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NEW DEVELOPMENTS IN ASTRODYNAMICS ALGORITHMS FOR AUTONOMOUS RENDEZVOUS

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At the core of any autonomous rendezvous guidance system must be two algorithms for solving Lambert's and Kepler's problems, the two fundamental problems in classical astrodynamics. Lambert's problem is to determine the trajectory connecting specified initial and terminal position vectors in a specified transfer time. The solution is the initial and terminal velocity vectors. Kepler's problem is to determine the trajectory that stems from a given initial state (position and velocity). The solution is the state at an earlier or later specified time.

To be suitable for flight software, astrodynamics algorithms must be totally reliable, compact, and fast. Although solving Lambert's and Kepler's problems has challenged some of the world's finest minds for over two centuries, only in the last year have algorithms appeared that satisfy all three requirements just stated.

This paper presents an evaluation of the most highly regarded Lambert and Kepler algorithms known to me. One Lambert and one Kepler algorithm are clear winners. All algorithms are available on request on floppy disks or by electronic mail.

Lagrange is credited with deriving the first analytic expression for the Lambert time of flight in 1778. In 1801, Gauss devised a method for solving Lambert's problem and used it to determine the orbit of the planetesimal Ceres from a 3° arc traversed in 41 days. He published solutions for both problems in his *theoria motus* in 1809. Hundreds of solutions have been published; improved methods continue to appear frequently.

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The outstanding recent contributions on Lambert's problem came from E. R. Lancaster and R. C. Blanchard[1], and on Kepler's problem from W. H. Goodyear[2,3], both in the 1960s. Recently, Robert H. Gooding[4] of the Royal Aerospace Establishment has made major improvements in the Lancaster-Blanchard algorithm. Francis M. Stienon[5] of the Jet Propulsion Laboratory has improved the Goodyear algorithm to handle high-energy hyperbolic trajectories, which caused the original Goodyear algorithm to overflow. I have made minor corrections for special cases to Gooding's and Stienon's solutions. The resulting algorithms are equal or superior, in every respect, to all other algorithms evaluated, hence are the clear winners. No cases have been found that the Gooding algorithm will not handle. The Stienon algorithm degrades only for extremely high-energy hyperbolic trajectories, more so for trajectories inbound with respect to the central body than for trajectories outbound (Stienon's original case is essentially outbound).

Other major contributions have been made by Richard H. Battin, his thesis students at the Massachusetts Institute of Technology, and Stanley W. Shepperd of the C. S. Draper Laboratory. Battin and Shepperd[6] improved the Lancaster-Blanchard algorithm in a number of ways. Most significantly, the Battin-Shepperd algorithm eliminates a singularity for transfer angles that are a multiple of 180°. Battin and Vaughan[7,8] improved Gauss' 1809 Lambert algorithm so that it converges for virtually all realizable trajectories. Battin and Loechler[9] extended the Gauss algorithm to handle multiple-orbit transfers, but this extension is not implemented in the algorithm evaluated here because the extension substantially complicates the algorithm. Shepperd[10] developed a universal-variable equivalent of the Goodyear Kepler algorithm. Battin and Fill[11,12] extended Gauss' 1809 Kepler algorithm to trajectories not necessarily reckoned from periapse in which the eccentricity and arc length are arbitrary. Improved versions of these algorithms were published in Battin's 1987 book[13]. I further improved the Battin-Vaughan Lambert algorithm, and extended the Battin-Fill Kepler algorithm to trajectories for which the original did not converge[14,15,16].

Unfortunately, each of the algorithms of the preceding paragraph falls short of those by Gooding and Stienon in one or more important ways. The Battin-Shepperd Lambert algorithm is slower than the Gooding algorithm, and less accurate in some extreme cases. The Battin-Vaughan Lambert algorithm requires up to 660 iterations and a great deal of time for minimum-energy orbits approaching 360° transfer angle, whereas the Gooding algorithm handles all single-orbit transfers in five iterations (more precisely, five evaluations of normalized time). Furthermore, the Gooding algorithm is much more compact, and, again, far more accurate in some extreme cases. Shepperd's Kepler algorithm, although slightly faster than Stienon's for more-common trajectories, is much slower and much less accurate for high-energy hyperbolic trajectories. My extension of the Battin-Fill Kepler algorithm is very robust, but the Stienon algorithm is just as robust, many times faster, and more compact.

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