N93-22222

SYNTHESIS OF NONLINEAR CONTROL STRATEGIES FROM FUZZY LOGIC CONTROL ALGORITHMS

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Abstract Fuzzy control has been recognized as an alternative to conventional control techniques in situations where the plant model is not sufficiently well known to warrant the application of conventional control techniques. Precisely what fuzzy control does and how it does what it does is not quite clear, however. This paper deals with this important issue and in particular shows how a given fuzzy control scheme can resolve into a nonlinear control law and that in those situations the success of fuzzy control hinges on its ability to compensate for nonlinearities in plant dynamics.

INTRODUCTION

Fuzzy logic control has been recognized as an alternative to conventional control techniques(primarily PID, or switching type control) for application in industrial process control and manufacturing automation(Sugeno 1985). More often than not, however, empirical observation provides the only means to a comparative study of performance of fuzzy controllers in relation to their conventional counterparts. While this fact is recognized and even appreciated by practitioners in the process control area, precisely what a fuzzy controller does, that is from an analytical standpoint, and how it does what it does is still of interest.

In order to investigate this issue, we will consider the notion of *parametrized* fuzzy sets and discuss its implication in analysis of fuzzy control algorithms. This idea, it turns out(Langari and Tomizuka 1990, Langari 1990, Langari 1992) gives rise to a framework for analysis and synthesis of *nonlinear* control strategies that emerge quite naturally from an initial statement of a given control strategy as a fuzzy linguistic control algorithm.

In this article, we will use this framework to explain how a given fuzzy control strategy deals with process nonlinearities that conventional controllers, for instance PID, generally do not. In particular, we apply this framework to the problem of control synthesis in a typical situation where asymmetric response characteristics of the process precludes, or severely encumbers the application of

conventional(linear) control theory. We further show how in this situation an appropriately designed fuzzy controller overcomes this difficulty and by in effect compensating for the underlying nonlinearities produces superior behavior.

We start with an overview of fuzzy control

FUZZY CONTROL SYSTEMS

The typical architecture of a fuzzy control systems in shown in Figure 1. As a rule based control strategy, fuzzy linguistic control is based on explicit representation of knowledge of operation of the process as condition action rules of the form

$$R_{j,i}: \text{ if } e(t) \text{ is } A_j \text{ and } de(t) \text{ is } B_i \text{ then } u(t) \text{ is } C_{j,i}$$
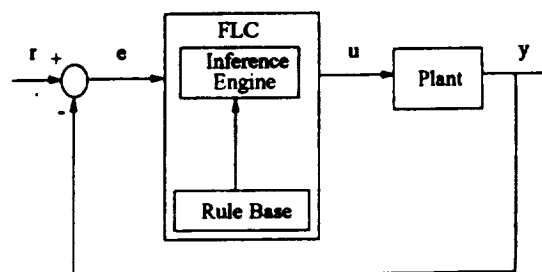
where $e(t)$ denotes the instantaneous value of the

Figure 1 Architecture of a Fuzzy Logic Control System

process error at time t and $de(t)$ is short for $\mathcal{D}(e;t)$, which stands for $\dfrac{de}{dt}$ or $\int e\, d\tau$. Further, $\tilde{A}_j$, $\tilde{B}_l$, and $\tilde{C}_{j,l}$ belong to collections $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ of fuzzy subsets defined over the domains of definition of the relevant variables, that is, E, DE, and U respectively and $R_{j,l}$ denotes the j,l^{th} rule in the rule set $\mathbf{R}$. In particular $R_{j,l}$ may be
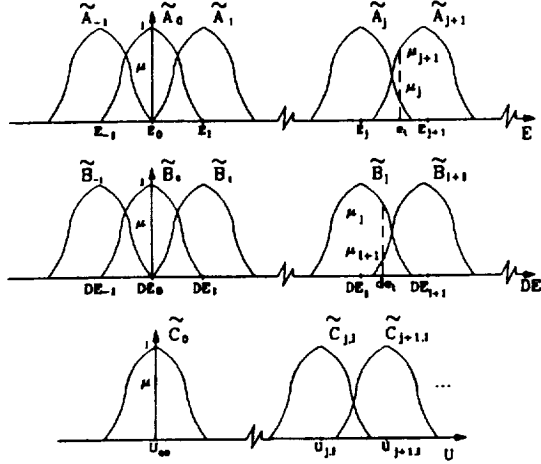

Figure 2. Fuzzy partitioning of the domains of definition.

be viewed as associating elements $\tilde{A}_j$ of $\mathcal{A}$ and $\tilde{B}_l$ of $\mathcal{B}$ with element $\tilde{C}_{j,l}$ of $\mathcal{C}$, thereby forming a *fuzzy relation*[1] $\tilde{R}_{j,l}$ over the Cartesian product space, $E \times DE \times U$. From this standpoint, the given fuzzy control algorithm in effect amounts to a disjunction of such associations, as in $\tilde{\mathcal{R}} = \underset{j,l}{\vee}\,\tilde{R}_{j,l}$, which Mamdani and Assilian(1975) refer to as the *fuzzy relation matrix*.

Control Computation

Suppose, at some instance t , as shown in Figure 2, the error $e(t)$ has positive grades of membership, $\mu_{\tilde{A}_j}(e(t))$ and $\mu_{\tilde{A}_{j+1}}(e(t))$ to some pair $\tilde{A}_j$ and $\tilde{A}_{j+1}$ in $\mathcal{A}$. Similarly, suppose $de(t)$ belongs to some pair $\tilde{B}_l$ and $\tilde{B}_{l+1}$ in $\mathcal{B}$. At this instant, the following control rules apply

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[1]Note that the distinction in the notation used, that is $R_{j,l}$ vs. $\tilde{R}_{j,l}$ reflects the distinction between rules and associations. **349**

$R_{j,l}$: if $e(t)$ is A_j and $de(t)$ is B_l then $u(t)$ is $C_{j,l}$

$R_{j+1,l}$: if $e(t)$ is A_{j+1} and $de(t)$ is B_l then $u(t)$ is $C_{j+1,l}$

$R_{j+1,l+1}$: if $e(t)$ is A_{j+1} and $de(t)$ is B_{l+1} then $u(t)$ is $C_{j+1,l+1}$

$R_{j,l+1}$: if $e(t)$ is A_j and $de(t)$ is B_{l+1} then $u(t)$ is $C_{j,l+1}$

with each rule satisfied to some degree. The corresponding *truth value* is defined, for instance for the first rule, by

$$\mu_{j,l} = \min\left(\mu_{\tilde{A}_j}(e(t)), \mu_{\tilde{B}_l}(de(t))\right) \quad (1)$$

or, alternatively by

$$\mu_{j,l} = \mu_{\tilde{A}_j}(e(t)) \cdot \mu_{\tilde{B}_l}(de(t)) \quad (2)$$

The truth values of other rules in the above set are similarly defined.

Note that the *product* instead of *min* results in interactivity between the truth values of the components of the antecedent clause. This fact is essential to our analytic treatment(Langari and Tomizuka 1990.)

Now, representing the consequent clause of each $R_{j,l}$ rule, that is , by its single representative, or *defuzzified*, value that is $U_{j,l}$, defined as

$$U_{j,l} = \frac{\int u\, \mu_{\tilde{C}_{j,l}}(u)}{\int \mu_{\tilde{C}_{j,l}}(u)} \quad (3)$$

the control action, $u(t)$, is computed as:

$$u(t) = \frac{\sum_{j,l} \mu_{j,l} U_{j,l}}{\sum_{j,l} \mu_{j,l}} \quad (4)$$

where j and l range over the indices of all applicable rules. Note that this approach is based on a variation of the Centroid of Area(COA) defuzzification rule(Zimmermann 1991), but has improved analytical properties(Langari and Tomizuka 1990).

ANALYSIS OF FUZZY LOGIC CONTROL ALGORITHMS.

Consider the single input, single output fuzzy linguistic control system shown in Figure 1. Here we develop an analytic description of the control law in the form, $u = FLC(e, de)$.

Definitions and Assumptions

Let us denote the domains of definition of e, de, and u by E, DE, and U respectively. Then, as shown in Figure 2, collections $\mathscr{A} = \{\tilde{A}_j\}$, $\mathscr{B} = \{\tilde{B}_i\}$, and $\mathscr{C} = \{\tilde{C}_{j,i}\}$ of unimodal, convex, and *normal* fuzzy subsets(Dubois and Prade 1980) effectively partition E, DE, and U, respectively, as follows.

Each element $\tilde{A}_j$ of $\mathscr{A}$ is centered at some $E_j \in E$ and is further characterized by a pair $L_j(\cdot)$ and $R_j(\cdot)$ of left and right characteristic functions(cf. Appendix A). Similarly, each $\tilde{B}_i \in \mathscr{B}$ is centered at some $DE_i \in DE$ and is characterized by $L_i'(\cdot)$ and $R_i'(\cdot)$. Moreover, each element $\tilde{C}_{j,i}$ is represented by its *defuzzified* value, $U_{j,i}$.

We further place some constraints on $\mathscr{A}$ and $\mathscr{B}$ as follows. First, we require that $\mathscr{A}$ and $\mathscr{B}$ form *true fuzzy partitions* of E and DE respectively.

Assumption 1. *Let* $\mathscr{A} = \{\tilde{A}_j\}$ *(and* $\mathscr{B} = \{\tilde{B}_i\}$) *be collection(s) of fuzzy subsets defined over E (and DE.) Then, for each element $e \in E$*

$$\sum_j \mu_{\tilde{A}_j}(e) = 1 \qquad (5)$$

(A similar condition holds for $\mathscr{B}$.)

The interpretation of Assumption 1 is that, externally, fuzzy classification must be compatible with feature based classification in terms of classical sets, where each element is categorized under one and only one class. This assumption is crucial to the development of our results and in effect amounts to *objectification* of the control law.

A sufficient condition for Assumption 1 to hold is that the characteristic functions of $\tilde{A}_j$(and $\tilde{B}_i$) be linear[2]:

Assumption 2. *For each j, let $\tilde{A}_j \in \mathscr{A}$ be defined in terms of a pair $L_j(\cdot)$ and $R_j(\cdot)$ of left and right characteristic functions. Then*

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[2]A generalization of this condition, where nonlinear characteristic functions are allowable, is possible. The present discussion, however, does not hinge on this fact. The interested reader may refer to Langari(1992).

$$R_j(e) = 1 - (e - E_j)/\beta_j \qquad (6)$$

$$L_j(e) = 1 - (E_j - e)/\alpha_j \qquad (7)$$

and given j, and $j+1$, the line segments defined by $R_j(\cdot)$ and $L_{j+1}(\cdot)$ intersect E precisely at E_j and E_{j+1} respectively. (A similar condition holds for $\mathscr{B}$.)
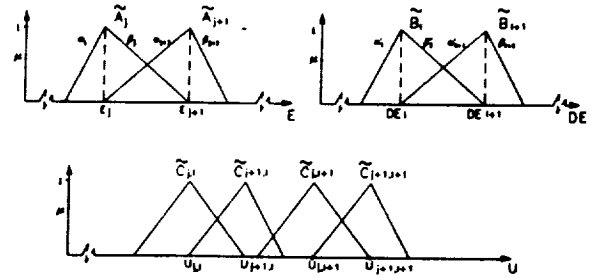
This assumption implies that, as shown in Figure 3,

Figure 3. True Fuzzy Partitioning.

β_j and α_{j+1}, respectively representing the inverse of the slopes of the line segments defined by $R_j(\cdot)$ and $L_{j+1}(\cdot)$, must be equal. Let us denote this unique slope by m_j:

$$m_j := \frac{1}{\beta_j} = \frac{1}{\alpha_{j+1}}. \qquad (8)$$

Similarly, α_{i+1}' and β_i', must also be equal; let us define $m_i' := \frac{1}{\alpha_{i+1}'} = \frac{1}{\beta_i'}$ to clearly indicate this fact as well. Consequently, we can define ΔE_j and ΔDE_i as follows:

$$\Delta E_j = E_{j+1} - E_j, \qquad (9)$$

$$\Delta DE_i = DE_{i+1} - DE_i. \qquad (10)$$

Let us also define $K_{j,i}$ and $K_{j,i}'$ as follows.

Definition 1. *Let us denote the functional relationship between $U_{j,i}$, E_j and E_{j+1} as:*

$$U_{j,i} = K_{j,i}E_j + K_{j,i}'DE_i. \qquad (11)$$

Then for each pair, j and i, $K_{j,i}$ and $K_{j,i}'$ are implicitly defined by (11).

350

Note that (11) simply relates $U_{j,l}$ to E_j and DE_l in a compact form and does not in any way constrain $U_{j,l}$.

We further define $\Delta K^j{}_{j+1,l}$...as follows:

$$\Delta K^j{}_{j+1,l} = K_{j+1,l} - K_{j,l},$$ (12)

$$\Delta K^l{}_{j,l+1} = K_{j,l+1} - K_{j,l},$$ (13)

$$\Delta K_{j+1,l+1} = K_{j+1,l+1} - K_{j,l}.$$ (14)

$$\Delta K'^l{}_{j,l+1} = K'_{j,l+1} - K'_{j,l},$$ (15)

$$\Delta K'^j{}_{j+1,l} = K'_{j+1,l} - K'_{j,l},$$ (16)

$$\Delta K'_{j+1,l+1} = K'_{j+1,l+1} - K'_{j,l}.$$ (17)

Now in view of the above assumptions the expression for $u(t)$, given by (4), resolves into

$$u(t) = K_{j,l}e(t) + m_j\left(e(t) - E_j\right)\left[\Delta K'_{j+1,l}E_{j+1} + \Delta K''_{j+1,l}DE_l\right] +$$
$$K'_{j,l}de(t) + m'_l\left(de(t) - DE_l\right)\left[\Delta K'_{j,l+1}E_j + \Delta K''_{j,l+1}DE_{l+1}\right] +$$
$$m_j m'_l\left(e(t) - E_j\right)\left(de(t) - DE_l\right)\begin{bmatrix}\left(\Delta K_{j+1,l+1} - \Delta K'_{j,l+1} - \Delta K'_{j+1,l}\right)E_{j+1} + \\ \left(\Delta K'_{j+1,l+1} - \Delta K''_{j,l+1} - \Delta K''_{j+1,l}\right)DE_{l+1} + \\ \left(\Delta K_{j+1,l+1} - \Delta K''_{j,l+1}\right)\Delta DE_{l+1} + \\ \left(\Delta K_{j+1,l+1} - \Delta K'_{j+1,l}\right)\Delta E_{j+1}\end{bmatrix}$$

(18)

The implication of the above formulation is that a given fuzzy logic control algorithm in effect amounts to a nonlinear control law that is further described in terms of three terms: one that is linear in each of $e(t)$ and $de(t)$, one that is linear in each of $e(t) - E_j$ and $de(t) - DE_l$, and finally one that is bilinear in the latter two terms. In effect the control law given by (18) reflects the capacity of fuzzy logic control to interpolate across the situations where individual control rules are directly applicable. We will see next how this capacity can be used to develop a control strategy that deals effectively with nonlinearities that commonly occur in process control.

APPLICATION

Let us consider the dynamic system:

$$\dot{x}_1 = a_1 x_1 + a_2 x_2 + bu,$$
$$\dot{x}_2 = x_1,$$ (19)
$$y = x_2 - x_1,$$

which reflects the behavior of a rather broad class of industrial processes; it is a relatively low order model, and has the somewhat dubious distinction of being *non-minimum phase*. The parameters, a_1, and a_2 are given by

$$a_1 = a_{10} + \delta a_1,$$ (20)

$$a_2 = a_{20} + \delta a_2,$$ (21)

where δa_1 and δa_2 reflect the variations in the plant parameters.

Suppose now, as it is commonly done in practice, we knew the process model and were to design a simple *proportional plus integral* control law:

$$u = k_p e + k_i \int_0^t e\, d\tau,$$ (22)

perhaps based on nominal values of the plant parameters, a_{10}, and a_{20}, as follows.

The plant and controller transfer functions are given by:

$$G_p(s) = \frac{1-s}{s^2 + a_{10}s + a_{20}},$$ (23)

$$G_c(s) = \frac{k_c(s+\gamma)}{s},$$ (24)

where $\gamma > 0$, $k_c = k_p$ is same as the proportional control gain, and $k_i = \gamma k_c$ is the equivalent integral gain.

Now, assuming that the closed loop system will behave as a dominantly second order system, the closed loop characteristic equation is given by

$$A(s) = (s+p)(s^2 + 2\xi\omega s + \omega^2),$$ (25)

where p is assumed large, we can use any number of ways of selecting ξ and ω— and thus k_c and γ — (Franklin, Powell, and Emami-Naeini 1991). For instance, we can simply pre-select γ and then choose ξ for desired response pattern and thus determine the gain k_c.

In practice, however, variations in the parameters of the plant, that is δa_1 and δa_2, affect the behavior of the process, and as a result the desired response is not reproduced as predicted. For instance, let us

351

suppose that these variations are function of the process error[3], e :

$$\delta a_1 = -\alpha\,\mathrm{sgn}(e), \qquad (26)$$

$$\delta a_2 = -\alpha\,\mathrm{sgn}(e), \qquad (27)$$

where $\alpha > 0$.

This situation happens in arc welding, for instance, where active heating and only passive cooling is available(Langari and Tomizuka 1988). A consequence of this change is that a fixed set of gains will not work well, no matter what values one chooses. Alternatively, one may resort to adaptive control. Generally, however, this approach requires slow variation in the plant parameters. One could also, in principle, rely on robust control, perhaps within the H_∞ framework. The drawback of this approach, however, is that while robust performance may be guaranteed, *uniformly* robust performance is not. These claims should not be surprising since neither adaptive control or robust control is really meant to compensate for strong nonlinearities in the plant model.

Given this fact, therefore, one should at least ideally consider nonlinear control— global or feedback linearization. Indeed if the nature and extent of nonlinearity is known reasonably well, through a reasonably accurate plant model, one would do just that. Moreover, even in the absence of a formal model, it is our conjecture that the human operator of the process, having learned the peculiarity of its behavior, develops response behavior that in practice amounts to a nonlinear control scheme that compensates for the dominantly nonlinear, and undesired, characteristics of the process. In effect s/he globally linearize the process *and* compensates for the deficiencies in its dynamic response characteristics.

In the context of the current example, in particular, it seems plausible that a human operator would be able to compensate for variations in the plant parameters, as required and as shown in Figure 4 produce response pattern superior to *any* linear control strategy.

Analysis of Response Pattern

Clearly, assuming that the control action of the human operator is described in linguistic form, the key factor would be the manner of definition of the rule set and its constitutive linguistic term set. This is evident, as shown in Figure 5, in the manner of

definition of the linguistic term set defined over the domain of definition of the process error.
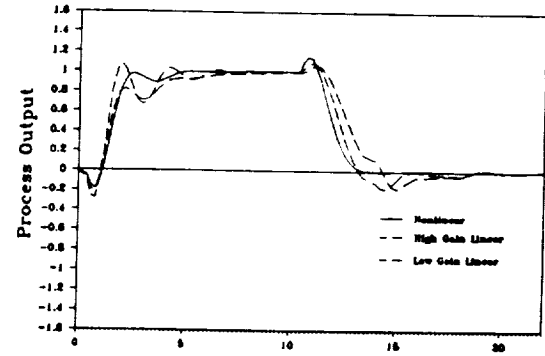

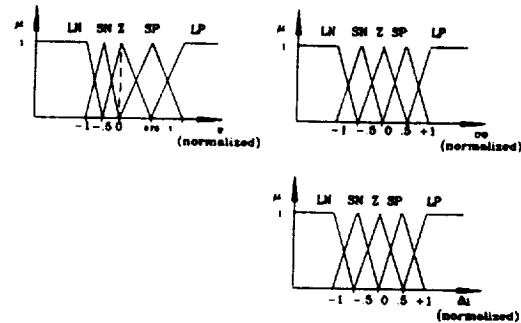Figure 4. Response patterns of fuzzy vs. linear control.

Figure 5. Definition of fuzzy membership functions.

In particular, the asymmetry in the definition of terms such as *small-positive* and *small-negative*, denoted in the figure by SP and SN respectively, reflects the variation in the *proportional* gain across the origin of the domain of definition of e .

Now, using the formalism presented earlier, one can show that the operator's action, interpreted above in linguistic terms, effectively amounts to a nonlinear control scheme

$$u = k_p e + k_i \int_0^t e\,d\tau, \qquad (28)$$

where k_p , is given by $k_p = k_{p0} - \alpha\,\mathrm{sgn}(e)/b$, which in the case of the regulation problem, in effect *cancels* the nonlinear terms which we attributed earlier to parametric variation[4].

CONCLUSION

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[3]Actually it would be more accurate to consider variation as a function of the process input so as to reflect the coupling between state and input variables, however, in closed loop control the input is itself a function of the error.

[4]In reality when the setpoint is changed, this cancellation does not hold in the exact sense, however, since the plant dynamics is still linearized and stable, treating the setpoint change effect as a disturbance which results in diminishing transients is a reasonable assumption.

In this paper we showed how fuzzy control can be viewed as a paradigm for designing nonlinear control strategies in situations where the plant model is not a priori known— at least sufficiently well— to warrant the application of conventional control theory. In particular, we made a point regarding the use of fuzzy control in situations that occur frequently in industrial process control where (nonlinear)dependence of the parameters of the plant on its state variables precludes the application of linear control theory and thus nonlinear control, albeit by means of fuzzy control, seems to be the most appropriate approach. The framework presented here, however, is somewhat restrictive in that it requires a specific form for parametrization of fuzzy sets(LR) and places some restrictions on the manner of definition of the control rules($\sum \mu = 1$). To be more widely applicable, this framework needs to to allow for a wider range of nonlinear control schemes and also to allow for nonparametrized fuzzy sets.

APPENDIX.

A. Parametrization

Although not absolutely essential, parametrization simplifies quantitative description of fuzzy subsets. In LR parametrization(Dubois and Prade 1980), a fuzzy subset $\tilde{A}$, defined on some universe of discourse U, is characterized, in terms of its membership function , as follows:

$$\mu_{\tilde{A}}(u) = \begin{cases} L((u_0 - u)/\alpha) & \text{if } u \leq u_0 \\ R((u - u_0)/\beta) & \text{if } u > u_0 \end{cases} \quad (29)$$

where, as shown in Figure 6, $L(\cdot)$ and $R(\cdot)$ characterize the left and right halves of $\tilde{A}$, relative to its *center value*, u_0, that is where the linguistic term that $\tilde{A}$ represents fully achieves its meaning, or is maximally satisfied. Moreover, α(and β) parametrize $L(\cdot)$(and $R(\cdot)$), which typically takes the form

$$L(x) = \begin{cases} \max(0, 1 - |x|^p), & \text{or} \\ e^{-|x|^p}, & \text{or} \\ \dfrac{1}{1 + |x|^p} \end{cases} \quad (30)$$
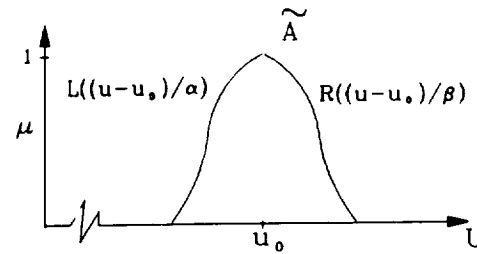
where $p \geq 1$ in all cases.

Figure 6. Parametrization of a fuzzy subset.

Finally, it is sometimes sufficient to use a simple linear form, based on $L(x) = \max(0, 1 - |x|)$, in which case, α(or β), discussed above, would represent the *inverse* of the slope of the characteristic function:

$$R(u) = 1 - (u - u_0)/\beta, \quad (31)$$

$$L(u) = 1 - (u_0 - u)/\alpha. \quad (32)$$

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