

# Truth-Valued-Flow Inference(TVFI) and Its Applications in Approximate Reasoning

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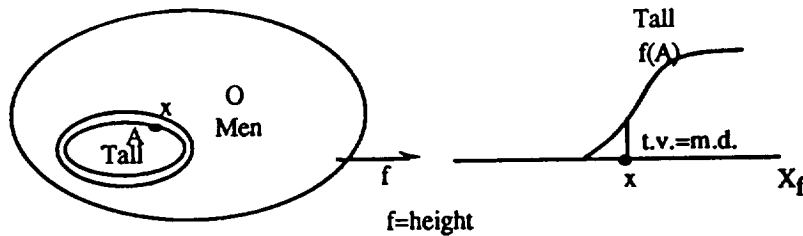
## Abstract

In this paper, we introduce the framework of the theory of Truth-valued-flow Inference(TVFI) which was presented by the authors and has been successfully made into products by Apronix, the Fuzzy Logic Technology Company. Even though there are dozens of papers presented out on fuzzy reasoning, we think it is still needed to explore a rather unified fuzzy reasoning theory which has the following two features: the one is that it is simplified enough to be executed feasibly and easily; and the other is that it is well structural and well consistent enough that it can be built into a strict mathematical theory and is consistent with the theory proposed by L.A.Zadeh. TVFI, introduced in this paper, is one of the fuzzy reasoning theories that satisfies the above two features. It presents inference by the form of networks, and naturally views inference as a process of truth values flowing among propositions.

## 1. What is inference?

Inference is truth values flowing among propositions. Here, the name 'truth value' is taken by logicians and stands for an abstract quantity who can be calculated by means of logical operations and used to evaluate the truth of propositions.

A proposition is a sentence "u is A" which can be viewed as has to be judged (may be fail). For example, " John is tall" or " John's height is tall" are propositions. Each proposition can be decomposed into two parts: A—a concept, a subset of a universe U; u—an object or its state respects to some factor, a point of U. If u stands for an object, like John, Mary,..., we usually denote the discussion universe U as O which consists of objects; if u stands for some state of an object, like height, weight,... we usually denote the discussion universe as  $X_f$ , which is the states space of the factor f.



A concept TALL, for example, can be represented as a fuzzy subset in an universe U. But U is not uniquely selected, it can be selected as O or  $X_f$  (shown in the above figure). Each concept can be represented as not only one but a class of membership functions; how to make a selection depends on what is the universe X or what is the variable x. So that, the combination of a concept A and a variable x, denoted as  $A(x)$ , determines a conceptual representation. When x is fixed, it is the proposition 'x is A'; when x is varying, it is called a predicate. A predicate corresponds to a fuzzy subset in X.

$A(x)$  offers us making judgment: What about the truth of it? It comes the truth value  $T(A(x))$ , the truth degree of proposition 'x is A'. It is equal to the membership degree  $\mu_A(x)$ . The form of truth values can be real numbers in  $[0,1]$  or linguistic values such as RATHER TRUE, VERY FAIL,... for examples, which are described as fuzzy subsets of  $[0,1]$ .

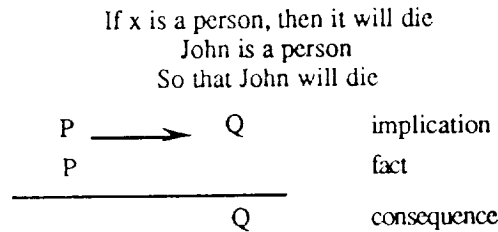
$A(x)$  also provides us a piece of information; since the concept  $A$  is usually a common sense, we are concerned chiefly with the variable  $x$ : where does it occur? In this sense, truth value  $T(A(x))$  is the possibility of  $x$  under the constraint  $A$ . It comes the possibility theory presented by L.A.Zadeh.

"John is tall" provides the information that the height of John is in the area of tall: it occurs at  $x$  with possibility  $T(A(x)) = \mu_A(x)$ .

By means of the Falling shadow theory, a possibility distribution is the covering function of a random set. While the probability distribution of a discrete random variable is also the covering function of it, so that we can view possibility as a generalization of probability as that: possibility is probability if variable  $x$  is to have exclusiveness.

## 2. Introduction of the Concept of Truth Valued Flow Inference

First let's see why can we see the inference processes as truth values flowing among propositions? That is how inference channels realize inference as logic system does. Let us consider the syllogism inference as follows:



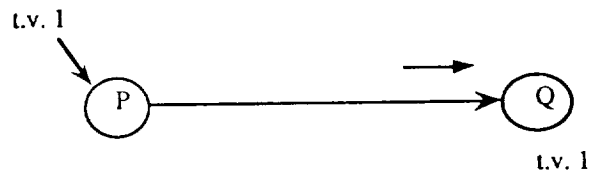
When we face an object,  $x=John$ . The fact is: "John is a person". i.e.,

$$T(P(x)) = T(Person(John)) = 1$$

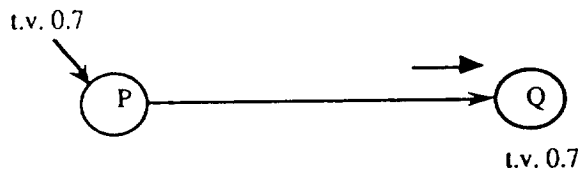
By means of the implication "If  $x$  is a person, then it will die", denoted as  $P \rightarrow Q$ , we get

$$T(Q(x)) = T(end\ in\ dead(John)) = 1.$$

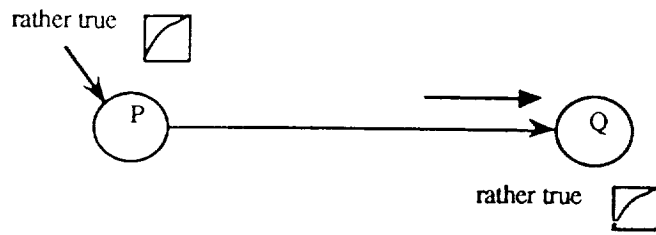
Then we get the consequence: John will die. Here, we can see that an implicate likes a channel transferring truth value from head to tail.



When the fact does not qualify the head  $P$  completely but partly support it with truth value 0.7 for example, then the consequence is not certainty, we don't accept  $Q$  with truth value 1 but 0.7. This is the uncertainty inference, it can be also viewed as the truth value of input transferred to the tail along a inference channel.



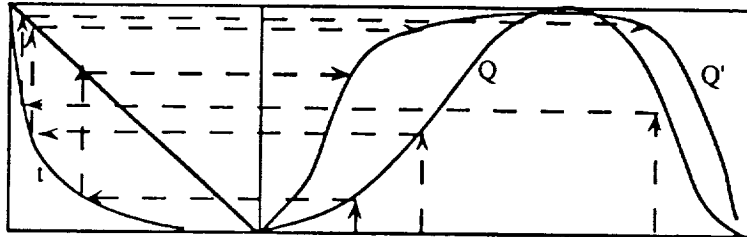
Of course, the truth values can be a linguistic value such as RATHER TRUE, VERY TRUE, ..., the inference channel also transfers them from its head to its tail.



In this case we need the theory of Truth valued qualification(Baldwen 1979):

$$(y \text{ is } Q) \text{ is } t = Y \text{ is } Q'$$

$$\mu_{Q'}(y) = t[\mu_Q(y)]$$



when the variables x, y are given, an implication

$$(\forall(x, y)) \text{ if } P(x) \text{ then } Q(y)$$

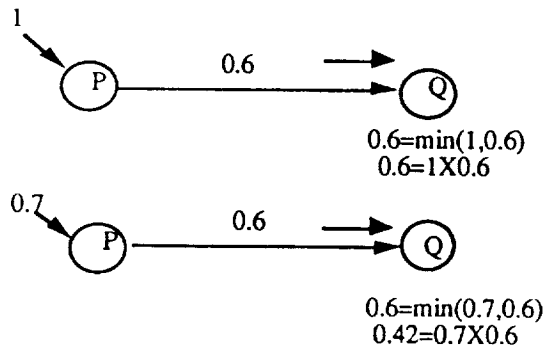
is determined by the pair of concepts P and Q. So an inference channel, through whom truth values can flow, can be denoted as [P,Q]. We call that the channel [P,Q] connects with concepts P and Q; P is its head and Q is its tail. A channel does not connect with propositions but concepts. The function of a channel is only transferring truth values, it is independent of how much truth value does its head have.

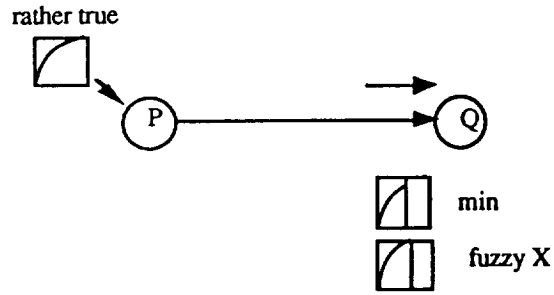
Inference channels have different qualities on transferring truth values. We call a channel [P,Q] has a quality coefficient q or call [P,Q] a q-quality channel if

$$t.v.\text{output } t' = t.v.\text{input } t \wedge^* q$$

Where  $\wedge^* = \times$  or min or others.

When  $\wedge^* = \times$ , we call channel has 1-q friction, when  $\wedge^* = \min$ , we call q the transfer capacity of the channel.

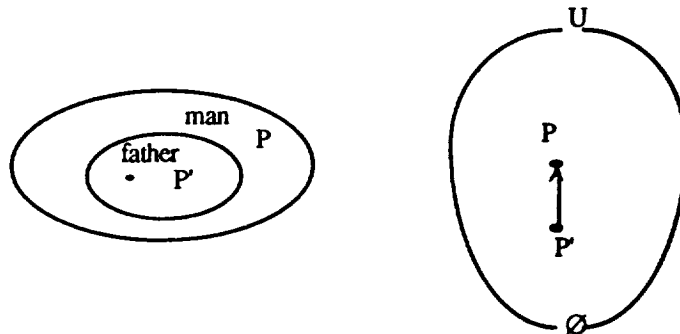




### 3. Properties of channels

For simple, we consider the head and tail of channels are all ordinary subsets. There are some basic properties of inference channels.

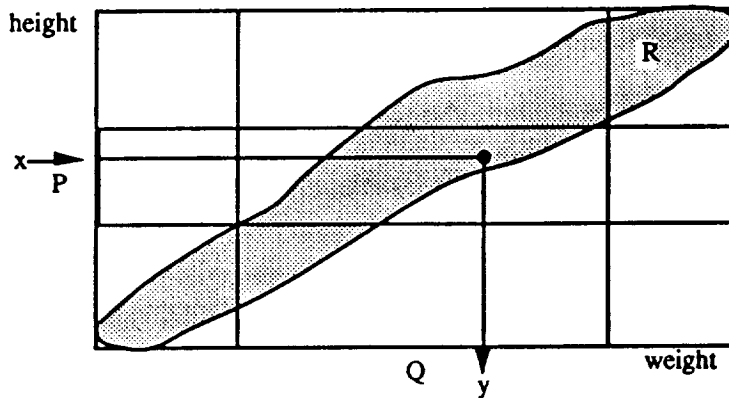
**PROPERTY 1.** If  $P \subseteq Q$  then  $[p, Q]$  is an 1-channel, called Natural channel



A concept in the Cartesian product space of  $X(x\text{-Universe})$  and  $Y(y\text{-Universe})$  is called a relation between  $x$  and  $y$ . For example,  $O =$  a group of people, factor  $f =$  height,  $g =$  weight,  $X=X_f$ ,  $Y=X_g$ . For any  $o \in O$ , define  $x=f(o)$ ,  $y=g(o)$ , and denote the set of  $(x,y)$  as

$$R = \{(x,y) \mid o \in O\}$$

$R$  is height-weight relation respect to  $O$



$R$  is the promised range of the point  $(x,y)$ . It means that  $(x,y)$  cannot occur outside of it. That is

$$(x, y) \in R = X \times Y \cap R$$

Because of

$$x \in P \Leftrightarrow (x,y) \in P \times Y \Leftrightarrow (x,y) \in P \times Y \cap R,$$

and

$$y \in Q \Leftrightarrow (x,y) \in X \times Q \Leftrightarrow (x,y) \in X \times Q \cap R$$

when

$$P \times Y \cap R \subseteq X \times Q \cap R$$

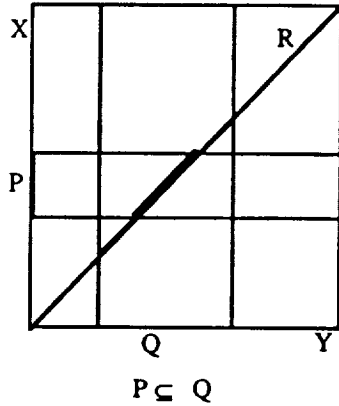
According to Property 1, we can say  $[P,Q]$  is an 1-channel. So we get the next property

**PROPERTY 2.** For a given relation  $R$  between  $X$  and  $Y$ , if

$$P \times Y \cap R \subseteq X \times Q \cap R$$

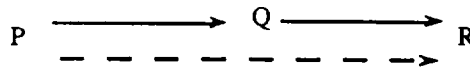
then  $[P,Q]$  is an 1-channel from  $X$  to  $Y$ . It is called a channel under relation  $R$ , and  $R$  is called the **ground relation** of the channel.

Property 1 is a special case of property 2. Indeed  $\subseteq$  is a binary-relation

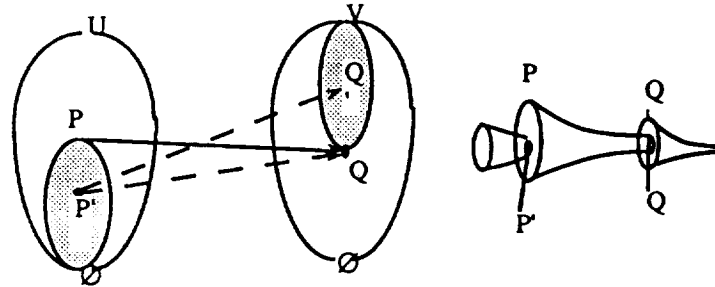


**Note:** A class of inference channels can be generated from a relation.

**PROPERTY 3.** If  $[P,Q]$  and  $[Q,R]$  are two 1-channels then  $[P,R]$  is a channel



**PROPERTY 3'.** If  $[P,Q]$  is a 1-channel,  $P' \subseteq P$  and  $Q \subseteq Q'$  then  $[P',Q']$  is a 1-channel.



For simplicity,  $[P,Q] \in C(X,Y)$  or  $C$  stands for  $[P,Q]$  is a 1-channel from  $X$  to  $Y$ .

**PROPERTY 4.**

$$[P_1,Q] \in C \text{ and } [P_2,Q] \in C \Rightarrow [P_1 \vee P_2, Q] \in C$$

$$[P,Q_1] \in C \text{ and } [P,Q_2] \in C \Rightarrow [P, Q_1 \wedge Q_2] \in C$$

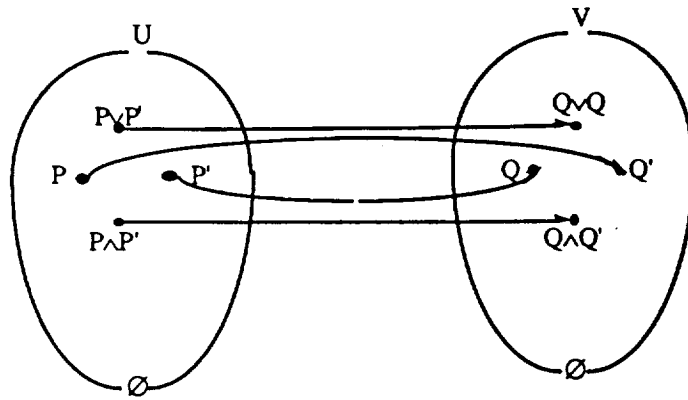
**PROPERTY 4'.**

$$[P_1,Q_1], [P_2,Q_2] \in C \Rightarrow [P_1 \vee P_2, Q_1 \vee Q_2], [P_1 \wedge P_2, Q_1 \wedge Q_2] \in C$$

**THEOREM.** Let  $c_1 = [P_1, Q_1]$ ,  $c_2 = [P_2, Q_2]$  define

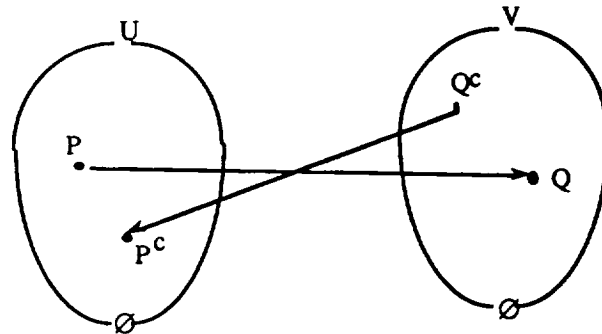
$$c_1 \vee c_2 = [P_1 \vee P_2, Q_1 \vee Q_2], c_1 \wedge c_2 = [P_1 \wedge P_2, Q_1 \wedge Q_2]$$

Then  $(C(X,Y), \wedge, \vee)$  forms a lattice, and it is called the channel lattice.



PROPERTY 5.

$$[P, Q] \in C(X, Y) \Rightarrow [Q^c, P^c] \in C(Y, X)$$



**DEFINITION** Let  $c_1=[P_1, Q_1]$ ,  $c_2=[P_2, Q_2]$  if  $P_1 \supseteq P_2$ ,  $Q_1 \subseteq Q_2$  then  $c_1$  is more valuable than  $c_2$ , denoted as  $c_1 \Rightarrow c_2$ . A channel  $c$  in  $C$  is called valuable channel if there isn't other channel  $c'$  in  $C$  such that  $c' \Rightarrow c$ . The subset of valuable channels is denoted as  $V$ .

About the concepts of "information value" and "belief degree" of a channel, the bigger the head and the smaller the tail, the more information the channel, and therefore the more valuable the channel; on the other hand, it has the smaller belief degree. They can be represented by the following formula.

Suppose  $P \rightarrow Q$  is a channel,  $P' \subseteq P$ ,  $Q' \supseteq Q$ , then we have know that  $P' \rightarrow Q'$  is also a channel. And

$$\text{belief-degree}(P' \rightarrow Q') \geq \text{belief-degree}(P \rightarrow Q),$$

$$\text{information-value}(P' \rightarrow Q') \leq \text{information-value}(P \rightarrow Q).$$

For any  $x \in X$ , define

$$Q_x = \bigcap \{Q \mid P \rightarrow Q \in C, x \in P\}$$

and assume that for any  $x \in X$ ,  $Q_x \neq \emptyset$ , then we have

**DEFINITION.** Define

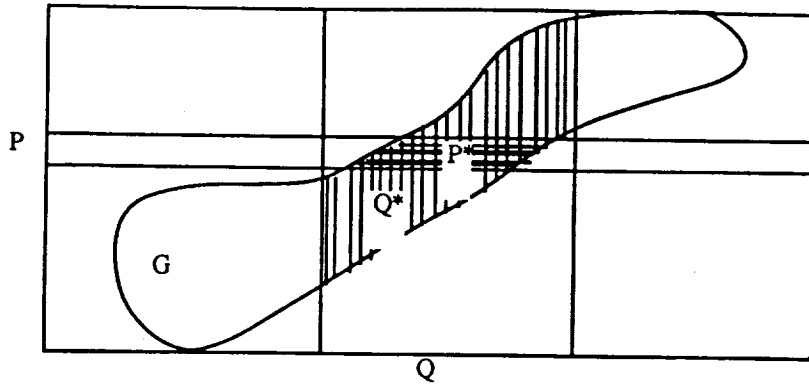
$$G = \bigcup \{Q_x \times \{x\} \mid x \in X\}$$

$G$  is called the background graph of lattice  $C$ .

**THEOREM.** Let  $C(X, Y)$  be the channel lattice generated from a ground relation  $R$ , let  $G$  be the ground graph of  $C(X, Y)$ , then we have that  $G=R$ .

**THEOREM.** Lattice  $C$  can be determined uniquely by its background graph  $G$ . That is to say that  $P \rightarrow Q$  is a channel in  $C$  if and only if  $P^* \subseteq Q^*$ .

where  $P^* = P \times Y \cap G$ ,  $Q^* = X \times Q \cap G$ . (As shown in the following figure)



**DEFINITION.** Giving channel  $c=[P,Q]$ ,

$$R(c) = P \times Q \cup P^c \times Y$$

is called inference relation of channel  $c$ .

**THEOREM**  $c=[P,Q]$   $(P \in X, Q \in Y) \in C$  if and only if  $R(c) \supseteq G$ .

**THEOREM.**  $c=[P,Q]$   $(P \in X, Q \in X) \in C$  if and only if  $Q \supseteq P$ .

**THEOREM.** About the relations of background graphs of channels, we have

$$R(c_1 \text{ and } c_2) = R(c_1) \cap R(c_2)$$

$$R(c_1 \text{ or } c_2) = R(c_1) \cup R(c_2)$$

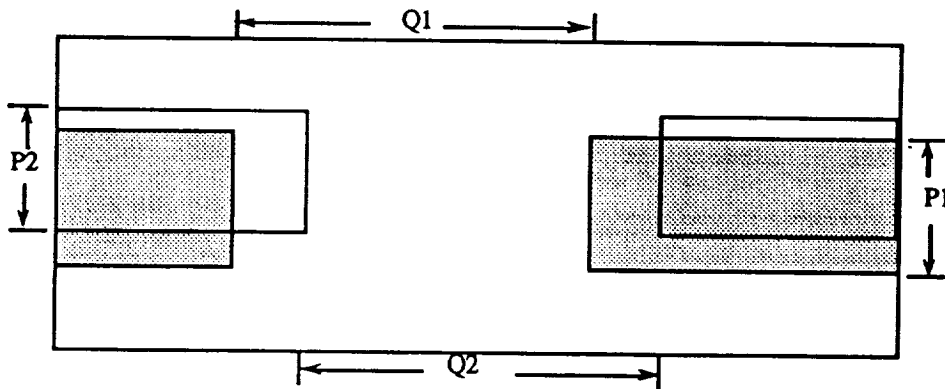
$$R([P, Q_1] \text{ and } [P, Q_2]) = R([P, Q_1 \wedge Q_2])$$

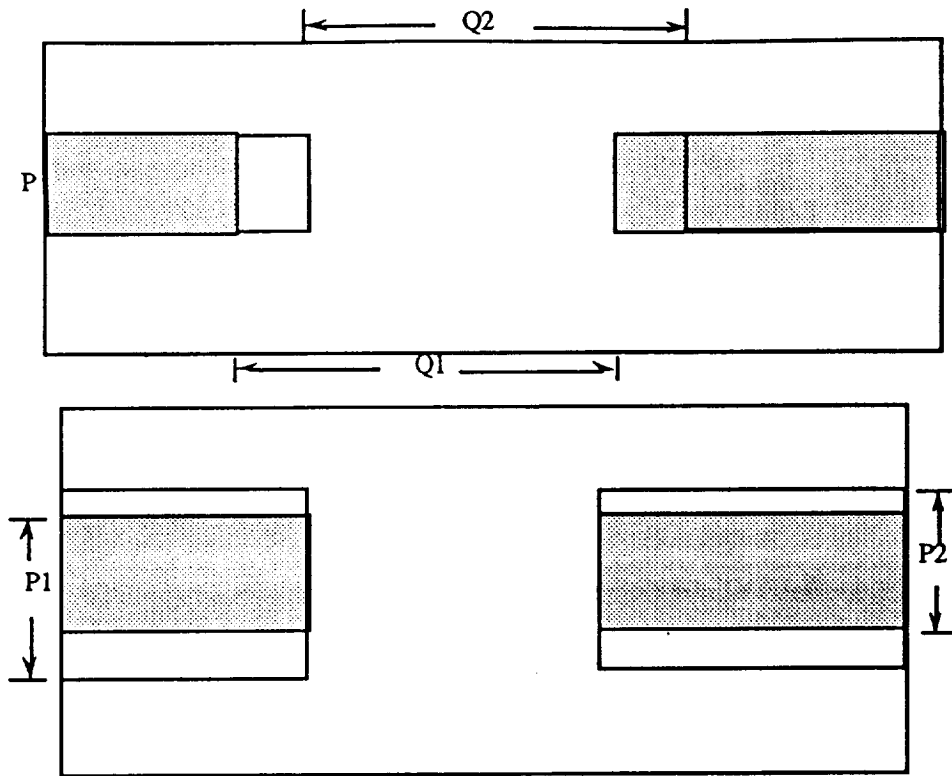
$$R([P, Q_1] \text{ or } [P, Q_2]) = R([P, Q_1 \vee Q_2])$$

$$R([P_1, Q] \text{ and } [P_2, Q]) = R([P_1 \vee P_2, Q])$$

$$R([P_1, Q] \text{ or } [P_2, Q]) = R([P_1 \wedge P_2, Q])$$

These can be shown in the following figure.





#### 4. Fuzzy channels Lattice

For given  $\lambda \in [0,1]$ , an  $\lambda$ -channel lattice  $L_\lambda$  consists of those channels who transfers truth value at least  $\lambda$  to the tail whenever the head is fulfilled with truth value 1.

For every definition of truth values operations  $\vee^*$  and  $\wedge^*$ , a channel  $[P, Q]$  is a  $\lambda$ -channel if and only if the qualify  $q$  of it is equal or larger than  $\lambda$ .

A  $\lambda$ -channel lattice satisfies axioms 1-5 as same as 1-channel lattice.

About the  $L_\lambda$  ( $\lambda \in [0,1]$ ), we obviously have the following proposition:

**PROPOSITION:** If  $\lambda \leq \mu$ , then  $L_\lambda \supseteq L_\mu$ .

Let  $L_\lambda$  ( $\lambda \in [0,1]$ ) be a  $\lambda$ -cut subset, then  $\{L_\lambda\}$  ( $\lambda \in [0,1]$ ) forms a fuzzy set on  $L$  called a fuzzy channel lattice, where  $L$  is the set of all channels.

Note that

$$\lambda \leq \mu \Rightarrow G_\lambda \supseteq G_\mu$$

$$\lambda \leq \mu \Rightarrow R_\lambda \supseteq R_\mu$$

where  $G_\lambda$ ,  $G_\mu$  and  $R_\lambda$ ,  $R_\mu$  are ground graph and ground relation of  $L_\lambda$ ,  $L_\mu$  respectively.

There is a difference between 1-channel lattice and  $\lambda$ -channel lattice ( $\lambda < 1$ ). In 1-channels, if  $[P, Q]$  and  $[P, Q']$  are both 1-channels then

$$Q \cap Q' \neq \emptyset$$

otherwise, we have  $[P, \emptyset] = [P, Q \cap Q']$  hold. From this, we have  $[P, R]$  (for any  $R$ ) hold, especially  $[P, Q^c]$ . Therefore, we have  $[P, Q]$  and  $[P, Q^c]$  are both hold in the same time, this is a contradiction in mathematics. But in  $\lambda$ -channels ( $\lambda < 1$ ),  $Q \cap Q' = \emptyset$  may be hold.

Principles of quality qualification:

1. Let  $[P, Q]$  is a  $q$ -channel and  $[P, Q] = [P, Q_1 \text{ or } Q_2 \text{ or } \dots \text{ or } Q_n]$ , then for  $i=1, \dots, n$ ,  $[P, Q_i]$  are all  $q/n$ -channels.



2. Let  $[P, Q]$  is a  $q$ -channel and  $[P, Q] = [P_1 \text{ and } P_2 \text{ and } \dots \text{ and } P_n, Q]$ , then for  $i=1, \dots, n$ ,  $[P_i, Q]$  are all  $q/n$ -channels.

Following we further discuss this problem from another view of point.

**DEFINITION:** Given a background graph  $G$  on  $X \times Y$ , which is a fuzzy subset with membership function  $G(x, y)$ . We can define two fuzzy subsets  $N$  and  $\Pi$  on  $P(X) \times P(Y)$  as follows:

$$N(P, Q) = 1 - \wedge \{G(x, y) \mid y \in Q \mid x \in P\}$$

$$P(P, Q) = \vee \{G(x, y) \mid y \in Q \mid x \in P\}$$

$P \rightarrow Q$  is called a  $\lambda$ -channel if  $N(P, Q) \geq \lambda$ .  $x \rightarrow y$  is called a  $\lambda$ -offshoot if  $\Pi(\{x\}, \{y\}) \geq \lambda$ .

**THEOREM:** For any fixed  $x \in X$ ,  $N_x = N(\{x\}, \cdot)$  and  $\Pi_x = \Pi(\{x\}, \cdot)$  are necessity measure and possibility measures on  $P(Y)$  respectively. That is:  $N_x(\emptyset) = 0$ ,  $\Pi_x(Y) = 1$ , and

$$N_x(P \cap Q) = \min(N_x(P), N_x(Q))$$

$$N_x(P \cup Q) \geq \max(N_x(P), N_x(Q))$$

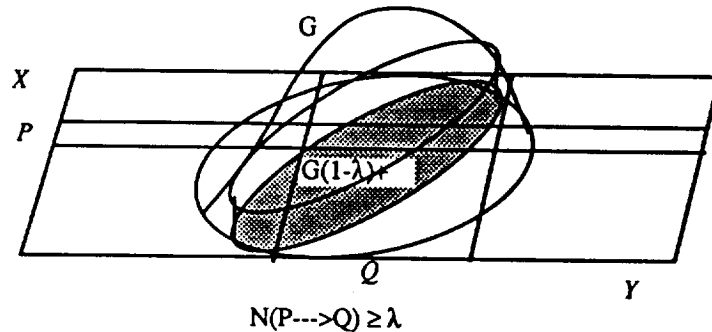
$$\Pi_x(P \cup Q) = \max(\Pi_x(P), \Pi_x(Q))$$

$$\Pi_x(P \cap Q) \leq \min(\Pi_x(P), \Pi_x(Q))$$

$$N_x(P) = 1 - \Pi_x(P^c)$$

**THEOREM:** For any  $\lambda (0 < \lambda \leq 1)$ ,  $N_\lambda$ , the  $\lambda$ -cut of  $N$ , is a channels lattice with respect to operations  $\cup$  and  $\cap$ . The corresponded background graph is  $G_{(1-\lambda)_+}$ , the  $1-\lambda$  open cut of  $G$ , i.e.

$$(P, Q) \in N_\lambda \Leftrightarrow P^* = P \times Y \cap G_{(1-\lambda)_+} \subseteq X \times Q \cap G_{(1-\lambda)_+} = Q^*$$



**THEOREM:** For any  $\lambda (0 < \lambda \leq 1)$ ,  $(P, Q) \in \Pi_\lambda$  if and only if for any  $x \in P$  there is a point  $y$  such that

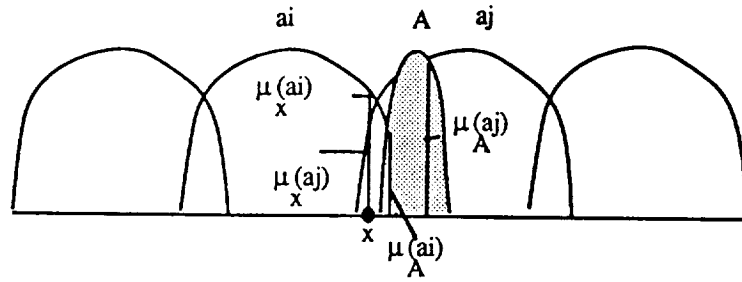
$$(x, y) \in (P \times Q) \cap G_{\lambda_+}$$

The membership degree of  $(x, y)$  with respect to  $G$  is equals to the necessity of offshoot  $x \rightarrow y$ :

$$G(x, y) = \Pi(\{x\} \rightarrow \{y\})$$

## 5. Truth Valued Flow Neural Networks

We call a Universe  $X$ , or corresponded variable  $x$ , is atomizable if there are only finite possible atoms  $a_i$  ( $i=1, \dots, n$ ) such that any information about  $x$  is stated through them in a problem.



Let  $X, Y$  are atomizable,  $X = \{a_i\} (i=1, \dots, n)$ ,  $Y = \{b_j\} (j=1, \dots, m)$ . The Cartesian product space  $X \times Y$  can be represented as an  $n \times m$  squares, and a ground relation (or graph) can be represented as a matrix  $R_{n \times m}$  with elements 0 or 1. For any head  $a_i$ , the valuable channel in the 1-channel lattice  $L_1$  is  $[a_i, B_j]$ , where the tail can be represented by atoms of  $Y$ :

$$B_i = \vee \{b_j \mid r_{ij}=1\}.$$

i.e.,

$$[a_i, B_j] = \text{OR}\{[a_i, b_j] \mid r_{ij}=1\}$$

$$= [a_i, b_{j1}] \text{ or } [a_i, b_{j2}] \text{ or } \dots \text{ or } [a_i, b_{j m_i}], \text{ where } r_{ij}=1.$$

According to the principle of quality qualification,  $[a_i, b_j]$  are  $1/m_i$ -channels.

For a given ground relation matrix  $R_{n \times m}$  of an 1-channel lattice  $L_1$ , normalizing each arrow of it, we get a matrix  $L_{n \times m}$  called TVF(truth valued flow) matrix of  $L_1$ :

$$l_{ij} = \begin{cases} r_{ij} / \sum_k r_{ik} & \text{if } \sum_k r_{ik} \neq 0 \\ 1/m & \text{else} \end{cases}$$

Truth values flow among the atoms from  $X$  to  $Y$  is a TVF Networks which consists of atom-channels(head and tail are atoms). The weight of  $[a_i, b_j]$  is  $l_{ij}$  and the Propagation rule is:

$$n_j = f(\vee^*(m_i \wedge^* l_{ij}))$$

where

$m_i$ -truth values at input;

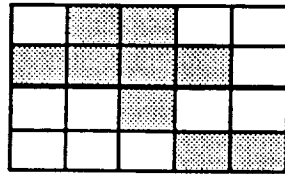
$n_j$ -truth values at output,

$f$ - threshold function,

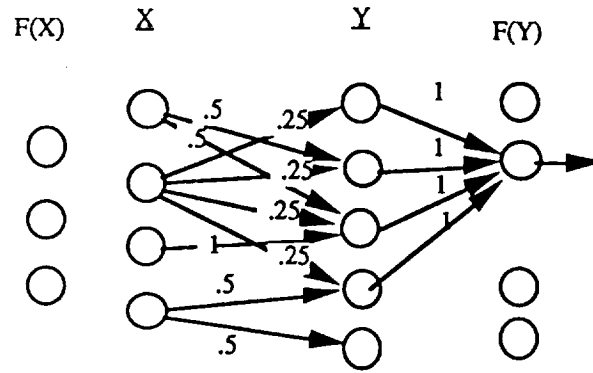
$(\vee^*, \wedge^*) = (\max, \min)$  or  $(+, \times)$  or other fuzzy operations.

From the following specific example, we can know the general TVF Networks structure.

**EXAMPLE:** Let  $X = \{a_1, a_2, a_3, a_4\}$  and  $Y = \{b_1, b_2, b_3, b_4, b_5\}$ , the ground graph is presented by the shadow area (left of the following Fig.), and ground relation  $R$  is presented by the  $L_{4 \times 5}$  matrix (right of the following Fig.), then this TVF network has the following structure (down of the following Fig.)



	.5	.5		
.25	.25	.25	.25	
		1		
			.5	.5



## 6. Applications of TVFI

### (1) TVFI Applications in AI

In the above section, we have gotten that for every ground graph, we can get a True-Value-Flow inference network. In AI field, the ground graph is just the database, and the Truth-Value-Flow inference network is just the knowledge base. So we actually realize the transferring from database to knowledge using Truth-Value-Flow inference. In practice, it is also very important to get ground graph from some kinds of database. In the following we will introduce several kinds of database, the ways to get database, and the ways to get knowledge base from database.

The kinds of database we often use are listed as follows:

- 1) statistical sample:  $\{(x_k, y_k)\}$ ;
- 2) relation data base:  $R(x_k, y_k, z_k, \dots)$ ;
- 3) causality rule:  $f=ma$ ;
- 4) experts experiences: if... then...;

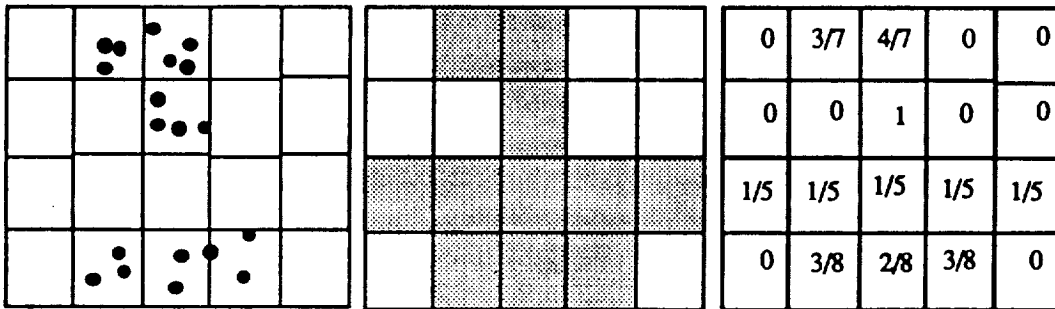
Below we will give a specific method how to get ground graph and ground relation from statistical samples, and how to get TVF neural networks (knowledge base) from ground graph (database).

For each  $i$ , get a distribution  $\{l_{ij}\}$

$$l_{ij} = \begin{cases} m_{ij}/m_i & \text{if } m_i \neq 0 \\ 1/m & \text{else} \end{cases}$$

where  $m_{ij} = \sum_k (m_{a_i}(x_k) \times m_{b_j}(y_k))$ ,  $m_i = \sum_j m_{ij}$ .

**Note:** When there is not point occurred in an arrow (for example, 3th arrow in the following Fig.) the relation or graph is not empty but full in  $Y$ , and  $l_{ij}$  are uniformly distributed.

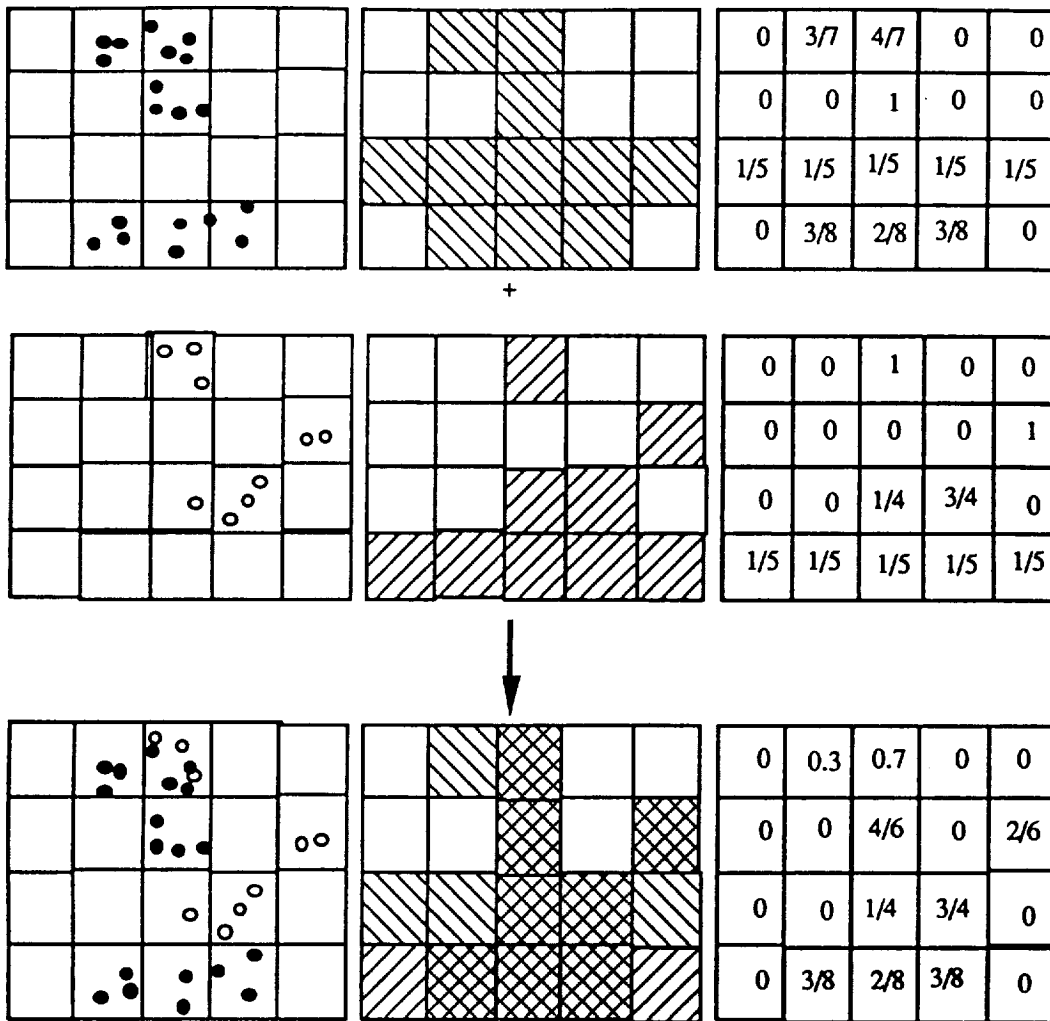


When our information (i.e. data base) is not complete, we can only get a sublattice of an unknown channel lattice.

**DEFINITION.** A channel lattice  $L'$  is called a sublattice of a channel lattice  $L$  if the ground graph of  $L'$  contains the ground graph of  $L$ .

In data base, the sample of statistics or the relation form corresponded to a sublattice  $L'$  is more incomplete than that of channel lattice  $L$ .

For an incomplete channel lattice, we can extend data base by adding any kind of information and knowledge.



**DEFINITION.** Let  $L_{n \times m}$  be the TVF matrix of channel Lattice L, then

$$l_j = \max_i l_{ij} \text{ and } l = \min_j l_j$$

are called the inductable degree of L at  $b_j$  and of L respectively. If  $l \geq l^*$  (or  $l_j \geq l^*$ ), we call L is  $l^*$ -sufficient (or for  $b_j$ ).  $l^*$ -sufficient is called completely sufficient.

To know which head is able to infer to  $b_j$ , we are natural to inversely search along the weightiest channel (whose quality equals to  $l_j$ ), if  $l_j$  is larger than the given threshold  $l^*$ , then we find out the head we want to know.

After adding information to L, if the inductable degree is still smaller than the given threshold  $l^*$ , It means that the factor concerned with x is not enough to infer y. We have to move X into another factor space.

Let F be the set of factors concerned with variable y. Let  $L_f$  be the channel lattice from  $x_f$  to y. Set  $X = X_f$ , the inductable degree is  $l_f$ . The more complex the factor f, the higher the inductable degree of  $L_f$ .

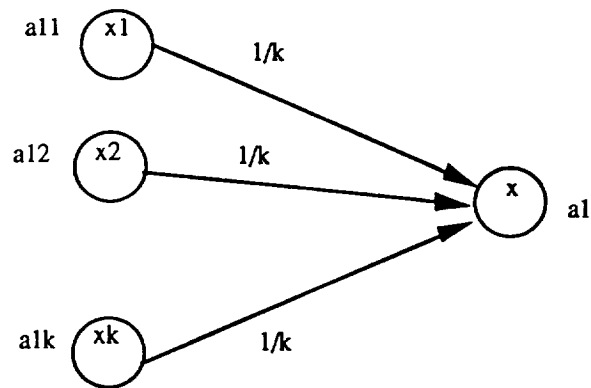
When  $l_f$  is enough, suppose that

$$f = f_1 \vee \dots \vee f_k$$

where  $f_1 \dots f_k$  are simple factors which concerned with variable  $x_1, \dots, x_k$  respectively, then an atom in x is in the form:

$$x_1 \text{ is } a_{11} \wedge \dots \wedge x_k \text{ is } a_{1k}$$

According to the principle of quality qualification, we can arrange a neural network as follows:

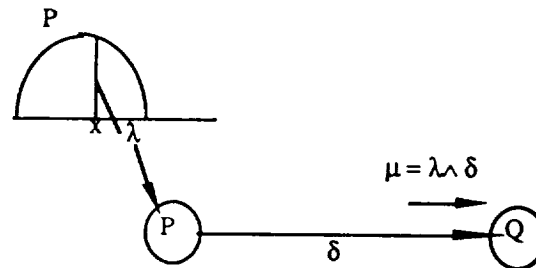


This is a TVFI neural network taken in factor spaces. It is actually the network representation of knowledge base. Thus we complete the transferring from database to knowledge base.

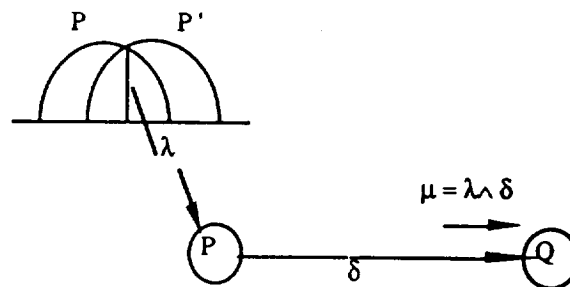
## (2) TVFI Applications in Approximate Reasoning

Suppose we have a channel  $P \rightarrow Q$ , then we may execute many kinds of approximate reasoning along this channel. Following we give the execution of two kinds of most often using approximate reasoning using TVFI channel.

1) The input is an element  $x$ , in this case we can do approximate reasoning as follows:



2) The input is a fuzzy set  $P'$  (i.e. concept), in this case we can do approximate reasoning as follows:



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