# Multi-layered Reasoning by means of Conceptual Fuzzy Sets

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**Key Words:** context dependency, fuzzy sets theory, fuzzy associative memory, approximate reasoning, neural network, knowledge representation, inductive learning, concept formation

## 1. Introduction

Real world consists of a very large number of instances of events and continuous numeric values. On the other hand, people represent and process their knowledge in terms of abstracted concepts derived from generalization of these instances and numeric values. Logic based paradigms for knowledge representation use symbolic processing both for concept representation and inference. Their underlying assumption is that a concept can be defined precisely. However, as this assumption hardly holds for natural concepts, it follows that symbolic processing cannot deal with such concepts. Thus symbolic processing has essential problems from a practical point of view of applications in the real world. In contrast,fuzzy set theory can viewed as a stronger and more practical notation than formal, logic based theories because it supports both symbolic processing and numeric processing, connecting the logic based world and the real world.

For example, in the case of an intelligent control system, control actions are determined not only by numeric processing but also integrated with the result of intellectual decision making at a more abstract level based on meaning understanding of numeric data. Using only numeric processing or describing simple correspondences of instances produces a black box effect and is difficult to integrate with symbolic, logic based information processing. For this reason, multi-layer structured frameworks have been proposed, where intellectual information processing based on meaning understanding and state recognition in upper layer supervises the data processing in lower layer [2]-[3]. The duality abstract/concrete of the real world is reflected in the intelligent/lack of intelligence duality at the intellectual level (Increasing Precision with Decreasing Intelligence principle, IPDI, [4] - [5]) To cope with this duality a knowledge representation paradigm must be able to hierarchically represent both aspects. Thus we are led to consider multi-layered structures representation.

A concept such as an operator's know-how in the upper abstracted layer is essentially vague. Moreover, it is difficult to eliminate this vagueness during the generalization process from control experiences. For this reason, fuzzy set theory can be expected to provide us with a strong notation for concept representation at different levels of granularity: lower, concrete concepts describe an upper, vague concept constructing thus a multi-layered structure and a capability connecting information processing in different layers.of abstraction.

However, simple notion using ordinary fuzzy sets cannot solve all the problems of (concept) knowledge representation because of the following:

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1. Lack of context dependency
2. Impossibility of explicit formulation of a concept.

These problems arise because the meaning of a concept changes depending on various situations and concrete events cannot always be generalized into logical notation explicitly. For example, a fuzzy controller of a car aims to realize intelligent control in terms of modeling the driver's know-how such as : "If the distance between cars is big, then the change of acceleration is big". Nevertheless, since the concepts such as "big" or "small" describing control rules are defined on a numeric axis absolutely using a simple formulation, the definition indicates only a simply unique meaning of a concept and cannot cover the variety of meanings (depending on the size of a car and road conditions). The fuzzy control does not achieve the intellectual information processing in the upper level nor the aims of intelligent control.

All these problems relate to the representation of **the meaning** of a concept. According to Wittgenstein [1], the meaning of a concept is represented by the totality of its uses. In this spirit we proposed [2] the notion of Conceptual Fuzzy Sets:( henceforth referred to as CFS). In the CFS the meaning of a concept is represented by the distribution of activation of labels naming concepts. Since the distribution changes depending on the activated labels to indicate a situation, CFS can represent context dependent meanings. CFS are realized using bidirectional associative memories implemented as neural networks. Since the propagation of activation realizes logical operations and inference as well as the representation of meanings, many advantageous features are obtained which are not realized by logic based representation alone.

Further, since the distribution of activation determined by the propagation of activation in CFS represents the meaning of a concept, the propagation of activations corresponds to reasoning. In particular, a multi-layer structured CFS represents the meaning of a concept in various expressions in each layer. Therefore, it follows that due to the capability of naturally realizing information processing in multi-layered structures, the CFS have the following features:

1. Because CFSs are realized and connected using a bi-directional associative memory, CFS can carry out information processing both in the upper layer and lower layer simultaneously exchanging information. Thus they provide us easily with a framework where the processing in the upper layer supervises the processing in the lower layer.
2. Since CFS are realized as a bi-directional associative memory, it can carry out both bottom-up processing from the lower layer to the upper layer, and top-down processing from the upper layer to the lower layer simultaneously.

In this paper, we propose Multi-layered Reasoning realized by using CFS and we discuss the above two features. In section 2, we show the general characteristics of CFS. In section 3, we discuss the structure where the upper layer supervises the lower layer and we illustrate it with examples. In section 4, we discuss the context dependent processing carried out by the simultaneous bottom-up processing and top-down processing.

## 2. Conceptual Fuzzy Sets

### 2.1. Conceptual Fuzzy Sets for Concept Representation

A label of a fuzzy set represents the name of a concept and a fuzzy set represents the meaning of the concept. Therefore, the shape of a fuzzy set should be determined from the meaning of the label depending on various situations. According to the theory of meaning representation from use proposed by Wittgenstein [7], the various meanings of a label (word) may be represented by other labels (words) and we can assign grades of activation showing compatibility degrees between different labels.

The Conceptual Fuzzy Set proposed in [8], achieves this by the distributions of activations. Since the distribution changes depending on the activated labels which indicate conditions, the activations resulted through CFS show a context dependent meaning. When more than two labels are activated CFS is realized by the overlapping propagations of activations. In CFS notations, operations and their controls are all realized by the distributions of activation and their propagations in associative memories.

We can say that the distribution determined by the activation of a label agrees with the region of thought corresponding to the word expressing its meaning. Since situations are also indicated by activations, the meaning is expressed by overlapping the regions of thought determined by these activations. Fig 2.1 illustrates the different meanings of the same label, L1, in different situations, S1 and S2.

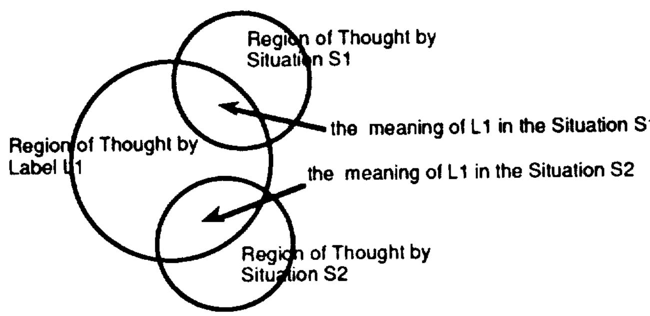


Fig.2.1 Different meanings in different situations

A CFS is realized as an associative memory, in which a node represents a concept and a link represents a strength of the relation between two (connected) concepts. Activations of nodes produce a reverberation and the system energy is stabilized to a local minimum where corresponding concepts are recollected as a result. The recollections are carried out through a weight matrix encoded from stimulus-response paired data.

In this paper we use Bidirectional Associative Memories (BAMs) [9] because of the clarity of constraints for their utilization. At the association in BAMs reverberations are carried out according to:

$$Y_t = \phi(M \cdot X_t), \quad X_{t+1} = \phi(M^T \cdot Y_t). \qquad (1)$$

where, $Xt=[x1, x2, ..., xm]^T$, $Yt=[y1, y2, ..., yn]^T$ are activation vectors on x and y layers at the reverberation step t, and $\phi(\cdot)$ is a sigmoid function of each neuron. BAMs memorize corresponding pairs of elements at each layer in terms of a synaptic weight matrix, M, to memorize CFS, and calculated from corresponding input/output pairs of Ai /Bi with coefficient $\alpha$ i:

$$M = \Sigma i \ \alpha \, iAi'Bi' \qquad (2)$$

**Example 2.1. CFS representing a composed concept which has multiple meanings depending on situations**

Let us consider the concept "tall", and its meaning according to whether it is applied to an American or Japanese person. The meaning of concept "tall" changes in these two situations. The distribution of activation of other labels explains the meaning of "tall" depending on these contexts. Fig. 2.2 shows the concept "tall American" which agrees with the meaning of "tall" in case of an American person. In this figure and throughout the remainder of this paper, "American" and "Japanese" refer to "American height" and "Japanese height" respectively. The activations of nodes which express "American" and "tall" make the distribution of activation in the middle layer which consists of numerical values.

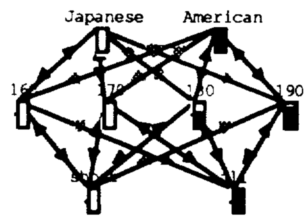


Fig.2.2 CFS representing "tall" American

In contrast, activating only "American", the different distribution from above in middle layer expresses its general meaning in the numeric support set. The propagated activation of "tall" in lowest layer indicates the perception of the height of an American, and it means "(an) American is tall". As we see in this latter example, the meaning of a label in CFS is expressed in multi-layers simultaneously and it is interpreted by each expression.

## 2.2. Construction of CFS by Learning

We proposed the method to inductively construct CFS as a representation of concepts using neural network learning [10]. It means that the construction is carried out in terms of instances.

### Inductive Construction of CFS

CFS are constructed inductively using Hebbian learning. CFS is realized using associative memories in which a link represents a strength of the relation between two concepts. Hebbian learning modifies the strength $m_{ij}$ of links by the product of the activations of two nodes $x_i$ and $y_j$ according to:

$$\dot{m}_{ij} = -m_{ij} + x_i y_j \qquad (3)$$

In this case the correlation matrix is obtained directly from instances such as

"The height of Mark is 175cm. He is tall with a grade 0.8"
"The height of George is 160cm. He is tall with a grade 0.2"

On the other hand CFS are also constructed by the previously proposed algorithm [2] from the fuzzy set.

"tall" = { 0.2/160cm, 0.8/175cm, ... }

generalized from the instances above.

### Structural Learning of Concepts

The proposed construction method also covers the structural learning. Since the proposed learning method makes negative correlation for the pairs of elements which are not relating to the concept in question, the obtained CFS does not make unnecessary elements activated. For this reason the proposed method can provide us with a desirable CFS even in support sets which contain verbose elements.

### Composition of Subdivided Knowledge

A complex CFS is realized by composing several pieces of associative memory structured individually. Further composition of pieces of knowledge makes the representation of the concept context dependent. In this procedure the constraints of associative memories are very important.

If $C_1, C_2, ..., C_n$, denote individual CFSs and $M_1, M_2, ..., M_n$ are their corresponding correlation matrices then we can combine them to obtain a CFS, C, whose correlation matrix, M, is given by:

$$M = M_1 + \cdots + M_n \qquad (4)$$

The following features of CFS allow for solving the shortcomings of purely symbolic knowledge representation paradigms:

1. CFS can represent the context dependent meaning of a concept. At the same time being built through simple combinations it avoids combinatorial explosion.
2. CFS can explicitly represent the concept whose logically explicit representation is impossible.
3. Since CFS can employ a multi-layered distributed structure, many kinds of expressions such as denotative and connotative can be mixed. Inference is performed by passing through layers and propagating activations.
4. As indicated in [11] propagations of activations realize approximate reasoning. Thus, associative memories lend CFS's the characteristics of intellectual information processing such as decrease of fuzziness, bidirectional inference, context dependent reasoning, etc..

## 3. Fusion of symbolic processing and numerical processing

### 3.1. Fuzzy Reasoning by means of CFS

As we see above, CFS represent the meaning of a concept in multiple layers. The meaning of the concept is translated into the expression indicated by the distribution of activation in each layer. Since the representation of the meaning in the input layer is translated into a representation in the output layer, the propagation of activation corresponds to reasoning. CFS can realize many kinds of reasoning which behave consistently with other reasoning methods (slight differences are due to different notation).

In particular, rule based approximate reasoning is realized as follows. Consider a rule of the form IF x is A then y is B. A layer consists of nodes representing premises A1,A2, ..., Am, describing x. Another layer consists of nodes representing the consequences B1,B2, ..., Bn, describing y. These layers are connected by a weight matrix M calculated from correspondences of premise Ai and consequence Bj. If the input is x=x*, the concepts A1,A2, ..., Am are activated with the activations being equal to the corresponding membership values of x*. The propagation of activation determined by the activation of the premise layer produces the distribution of activations in the consequence layer, that is B1,B2, ..., Bn. As each activation corresponds to the truth value of each concept, approximate reasoning is realized [12].

As CFS behave beyond the limitation of logic based notation, the following reasoning can be realized using CFS:

1. Propagations which arise from the activation of an abstracted concept show its meaning in the concrete layer. This corresponds to answering the question asking the meaning of the concept.
2. In contrast, the activation of a lower concept determines the activations of an upper concept and it

corresponds to recognition or understanding.

Further, due to its bidirectional features, the reasoning in CFS has various characteristics which cannot be achieved by the logic based paradigm[11].

### 3.2. Multi-layered Reasoning

Consider a simple example of predicting the currency exchange rate. In the case of a war happening, we use concrete examples from past experience, such as the Gulf War, to predict a precise value. At the same time, we refer the macroscopic knowledge such as "dollar rises in case of emergency" and make rough prediction such that dollar rises up. We can say that the abstracted knowledge described in the upper layer supervises the generous reasoning path and corrects the result of reasoning in the lower layer in terms of concrete knowledge such as numeric data and event data.

In general, quantitative processing or neural network deal with numeric data and are not capable of integrating symbolic semantics. In contrast, symbolic processing suits intellectual information processing, but does not suit numeric processing. Since both processing methods take completely different approaches to knowledge processing and knowledge acquisition, the effective integration of these methods, while desirable, is difficult to achieve in a way of which combines their best features.

A reasoning in a multi-layer structured CFS realizes, to some extent, the integration of these two paradigms. The upper layer is meant to carry out symbolic processing using abstracted concepts while the lower layer to process numeric data and instances. If only the reasoning in the lower layer is used, it gives us precise results, but possibly a wrong reasoning path from macroscopic view point. On the other hand, the reasoning in upper layer alone cannot provide a precise result. Bidirectional association connecting two layers enable us to fuse the simultaneous processing in upper and lower layers to obtain a semantic guide supported by the upper layer and the precise processing supported by lower layer. The correspondences of concepts in upper layer represent the abstracted knowledge and the correspondences of examples or numeric data in the lower layer represent concrete knowledge. Since the concepts in the upper layer are connected with examples in lower layer, these connections result in the fusion of two differently abstracted layers. In the case when more than two layers exist various abstracted processes are carried out at the same time.

The reasoning in a multi-layer structured CFS is carried out according to the following procedure: The activation of the node in premises activates the corresponding several nodes in consequences in the lower layer. At the same time, the result of the semantic information processing in the upper layer propagated by the activation of the node in the premises in lower layer affects the consequences in the lower layer. As a result, the nodes affected by both the direct propagation in the lower layer and the semantic propagation in the upper layer remains to be activated. Finally, a concrete result is obtained in the lower layer and abstracted results are obtained in the upper layer simultaneously. We call **Semantic Guide Line** the supervision of the processing in lower layer by the intellectual information processing in upper layer.

**Example 3.1. Decision regarding the amount to steering**
When driving a car the amount to steering changes depending on situations. In the case that parking spaces are indicated by a painted line, we usually park the car passing the line. If the spaces are surrounded by borders or walls (as in a garage), another trajectory is considered (to avoid the collision with the wall as in Fig.3.1).

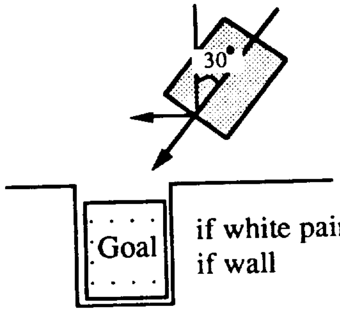


Fig.3.1 Parking Conditions

Consider the case that we decide the amount to steering besides parking space and the direction of the car is placed at 30 degree with the direction of parking space as indicated in Fig 3.2.

We decide the amount to steering using generous rule such as "steer to right to make right turn". The "right" is a concept generalized from various driving experiences and:

1. This kind of symbolic representation is effective to describe explicit and semantic knowledge.
2. However, its indications are vague and can not determine the amount to steering precisely.
3. Its meaning changes depending on the situations such as the position of a car.

On the other hand, cases such as "when the car makes x degree, we steered y degree" are described by concrete numeric values and:

1. The concrete experience indicates the precise amount to steering.
2. However, purely quantitative correspondence of conditions and actions does not suit logical information arising from varieties of conditions.

The CFS fuse both representations consisting of two layers. The lower layer memorizes the correspondences of the numerically described direction of the car and decided amount of steering. Since the lower layer consists of superficial numeric correspondences, it does not recognize the difference between the cases "with wall" and "without wall". In the upper layer, the conditions described by the symbolic notation such as "direction of the car" correspond to the actions such as "with wall" or "without wall". The correspondences of symbols are equivalent to the semantic control rules generalized from experiences . The nodes in the lower layer represent: direction of the car (left nodes) and decided amount of steering (right nodes). The nodes in the upper layer represent: the concept associating with the degree of the car such as "about 45 degree" and "about 90 degree (parallel to the front wall), two nodes on the left, and the conditions "wall" and "no wall" , the remaining two nodes on the left. The nodes on the right side of the upper layer represent the resulted actions such as "Turn left","Go Straight" and "Turn Right". Further, the concepts of the upper layer are connected to the concrete nodes of the lower layer, thus realizing meaning representation.

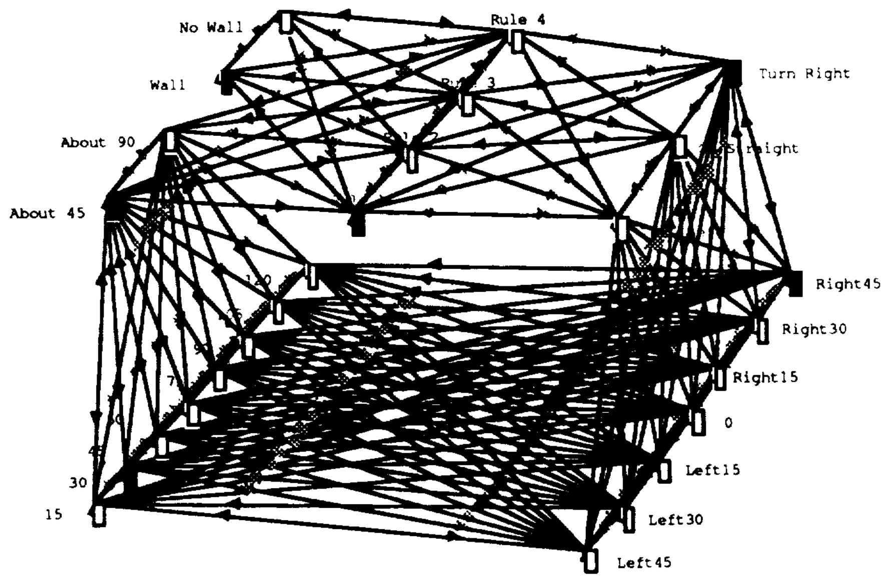


Fig. 3.2 Decision of the amount to steering by two-layered reasoning

Fig.3.2 also shows the conditions and the decided action when the car is placed in 30 degrees with a parking space having a wall. The condition "30 degrees" results in two kinds of actions depending on the cases "with wall" and "without wall". Because the lower layer simply memorizes both actions "15 degree to left" and "45 degree to right" corresponding to the conditions 30 degrees, the correct result cannot be recollected by using only the lower layer.

In the upper layer the recognition of a close wall activates "Turn Right" and it produces the activation of "turn right by 45 degrees" in the lower layer. The results of this multi-layered reasoning are "Turn right" in the upper layer and "turn right by 45 degrees" in the lower layer. This process of determining the actions indicates the successful supervision by the macroscopic views in the upper layer of the lower layer. Moreover, the results of the reasoning are equivalent to the meaning of "right" depending on different conditions.

## 4. Fusion of top-down and bottom-up processing

Usually natural language processing consists of two steps: (1) parsing and (2) semantic analysis. A lot of meaningless results are obtained by parsing alone. If semantic information could be used simultaneously in the step of parsing it would lead to a more efficient parsing. In image processing, recognition is carried out using characteristic values which are already obtained by low image processing. The fusion of referring a model of an object or the context with the image processing makes the image recognition more efficient. We can say that people simultaneously realize both image processing and recognition.

For the reasons indicated above substantial work has been focused on replacing serial processing by parallel processing [2]. However, this work fails to achieve a real fusion of bottom-up and top-down processing supported by simultaneous information exchange and parallel processing, as it makes use of external procedures (such as for deciding the priority of layers or looping algorithms).

CFS can realize the parallel processing to support the fusion of bottom-up and top-down processing in terms of combining the semantic information processing in upper layer and local processing in lower layer. For example, in image recognition, the upper layer describes the knowledge on a context while the lower layer describes primitive concepts. The concepts in the upper layer are explained by the primitives in the lower layer. The characteristic values activate the primitives in the lower layer. This results in the activation of the concept in the upper layer. At that time the context described in the upper layer depresses the meaningless patterns of distribution of activation and promotes the meaningful patterns of activations in lower layer. Thus the primitives activated are those affected by the characteristic values and also satisfying the context. This **context sensitive processing** provides us with an accurate result. It uses the context to eliminate vagueness which may come from noisy and vague data and which could otherwise cause misunderstandings.

**Example 4.1. Recognition of "THE CAT"**
We recognize the words "THE CAT" in Fig. 4.1. Actually the characters in the middle of THE and CAT have exactly the same shape, and the shape can be recognized as either A or H. Therefore if the recognition of the characters is carried out before the recognition of words, it cannot be decided what the character is: A or H. Our actual response recognizing THE CAT indicates the simultaneous processing of character recognition and word recognition (context). CFS can realize this recognition supported by the fusion of bottom-up recognition process and top-down context sensitive processing as in Fig.4.2.

Fig.4.1 THE CAT

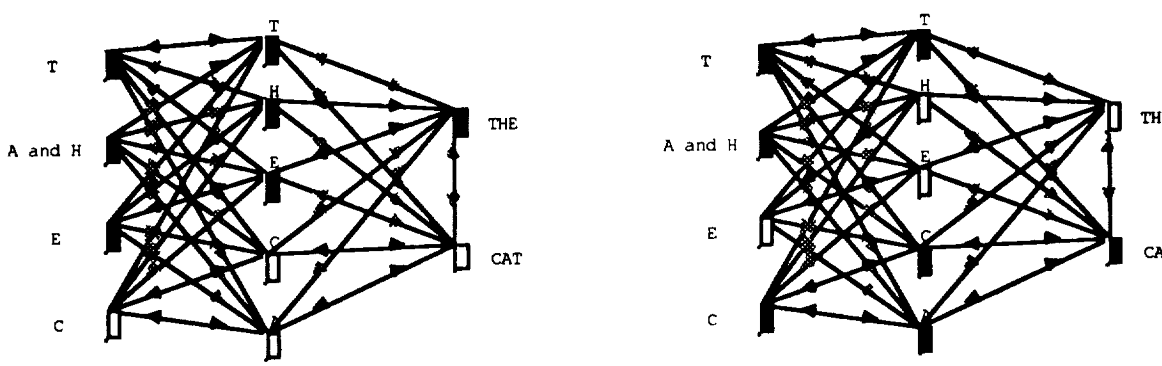


Fig.4.2 The recognition of THE CAT using CFS

The CFS in Fig.4.2 consists of the nodes indicating each character in the lowest layer, alphabets as results of character recognition in the middle layer, and correct words as a context in the upper layer. The lower half of CFS indicates how each character looks like and the upper half indicates the alphabets constructing word. Although the character T, E and C are recognized without vagueness and are connected to corresponding places in the alphabets in the middle layer, the characters of interest which have the shape between A and H are connected to both alphabets to indicate the possibility to be recognized as A or H.

The activation of T, E and the ambiguous character in the lowest layer carry out the recognition. As a result of the propagation of activations, T, H and E are activated in the middle layer and node "THE" is activated in upper layer. The simultaneous recognition indicates that the character in the middle of the word is H and the word is "THE". It should be noticed that context sensitive recognition supported by the upper layer and bottom-up recognition from the lower layer are processed simultaneously.

**Example 4.2. Recognition of facial expressions**

A facial expression is a vague concept: it is difficult of explicitly describing a facial expression: any descriptions have vague boundaries. In this example, the recognition of facial expression is discussed using multi-layered reasoning by means of CFS. The CFS for facial expressions consists of three layers: the upper layer contains facial expressions, the middle layer contains characteristics of the components of a face and the lower layer contains attributive characteristic values. The facial expressions are described in terms of the following characteristics:the condition of both eyes (UP:upward, HZ:horizontal, DW:downward), and of the mouth (UP, HZ, DW). The above characteristics are described by the following characteristic values: the angle of the edge of both eyes (RA, LA) and the angle of mouth (M) in Fig.4.3. Fig 4.4 shows the object face. The recognition of facial expressions is carried out by activating the node in the lowest layer describing characteristic values.

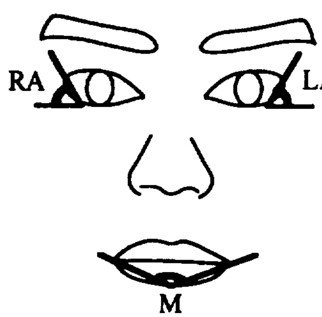


Fig. 4.3 Face characteristic value

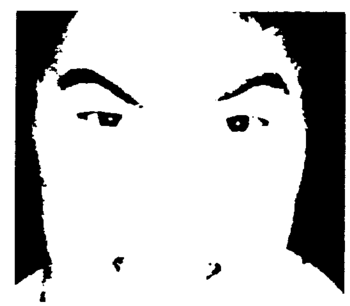

Fig. 4.4 Object image

We can say that humans recognize objects using generous (global) characteristics instead of detecting precise numerical characteristic values. Also, the context constructed by several patterns of facial expressions improves the efficiency and accuracy of recognition. In this section we illustrate the context sensitive image processing by describing general patterns of facial expressions in the middle and upper layers. Fig.4.5 shows the constructed CFS to recognize facial expressions. The general patterns of facial expressions are described by promoting links connecting the characteristics to represent the facial expressions in the middle layer. These patterns are connected to the node in the upper layer standing for facial expressions. The patterns in the middle layer are connected by depressing links. We investigated the recognition using vague characteristic values, which are described by fuzzy sets, to simulate the recognition process by humans without using accurate characteristic values. The object face is recognized as "Angry" and the result is in agreement with our recognition.

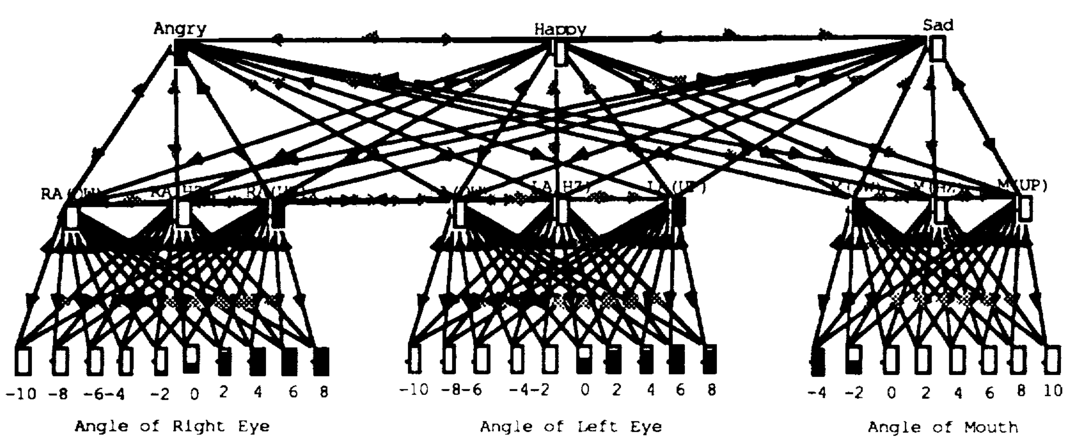


Fig. 4.5 Recognition of facial expressions by means of multi-layered reasoning

In contrast, the recognition using simple logical notation was "Happy" as shown in the following example: facial expression are determined by:

Angry = (Angle of right eye is big) and (Angle of left eye is big) and (Angle of mouth is big)
Happy = (Angle of right eye is small) and (Angle of left eye is small) and (Angle of mouth is small)
Sad = (Angle of right eye is medium) and (Angle of left eye is medium) and (Angle of mouth is big)

Each truth value is calculated as:

Tv(Angry)= min(1.00, 1.00, 0.62) = 0.62
Tv(Happy)= min(0.73, 0.82, 1.00) = 0.73
Tv(Sad) = min(0.92, 0.82, 0.62) = 0.62

Taking the facial expression which has maximum truth value produces the result "Happy".

We also investigated the face recognition of 28 people as shown below and the results show the advantage of context sensitive recognition using CFS.

| | |
|---|---|
| CFS: | 14.3 % fail |
| logic based: | 21.4 % fail |

The results show the advantage of context sensitive recognition which is supported by the fusion of bottom-up and top-down processing, in particular, when the recognition starts with error containing vague characteristic values. It also implies the possibility of CFS for image understanding to eliminate the need for precise image processing

## 5. Conclusion

Fuzzy set theory can be viewed as a stronger and more practical notation than purely symbolic information processing paradigms, connecting the logic based world and the real world. The duality abstract/concrete of the real world is reflected in the intelligent/lack of intelligence duality at the intellectual level. To cope with this duality a knowledge representation paradigm must be able to hierarchically represent both aspects.

Previously we proposed Conceptual Fuzzy sets (CFS) based on the meaning representation of a concept: the meaning of a concept is represented by the distribution of activations of labels naming concepts. In particular, a multi-layer structured CFS represents the meaning of a concept in various expressions in each layer.

In this paper, we proposed Multi-layered Reasoning in CFS. Since the propagation of activations corresponds to reasoning, multi-layer structured CFS can realize multi-layered reasoning which has following features:

1. capable of simultaneous symbolic and quantitative processing (semantic guide line)
2. capable of simultaneous top-down and bottom-up processing (context sensitive processing)

We also showed its effectiveness through illustrative examples.

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