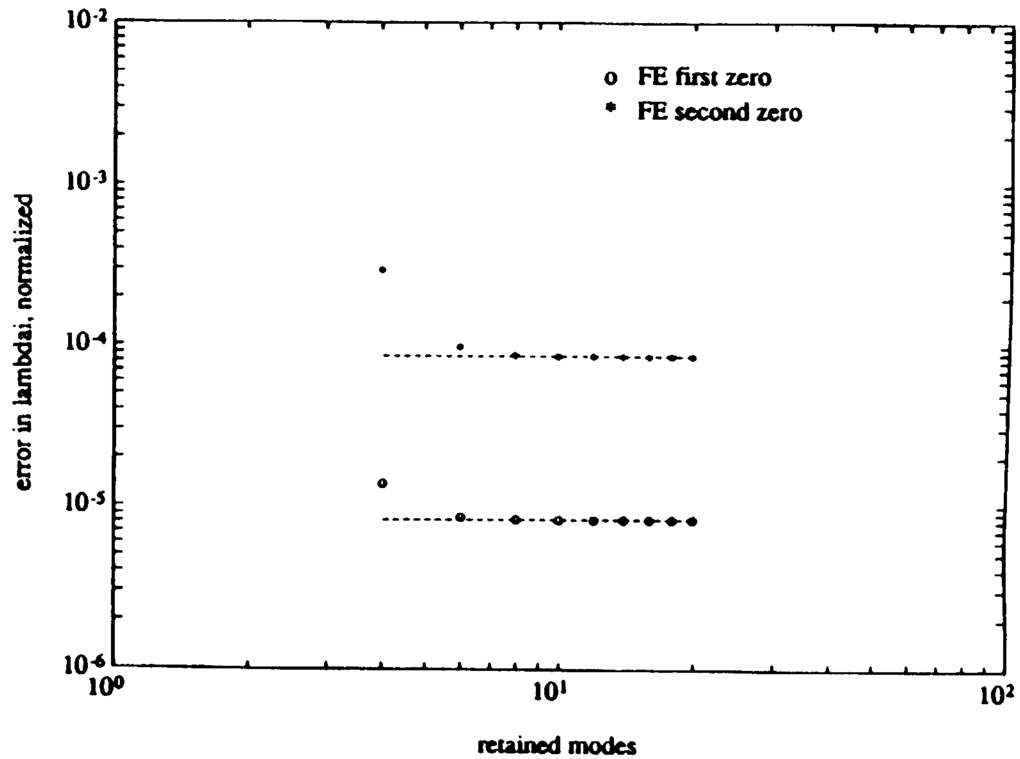


A UNIFORM CLAMPED-FREE BEAM WITH COLLOCATED TIP SENSOR AND ACTUATOR IDENTICAL TO THE OTHER TEST WAS USED HERE. THE OBJECTIVE WAS TO CONSTRUCT A FINITE ELEMENT MODEL WITH A CERTAIN DISCRETIZATION LEVEL AND THEN SUCCESSIVELY RETAIN AN EVER LARGER NUMBER OF THE MODES IN THIS MODEL. IN THIS WAY, THE EXTENT OF ADDING MORE MODES TO A MODEL WITH A CERTAIN DISCRETIZATION LEVEL COULD BE UNDERSTOOD. AS THE MODES WERE TRUNCATED, THEIR STATIC CONTRIBUTION WAS KEPT IN THE MODEL.

THE RESULTS OF THIS STUDY SHOWED THAT ONLY SIX OF THE INITIAL TWENTY MODES WERE NECESSARY TO OBTAIN ALMOST THE PEAK ACCURACY AVAILABLE IN THE MODEL. THE HORIZONTAL LINES DENOTE THE ACCURACY IN THESE ZEROES OF THE COMPLETE TWENTY MODE MODEL.

High Fidelity Finite Element Modes Plus the Static Finite Element Response

- Clamped-free beam with collocated tip sensor and actuator. Initial model has 20 finite element calculated modes.
- Zeroes are calculated with static response plus N_r-1 retained modes.
- Only 6 modes are needed to capture the zeroes to an accuracy almost equivalent to a model with 20 modes.



Conclusions

- Pole-zero patterns may be influenced before any control design is performed by means of the choice of actuator and sensor attributes.
- Pole-zero patterns for collocated and dual sensor and actuator pairs are always alternating. The bounds made by the poles are not reached by the zeroes when the system is unconstrained.
- As the sensor and actuator pair become non-collocated, the zeroes increase in frequency, and for certain structures become non-minimum phase. The non-minimum phase zeroes migrate to lower real frequency as the non-collocation distance becomes large.
- The zeroes have zero sensitivity to variations in actuator or measurement influence in the vicinity of a pole-zero cancellation for collocated and dual cases. The sensitivity is non-zero near a pole-zero cancellation of a non-collocated system.

Conclusions

- The prediction of zero frequencies is dependent on the modelling scheme used.
- Improvement of the zero predictions for a given number of modelled modes when truncation occurs above the bandwidth may be accomplished by including the static deflection information into the model.
- High order elements improve the zero predictions for a given number of degrees of freedom. Hence, higher order interpolation functions or more nodes should be used, where possible.
- The recommended procedure for accurate zero prediction is:
    - Construct a high order model of high order elements (if possible)
    - Include method to retain static response of truncated modes.
    - Truncate higher frequency modes until zero prediction accuracy becomes unacceptable.
    - Be cognizant that truncation of modes in the bandwidth will cause errors in zero above the frequency of the truncated pole.

THE APPROACH TAKEN IN THE CURRENT RESEARCH IS MOTIVATED BY STATISTICAL ENERGY ANALYSIS. SEA IS USED PRIMARILY BY MECHANICAL ENGINEERS IN VIBRATION AND ACOUSTIC ANALYSIS OF COMPLICATED STRUCTURES, OFTEN WITH HUNDREDS OR THOUSANDS OF MODES IN THE FREQUENCY RANGE OF INTEREST. ONCE AGAIN, THE STRUCTURE WILL BE MODELLED WITH THE DEREVERBERATED DRIVING POINT MOBILITY. THE MAIN DIFFICULTY WITH THE H_2 (WIENER-HOPF) APPROACH EARLIER IS THAT THE LOCAL MODEL DOES NOT INCLUDE THE INFORMATION THAT THE STRUCTURE IS CONSERVATIVE. THAT IS, IN THE REAL STRUCTURE, DEPARTING ENERGY WILL EVENTUALLY RETURN. IN THIS APPROACH, THIS INFORMATION IS PUT BACK INTO THE PROBLEM IN THE DERIVATION OF THE COST FUNCTIONAL. THE COST IS THE FREQUENCY WEIGHTED VIBRATIONAL ENERGY IN THE STRUCTURE. FINITE COST THEREFORE IMPLIES FINITE ENERGY, WHICH IMPLIES THAT THE STRUCTURE MUST BE STABLE. FURTHERMORE, USING SEA, THE DESIRED RMS COST CAN BE RELATED TO THE ENERGY IN THE STRUCTURE. THUS THIS DESIRED COST CAN BE MINIMIZED, BY EXPRESSING IT IN TERMS OF THE POWER FLOW INTO THE STRUCTURE.

MIT
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Active Broadband Impedance Matching Using Statistical Energy Analysis

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SERC Steering Committee Workshop
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THE PROBLEM WE ARE STUDYING IS TO DEVELOP CONTROL METHODOLOGIES FOR DESIGNING LOW-AUTHORITY BROADBAND ACTIVE CONTROL SYSTEMS FOR UNCERTAIN STRUCTURES. IN ORDER TO CONTROL MANY MODES OF SUCH A STRUCTURE,COLLOCATED FEEDBACK IS REQUIRED. FOR NONCOLLOCATED SENSORS AND ACTUATORS, THERE IS ALMOST NO PHASE INFORMATION ABOUT THE TRANSFER FUNCTION IN THE MODALLY DENSE REGIME, AND THEREFORE, NOTHING CAN BE DONE TO CONTROL THE STRUCTURE. HOWEVER, USING COLLOCATED AND DUAL FEEDBACK, ONE CAN TAKE ADVANTAGE OF THE FACT THAT THE STRUCTURE IS POSITIVE REAL, AND, HENCE, THAT ANY POSITIVE REAL COMPENSATOR WILL NOT DESTABILIZE THE STRUCTURE.

THE APPROACH TAKEN HERE IS TO USE A STATISTICAL MODEL OF THE LOCAL DYNAMICS NEAR THE SENSOR/ACTUATOR PAIR, RATHER THAN TRYING TO DEVELOP A MODEL THAT CONTAINS ACCURATE INFORMATION ABOUT EVERY MODE OF THE STRUCTURE. BECAUSE THE MODEL ONLY CONTAINS LOCAL INFORMATION, IT IS ROBUST TO UNCERTAINTIES ELSEWHERE IN THE STRUCTURE. CONTROL DESIGN FOR THIS MODEL IS BASED ON MAXIMIZING THE POWER DISSIPATED BY THE CONTROLLER. THIS IS EQUIVALENT TO AN IMPEDANCE MATCHING PROBLEM. IN GENERAL, A NONCAUSAL COMPENSATOR IS REQUIRED IN ORDER TO MATCH THE IMPEDANCE EXACTLY AT EVERY FREQUENCY. INSTEAD, THE CONTROL DESIGN MUST FIND AN OPTIMAL, CAUSAL APPROXIMATION TO THIS COMPENSATOR.

Problem Statement

- Uncertain modally dense structure.
- Collocated feedback — Structural transfer function is positive real.

Approach

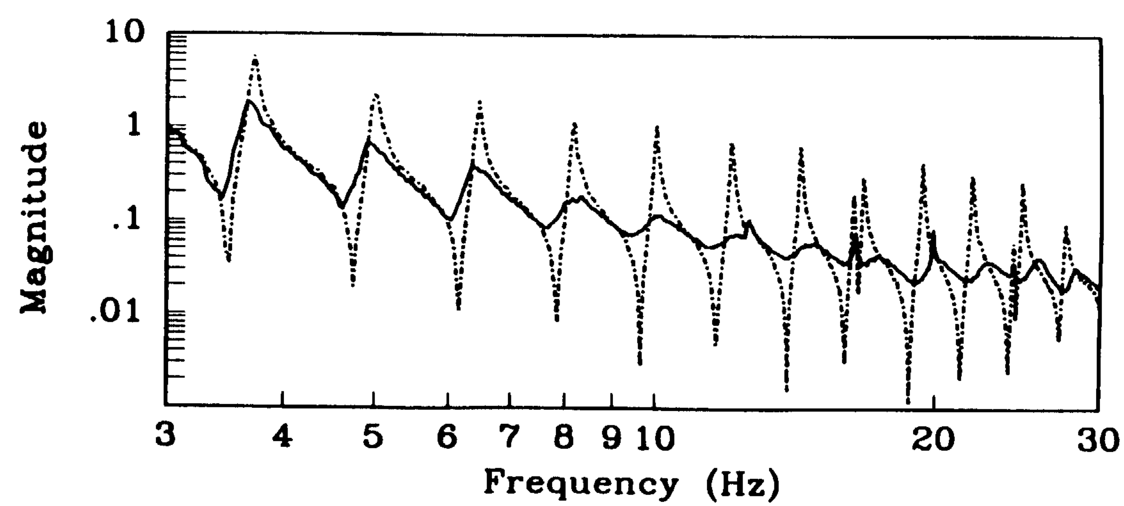
- Local model of structure only. (Statistical/Acoustic)
  - Robust to uncertainties elsewhere in structure.
- Maximize power dissipation.
  - Equivalent to impedance matching.
  - Cannot match impedance exactly due to causality constraint.

MILLER ET AL. USED A WAVE APPROACH TO MODEL THE STRUCTURE. THE STRUCTURE IN THIS CASE IS MODELLED AS A NETWORK OF ONE DIMENSIONAL WAVE-GUIDES WHICH MEET AT JUNCTIONS. THIS APPROACH RESULTS IN A LOCAL MODEL OF THE STRUCTURE AT THE SENSOR AND ACTUATOR LOCATION, AND A MODEL OF THE POWER DISSIPATED BY THE CONTROL. THE H_2 NORM OF THIS POWER FLOW WAS MINIMIZED, USING A WIENER-HOPF TECHNIQUE TO GUARANTEE CAUSALITY. THE PROBLEM WITH THIS APPROACH IS THAT IT DOES NOT GUARANTEE CLOSED-LOOP STABILITY WHEN THE COMPENSATOR IS APPLIED TO THE REAL STRUCTURE. THE COMPENSATOR IS DERIVED BASED ONLY ON THE LOCAL MODEL, WHICH DOES NOT INCLUDE THE INFORMATION THAT ENERGY WHICH DEPARTS THE JUNCTION WILL EVENTUALLY RETURN. AS A RESULT, THE COMPENSATOR MAY ALLOW POWER TO BE GENERATED AT SOME FREQUENCIES, IN ORDER TO ACHIEVE GREATER POWER DISSIPATION AT OTHER FREQUENCIES.

APPROXIMATING THE OPTIMAL H_2 COMPENSATOR WITH A POSITIVE REAL FORM WILL GUARANTEE CLOSED-LOOP STABILITY. USING THIS AD HOC APPROACH, COMPENSATORS WERE DESIGNED AND IMPLEMENTED EXPERIMENTALLY ON A 24 FOOT BRASS BEAM SUSPENDED IN THE LABORATORY. WITH A COMPENSATOR FROM A SLOPE RATE SENSOR TO A COLLOCATED MOMENT ACTUATOR AT ONE END OF THE BEAM, THE OPEN AND CLOSED-LOOP TRANSFER FUNCTIONS FROM FORCE TO COLLOCATED VELOCITY AT THE OTHER, FREE END OF THE BEAM WERE OBTAINED, AND ARE SHOWN HERE. SIGNIFICANT DAMPING WAS ADDED TO MANY MODES OF THE BEAM, AND THE DAMPING ACHIEVED WAS MUCH HIGHER THAN THAT WHICH COULD BE OBTAINED WITH RATE FEEDBACK.

I: Wave Model/Wiener-Hopf Approach

- Model structure with waves.
- Minimize $\mathcal{H}_2$ norm of power flow using Wiener-Hopf techniques.
- Problem: No guarantee of stability.
  - Local model is not conservative; departing energy does not return.
- Experimental results:



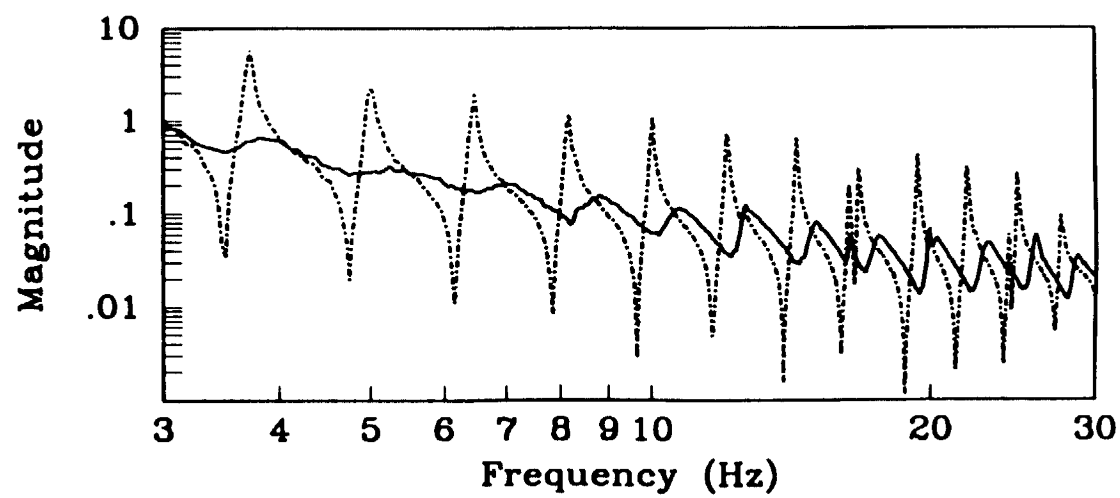
AN ALTERNATIVE APPROACH FOR OBTAINING A LOCAL MODEL OF THE STRUCTURE IS TO USE THE DEREVERBERATED DRIVING POINT MOBILITY. THE STRUCTURAL RESPONSE AT A POINT CAN BE CONSIDERED TO BE THE SUM OF TWO PARTS: A DIRECT FIELD, DUE TO THE LOCAL DYNAMICS; AND A REVERBERANT FIELD, WHICH IS CAUSED BY ENERGY REFLECTED BACK FROM OTHER PARTS OF THE STRUCTURE. THE TERM "DEREVERBERATED" IMPLIES THAT THE "REVERBERANT" PART OF THE RESPONSE HAS BEEN REMOVED BEFORE COMPUTING THE MOBILITY. THIS MODEL CAN BE OBTAINED DIRECTLY FROM AN EXPERIMENTAL TRANSFER FUNCTION BY A LOGARITHMIC AVERAGE, AND THUS CAN BE APPLIED TO ANY ARBITRARY STRUCTURE.

IF THE WORST CASE POWER FLOW IS MINIMIZED, RATHER THAN THE RMS POWER, THEN THE CONTROLLER WILL NOT ADD POWER TO THE STRUCTURE AT ANY FREQUENCY, AND HENCE THE CLOSED LOOP SYSTEM IS GUARANTEED TO BE STABLE. THIS ALSO GUARANTEES A POSITIVE REAL COMPENSATOR. THE DRAWBACK TO THIS APPROACH IS THAT IN GENERAL, THE DESIRED PERFORMANCE METRIC FOR THE STRUCTURE IS THE RMS LEVEL OF SOME QUANTITY, RATHER THAN THE WORST CASE LEVEL. THUS WHILE THIS APPROACH GUARANTEES STABILITY, IT IS DIFFICULT TO MINIMIZE THE DESIRED PERFORMANCE INDEX.

COMPENSATORS DERIVED USING THIS APPROACH WERE IMPLEMENTED EXPERIMENTALLY ON THE SAME 24 FOOT BEAM AS THOSE OBTAINED USING THE WAVE/WIENER-HOPF APPROACH. THE SAME OPEN AND CLOSED-LOOP TRANSFER FUNCTIONS ARE SHOWN. AGAIN, SIGNIFICANT DAMPING WAS ADDED TO MANY MODES.

II: Dereverberated Mobility/$\mathcal{H}_\infty$ Approach

- Model structure with dereverberated mobility.
  - Total Response = Direct Field + Reverberant Field
- Minimize $\mathcal{H}_\infty$ norm of power flow. (Worst case)
  - Guaranteed dissipation $\Rightarrow$ Guaranteed stability.
- Problem: Not minimizing desired rms performance metric.
- Experimental results:



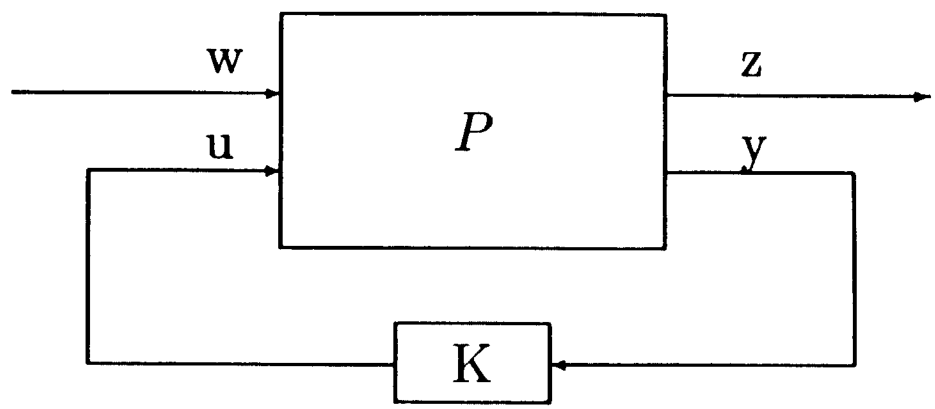
THE APPROACH TAKEN IN THE CURRENT RESEARCH IS MOTIVATED BY STATISTICAL ENERGY ANALYSIS. SEA IS USED PRIMARILY BY MECHANICAL ENGINEERS IN VIBRATION AND ACOUSTIC ANALYSIS OF COMPLICATED STRUCTURES, OFTEN WITH HUNDREDS OR THOUSANDS OF MODES IN THE FREQUENCY RANGE OF INTEREST. ONCE AGAIN, THE STRUCTURE WILL BE MODELLED WITH THE DEREVERBERATED DRIVING POINT MOBILITY. THE MAIN DIFFICULTY WITH THE H_2 (WIENER-HOPF) APPROACH EARLIER IS THAT THE LOCAL MODEL DOES NOT INCLUDE THE INFORMATION THAT THE STRUCTURE IS CONSERVATIVE. THAT IS, IN THE REAL STRUCTURE, DEPARTING ENERGY WILL EVENTUALLY RETURN. IN THIS APPROACH, THIS INFORMATION IS PUT BACK INTO THE PROBLEM IN THE DERIVATION OF THE COST FUNCTIONAL. THE COST IS THE FREQUENCY WEIGHTED VIBRATIONAL ENERGY IN THE STRUCTURE. FINITE COST THEREFORE IMPLIES FINITE ENERGY, WHICH IMPLIES THAT THE STRUCTURE MUST BE STABLE. FURTHERMORE, USING SEA, THE DESIRED RMS COST CAN BE RELATED TO THE ENERGY IN THE STRUCTURE. THUS THIS DESIRED COST CAN BE MINIMIZED, BY EXPRESSING IT IN TERMS OF THE POWER FLOW INTO THE STRUCTURE.

III: Statistical Energy Analysis (SEA) Approach

- Model structure with dereverberated mobility.
- Include knowledge that structure conserves energy.
- Guaranteed finite energy $\Rightarrow$ Guaranteed stability.
- Minimize desired rms cost, expressed in terms of power flow.

PREVIOUS RESULTS ALLOW THE POWER REFLECTED BACK INTO THE STRUCTURE TO BE REPRESENTED AS A TRANSFER FUNCTION. THE 2X2 TRANSFER MATRIX P CAN BE OBTAINED IN TERMS OF STATE SPACE REPRESENTATIONS FOR THE DEREVERBERATED MOBILITY G(S). IF H(S) IS THE TRANSFER FUNCTION FROM THE NORMALIZED DISTURBANCE INPUT W TO THE OUTPUT Z, THEN H^*H IS THE REFLECTED POWER FLOW AT EACH FREQUENCY FOR UNIT INPUT. THE DISSIPATED POWER PER UNIT INPUT IS THEN $(I-H^*H)$. A COMPENSATOR K IS DESIRED WHICH MINIMIZES, IN AN APPROPRIATE SENSE, THE TRANSFER FUNCTION H(S).

Power Flow:



- Define $H(s) = \frac{z(s)}{w(s)}$, then H^*H is the reflected power at each frequency for unit input, $I - H^*H$ is the relative power dissipated.

STATISTICAL ENERGY ANALYSIS ATTEMPTS TO MODEL ONLY CERTAIN STATISTICAL PROPERTIES OF THE STRUCTURE, RATHER THAN THE DETAILS OF EVERY MODE. THE SYSTEM IS DIVIDED INTO SUBSYSTEMS, AND THE AVERAGE MODAL ENERGY IN EACH OF THESE IS PREDICTED BASED ON A MODEL OF THE COUPLING POWER FLOW BETWEEN SUBSYSTEMS. SEA MAKES TWO BASIC ASSUMPTIONS. THE FIRST, EQUIPARTITION, ASSUMES THAT EVERY MODE IN A GIVEN FREQUENCY REGION HAS APPROXIMATELY THE SAME ENERGY $E(\omega)$. THE SECOND, INCOHERENCE, ASSUMES THAT THE AMPLITUDES OF INDIVIDUAL MODES ARE UNCORRELATED. THIS ASSUMPTION IS OFTEN A POOR ONE TO MAKE, SINCE IF A DISTURBANCE ENTERS THE STRUCTURE AT A POINT, EVERY MODE WITH A NODE THERE WILL CLEARLY HAVE CORRELATED AMPLITUDES. SEA RESEARCH, HOWEVER, INDICATES THAT MOST OF THE PREDICTIONS OF SEA REMAIN VALID EVEN WHEN THIS ASSUMPTION FAILS. USING THESE TWO ASSUMPTIONS, THE RMS LEVEL OF ANY OF THE SYSTEM STATES CAN BE RELATED TO THE AVERAGE MODAL ENERGY. HENCE A GLOBAL COST CRITERION CAN BE REPRESENTED ONLY IN TERMS OF THE ENERGY LEVEL.

IF THERE IS SUFFICIENT UNCERTAINTY ABOUT THE STRUCTURE, THEN THE AMPLITUDES OF INDIVIDUAL MODES, AND THE CORRELATION BETWEEN THE AMPLITUDES OF DIFFERENT MODES, WILL BE DIFFICULT OR IMPOSSIBLE TO CALCULATE. AS A RESULT, THE EQUIPARTITION AND INCOHERENCE ASSUMPTIONS ARE REASONABLE, AND THE RELATIONSHIP BETWEEN THE COST AND THE ENERGY REPRESENTS THE BEST AVAILABLE ESTIMATE OF THE COST.

Statistical Energy Analysis (SEA)

- Describe statistical properties of system only.
- Compute expected energy levels based on modelling power flow between subsystems.
- Assumptions:
  - Equipartition: All modes in each frequency region have the same energy, hence can describe system by $E(\omega)$.
  - Incoherence: Modal amplitudes uncorrelated.
- Cost:

$$\langle y^T y \rangle \simeq \int_{-\infty}^{\infty} C(\omega) E(\omega)\, \mathrm{d}\omega$$

USING THE DEREVERBERATED MOBILITY POWER FLOW MODEL DISCUSSED EARLIER, THE RELATIVE DISSIPATION ASSOCIATED WITH THE CONTROL IS GIVEN BY THE TRANSFER FUNCTION (I-H*H). UNDER THE EQUIPARTITION ASSUMPTION FROM SEA, THE POWER IN THE WAVES INCOMING TO THE ACTUATOR LOCATION IS PROPORTIONAL AT EACH FREQUENCY TO THE TOTAL ENERGY IN THE STRUCTURE. THUS THE TOTAL POWER DISSIPATED IS RELATED TO THIS AVERAGE MODAL ENERGY. THE KEY PROBLEM EARLIER IN USING THE DEREVERBERATED MOBILITY WAS THAT THIS LOCAL MODEL DID NOT INCLUDE THE FACT THAT THE ACTUAL STRUCTURE IS APPROXIMATELY CONSERVATIVE. INVOKING CONSERVATION OF ENERGY, THEN, YIELDS THAT THE POWER DISSIPATED BY THE CONTROL SYSTEM, Π_{DISS} IS EQUAL TO THE POWER INPUT FROM DISTURBANCE SOURCES, Π_{IN}, WHICH CAN BE EASILY CHARACTERIZED IN TERMS OF THE POWER SPECTRAL DENSITY OF THE DISTURBANCES. COMBINING THIS WITH THE PREVIOUS RESULT THAT THE COST IS RELATED THROUGH AN INTEGRAL OVER FREQUENCY TO THE AVERAGE MODAL ENERGY YIELDS A COST FUNCTIONAL OF THE POWER FLOW H*H THAT IS EQUIVALENT TO MINIMIZING THE DESIRED COST.

THIS COST COMBINES FEATURES OF BOTH THE H_2 AND H_∞ COSTS USED IN EARLIER RESEARCH. IT GUARANTEES AN H_∞ NORM BOUND, AND AS A RESULT, GUARANTEES THAT THE COMPENSATOR DISSIPATES POWER AT ALL FREQUENCIES, AND THEREFORE STABILIZES THE ACTUAL STRUCTURE. IT ALSO OVERBOUNDS THE H_2 COST MINIMIZED ORIGINALLY WITH THE WIENER-HOPF APPROACH.

Cost Functional

- Dereverberated mobility power flow model for dissipation:

$$\Pi_{diss} = \underbrace{\left(I - H(j\omega)^* H(j\omega)\right)}_{Relative\ Dissipation} E(\omega)$$

- Conservation of energy: $\Pi_{diss} = \Pi_{in}$
- Use SEA cost equation to give cost functional:

$$J = \int_{-\infty}^{\infty} C(\omega) \left[(I - H^*H)^{-1} H^*H\right] \Pi_{in}(\omega)\, d\omega$$

- Guarantees $\mathcal{H}_\infty$ norm bound, $\|H\|_\infty < 1$.
  - Guarantees $\begin{cases} \text{positive real} \\ \text{dissipates power} \\ \text{stable closed loop} \end{cases}$
- Overbounds weighted $\mathcal{H}_2$ cost.
  - Minimizes desired performance metric.

FOR A STATE SPACE SYSTEM H(S), THE COST JUST DERIVED CAN BE EVALUATED USING THE SOLUTION TO A SINGLE RICCATI EQUATION AND A LYAPUNOV EQUATION. THE INTEGRAND OF THE COST CAN BE FACTORED AS M*M USING A RICCATI EQUATION, AND THE COST IS THEN THE H_2 NORM OF THE SYSTEM M, WHICH CAN BE EVALUATED WITH A LYAPUNOV EQUATION. THESE TWO EQUATIONS CAN BE APPENDED TO THE COST WITH LAGRANGE MULTIPLIERS IN ORDER TO OPTIMIZE THE COST WITH RESPECT TO THE PARAMETERS OF THE CONTROL SYSTEM. THIS YIELDS 4 COUPLED RICCATI AND LYAPUNOV EQUATIONS. IN THE STATE FEEDBACK CASE, THESE ARE EASILY SOLVED BY AN ITERATIVE TECHNIQUE, HOWEVER, THIS IS NOT THE CASE FOR DYNAMIC FEEDBACK. THE STRUCTURE OF THE DYNAMIC PROBLEM IS SIMILAR TO THAT OF A VARIETY OF OTHER IMPORTANT, UNSOLVED CONTROL PROBLEMS, WHICH ARE ALSO BEING STUDIED. ONE SUCH IS MULTIPLE MODEL CONTROL DESIGN -- THE DESIGN OF A SINGLE CONTROL SYSTEM TO BE APPLIED TO SEVERAL DIFFERENT MODELS. ANOTHER IS THE DESIGN OF A STABLE H_2-OPTIMAL CONTROLLER. THE COST CAN ALSO BE RELATED TO THE H_2/H_∞ COST OF ZHOU ET AL., AND, AS IN THE STANDARD H_∞ PROBLEM, TO DIFFERENTIAL GAMES.

Using Cost Functional

- Can evaluate cost for a state space system with solution to a Riccati equation and a Lyapunov equation.
- Can optimize cost using Lagrange multiplier approaches: yields coupled Riccati and Lyapunov equations.
- State feedback case easily solved.
- Dynamic feedback case is difficult to solve exactly:
  - yields a problem similar to multiple model estimation or control problems, optimization with a stable controller,...
- Cost is related to other $\mathcal{H}_2/\mathcal{H}_\infty$ approaches, and to differential games.

FOR THE CONTROL OF MANY MODES OF A STRUCTURE, A GLOBAL MODEL THAT CONTAINS DETAILED INFORMATION ABOUT EACH MODE IS LIKELY TO BE HIGHLY INACCURATE. A LOCAL MODEL, HOWEVER, WILL BE ROBUST TO UNCERTAINTIES ELSEWHERE IN THE STRUCTURE. TWO APPROACHES HAVE BEEN STUDIED IN SERC FOR DEVELOPING LOCAL MODELS FOR CONTROL DESIGN. A WAVE-BASED APPROACH IS EASILY APPLIED TO STRUCTURES COMPOSED OF ONE-DIMENSIONAL WAVEGUIDES. A MORE GENERAL APPROACH IS TO USE THE DEREVERBERATED MOBILITY, WHICH CAN BE OBTAINED FOR ANY STRUCTURE BY AVERAGING THE EXPERIMENTAL TRANSFER FUNCTION, OR FROM A WAVE MODEL OF THE STRUCTURE.

CONTROL DESIGN USING THESE LOCAL MODELS IS BASED ON THE MINIMIZATION OF THE POWER FLOWING INTO THE STRUCTURE. AN OPTIMAL CONTROLLER CAN YIELD SIGNIFICANTLY BETTER PERFORMANCE THAN RATE FEEDBACK. MINIMIZING THE H_2 COST, OR RMS POWER FLOW DOES NOT GUARANTEE CLOSED-LOOP STABILITY WHEN THE RESULTING COMPENSATORS ARE APPLIED TO THE ACTUAL STRUCTURE. MINIMIZING THE POWER IN AN H_∞ SENSE GUARANTEES STABILITY, BUT IT IS MORE DIFFICULT TO MINIMIZE THE TRUE PERFORMANCE INDEX, WHICH IS GENERALLY AN RMS QUANTITY. A STATISTICAL MODEL OF THE STRUCTURE, WITH ASSUMPTIONS MOTIVATED BY SEA, CAN BE USED TO DEVELOP A COST THAT WILL GUARANTEE STABILITY, AS WELL AS OPTIMIZING AN RMS PERFORMANCE INDEX OF THE STRUCTURE.

Summary

- Local model is robust to uncertainties elsewhere in structure.
  - Wave model.
  - Dereverberated mobility model.
- Power flow minimization can yield good performance.
  - $\mathcal{H}_2$ cost: no guarantee of stability.
  - $\mathcal{H}_\infty$ cost: guaranteed stability, but doesn't minimize rms performance.
  - SEA cost: guarantees stability, as well as optimizing rms performance.

Cost Averaging Techniques for Robust Control Synthesis for Parametrically Uncertain Systems

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THIS WORK ADDRESSES THE PROBLEM OF ADDING ROBUSTNESS TO CONTROL DESIGNS WHERE THE PLANT UNCERTAINTIES ARE CHARACTERIZED BY UNKNOWN PARAMETERS. IN PARTICULAR, THE PROBLEM OF STABILITY ROBUSTNESS IS EXAMINED. WHAT IS DESIRED IS A CONTROLLER DESIGN METHODOLOGY WHICH WILL STABILIZE A PARAMETERIZED SYSTEM OVER A PRE-SPECIFIED RANGE OF REAL PARAMETER VALUES. AT EACH POINT OVER THIS RANGE (AT EACH SPECIFIED SET OF PARAMETER VALUES) A DIFFERENT SYSTEM IS DESCRIBED. THE COLLECTION OF ALL THE SYSTEMS DESCRIBED BY THE PARAMETERIZATION, IS CALLED THE SET OF SYSTEMS. THE GOAL IS THUS TO DESIGN A CONTROLLER WHICH WILL ACHIEVE STABILITY OVER THE SET OF SYSTEMS.

THE UNCERTAINTY IS CHARACTERIZED BY REAL UNCERTAIN PARAMETERS. THIS TYPE OF UNCERTAINTY IS REPRESENTATIVE OF THE TYPES OF STRUCTURED UNCERTAINTIES FOUND IN TYPICAL AEROSPACE APPLICATIONS, IN PARTICULAR CONTROLLED STRUCTURES. THE UNCERTAIN PARAMETER CAN REPRESENT INCOMPLETE KNOWLEDGE OF THE SYSTEM NATURAL FREQUENCIES, DAMPING, OR MODE SHAPE AMPLITUDES AT THE SENSOR AND ACTUATOR LOCATIONS. AT A MORE BASIC LEVEL, THE UNCERTAIN MATERIAL PROPERTIES SUCH AS STIFFNESS AND MATERIAL DAMPING CAN BE REPRESENTED AS PARAMETRIC UNCERTAINTIES. THUS ACHIEVING ROBUSTNESS IN THE PRESENCE OF REAL PARAMETRIC UNCERTAINTIES IS AN IMPORTANT PROBLEM IN THE DESIGN OF CONTROLLED STRUCTURES.

Problem Statement

- Consider the set of systems G parameterized by bounded *real valued* variables.

$$\boldsymbol{G} = \left\{ G(s,\alpha) \quad : \alpha \in \boldsymbol{R}^n, -\delta_i \le \alpha_i \le \delta_i \right\}$$

where

$$G(s,\alpha) \Leftrightarrow \begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left[\left[\begin{array}{c|cc} A_0 & B_{01} & B_{02} \\ \hline C_{01} & 0 & D_{12} \\ C_{02} & D_{21} & 0 \end{array} \right] + \sum_i \alpha_i \left[\begin{array}{c|cc} A_i & 0 & B_{i2} \\ \hline 0 & 0 & 0 \\ C_{i2} & 0 & 0 \end{array} \right] \right] \begin{bmatrix} x \\ w \\ u \end{bmatrix}$$

- The ultimate goal is to design a controller, y = C(s) u, which provides stability throughout the set while minimizing some measure of performance over the set.

- Motivation: Controlled structures typically have real parametric uncertainty.

  Examples: Uncertain stiffness, natural frequency, or damping.

THE IMPORTANCE OF THE PROBLEM OF ROBUST STABILITY IN THE FACE OF REAL PARAMETRIC UNCERTAINTIES HAS PROMPTED MUCH WORK IN THE FIELD. IN GENERAL THIS BODY OF WORK CAN BE DIVIDED INTO TWO AREAS BASED ON THE TECHNIQUE USED TO GENERATE ROBUSTNESS PROPERTIES IN THE CONTROLLERS. THE FIRST GROUP WILL BE THOSE RELYING ON ANALYTICAL BOUNDS TO GUARANTEE STABILITY, AND THE SECOND GROUP WILL BE THOSE WHICH SENSITIZE THE COST TO THE UNCERTAINTY BY ADDING TERMS WHICH PENALIZE UNCERTAINTY. THE GROUPS ARE KNOWN AS BOUNDING METHODS AND SENSITIZING METHODS, RESPECTIVELY.

THE TECHNIQUE USED IN BOUNDING METHODS IS TO MODIFY THE COST BY USING AN ANALYTICAL BOUND SUCH THAT IF THE BOUND IS SATISFIED ROBUST STABILITY OVER THE SET OF SYSTEMS IS GUARANTEED. THERE ARE SEVERAL ANALYSIS TESTS WHICH CAN BE USED BASED ON, FOR INSTANCE, LYAPUNOV STABILITY THEORY, KARITONOV'S THEOREM, ETC. THE IDEA IS THEN TO TAKE THE DESIGNER'S FAVORITE ANALYSIS TEST AND APPEND THIS TO THE COST IN SUCH A WAY THAT A CONTROLLER THAT MINIMIZES THE MODIFIED COST AUTOMATICALLY STABILIZES THE SET OF SYSTEMS. ALTHOUGH THIS DESIGN METHOD GUARANTEES STABILITY, THE DRAWBACK IS THAT MOST ANALYSIS TECHNIQUES SIMPLE ENOUGH TO BE USED TO MODIFY THE COST ARE ALSO VERY CONSERVATIVE FOR REAL PARAMETRIC UNCERTAINTY. THIS CONSERVATISM MEANS THAT THE DESIGN TECHNIQUE FORCES THE CONTROLLER TO STABILIZE A MUCH LARGER SET OF SYSTEMS THAN THE HIGHLY STRUCTURED SET USED IN THE DESIGN. STABILIZING A LARGER SET OF SYSTEMS CAN LEAD TO UNNECESSARILY HIGH COSTS AND CONTROL GAINS.

Background in Robust Control Synthesis

Bounding Methods

- Develop robustness analysis test: Lyapunov, Karitonov, etc.
- Incorporate robustness analysis test into cost (eg. modify Lyapunov equation, stability index)
- Find gains which minimize cost subject to constraints: Yedavalli, Siljak, Petersen, Bernstein, et al.

Sensitivity Methods

- Make the cost reflect the parameter uncertainties but not necessarily guarantee robustness.
- Multiplicative White Noise: Hyland, Bernstein
- Cost Sensitivity Minimization: Skelton.
- Multi-Model Techniques: Bryson, Li

THE SECOND TECHNIQUE IS TO MODIFY THE COST BY SENSITIZING THE COST TO PARAMETRIC UNCERTAINTIES. THE THREE MAJOR TECHNIQUES IN THIS AREA ARE THE MULTIPLICATIVE WHITE NOISE MODELS OF HYLAND AND BERNSTEIN, THE SENSITIVITY SYSTEM MODELS OF SKELTON, AND THE MULTI MODEL TECHNIQUES OF BRYSON AND LI. IN THE MULTIPLICATIVE WHITE NOISE MODEL, THE COST EQUATIONS ARE MODIFIED TO REFLECT THE FACT THAT KNOWLEDGE OF THE PARAMETERS IS LIMITED BECAUSE THEY ARE CONSIDERED TO BE NOISE PRECESSES. THE SENSITIVITY SYSTEM TECHNIQUE AUGMENTS THE PLANT STATED BY SENSITIVITY STATES WHICH REFLECT THE FIRST ORDER SENSITIVITIES OF THE STATE TRAJECTORIES TO PARAMETER VARIATION. OF PARTICULAR INTEREST TO THE PRESENT WORK ARE THE MULTI-MODEL TECHNIQUES IN WHICH THE COSTS ASSOCIATED WITH A DISCRETE SET OF SYSTEMS, REPRESENTING KEY ELEMENTS OF THE CONTINUOUS SET OF SYSTEMS, ARE AVERAGED. THE AVERAGE COST IS MINIMIZED TO OBTAIN A CONTROLLER WHICH ADDS ROBUSTNESS TO THE CONTROLLER. THERE ARE OPEN ISSUES AS TO THE SELECTION OF THE FINITE SET OF SYSTEMS TO BE USED IN THE CONTROL DESIGN.

THE APPROACH USED IN THE MULTI-MODEL ROBUST DESIGN TECHNIQUE IS VERY SIMILAR TO THE APPROACH TAKEN IN THIS WORK. THE KEY DIFFERENCE IS THAT IN THIS WORK THE COST TO BE MINIMIZED IS THE AVERAGE OF THE QUADRATIC (H2) COST OVER A CONTINUOUS RATHER THAN DISCRETE SET OF PLANTS. THIS COST CAN BE COMPUTED BY CONSIDERING A PARAMETERIZED LYAPUNOV EQUATION. AT ANY PARAMETER VALUE, THE SOLUTION OF THIS EQUATION CAN BE USED TO GIVE THE QUADRATIC COST AT THAT VALUE. THEREFORE, THE PARAMETERIZED LYAPUNOV EQUATION HAS A PARAMETERIZED SOLUTION MATRIX WHICH CAN BE AVERAGED TO OBTAIN THE AVERAGE COST. SINCE A CONTINUOUS SET OF SYSTEMS IS CONSIDERED, THIS AVERAGE IS REPRESENTED AS AN INTEGRAL OF THE QUADRATIC COST OVER THE DOMAIN OF THE PARAMETER VALUES.

Approach

- For each parameter value, the cost can be defined as the $\mathcal{H}_2$ norm of the system which is equivalent to the standard quadratic cost.

$$J(\alpha) = \|G(s,\alpha)\|_2 = \mathrm{tr}\left[C_1 Q(\alpha) C_1^T\right]$$

- For each parameter value, the cost is dominated by the solution of a parameterized Lyapunov equation.

$$0 = \left[A_{cl} + \sum_i \alpha_i A_{cli}\right] Q(\alpha) + Q(\alpha)\left[A_{cl} + \sum_i \alpha_i A_{cli}\right]^T + B_1 B_1^T$$

- The approach used is to minimize the cost averaged over the set of systems.

$$J = \int_\alpha J(\alpha)\, d\mu(\alpha) = \mathrm{tr}\left[C_1 \langle Q(\alpha)\rangle C_1^T\right]$$

THE MOTIVATION FOR MINIMIZING THE AVERAGE COST IS THAT BOUNDED AVERAGE COST IMPLIES SYSTEM STABILITY OVER THE MODEL SET. IF ANY ELEMENT OF THE SET OF SYSTEMS WERE UNSTABLE, THIS WOULD IMPLY UNBOUNDED COST AT THAT POINT AND ALSO UNBOUNDED AVERAGE COST. AS A CONSEQUENCE OF THIS PROPERTY OF THE AVERAGE COST, ANY CONTROLLER DESIGNED BY MINIMIZING THE AVERAGE COST WILL GUARANTEE STABILITY FOR ALL OF THE CLOSED LOOP SYSTEMS IN THE SET. THIS IS NOT A PROPERTY DEMONSTRATED BY THE DISCRETE SET AVERAGES SINCE ONLY THE POINTS ACTUALLY INCLUDED IN THE AVERAGE COULD BE GUARANTEED STABLE AND NOT THOSE PARAMETER VALUES THAT LAY BETWEEN THE CHOSEN POINTS.

THERE IS A PROBLEM WITH USING THE AVERAGE COST AS A DESIGN TOOL WHICH IS ASSOCIATED WITH THE COMPUTATION OF THE AVERAGE COST. THE AVERAGE COST CAN RARELY BE CALCULATED EXACTLY. THIS PROBLEM ARISES BECAUSE OF THE HIGH DIMENSIONALITY OF THE PARAMETER SPACE. IF THE PROBLEM HAS FEW UNCERTAINTIES OR PARAMETER DEPENDENCE OF THE SOLUTION OF THE PARAMETERIZED LYAPUNOV EQUATION CAN BE FOUND EXACTLY, THEN THE EXACT AVERAGE COST CAN BE COMPUTED BY DIRECT NUMERICAL INTEGRATION TECHNIQUES. THIS IS NOT TYPICALLY THE CASE FOR CONTROLLED STRUCTURES WHERE THE NUMBER OF UNCERTAINTIES AND THE SIZE OF THE PLANT ARE BOTH LARGE. IN THIS CASE, THE AVERAGE MUST BE FOUND BY APPROXIMATION TECHNIQUES. AMONG THE POSSIBLE OPTIONS ARE DIRECT MONTE-CARLO INTEGRATION TO FIND THE AVERAGE. THIS REQUIRES A LARGE NUMBER OF SAMPLE CASES TO FIND THE AVERAGE AND IS THEREFORE INEFFICIENT. THERE ARE

Motivation

- Bounded average H_2-norm implies stability over the set of systems.
- A controller designed to minimize the average cost will guarantee stability for all the elements of the model set.
- Problem: Difficult to compute the exact average cost since it involves the average solution to a parameterized Lyapunov equation.
- Computational Options:
  - Monte-Carlo Simulation
  - Propabalistic Structural Analysis.
  - Stochastic Operator Methods

NEW TECHNIQUES IN THE AREA OF PROBABILISTIC STRUCTURAL ANALYSIS FOR COMPUTING THE MEAN OF THE SOLUTIONS TO RANDOM LINEAR EQUATIONS. THESE HAVE BEEN FOUND TO BE INAPPLICABLE TO THE TYPE OF PROBLEM FOUND IN ROBUST CONTROL DESIGN BECAUSE OF THE STRUCTURE OF THE PROBLEM. FINALLY SOME APPROXIMATION METHODS FROM THE FIELD OF STOCHASTIC OPERATOR THEORY WERE CHOSEN AS THE MOST PROMISING METHODS TO OBTAIN THE AVERAGE SOLUTIONS TO PARAMETERIZED LYAPUNOV EQUATIONS.

THERE ARE SEVERAL STOCHASTIC OPERATOR TECHNIQUES FOR CALCULATING APPROXIMATIONS TO THE AVERAGE COST WHICH CAN BE BORROWED FROM THE FIELDS FOR WAVE PROPAGATION IN RANDOM MEDIA AND TURBULENCE MODELLING. IN THESE FIELDS THE STOCHASTIC OPERATOR TECHNIQUES ARE USED TO CALCULATE THE AVERAGE INTENSITY OF A BEAM TRAVELING THROUGH A MEDIA OF RANDOMLY VARYING TRANSMISSIBILITY OR THE AVERAGE MOMENTUM TRANSFER IN A TURBULENT FLOW. THE CORE OF THE TECHNIQUES RELY ON OPERATOR DECOMPOSITION. THE GOVERNING OPERATOR FOR THE PARTICULAR PROBLEM IS DECOMPOSED INTO A NOMINAL PART AND A PARAMETER DEPENDENT PART. FOR ROBUST CONTROL, THE PARAMETERIZED LYAPUNOV EQUATION CAN BE CONSIDERED AS A STOCHASTIC LINEAR OPERATOR WHICH IS DECOMPOSED INTO A NOMINAL AND PARAMETER DEPENDENT PART. ONCE THE OPERATOR IS DECOMPOSED, THERE ARE TWO POSSIBLE TECHNIQUES USED TO EXPRESS THE EXACT AVERAGE AS AN INFINITE SERIES. THE TRUNCATION OF THESE TWO SERIES LEADS TO TWO POSSIBLE APPROXIMATIONS TO THE EXACT AVERAGE WHICH ARE CALLED THE PERTURBATION EXPANSION APPROXIMATION AND THE DYSON APPROXIMATION.

Stochastic Operator Methods

- Can borrow techniques from field of random wave propagation and turbulence modelling.

- Parameterized Lyapunov equation can be decomposed into a sum of nominal and parameter dependent operators

$$0=\left[A_{cl}+\sum_i \alpha_i A_{cli}\right]Q(\alpha)+Q(\alpha)\left[A_{cl}+\sum_i \alpha_i A_{cli}\right]^T+B_1B_1^T$$

  becomes

$$\mathbf{L}_0(Q)+\mathbf{R}(Q)=-B_1B_1^T$$

  where

$$\mathbf{L}_0(Q):\quad Q\rightarrow A_{cl}Q+QA_{cl}^T$$

$$\mathbf{R}(Q):\quad Q\rightarrow\left(\sum_i \alpha_i A_{cli}\right)Q+Q\left(\sum_i \alpha_i A_{cli}\right)^T$$

- Two methods for computing the exact average:

  Perturbation Expansion
  Dyson Equation

THE SOLUTION OF A STOCHASTIC LINEAR OPERATOR EQUATION CAN BE FOUND FOR ALL PARAMETERS BY EXPANDING THE SOLUTION ABOUT THE NOMINAL SOLUTION IN POWERS OF THE PARAMETER DEPENDENT PART OF THE OPERATOR--THIS EXPANSION IS CALLED THE PERTURBATION EXPANSION BECAUSE THE SOLUTION IS FOUND BY CONSIDERING THE PARAMETER DEPENDENT SOLUTION TO BE A SMALL PERTURBATION ABOUT THE NOMINAL. THE EXACT AVERAGE SOLUTION CAN BE FOUND BY INTEGRATING EACH TERM IN THE SERIES. AN APPROXIMATION TO EXACT AVERAGE CAN BE DERIVED BY ONLY CONSIDERING A FINITE NUMBER (TYPICALLY TWO) TERMS IN THE SERIES FOR THE AVERAGE. THE AVERAGE SOLUTION CAN BE CONSIDERED AS A SUM OF NOMINAL SOLUTION AND A TERM DEPENDING ON THE SECOND POWER OF THE PARAMETER DEPENDENT PART. THERE ARE SEVERAL LIMITATIONS TO THE PERTURBATION EXPANSION APPROXIMATION. FIRST, SINCE THE SERIES IS ABOUT THE NOMINAL SOLUTION WHICH CAN BE VERY FAR REMOVED FROM THE AVERAGE SOLUTION, THE PERTURBATION EXPANSION CAN TAKE MANY TERMS TO CONVERGE. THIS CAUSES DIFFICULTIES BECAUSE EACH HIGHER TERM IN THE SERIES IS MORE DIFFICULT TO AVERAGE THAN THE PREVIOUS. THE PERTURBATION EXPANSION TRUNCATION ALSO HAS LIMITATIONS IN ITS USE AS AN APPROXIMATE AVERAGE COST FOR CONTROL DESIGN. THE REASON FOR THIS IS THAT THE APPROXIMATION WILL NEVER "BLOW UP" AS LONG AS ONLY A FINITE NUMBER OF TERMS ARE USED IN THE APPROXIMATION. THE APPROXIMATION WILL THEREFORE NEVER INDICATE THAT THE CLOSED LOOP SYSTEM IS UNSTABLE. CONTROL DESIGN USING THIS APPROXIMATION MAY LEAD TO CONTROLLERS WHICH ARE UNSTABLE OVER A LARGE PART OF THE SET.

Perturbation Expansion

- The decomposed equation can be solved by successive substitution.

$$Q = Q_0 - \mathbf{L}_0^{-1}\mathbf{R}Q_0 + \mathbf{L}_0^{-1}\mathbf{R}\mathbf{L}_0^{-1}\mathbf{R}Q_0 - \cdots \quad = \sum_{i=0}^{\infty}(-\mathbf{L}_0^{-1}\mathbf{R})^i\, Q_0$$

- The average can be found by averaging each term.

$$\langle Q\rangle = Q_0 + \mathbf{L}_0^{-1}\langle \mathbf{R}\mathbf{L}_0^{-1}\mathbf{R}\rangle Q_0 + \text{higher order terms}$$

- Problems with the formal perturbation expansion:
  - Average converges very slowly
  - Any finite truncation never "blows up"

AN ALTERNATE METHOD FOR COMPUTING THE AVERAGE SOLUTION OF A STOCHASTIC LINEAR OPERATOR IS THE DYSON EQUATION. THE DYSON EQUATION IS ALSO AN INFINITE SERIES FOR THE AVERAGE BUT INSTEAD OF EXPANDING ABOUT THE NOMINAL, THE SERIES IS EXPANDED ABOUT THE AVERAGE SOLUTION. THEREFORE THE AVERAGE SOLUTION IS GIVEN AS A FUNCTION OF THE PARAMETER DEPENDENT PART OF THE OPERATOR AS WELL AS THE AVERAGE SOLUTION ITSELF. THE DYSON EQUATION HAS SOME USEFUL PROPERTIES. THE FIRST IS THAT THE DSYON EQUATION IS A LINEAR EQUATION FOR THE AVERAGE SOLUTION. THIS EASES THE SOLUTION PROCEDURES FOR THE AVERAGE. THE SECOND IS THAT A FINITE TRUNCATION OF THE DYSON INFINITE SERIES IS STILL EFFECTIVELY AN INFINITE TERM EXPANSION SINCE THE AVERAGE SOLUTION APPEARS ON BOTH SIDES OF THE EQUATION. THIS PROPERTY ENABLES THE DYSON EQUATION TO "BLOW UP" AND THEREBY INDICATE AREAS OF INSTABILITY IN THE PARAMETER DOMAIN.

Dyson Equation

- Define the Averaging Operator, **A**, which separates the random solution into the *mean* and the *fluctuating* part.

$$\langle Q \rangle = \mathbf{A}(Q), \quad \partial Q = (\mathbf{I} - \mathbf{A})Q$$

- By manipulation, can remove the dependance on the fluctuating field and derive an operator equation for the mean, called the Dyson Equation.

$$\langle Q \rangle = Q_0 + \mathbf{L}_0^{-1}\mathbf{M}\langle Q \rangle$$

$$\mathbf{M} = -\sum_{n=1}^{\infty} \mathbf{A}\mathbf{R}\left[-\mathbf{L}_0^{-1}(\mathbf{I} - \mathbf{A})\mathbf{R}\right]^n \mathbf{A}$$

- Properties:
  - Linear equation for <Q>
  - The infinite series is about the mean solution instead of the nominal.
  - Truncation of **M** still implies and infinite series expansion
  - Truncation of **M** will still "blow up".

NOW THAT THE TWO METHODS OF COMPUTING THE AVERAGE COST HAVE BEEN DERIVED, ITS USEFUL TO STEP BACK AND EVALUATE THE OPTIONS AVAILABLE FOR CONTROL DESIGN. THESE OPTIONS CAN AGAIN ROUGHLY BE DIVIDED INTO BOUNDING AND APPROXIMATING METHODS.

THE AVERAGE COST CAN BE BOUNDED IN TWO WAYS. FIRST THE PERTURBATION EXPANSION SERIES FOR THE SOLUTION AT ANY PARAMETER VALUE CAN BE BOUNDED TERM BY TERM. THIS IS CALLED THE "WORST CASE OVERBOUND" SINCE THIS OVERBOUND GIVES A COST HIGHER THAN THE COST ASSOCIATED WITH ANY PLANT IN THE SET. THE SECOND METHOD IS TO BOUND THE AVERAGED PERTURBATION EXPANSION SERIES TERM BY TERM. INSTEAD OF BOUNDING EACH ELEMENT OF THE SET, THIS OVERBOUND IS ONLY GUARANTEED TO BOUND THE AVERAGE SOLUTION AND CAN THEREFORE BE CONSIDERED LESS CONSERVATIVE. THIS TECHNIQUE IS KNOWN AS THE AVERAGE OVERBOUND.

APPROXIMATION FOR THE AVERAGE COST CAN BE DERIVED EITHER FROM A TRUNCATION OF THE PERTURBATION EXPANSION FOR THE AVERAGE OR THE DYSON EQUATION FOR THE AVERAGE. THESE TECHNIQUES WILL BE EXAMINED IN MORE DETAIL IN THE COMING PAGES.

Options

- Both techniques for calculating the average involve infinite series of operators which cannot be computed.
- There are therefore two options for continuing:

Overbounding

- Worst Case Overbound
  - Overbound the unaveraged Perturbation Expansion.
- Average Overbound
  - Overbound the averaged Perturbation Expansion.

Approximating

- Perturbation Expansion Truncation
  - First two term in the Averaged Perturbation Expansion
- Dyson Equation Truncation- Bourret Equation.
  - First term of the **M** operator retained.

THE PERTURBATION EXPANSION APPROXIMATION TO THE AVERAGE COST IS OBTAINED BY TRUNCATING THE PERTURBATION EXPANSION INFINITE SERIES FOR THE AVERAGE AFTER THE SECOND TERM LEAVING ONLY THE QUADRATIC TERM TO REFLECT THE PARAMETER DEPENDENCE. WHEN APPLIED TO THE PROBLEM OF APPROXIMATING SOLUTIONS TO THE PARAMETERIZED LYAPUNOV EQUATION, TWO SETS OF LYAPUNOV EQUATIONS ARE OBTAINED. THE EQUATIONS ARE COUPLED HIERARCHICALLY SO THAT ONCE THE SECOND EQUATION IS SOLVED ITS SOLUTION CAN BE SUBSTITUTED INTO THE FIRST EQUATION TO OBTAIN THE APPROXIMATE AVERAGE.

Perturbation Expansion Approximation

- Perturbation expansion can be approximated by taking only the first two terms.

$$Q^P = Q_0 + \mathbf{L}_0^{-1}\langle \mathbf{R}\mathbf{L}_0^{-1}\mathbf{R}\rangle Q_0$$

- For the parameterized Lyapunov equation, this gives a set of equations for the approximate mean.

$$Q^P \cong \langle Q\rangle$$

where:

$$A_{cl}Q^P + Q^P A_{cl}^T = -B_1^T B_1 + \sum_i \sigma_i \left(A_{cli}Q_i + Q_i A_{cli}^T\right)$$

$$A_{cl}Q_i + Q_i A_{cl}^T = \sigma_i \left(A_{cli}Q_0 + Q_0 A_{cli}^T\right)$$

- The two sets of Lyapunov equations are coupled hierarchically and easily solved.

THE BOURRET APPROXIMATION TO THE AVERAGE COST IS OBTAINED BY TRUNCATING THE INFINITE SERIES IN THE DYSON EQUATION AFTER THE SECOND TERM AND THEREFORE INVOLVES ONLY SECOND ORDER CORRELATIONS AMONG THE PARAMETERS. THE EQUATION CONTAINS ONLY THE QUADRATIC TERM IN THE PARAMETERIZED OPERATOR, R; BUT, BECAUSE THE AVERAGE APPEARS ON BOTH THE RIGHT AND LEFT HAND SIDES OF THE EQUATION, THE BOURRET EQUATION IS EQUIVALENT TO AN INFINITE EXPANSION FOR THE AVERAGE. IN THE CASE OF COMPUTING THE AVERAGE TO THE PARAMETERIZED LYAPUNOV EQUATION, TWO COUPLED PARAMETER INDEPENDENT LYAPUNOV EQUATIONS ARE OBTAINED FOR THE APPROXIMATION OF THE SOLUTION OF THE AVERAGE COST. THESE TWO EQUATIONS CAN BE WRITTEN AS A SINGLE EQUATION AND SOLVED EXPLICITLY USING KRONECKER MATH TECHNIQUES.

Dyson Equation Approximation - Bourret Equation

- Dyson Equation can be approximated by taking only the first term of **M**. Called the Bourret or first order smoothing approximation.

$$Q^B = Q_0 + \mathbf{L}_0^{-1} \langle \mathbf{R} \mathbf{L}_0^{-1} \mathbf{R} \rangle Q^B$$

- For the parameterized Lyapunov equation, this gives a set of equations for the approximate mean.

$$Q^B \cong \langle Q \rangle$$

where:

$$A_{cl} Q^B + Q^B A_{cl}^T = -B_1^T B_1 + \sum_i \sigma_i \left(A_{cli} Q_i + Q_i A_{cli}^T \right)$$

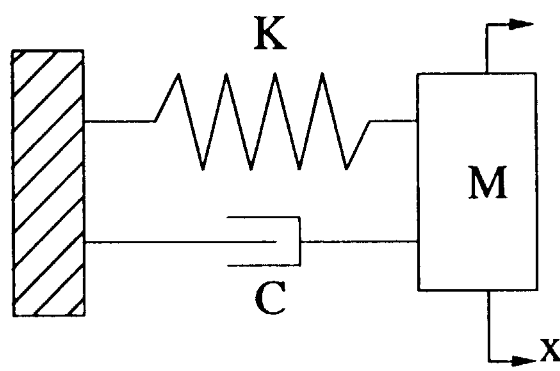
$$A_{cl} Q_i + Q_i A_{cl}^T = \sigma_i \left(A_{cli} Q^B + Q^B A_{cli}^T \right)$$

- Can be written as a single linear equation for the coefficients of Q^B and solved explicitly using Kronecker math.

A SIMPLE EXAMPLE IS CONSIDERED TO ILLUSTRATE AND COMPARE SOME OF THESE OPTIONS FOR BOUNDING AND APPROXIMATING THE AVERAGE COST. THE PROBLEM CONSIDERED IS A SIMPLE SPRING MASS DAMPER SYSTEM WHERE THE DAMPER IS UNCERTAIN AND VARIES ABOUT SOME NOMINAL VALUE. FOR THE PURPOSES OF THIS EXAMPLE THE NOMINAL VALUES OF THE CONSTANTS WERE TAKEN TO BE $K = 1$, $M = 1$, AD $C = 0.25$. THE BOUNDS AND APPROXIMATIONS TO THE COST WERE THEN COMPUTED AS A FUNCTION OF THE UNCERTAINTY BOUND, THE RANGE ABOUT THE NOMINAL WITHIN WHICH THE DAMPER VALUES CAN VARY. THIS FUNCTION IS PLOTTED IN THE NEXT PAGE.

Illustrative Example

- Consider a simple spring-mass damper system (K=1, M=1, C=0.25):



- Allow the damper to vary about the nominal value, C_0, within bounds, δ.

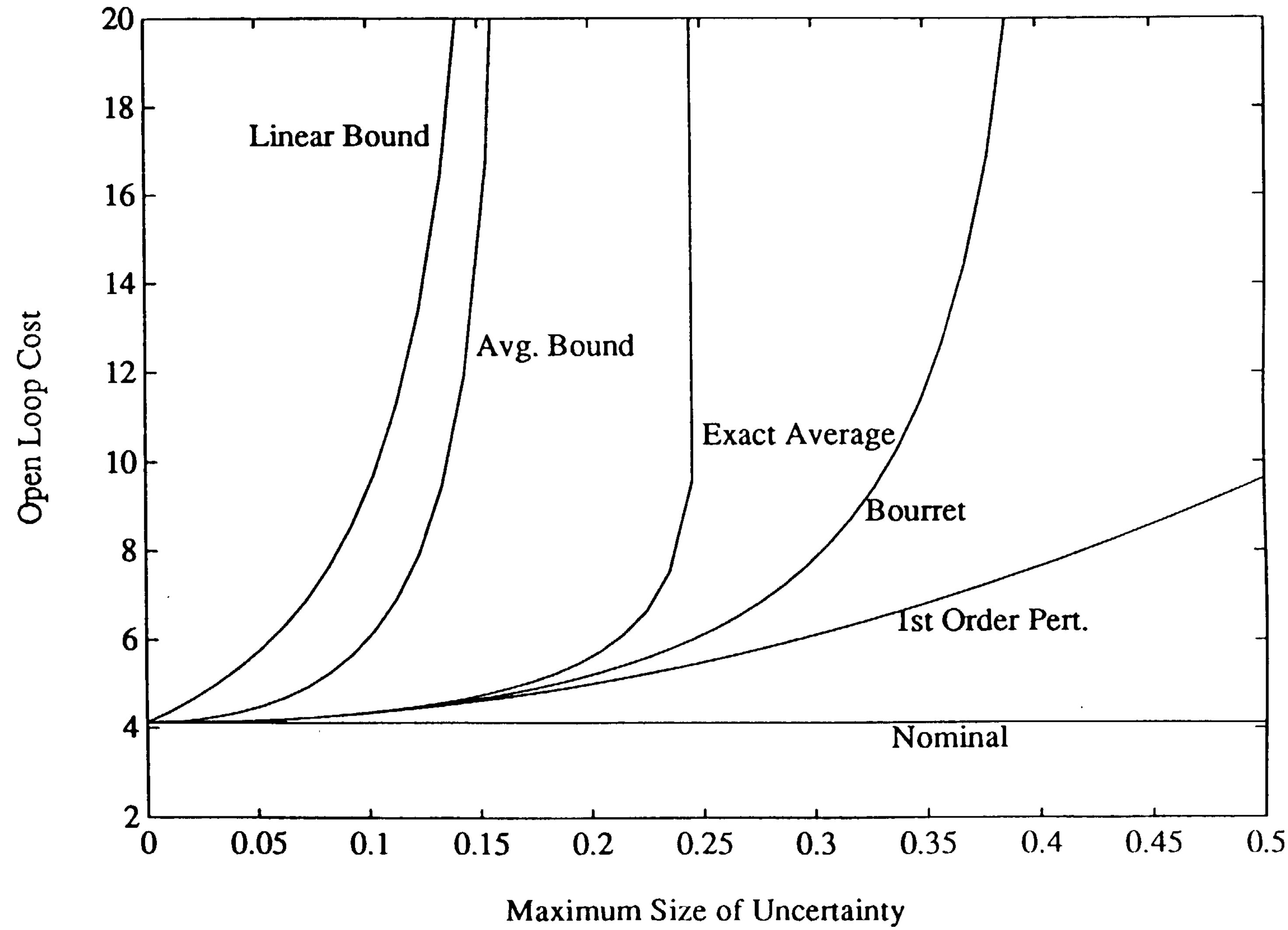
$$C(\alpha) = C_0 + \alpha \quad \alpha \in [-\delta, \delta]$$

- Assume a uniform weighting over the set of systems.

VALUES OF THE BOUNDS AND APPROXIMATIONS TO AVERAGE QUADRATIC COST OF THE SYSTEM DESCRIBED IN THE PREVIOUS PAGE ARE COMPARED. NOTE THAT AS SIZE OF THE UNCERTAINTY BOUND EXCEEDS NOMINAL DAMPER VALUE (IE., SOME ELEMENTS OF THE SET OF SYSTEMS ARE UNSTABLE) THE EXACT AVERAGE COST GOES TO INFINITY. THIS SUPPORTS THE CLAIM THAT BOUNDED AVERAGE COST INDICATES STABILITY OVER THE SET OF SYSTEMS. THE REMAINING CURVES CAN BE DIVIDED INTO THOSE THAT BOUND THE AVERAGE AND THOSE THAT APPROXIMATE IT. OF THE APPROXIMATING CURVES (THOSE THAT LIE BELOW THE EXACT AVERAGE COST CURVE) THE BOURRET APPROXIMATION IS SUPERIOR TO THE FIRST ORDER PERTURBATION EXPANSION APPROXIMATION. THE BOURRET APPROXIMATION "BLOWS UP" WHEN THE BOUND EXCEEDS ± 0.4 INDICATING THAT THE DESIGN TECHNIQUE THINKS THAT THE SYSTEM COULD HAVE UNSTABLE ELEMENTS WHEN THIS SIZE UNCERTAINTY BOUND IN USED. APPROXIMATE TECHNIQUES WILL NOT GUARANTEE STABILITY OVER THE DESIGN MODEL SET SINCE THE COST DOES NOT ASYMPTOTE TO INFINITY UNTIL THERE ARE ALREADY UNSTABLE SYSTEMS WITHIN THE SET. THE COST BOUNDING CURVES ARE GIVEN BY LINEAR BOUND AND AVERAGE BOUND. THE LINEAR BOUND IS A WORST CASE OVERBOUND AS DISCUSSED BEFORE SINCE IT BOUNDS THE WORST CASE COST OVER THE SET OF SYSTEMS. BOTH THE BOUNDS ASYMPTOTE TO INFINITY AT A SMALLER BOUND THAN DOES THE EXACT AVERAGE INDICATING CONSERVATISM IN THE BOUND. THE BOUNDS "THINK" THE SYSTEM WILL HAVE UNSTABLE ELEMENTS BEFORE IT DOES. WHEN USED IN DESIGN, THESE TECHNIQUES WILL COMPENSATE FOR THESE IMAGINARY UNSTABLE SYSTEMS AND GIVE CONTROLLERS WITH HIGHER CONTROL GAINS.

Cost Versus Uncertainty

- The value of various parameter dependent cost equations.



NOW THAT TECHNIQUES HAVE BEEN ESTABLISHED FOR DEALING WITH THE UNCERTAINTY IN THE CONTROL DESIGN, A SYSTEM DESIGN METHODOLOGY CAN BE DEVELOPED TO COMPLEMENT THE ROBUST CONTROL DESIGN PROBLEM. THE FIRST STEP IS TO FORMULATE THE COMPLETE PARAMETRIC ERROR MODEL USING WHATEVER INFORMATION IS AVAILABLE CONCERNING THE STRUCTURE OF THE UNCERTAINTIES. THERE ARE SEVERAL OPTIONS AVAILABLE FOR FORMULATING THIS MODEL. THE ERROR MODEL AND STRUCTURE CAN BE OBTAINED BY COMPARING A FINITE ELEMENT MODEL'S NATURAL FREQUENCIES,DAMPING RATIOS AND MODE SHAPES TO THOSE DETERMINED EXPERIMENTALLY. IF A MEASUREMENT BASED MODEL IS USED IN THE CONTROL DESIGN, THE RELATIVE ACCURACIES OF THE IDENTIFIED PARAMETERS CAN BE INCLUDED IN THE ERROR MODEL. IF AN ANALYTICAL MODEL IS USED FOR THE CONTROL DESIGN THE ERROR MODEL AND STRUCTURE CAN BE DETERMINED ANY CALCULATING THE CRITICAL PARAMETER UNCERTAINTIES BASED ON UNCERTAINTIES IN THE MATERIAL PROPERTIES, LENGTHS, ETC. ONCE THIS COMPLETE ERROR MODEL HAS BEEN DERIVED, THE NUMBER OF UNCERTAINTIES CONSIDERED IN THE CONTROL DESIGN MUST BE REDUCED. THE PRIMARY MOTIVATION FOR THIS IS TO SIMPLIFY THE CONTROL DESIGN PROCEDURE SINCE EACH UNCERTAINTY GREATLY COMPLICATES THE DESIGN PROCESS. THE UNCERTAINTY REDUCTION STEP IS KNOWN AS MODEL UNCERTAINTY TRUNCATION. THE UNCERTAINTIES CAN BE TRUNCATED FROM THE MODEL ON THE BASIS OF THE SIZE OF THEIR CONTRIBUTION TO THE AVERAGED COST. IF AN UNCERTAINTY CONTRIBUTES LITTLE TO THE COST, IT CAN BE IGNORED IN THE DESIGN PROCESS.

System Design Philosophy

- Formulate Complete Parametric Error Model
  - From analytical model (Finite Element) via error with measurement model or parameter uncertainties.
  - From measurement model via quantification of id errors.

- Model Uncertainty Truncation
  - Uncertainties are computationally expensive in control system design.
  - Can truncate uncertainties on the basis of how they influence the cost.

- Robust Controller Design
  - Use any of the average related costs (Overbounds, Exact, Approximates) to design static or dynamic controllers which are sensitive to magnitude and direction of uncertainties.

- Design Validation on Complete Error Model
  - evaluate design robustness in the presence of all model errors.

FOLLOWING THE UNCERTAINTY REDUCTION STEP, THE NEXT STEP IS TO DESIGN THE CONTROLLER. THIS CAN BE ACCOMPLISHED BY MINIMIZING ANY OF THE TWO OVERBOUNDING OR TWO APPROXIMATING AVERAGE COST EQUATIONS. THE CHOICE OF WHICH TO USE IS OFTEN DICTATED BY THE REQUIREMENTS OF GUARANTEED STABILITY OR THE NEED FOR EFFICIENT COMPUTATIONAL SCHEMES. THE CONTROLLER DESIGN STEP IS FOLLOWED BY A DESIGN VALIDATION STEP. THE CONTROLLER IS APPLIED TO THE COMPLETE ERROR MODEL AND STABILITY IS CHECKED USING THE MOST APPLICABLE STABILITY ANALYSIS TOOL, FOR EXAMPLE, THE HURWITZ CRITERIA. FAILURE AT THIS STEP NECESSITATES ANOTHER ITERATION IN THE DESIGN PROCESS.

THE ESSENCE OF UNCERTAINTY TRUNCATION IS TO DETERMINE WHICH PARAMETERS MUST BE INCLUDED IN THE CONTROL DESIGN AND WHICH CAN BE NEGLECTED. THE DISCRIMINATION CAN BE ACHIEVED BY RANKING THE UNCERTAINTIES ON THE BASIS OF HOW THEY CONTRIBUTE TO THE COST. THIS PROCESS IS CALLED UNCERTAINTY COST ANALYSIS AND HAS ITS ROOTS IN SKELTON'S COMPONENT COST ANALYSIS. THE KEY STEP IS TO DECOMPOSE THE COST INTO ITS NOMINAL VALUE AND A PART REFLECTING THE ADDED COST OF EACH UNCERTAINTY. EACH OF THE AVERAGE BOUNDING AND APPROXIMATING COSTS CAN BE DECOMPOSED IN THIS MANNER. THOSE PARAMETERS WHICH CONTRIBUTE LITTLE TO THE COST ARE THEN IGNORED. THE ONLY DIFFICULTY IN THIS PROCESS STEMS FROM THE FACT THAT THE IMPORTANT OPEN LOOP PARAMETERS ARE NOT NECESSARILY THE IMPORTANT CLOSED LOOP PARAMETERS. A GUESS AT THE CONTROLLER MUST BE MADE BEFORE THE TRUNCATION, AND THE UNCERTAINTY COSTS MUST BE EVALUATED AFTER THE CONTROL DESIGN TO DETERMINE IF IN FACT THE IMPORTANT PARAMETERS WERE INCLUDED IN THE DESIGN.

ONCE WHICH UNCERTAIN PARAMETERS WILL BE INCLUDED IN THE CONTROL DESIGN HAS BEEN DETERMINED A CONTROLLER DESIGN PROCEDURE BASED ON MINIMIZING THE EXACT AVERAGE, APPROXIMATE AVERAGE, OR OVERBOUNDING AVERAGE COST CAN BE DEFINED. THE FIRST STEP IS TO DEFINE THE COST BASED ON THE CHOSEN FUNCTION, FOR INSTANCE THE BOURRET APPROXIMATION. THIS EQUATION IS THEN APPENDED TO THE COST USING A SYMMETRIC MATRIX OF LAGRANGE MULTIPLIERS. THE NECESSARY CONDITIONS FOR MINIMIZATION ARE THEN DETERMINED USING MATRIX CALCULUS. THESE NECESSARY CONDITIONS ARE THEN USED FOR GRADIENT INFORMATION IN THE COST MINIMIZATION USING FOR EXAMPLE A QUASI-NEWTON MINIMIZATION ROUTINE.

C-4

Uncertainty Truncation

- What parameter errors contribute most to the cost and must therefore be included in the robust controller design?

- Uncertainty Cost Analysis: Can separate each cost into cost due to nominal system and cost due to parameter uncertainty.

$$Q = Q_0 + \sum_i \sigma_i^2 Q_i$$

for example, for the Bouret Cost.

$$A_{cl} Q_i + Q_i A_{cl}^T = A_{cli} G_i + G_i A_{cli}^T$$

$$A_{cl} G_i + G_i A_{cl}^T = A_{cli} Q^B + Q^B A_{cli}^T$$

- Remove uncertainties based on how they contribute to the cost.

- Related to Skelton's Component Cost Analysis based on sensitivity system model (first order perturbation expansion).

Robust Control Synthesis

- Can develop controllers based on the Exact Average Cost or any of the overbounding or approximating costs.

  Step 1: Define cost based upon overbounding or approximating the solution to the parameterized Lyapunov equation.

  Step 2: Append the appropriate equation to the cost via Lagrange multipliers.

  Step 3: Determine the necessary conditions for optimality using matrix calculus.

  Step 4: Minimize appropriate cost taking advantage of necessary conditions.

- Compare controllers based on an exact solution with those arrived at by approximation or overbounding techniques.

THE PROCESS OF ACTUALLY FINDING THE OPTIMAL CONTROLLER GAINS INVOLVES A TECHNIQUE KNOWN AS HOMOTOPIC CONTINUATION. THE GENERAL TECHNIQUE IS TO START WITH NO UNCERTAINTY AND GRADUALLY INCREASE THE AMOUNT OF UNCERTAINTY IN THE MODEL UNTIL THE DESIRED LEVEL IS REACHED. WITH NO UNCERTAINTY IN THE MODEL, THE STANDARD LQG DYNAMIC COMPENSATOR IS READILY AVAILABLE AS AN INITIAL GUESS. THE UNCERTAINTY IS THEN INCREASED A SMALL AMOUNT AND A NEW CONTROLLER IS DERIVED STARTING AT THE LQG CONTROLLER. THIS NEW CONTROLLER IS THEN USED AS THE INITIAL GUESS IN THE NEXT STEP INCREASE IN THE UNCERTAINTY. THE PROCESS IS REPEATED USING EACH SUCCESSIVE CONTROLLER AS THE STARTING POINT FOR THE NEXT ONE UNTIL THE DESIRED AMOUNT OF UNCERTAINTY IS INCORPORATED IN THE DESIGN. AT EACH STEP THE ACTUAL COST MINIMIZATION IS PERFORMED BY A STANDARD QUASI-NEWTON SCHEME.

Solution Techniques for Controller Gains

- "Sneak up" on optimal controller by slowly increasing the problem uncertainty.

  Step 1: With no uncertainty, LQG or LQR controller is readily available as an initial guess.

  Step 2: Increase uncertainty and find new optimal compensator.

  Step 3: Repeat Step 2, using the new compensator as a starting point for the minimization.

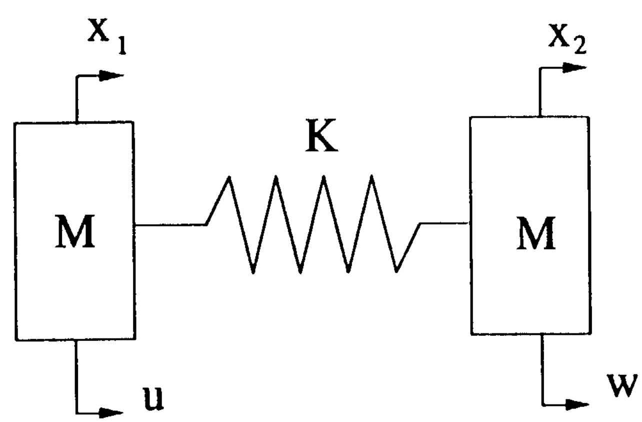
  Step 3: Continue increasing uncertainty until desired robustness is achieved.

- Cost minimization preformed using a quasi-newton scheme incorporating the gradient information provided by the necessary conditions.

THE COMPENSATOR DESIGN PROBLEM CAN BE ILLUSTRATED ON A SIMPLE EXAMPLE KNOWN AS THE ROBUST CONTROL SAMPLE PROBLEM. THIS PROBLEM HAS BEEN USED AS A TEST CASE FOR MANY CONTROL DESIGNS ATTEMPTING TO DEAL WITH REAL PARAMETER UNCERTAINTY. THE PLANT CONSISTS OF TWO MASSES CONNECTED BY A SPRING WITH UNCERTAIN STIFFNESS. THE NOMINAL VALUES ARE $M = 1$, WITH THE STIFFNESS VARYING IN THE RANGE $0.5 < K < 2$. THE GOAL IS TO DESIGN A CONTROLLER TO REGULATE OUTPUT X2 USING A NON-COLOCATED CONTROL FORCE AT X1 IN THE PRESENCE OF SOME SENSOR NOISE AND PLANT DISTURBANCE WHICH IS ROBUST TO THE SPRING UNCERTAINTY.

Robust Control Sample Problem

- Consider a simple 2 mass/spring system representing a generic flexible spacecraft with uncertain stiffness:



- Design dynamic compensator for stability in the presence of the uncertain spring.

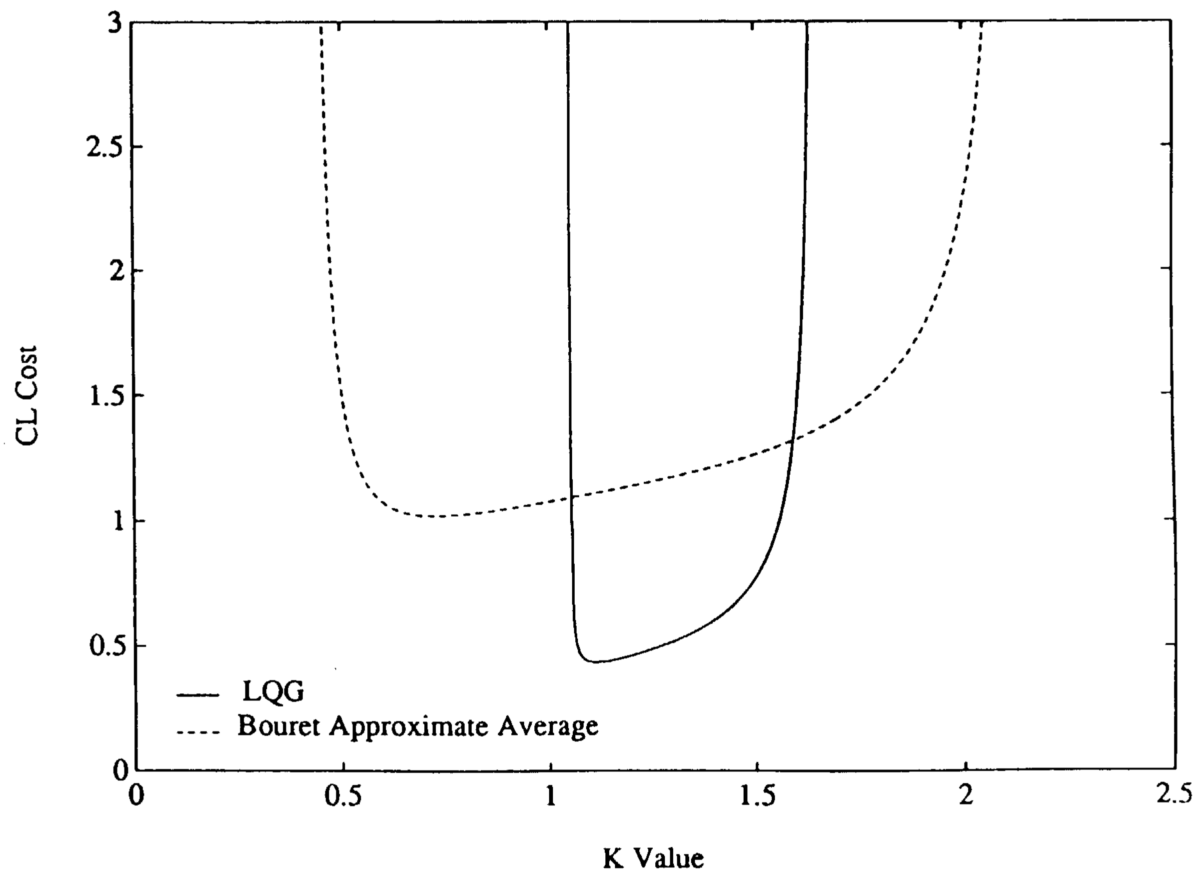
$$0.5 \leq K \leq 2$$

- Design for settling time of nominal plant (K = 1.1) to be < 15 seconds.

THE CONTROLLERS DERIVED EITHER FROM STANDARD LQG TECHNIQUES OR FROM MINIMIZATION OF THE BOURRET APPROXIMATE COST CAN BE COMPARED ON THE ROBUST CONTROL SAMPLE PROBLEM. THE CLOSED LOOP QUADRATIC COST (UNAVERAGED) CAN BE PLOTTED AS A FUNCTION OF THE ACTUAL STIFFNESS OF THE SPRING. THE STIFFNESS VALUES FOR WHICH THE CURVES ARE UNBOUNDED INDICATE UNSTABLE CLOSED LOOP SYSTEM USING THE INDICATED CONTROLLER. THE LQG DESIGN HAS A RELATIVELY NARROW TOLERANCE FOR K VARIATION AS INDICATED BY THE WIDTH OF THE COST CURVE. THE BOURRET DESIGN CLEARLY ADDS ROBUSTNESS, IN FACT ACHIEVING STABILITY THROUGHOUT THE DESIRED RANGE. THIS ROBUSTNESS IS INDICATED BY A BROADER COST "BUCKET." THE COST FOR THIS WIDTH IS LOSS OF PERFORMANCE AT THE NOMINAL K VALUE, THE DEPTH OF THE BUCKET. IT IS ALSO INTERESTING TO NOTE THAT WHILE THE LQG COST CURVE ACHIEVES A MINIMUM AT THE NOMINAL K VALUE ($K = 1.1$ WAS USED IN THE DESIGN), THE BOURRET DESIGN ACHIEVES A MINIMUM AT $K = 0.6$. THIS IS BECAUSE THE BOURRET DESIGN MINIMIZES THE AVERAGE COST AND NOT THE COST ASSOCIATED WITH THE NOMINAL PLANT.

Bourret Dynamic Compensation

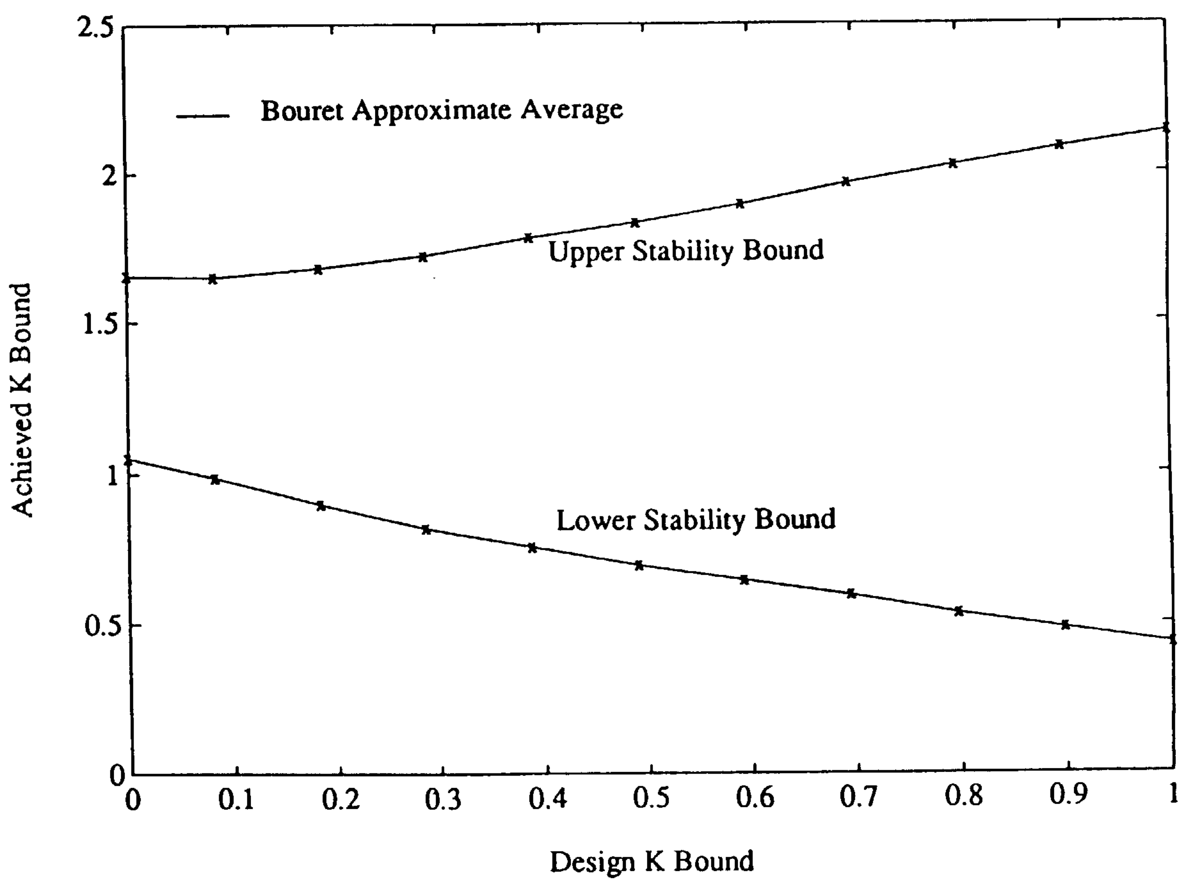
- Closed loop H2 -norm vs. actual spring value for LQG compensator and compensator designed by minimizing the Bourret approximate average.



THE BOURRET DESIGN DOES NOT NECESSARILY ACHIEVE STABILITY THROUGHOUT THE SYSTEM ON WHICH IT IS DESIGNED BECAUSE THE BOURRET COST IS ONLY AN APPROXIMATION TO THE EXACT AVERAGE COST. THE BOURRET DESIGN TECHNIQUE OBVIOUSLY ADDS ROBUSTNESS BUT IT IS IMPORTANT TO DETERMINE JUST HOW MUCH ROBUSTNESS THE DESIGNER GETS FOR A GIVEN SIZE OF THE SET USED. THE ACHIEVED STABILITY RANGE AS A FUNCTION OF THE DESIGN SET RANGE IS ONE MEASURE OF THE RESPONSIVENESS OF THE CONTROLLER. FROM THE FIGURE, INCREASING THE DESIGN K BOUND INCREASES THE RANGE OF SYSTEMS WHICH ARE CLOSED LOOP STABLE. THESE SYSTEMS ARE THOSE WITH K VALUES BETWEEN THE UPPER AND LOWER STABILITY BOUNDS. WITH NO ASSUMED UNCERTAINTY THE BOURRET DESIGN GIVES ONLY THE LQG STABILITY BOUNDS BUT IS QUITE RESPONSIVE TO INCREASING THE DESIGN BOUND. THE SMOOTH RELATIONSHIP BETWEEN TER DESIGNED AND ACHIEVE STABILITY RANGE OPENS OPTIONS FOR THE CONTROL DESIGNER. A LARGE THAN EXPECTED RANGE CAN BE USED IN THE DESIGN TO ACHIEVE ROBUSTNESS OVER A SMALLER BUT ACCEPTABLE RANGE.

Bourret Achieved Robustness

- Achieved upper and lower stability bounds as a function of parameter range used in Bouret approximate average design.



IN CONCLUSION A NEW CLASS OF CONTROLLERS HAS BEEN INVESTIGATED WHICH IS BASED ON MINIMIZING QUANTITIES RELATED TO THE QUADRATIC COST AVERAGED OVER A PARAMETERIZED SET OF PLANTS. SINCE THE EXACT AVERAGE QUADRATIC COST IS DIFFICULT TO OBTAIN, APPROXIMATIONS AND BOUNDS TO IT HAVE BEEN DEVELOPED USING OPERATOR DECOMPOSITION TECHNIQUES APPLIED TO THE PARAMETERIZED LYAPUNOV EQUATION WHICH DESCRIBES THE QUADRATIC COST AT EACH PINT IN THE MODEL SET. IN THE DESIGN PROCESS, THE UNIMPORTANT UNCERTAINTIES ARE FIRST IDENTIFIED AND REMOVED FOR THE PLANT SET FOR THE PURPOSES OF CONTROL DESIGN. CONTROLLERS ARE THEN DERIVED USING STANDARD MINIMIZATION TECHNIQUES WHICH EMPLOY THE EXPLICIT NECESSARY CONDITIONS DERIVED FOR THE PARTICULAR APPROXIMATION CHOSEN. THE FINAL CONTROLLER IS FOUND GRADUALLY BY SUCCESSIVELY INCREASING THE UNCERTAINTY IN THE CONTROL DESIGN AN STARTING WITH THE LQG CONTROLLER AS AN INITIAL GUESS. THIS CONTROL DESIGN METHODOLOGY HAS BEEN ILLUSTRATED FOR THE BOURRET APPROXIMATE AVERAGE COST ON A SIMPLE SAMPLE PROBLEM. FUTURE WORK WILL CONCENTRATE ON MORE COMPLICATED PROBLEMS WITH LARGER NUMBERS OF UNCERTAINTIES.

Conclusions

- Have investigated a new class of controllers based on minimizing quantities related to the cost averaged over a parameterized set of systems.
- Many difficulties in determining the exact average cost of a parameterized system.
- Used stochastic operator methods to determine approximations and bounds for the exact average cost.
- Can use these approximating or overbounding costs to determine the most important uncertainties to keep for the control design.
- Can use approximating or overbounding costs to derive static and dynamic controllers which are robust to the parametric model error.
- Future work to focus on applying these techniques to more realistic systems (Interferometer).

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CONTROL OF A CLASS OF SYSTEMS WITH PARAMETER UNCERTAINTY

Joel Douglas
Michael Athans

January 15, 1991

THE MOTIVATION OF THIS RESEARCH IS TO LEARN HOW TO CONTROL SYSTEMS WITH UNCERTAINTY. WE WILL CONCENTRATE ON SYSTEMS WITH PARAMETRIC UNCERTAINTY, I.E. UNCERTAIN PARAMETERS IN THE STATE VARIABLE DIFFERENTIAL EQUATIONS SUBJECT TO A PRIORI KNOWN BOUNDS ON THEIR NUMERICAL RANGE. THIS INCLUDES EXPLORING METHODS WHICH ALREADY EXIST, AS WELL AS DEVELOPING NEW METHODS.

THE ULTIMATE GOAL IS TO ROBUSTLY CONTROL THE INTERFEROMETER. THIS DEPENDS ON UNDERSTANDING HOW THE MODELING ERRORS, WHICH NECESSARILY WILL ARISE IN CREATING A MODEL OF THE SYSTEM, INFLUENCE THE CONTROL DESIGN AND THE ROBUSTNESS OF THE SYSTEM. WE NEED A CONTROLLER WHICH IS ROBUST TO THE UNCERTAIN PARAMETERS SUCH AS DAMPING, STIFFNESS, AND MODAL FREQUENCIES, AS WELL AS ACHIEVING ROBUSTNESS TO THE UNSTRUCTURED HIGH FREQUENCY MODELING ERRORS. WE NEED TO GUARANTEE THE INTERFEROMETER MEETS THE NECESSARY STABILITY AND PERFORMANCE CRITERIA REGARDLESS OF THE UNCERTAINTY IN THE SYSTEM.

MOTIVATION

- Understand existing design tools and develop new methodologies for robust control in the area of parameter (structured) uncertainty.

- Why? Control of the interferometer testbed will ultimately depend upon understanding how the modeling errors influence our control design, stability-robustness, and performance-robustness.
  - Structured parameter uncertainty (stiffness, damping)
  - High frequency modeling errors (unstructured uncertainty)
  - Other

OUR APPROACH WILL BE TO FIRST ASSUME WE HAVE COMPLETE KNOWLEDGE OF THE STATE OF THE SYSTEM. THOUGH THIS IS NOT REALISTIC IN REAL STRUCTURES SUCH AS THE INTERFEROMETER, UNDERSTANDING THE UNDERLYING FUNDAMENTALS IN THIS FRAMEWORK WILL HELP DIRECT US WHEN WE ASSUME KNOWLEDGE OF ONLY THE OUTPUT VARIABLES.

WE WILL FOCUS ON STRUCTURED UNCERTAINTY IN THE OPEN LOOP "A" MATRIX. THIS IS REPRESENTATIVE OF A STRUCTURAL SYSTEMS WHERE MODE FREQUENCIES AND DAMPING VALUES, WHICH APPEAR IN THE "A" MATRIX, ARE UNKNOWN.

WE WISH TO DESIGN A CONTROLLER FOR AN UNCERTAIN SYSTEM WHICH WILL GUARANTEE STABILITY OF THE CLOSED LOOP SYSTEM, AND ADDITIONALLY PROVIDE ROBUSTNESS GUARANTEES. WE WISH TO PROVIDE THE SAME ROBUSTNESS AS THOSE GUARANTEED BY THE CLASSICAL LQR DESIGN, NAMELY AN INFINITE UPWARDS GAIN MARGIN AND A DOWNWARDS GAIN MARGIN OF .5 INDEPENDENTLY AND SIMULTANEOUSLY IN ALL CONTROL CHANNELS, AND A PHASE MARGIN OF $\pm 60°$ INDEPENDENTLY AND SIMULTANEOUSLY IN ALL CONTROL CHANNELS.

APPROACH

- Assume full-state feedback (will relax later)
- Focus on uncertainty in open-loop dynamic parameters
- Try to guarantee stability and performance robustness of classical LQR design
    - Guaranteed stability
    - Reasonable guaranteed robustness (gain and phase margin properties)

We assume all uncertainty is in the "A" matrix, as shown in equation (1). Here, A_0 is the "nominal" A matrix, and each uncertain parameter is known to be in a bounded region; we assume we have p unknown parameters. The E_i matrices represent the structure of the uncertainty, and are scaled so that the magnitudes of the scalars q_i are less than 1. We further assume that the rank of each E_i is equal to 1.

The nominal cost functional for an LQR design is shown in equation (3). Here, Q_0 and ρ are free weighting parameters, which are chosen by loop-shaping or other techniques.

LQR DESIGN WITH UNCERTAINTY

Uncertain system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$A = A_0 + \Sigma q_i E_i ; \qquad |q_i| \le 1$$

The LQR design cost functional is

$$J = \int_0^\infty \left(x^T Q_0 x + \rho u^T u \right) dt$$

OUR GOAL IS TO PICK A WEIGHTING MATRIX Q TO REPLACE Q_0 IN THE COST FUNCTIONAL SUCH THAT AN LQR DESIGN WITH THIS NEW COST FUNCTIONAL GUARANTEES STABILITY AND STABILITY ROBUSTNESS OF THE UNCERTAIN SYSTEM. IN THE MASTERS THESIS RESEARCH OF JOEL DOUGLAS IT IS SHOWN THROUGH A FREQUENCY DOMAIN EQUALITY THAT A SUFFICIENT CONDITION FOR SUCH A MATRIX TO EXIST IS THAT $Q+P(A_0-A)+(A_0-A)^TP \geq 0$. SINCE THIS IS AN LQR DESIGN, P MUST BE THE SOLUTION OF THE STANDARD RICCATI EQUATION.

WE FIND AN APPROPRIATE Q BY USING AN OVERBOUNDING MATRIX PROPOSED BY PETERSEN AND HOLLOT IN "A RICCATI EQUATION APPROACH TO THE STABILIZATION OF UNCERTAIN LINEAR SYSTEMS", AUTOMATICA, VOL 22, NO. 4, 1986. WE SOLVE A MODIFIED RICCATI EQUATION FOR P AS SHOWN IN EQUATION (8). THEN WE PICK Q AS IN EQUATION (9). NOTE THAT AS THE UNCERTAINTY REDUCES TO ZERO, THIS ROBUST DESIGN REDUCES TO THE STANDARD LQR DESIGN.

THE MODIFIED RICCATI EQUATION (8) LOOKS VERY SIMILAR TO AN H_∞ RICCATI EQUATION, EXCEPT FOR THE ADDITIONAL NN^T TERM. NOTE THAT THE MATRICES L AND N ARE COUPLED THROUGH A SCALING FACTOR, γ, WHICH IS NOT UNIQUE.

TECHNICAL FRAMEWORK

If we pick Q such that

$$Q+P\left(A_0-A\right)+\left(A_0^T-A^T\right)P\geq 0$$

where $\quad Q+PA_0+A_0^TP-\frac{1}{\rho}PBB^TP=0$

Then we proved that we will guarantee the same stability robustness in the uncertain system as in standard LQR designs.

Choose Q using "Petersen-Hollot Bounds":

Recall 'A' matrix of uncertain system:

$$A=A_0+\sum q_iE_i; \qquad |q_i|\leq 1$$

Define

$$E_i=l_in_i^T \qquad N=\left[n_1 \dots n_p\right] \qquad L=\left[l_1 \dots l_p\right]$$

Then find P as the solution of the modified Riccati equation

$$PA_0+A_0^TP+Q_0+\gamma NN^T-\frac{1}{\rho}PBB^TP+\frac{1}{\gamma}PLL^TP=0$$

Pick Q as: $\quad Q=-PA_0-A_0^TP+\frac{1}{\rho}PBB^TP$

To summarize the robust design, we first take our system matrix, and separate a "nominal" matrix from the structured uncertainty. We factor the uncertainty matrices, and define the coupled matrices L and N. We then solve a modified Riccati equation (14) and compute the control according to equations (12) and (13).

Note that although we desire a matrix Q for the cost functional, it is not necessary to compute it in order to determine the controller gains.

The robust controller has guaranteed stability, and stability robustness for the uncertain system. Some unexpected properties of this design can be seen using simulations on an uncertain system.

SUMMARY OF ROBUST DESIGN

System

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad A = A_0 + \Sigma q_i E_i;$$

$$E_i = l_i n_i^T \qquad N = \left[n_1 \ldots n_p \right] \qquad L = \left[l_1 \ldots l_p \right]$$

Control

$$u(t) = -Gx(t)$$

$$G = \frac{1}{\rho} B^T P$$

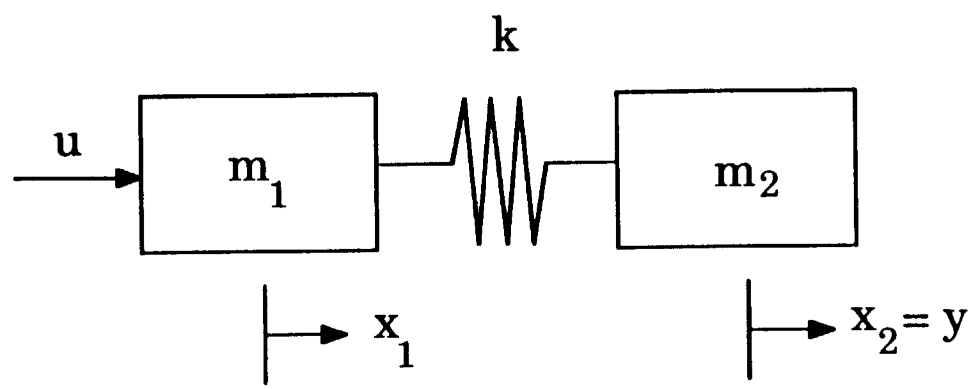
$$PA_0 + A_0^T P + Q_0 + \gamma NN^T - \frac{1}{\rho} PBB^T P + \frac{1}{\gamma} PLL^T P = 0$$

This will guarantee stability of the uncertain system, and a certain level of stability robustness.

This design technique also displays some unexpected interesting properties which can be seen with simulations.

THE UNCERTAIN SYSTEM IS A MODIFIED VERSION OF BERNSTEIN'S AND WEI'S BENCHMARK PROBLEM IN "A BENCHMARK PROBLEM FOR ROBUST CONTROLLER SYNTHESIS," PROC. AMERICAN CONTROL CONFERENCE, PP. 961-962, JUNE 1990. THE SYSTEM CONTAINS TWO KNOWN MASSES COUPLED BY A SPRING WHOSE STIFFNESS, K, IS UNCERTAIN. WE HAVE ADAPTED THE BENCHMARK PROBLEM BY ASSUMING KNOWLEDGE OF ALL 4 STATES OF THE SYSTEM. THE UNKNOWN SPRING CONSTANT CAN VARY FROM .5 TO 2, AND WE PICK A NOMINAL VALUE OF 1.25. HENCE THE KEY UNCERTAINTY IS THE POTENTIAL ENERGY STORED IN THIS SPRING $.5 \bullet K \bullet (x_1 - x_2)^2$.

UNCERTAIN SYSTEM



Masses are constant

$$m_1 = m_2 = 1$$

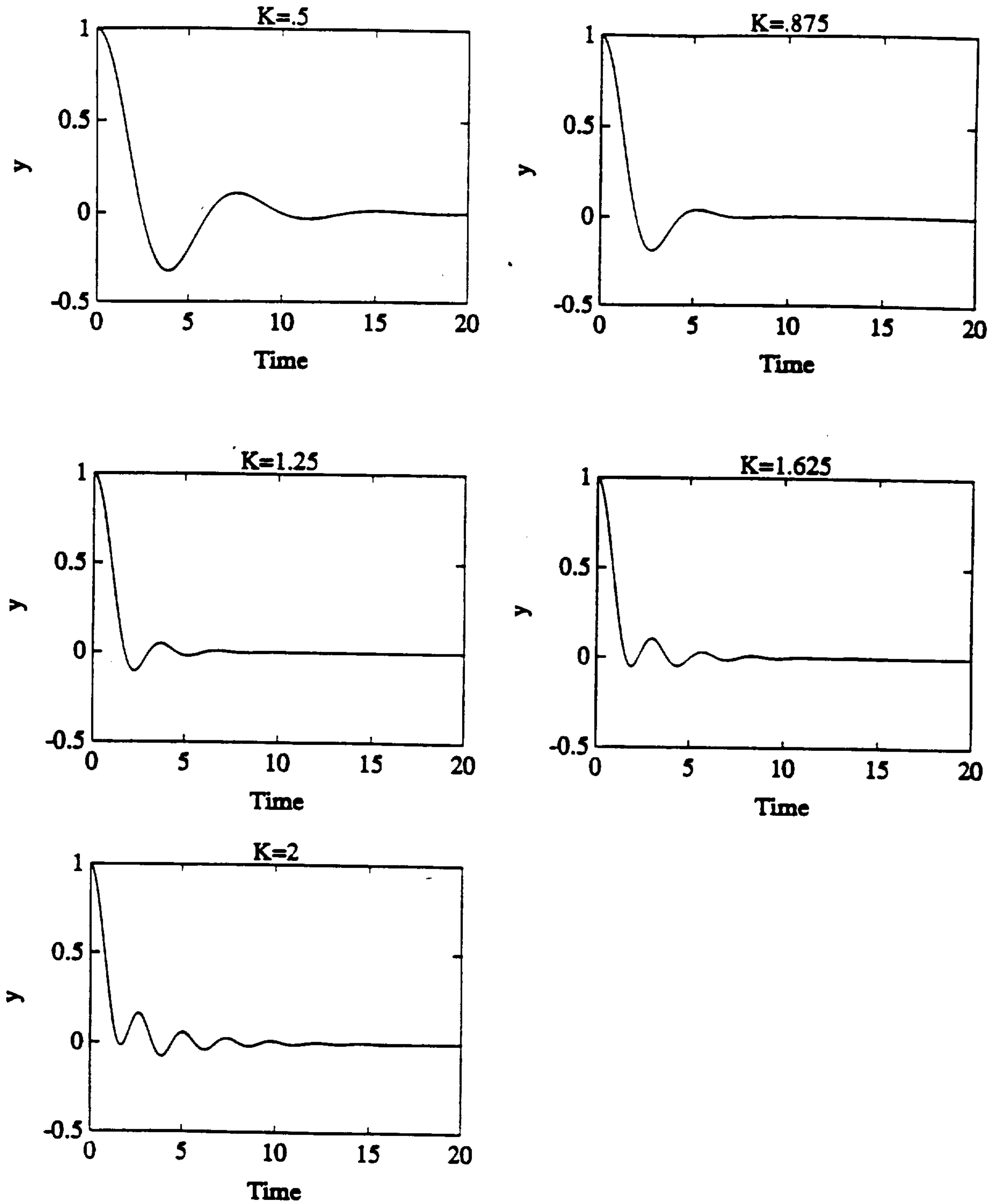
Uncertainty

$$.5 < k < 2$$

As a basis for comparison, we designed an LQR control for the nominal system, and applied the control to the system with different values of the spring constant. Thus the plot where K=1.25 is the system for which we designed the control and computed the LQR gain, whereas the control is mismatched in the other plots.

Note that the response can vary widely depending on the value of the spring. The "differences" in the shape of the transient responses are an indication of the "performance unrobustness" in this case. In this example, the system always remains stable, although this is not guaranteed in a mismatched design.

MISMATCHED LQR DESIGN



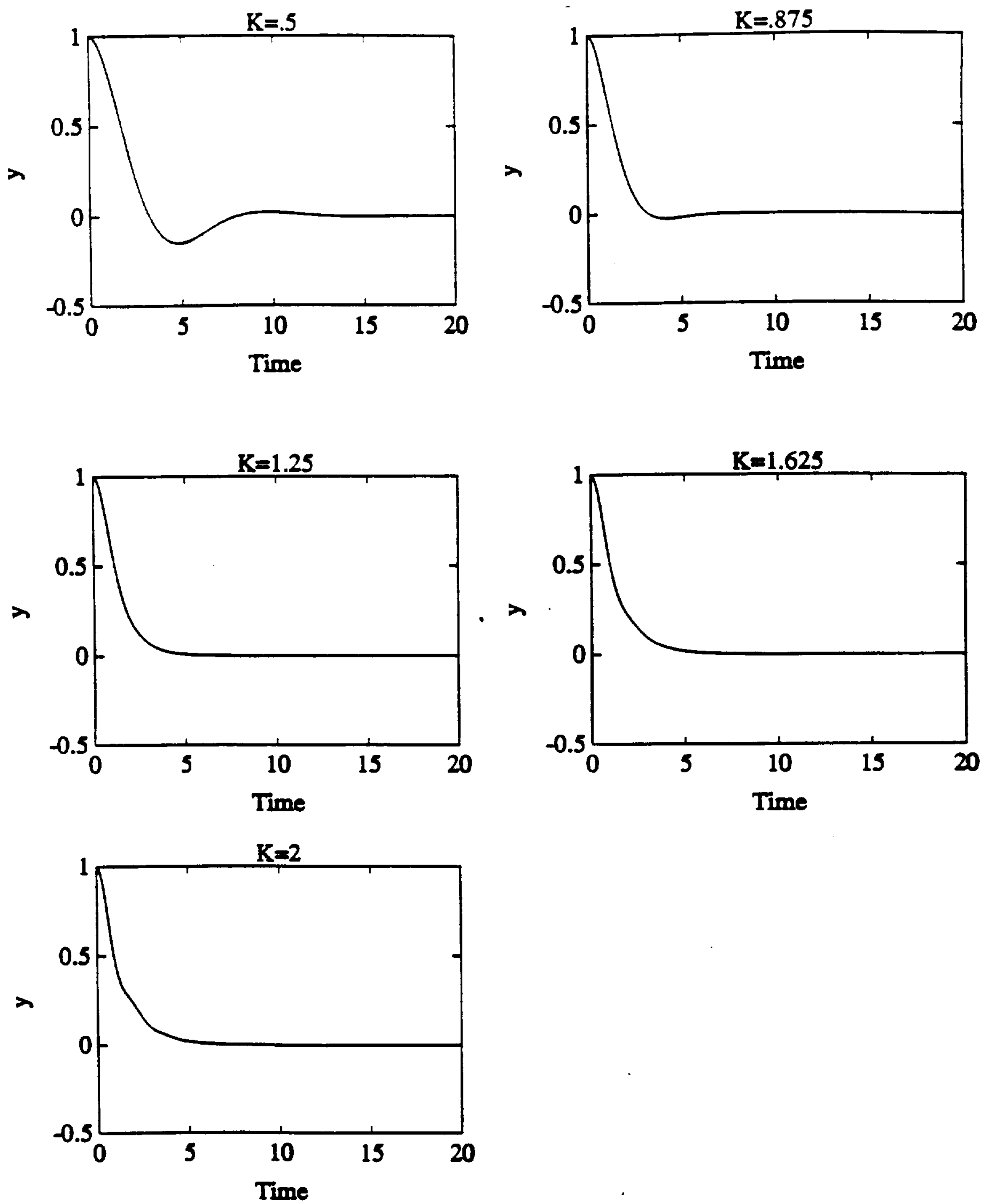
NOW WE APPLIED THE ROBUST CONTROLLER, WITH $\gamma=1$, TO THE SAME SYSTEM. NOTE THAT THIS TIME, THE PLOTS FOR ALL VALUES OF THE SPRING CONSTANT LOOK VERY SIMILAR. THIS IMPLIES A CERTAIN LEVEL OF PERFORMANCE ROBUSTNESS TO PARAMETER VARIATIONS.

ADDITIONALLY, THERE WERE SOME OTHER INTERESTING PROPERTIES OF THESE SIMULATIONS. THE CONTROL RESPONDED, IN ALL CASES, SO AS TO MOVE THE TWO MASSES SO THAT THE SPRING WAS AT ITS EQUILIBRIUM LENGTH, IN WHICH CASE THERE IS NO UNCERTAINTY IN POTENTIAL ENERGY. THEN IT WOULD MOVE THE TWO MASSES TOGETHER SLOWLY BACK TO THEIR ZERO POSITION. THIS WAS QUITE DIFFERENT THAN BEFORE WHEN THE SYSTEM MOVED THE MASSES TOWARDS THEIR ZERO POSITION AND THEN REDUCED THE SPRING LENGTH TO EQUILIBRIUM. THUS THE ROBUST DESIGN ACTED AS IF IT KNEW THAT THE UNCERTAINTY WAS IN THE SPRING CONSTANT, AND IT WORKED TO KEEP THE UNCERTAINTY IN THE POTENTIAL ENERGY OUT OF THE DYNAMICS OF THE MOTION.

IT WAS SHOWN THAT THE CHOICE OF THE STATE WEIGHTING MATRIX Q IN THE ROBUST DESIGN PENALIZED THE ENERGY OF THE SPRING AS WELL AS THE POSITION OF THE MASSES. THUS THE KEY TO THE ROBUST PERFORMANCE OF THIS DESIGN WAS A CONTROLLER THAT MINIMIZED THE UNCERTAIN ENERGY OF THE SPRING.

THE ROBUST CONTROLLER REQUIRED HIGHER CONTROL GAINS THAN THE NOMINAL LQR DESIGN. HENCE, IN A PRACTICAL SETTING, ONE MUST WORRY ABOUT STABILITY ROBUSTNESS TO HIGH FREQUENCY UNMODELLED DYNAMICS (UNSTRUCTURED UNCERTAINTY).

ROBUST DESIGN



THERE ARE SEVERAL AVENUES FOR FUTURE WORK IN UNDERSTANDING THIS CONTROLLER. ONE OF THE PROBLEMS WITH THIS CONTROL DESIGN IS THAT IT IS A CONSERVATIVE DESIGN. THAT IS, WE ARE GUARANTEEING STABILITY ROBUSTNESS AND PRODUCE HIGH CONTROLLER GAINS, WHICH MAY BE UNDESIRABLE. WE WOULD LIKE TO ADAPT THE DESIGN SO THAT WE STILL GUARANTEE THE SAME ROBUSTNESS, BUT DO NOT HAVE SUCH HIGH GAINS. A WAY IN WHICH WE MIGHT DO THIS IS TO UNDERSTAND THE ROLE OF THE FACTORIZATION OF E_I. THIS FACTORIZATION IS NOT UNIQUE, AND THUS THE CHOICE OF THE MATRICES L AND N ARE NOT UNIQUE. WE WOULD LIKE TO DETERMINE IF ONE FACTORIZATION IS (IN SOME SENSE) BETTER THAN ANOTHER. ONE OF THE ULTIMATE GOALS OF THIS RESEARCH IS TO BE ABLE TO USE THESE DESIGN TECHNIQUES ON REAL SYSTEMS. THIS WILL REQUIRE THE EXTENSION OF THE RESULTS TO THE CASE WHERE ONLY THE OUTPUT VARIABLE ARE AVAILABLE FOR FEEDBACK. IT SHOULD BE NOTED THAT THIS IS A MUCH HARDER PROBLEM. ANOTHER INTERESTING PROBLEM IS HOW WE MIGHT INTEGRATE THIS TECHNIQUE WITH METHODS FOR ROBUSTIFYING THE SYSTEM TO UNSTRUCTURED UNCERTAINTY. IN A REAL SYSTEM SUCH AS THE INTERFEROMETER, BOTH STRUCTURED AND UNSTRUCTURED UNCERTAINTY WILL RESULT FROM MODELING ERRORS AND DISTURBANCES. WE NEED TO BE ABLE TO CONTROL THE SYSTEM UNDER SUCH CONDITIONS. FINALLY, WE WOULD LIKE TO DETERMINE THE USEFULNESS OF REDUNDANT ACTUATORS AND SENSORS IN THIS DESIGN TECHNIQUE. FOR INSTANCE, HOW MUCH EXTRA ROBUSTNESS CAN WE OBTAIN IF THERE WAS A CONTROL ON MASS 2 IN THE ABOVE EXAMPLE? THIS TYPE OF QUESTION IS IMPORTANT IN CONTROL DESIGN.

FUTURE DIRECTIONS

- Understand and reduce conservatism in design procedure (different cost functional?)
  - concerned with high control gains
- Understand the factorization of E_i into l_i and n_i and their role in design.
- Extend to output feedback.
- Integrate with high-frequency (unstructured) modeling errors.
- Role of redundant sensors/actuators.

MIT
Space
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Preliminary Design of Controlled Structures

SERC Review

Robert N. Jacques

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CURRENT TRENDS IN FLEXIBLE SPACE STRUCTURES ARE TOWARD LARGER STRUCTURES WITH FUNDAMENTAL FREQUENCY MEASURED IN FRACTIONS OF HERTZ. UNFORTUNATELY, THE MISSIONS FOR WHICH THE SAME SPACECRAFT ARE PLANNED WILL HAVE VERY TIGHT POINTING AND ALIGNMENT REQUIREMENTS ON VARIOUS PARTS OF THE STRUCTURE. TO MEET THESE REQUIREMENTS WILL DEMAND ACTIVE CONTROLLERS WITH BANDWIDTHS FAR ABOVE THE FUNDAMENTAL FREQUENCIES. THE RESULTING SYSTEM WILL HAVE MANY FLEXIBLE MODES OF THE STRUCTURE INTERACTING WITH THE ACTIVE CONTROLLERS. THE LEVEL OF COMPLEXITY THIS INTRODUCES INTO THE DYNAMIC BEHAVIOR OF THE SYSTEM GREATLY COMPLICATES THE OVERALL DESIGN PROCESS. A METHOD IS NEEDED TO EFFECTIVELY DESIGN THE CONTROLLER AND THE STRUCTURE TO WORK WELL TOGETHER.

Basic Problem

- Flexible space structure has large number of modes, many at low frequencies
- Severe dynamic constraints require control with high bandwidth
- Interaction of flexible modes with controller results in complex system which is very difficult to design

ONE APPROACH TO THE PROBLEM OF DESIGNING A STRUCTURE AND CONTROLLER WELL SUITED TO EACH OTHER IS KNOWN AS CONTROL/STRUCTURE OPTIMIZATION. IN A NUTSHELL, THE APPROACH HERE IS TO REDUCE THE DESIGN PROBLEM TO DETERMINING THE SIZING OF VARIOUS STRUCTURAL AND CONTROL PARAMETERS. FOR EXAMPLE, IN THE DESIGN OF A SPACE TRUSS, ONE MIGHT SELECT THE CROSS SECTIONAL DIMENSIONS OF TRUSS MEMBERS AS STRUCTURAL PARAMETERS, AND FEEDBACK GAINS AS CONTROL PARAMETERS. A COST FUNCTION WHICH MEASURES THE "GOODNESS" OF A DESIGN IS THEN FORMULATED. OFTEN THESE COST FUNCTIONS DO NOT REFLECT THE MISSION REQUIREMENTS EXACTLY. RATHER, THEY ARE APPROXIMATIONS OF THE DESIGN REQUIREMENTS WHICH PERMIT MORE EFFICIENT SOLUTION OF THE PROBLEM. A COMPUTER PROGRAM SEARCHES OVER THE SPACE OF ALL ALLOWABLE DESIGN PARAMETERS FOR A DESIGN WHICH MINIMIZES THIS COST FUNCTION. THE PERFORMANCE OF THIS RESULTING "OPTIMAL" DESIGN IS THEN CHECKED AGAINST THE ACTUAL DESIGN REQUIREMENTS. IF THE PERFORMANCE IS NOT SATISFACTORY, THE COST FUNCTION IS USUALLY MODIFIED, AND THE OPTIMIZATION PROCESS IS REPEATED UNTIL A SATISFACTORY DESIGN IS OBTAINED.

Current Solution: Control/Structure Optimization

- Reduce problem to sizing of various structural parameters and control gains
- Select some cost function which represents the "goodness" of a design
- Use computer program to search over space of allowable designs for optimal solution

THERE ARE SEVERAL PROBLEMS WITH THE ABOVE METHOD FOR DEALING WITH THE SIMULTANEOUS DESIGN OF A STRUCTURE AND ITS CONTROL. FIRST OF ALL, THE COMPUTER PROGRAM PRODUCES A LIST OF NUMBERS WHICH REPRESENT A DESIGN. THIS POINT DESIGN IS COMPLETELY DEVOID OF UNDERSTANDING OF WHAT WENT INTO MAKING THIS PARTICULAR DESIGN OPTIMAL. ONE IS FORCED TO "TAKE THE COMPUTERS WORD FOR IT". A SECOND PROBLEM, IS THAT IN ORIGINALLY SPECIFYING THE CONTROL AND STRUCTURAL PARAMETERS FOR THE COMPUTER PROGRAM, ONE HAS ESSENTIALLY PERFORMED AN INITIAL PRE-DESIGN. FOR EXAMPLE, ONE NEEDS TO DECIDE THAT THE DESIGN WILL BE A TRUSS, AND SPECIFY SOME NOMINAL GEOMETRY BEFORE ONE CAN SPECIFY THE DIMENSIONS OF MEMBERS OF THAT TRUSS AS DESIGN PARAMETERS. IF THIS PRE-DESIGN IS NOT DONE WELL, THEN THE RESULTING "OPTIMAL" DESIGN WILL BE VERY POOR. FINALLY, CURRENT WORK IN THIS AREA HAS IGNORED SOME KEY ISSUES. AMONG THESE ARE THE IMPORTANCE OF DAMPING IN THE PROBLEM AND THE ROLE THAT ROBUSTNESS OF THE SYSTEM TO POORLY MODELLED DYNAMICS SHOULD PLAY IN THE DESIGN PROCESS.

Problems with Control/Structure Optimization

- Computer program produces point design
  - no insight into why design is optimal
  - having one optimal design does not give useful information about optimal designs for related problems
- Selection of parameters requires predesign step
  - poor selection of parameters will cause computer program to produce "optimal" but bad design
- Current work has concentrated on algorithms and has not yet included important issues
  - Robustness to poorly modelled dynamics
  - Damping

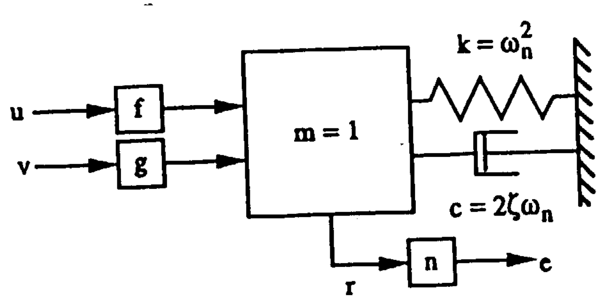
Preliminary Design of Controlled Structures

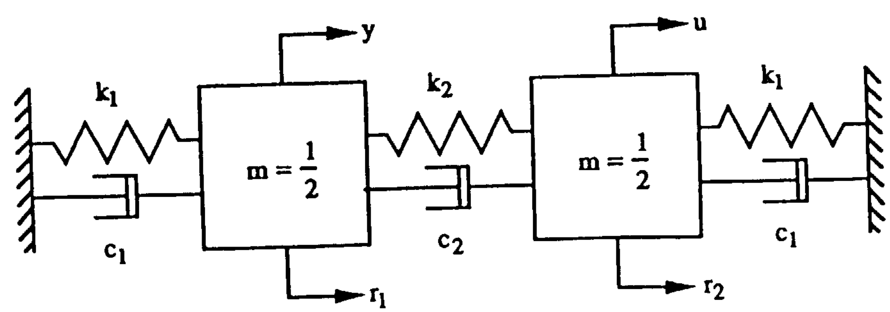
- A method for obtaining insight into behavior of controlled structures, useful in the very early stages of design
- Approach
    - Examine sensitivity of dynamic performance to various parameters for very simple systems
    - Use the results of this analysis to develop design rules for controlled structures
    - Analyze more complex systems to ascertain validity of design rules

ONE METHOD FOR GAINING INSIGHT INTO HOW A CONTROLLED STRUCTURE SHOULD BE DESIGNED IS THROUGH THE USE OF CONTROLLED STRUCTURE TYPICAL SECTIONS. THESE ARE SIMPLE ONE OR TWO MODE MASS-SPRING-DASHPOT SYSTEMS. THE IDEA IS THAT BY STUDYING HOW CHANGING THE STRUCTURAL PARAMETERS IN THESE SYSTEMS INFLUENCES THEIR CONTROLLED PERFORMANCE, ONE MIGHT BETTER UNDERSTAND HOW CHANGES IN MORE COMPLEX STRUCTURES WILL INFLUENCE THEIR CONTROLLED PERFORMANCE. IN THIS WORK, TWO TYPICAL SECTIONS WERE USED. THE FIRST WAS A SINGLE SPRING, MASS, AND DASHPOT WHICH EMPLOYED AN OPTIMAL LINEAR QUADRATIC REGULATOR (LQR) AS ITS CONTROLLER. THIS SIMPLE SYSTEM PERMITTED SOLUTION OF THE PERFORMANCE COST IN CLOSED FORM AND PROVIDED A GOOD MODEL FOR HOW A CONTROLLER MIGHT INTERACT WITH A SINGLE MODE OF A STRUCTURE. THE DRAWBACK TO THIS MODEL IS THAT IT IGNORES COUPLING BETWEEN MODES. THE SECOND TYPICAL SECTION USED CONSISTED OF TWO MODES, HOWEVER ITS CONTROLLER WAS IDENTICAL TO THE CONTROLLER DESIGNED FOR THE SINGLE MODE MODEL. ITS PURPOSE WAS TO STUDY THE EFFECT OF UNMODELLED DYNAMICS ON CLOSED-LOOP PERFORMANCE.

Typical Sections

- Single mass typical section
  - LQR controlled
  - cost can be solved for in closed form
  - good for understanding how controller interacts with single mode of a structure
  - ignores coupling effects
- Two mass typical section
  - LQR controller designed using single mode model
  - performance evaluated on full model
  - good for understanding role of certain paramters in reducing effects of "spillover"





THIS TABLE SHOWS SEVERAL APPROACHES ONE MIGHT USE IN IMPROVING THE PERFORMANCE OF A CONTROLLED STRUCTURE. THE COLUMN HEADINGS INDICATE FIVE BASIC APPROACHES, WHILE THE ENTRIES IN THE COLUMNS DESCRIBE EXAMPLES OF HOW ONE MIGHT GO ABOUT FOLLOWING THESE APPROACHES.

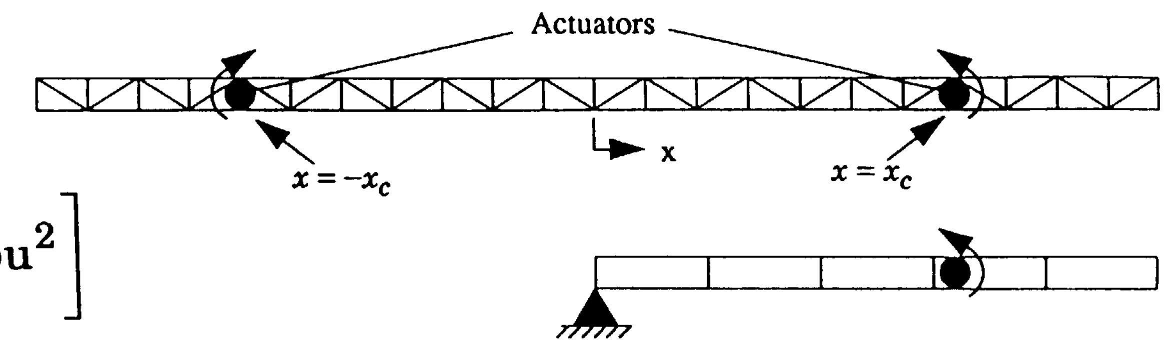
Methods for Improving Controlled Performance

Reduce Disturbability	Reduce Observability	Increase Controllability	Reduce Open Loop Time Constant	Increase Robustness
Remove Disturbance		Reduce Influence of control on initially quiet modes *Examples: 4*		
Active Isolation	Active Isolation			
Passive Isolation	Passive Isolation			
Postition disturbance near nodes	Position output point near nodes	Locate actutors/senser near anti-nodes of modelled modes *Examples: 3*		Move acuator/sensor near nodes of modelled modes
Position nodes near disturbances *Examples: 3, 5*	Position nodes near output point	Locate anti-nodes of modelled modes near actuators/sensors *Examples: 4*	Use convolving sensors	Move nodes of modelled modes near acuators/sensors
Stiffen system against deflection/step forces *Examples: 1, 2*		Remove Stiffness	Add stiffness *Examples: 3, 4, 5*	
		Remove Damping	Add damping	Add damping
				Use Postive Real Control *Examples:*

AS AN EXAMPLE OF HOW THE TYPICAL SECTIONS CAN BE USEFUL, AN EXAMPLE OF CONTROL/STRUCTURE OPTIMIZATION ALREADY DONE IN THE LITERATURE IS PRESENTED. HAFTKA AND ONADA CONSIDERED THE OPTIMAL DESIGN OF BEAM-LIKE SPACE CRAFT. THERE DESIGN MODEL CONSISTED OF A FIVE ELEMENT BERNOULLI-EULER BEAM THAT WAS PINNED ON ONE END AND FREE ON THE OTHER. THE ACTUATOR WAS A TORQUE ACTUATOR. THE DESIGN VARIABLES WERE THE THICKNESSES OF THE ELEMENTS AND THE POSITION OF THE ACTUATOR. THE DESIGN OBJECTIVE WAS TO MINIMIZE THE COST FUNCTIONAL SHOWN IN THE FIGURE WHEN THE SYSTEM WAS SUBJECTED TO A DISTRIBUTED WHITE NOISE DISTURBANCE WHICH, AT ANY POINT ON THE BEAM, WAS PROPORTIONAL TO THE DISTANCE FROM THE PINNED END. HAFTKA AND ONADA FOUND TWO FUNDAMENTALLY DIFFERENT TYPES OF OPTIMAL DESIGN. THE FIRST OCCURRED WHEN A LARGE PENALTY WAS PLACED ON CONTROL EFFORT RELATIVE TO THE STATE PENALTY. THE RESULTING OPTIMAL DESIGN PLACED MOST OF THE MASS TOWARD THE MIDDLE OF THE BEAM. AS THE PENALTY ON CONTROL WAS DECREASED, AN OPTIMAL DESIGN WAS FOUND TO PLACE MOST OF THE AVAILABLE MASS AT THE FREE TIP.

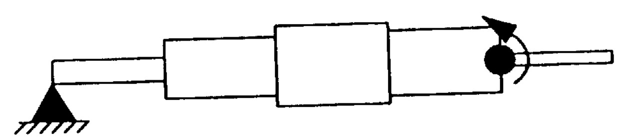
An Example: Beam Problem of Haftka and Onada

- Nominal system

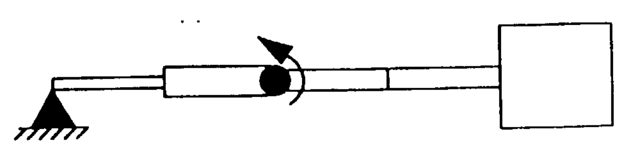


$$J = \lim_{t \to \infty} E\left[\int_0^L y^2 dx + \rho u^2\right]$$

- Optimal system (expensive control)



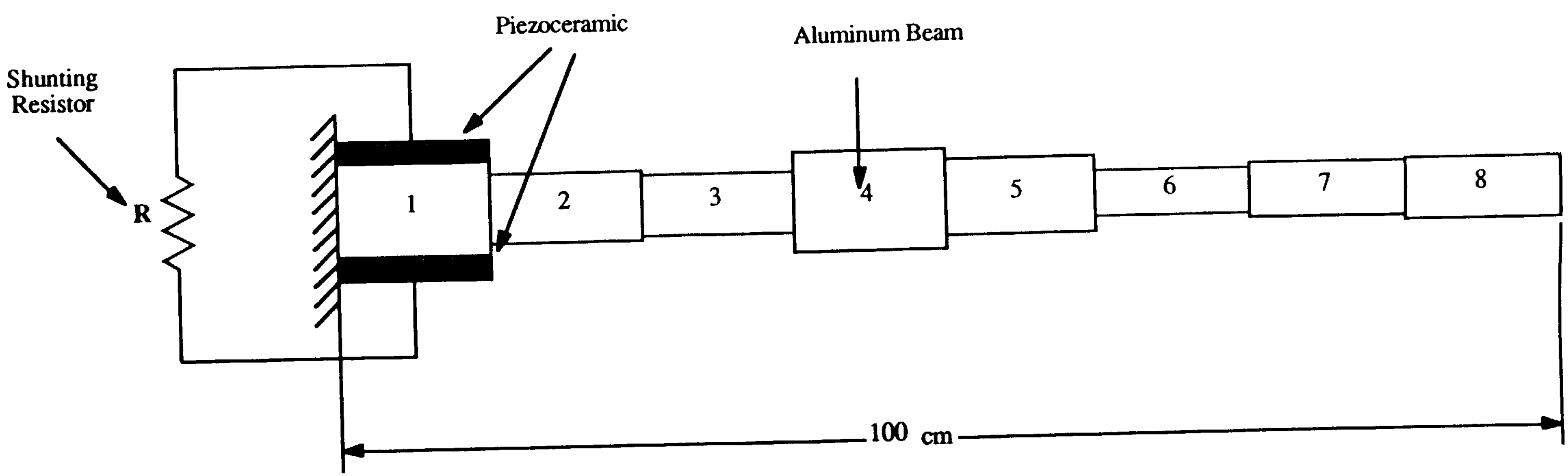
- Optimal system (cheap control)



IN FUTURE WORK, IT WILL BE NECESSARY DO STUDY THE EFFECTIVENESS OF DAMPING IN MORE COMPLEX SYSTEMS. ONE STUMBLING BLOCK TO ADDING DAMPING TO A MODEL IN CONTROL/STRUCTURE OPTIMIZATION HAS BEEN THE LACK OF GOOD DAMPING MODELS. GOOD MODELS ARE IMPORTANT IN QUANTIFYING THE RELATIVE COST OF ADDING DAMPING TO THE SYSTEM AS OPPOSED TO ADDING MASS AND/OR STIFFNESS. THIS PROBLEM IS SURMOUNTED BY STUDYING A CANTILEVERED BEAM WHICH USES RESISTIVELY SHUNTED PIEZO-CERAMICS AS A DAMPING TREATMENT. WORK DONE BY HAGOOD WILL PERMIT PRECISE MODELLING OF THE DAMPING IN THE PROBLEM.

Damping

- Beam model with shunted piezo-ceramic damping
  - Can be modelled exactly by state space equations



Properties of Aluminum

$E_{al} = 73\,\mathrm{gPa}$

$\rho_{al} = 2700\,\mathrm{kg/m^3}$

Properties of Piezoceramic

$E^{oc}_{piezo} = 60\,\mathrm{gPa}$

$\rho_{piezo} = 7650\,\mathrm{kg/m^3}$

$\varepsilon^T = 3300\varepsilon_0$

$k_{31} = .75$