FAULT DETECTION AND ISOLATION

Greg Bernath
Ohio University
Athens, Ohio

SUMMARY

Erroneous measurements in multisensor navigation systems must be detected and isolated. A recursive estimator can find fast growing errors; a least squares batch estimator can find slow growing errors. This process is called fault detection. A protection radius can be calculated as a function of time for a given location. This protection radius can be used to guarantee the integrity of the navigation data. Fault isolation can be accomplished using either a snapshot method or by examining the history of the fault detection statistics.

BACKGROUND

The objective of fault detection and isolation is to use inconsistencies in redundant sensor measurement data to detect and isolate sensor malfunctions. One criterion for determining whether a system can be used as a sole means of navigation is the percentage of time that the position error is greater than a given maximum. If a given single measurement is in error, it will cause the navigation solution to be in error, possibly outside the allowable error threshold. Outside sources may not be able to broadcast in a timely manner that the signal is in error. For instance, if a single GPS satellite starts to broadcast erroneous signals, it could be from 15 minutes to several hours before this is broadcast in the satellite health data. Therefore, it is imperative for Fault Detection and Isolation (FDI) algorithms to be able to detect and isolate instrument errors using only data from the instruments themselves.

Previous work in FDI has mainly centered around inertial navigation systems (refs. 2 through 4). However, FDI can be implemented in any multisensor navigation system with redundant measurements. Current work is focusing on satellite navigation using the Global Positioning System (GPS), along with hybrid systems such as GPS integrated with the Long Range Navigation System (Loran-C) or GPS integrated with an Inertial Reference System (IRS) (ref. 6). FDI used specifically with GPS is also known as Receiver Autonomous Integrity Monitoring (RAIM) (ref. 7).

Kalman filters are becoming standard as part of the navigation solution in most GPS receivers. The Kalman filter can look at the difference between a predicted state

estimate and the actual measured state, and declare a fault if this difference is too large. This works well for detecting step errors or fast growing ramp errors. However, this will not detect a slow growing ramp error, such as might be caused by a GPS satellite clock drift. To detect slow growing errors, the Kalman filter algorithm must be run in parallel with a least squares estimator algorithm. The least- squares algorithm requires at least one redundant measurement for fault detection, and at least two redundant measurements for isolation.

PARITY SPACE AND ESTIMATION SPACE

Estimation space contains the actual horizontal measurement error and the alarm threshold for a given positioning error. However, actual positions and actual errors cannot be known given that the only measurement data is coming from imperfect sensors. Therefore, the work of detecting and isolating errors is done in parity space. Parity space is a mathematical tool where measurement noise and biases are used to create a parity vector. The parity vector determines a detection statistic, d, which is compared to a detection threshold, T_D, in order to determine whether an alarm condition exists. The parity vector is also used as a tool for fault isolation.

Errors and biases in parity space and estimation space are related, but it is not a one to one correspondence. The exact correspondence will be determined by measurement geometries. For instance, with a good geometry, a large measurement error (parity space) will result in only a small position error (estimation space). The reverse can also be true. Figure 1 illustrates two different slow growing ramp errors plotted in parity space versus estimation space. In case I, the detection threshold is crossed before the alarm threshold is reached, yielding a false alarm. As the error continues to grow, the alarm threshold is crossed, turning it into a correct fault detection. In case II, the alarm threshold is crossed before the detection threshold is reached, resulting in a missed detection. As the error continues to grow, the detection threshold is crossed, turning it into a correct fault detection. An ideal algorithm would minimize both the number of false alarms and missed detections.

LEAST SQUARES ESTIMATOR ALGORITHM

In a least-squares approach to fault detection, the relationship between the measurements and the user state (position) is given by:

$$\mathbf{y} = \mathbf{H}\boldsymbol{\beta} \tag{1}$$

where: $\mathbf{y}$ = measurement vector (n-by-1)
          $\mathbf{H}$ = data matrix (n-by-m)
          $\boldsymbol{\beta}$ = user state vector (m-by-1)

$\mathbf{y}$ is a vector containing n measurements, one from each instrument. In the case

of using only GPS satellites, it would consist of the pseudoranges. $\underline{\beta}$ is the m-element user state vector, consisting of the user position coordinates and other navigation state elements such as clock offset with respect to GPS time. H is an n-by-m matrix which relates the measurements to the user states.

There are three possible cases:

    1) n < m : Underdetermined system
    2) n = m : Exactly determined system
    3) n > m : Overdetermined system

In the underdetermined case, a navigation solution is not possible. In the exactly-determined case, a navigation solution is possible, but fault detection is not.

Algorithms for managing the redundant measurements in an overdetermined system form the basis of fault detection. A parity equation can be derived from equation (1), starting with a mathematical manipulation called the QR factorization on the data matrix H (ref. 4):

$$H = QR \tag{2}$$

H is factored into an n-by-n orthogonal matrix Q ($Q^T Q = I$) and an n-by-m upper triangular matrix R. R contains (n-m) rows of zeros along the bottom, due to the n-m redundant measurements in H. Substituting QR for H in equation (1) gives:

$$\underline{y} = QR\underline{\beta}$$

$$Q^T \underline{y} = Q^T QR\underline{\beta} \tag{3}$$

$$Q^T \underline{y} = R\underline{\beta}$$

Next partition R into an m-by-m upper triangular matrix U and (n-m) rows of zeros, denoted by 0. Similarly, partition Q^T into Q_1 (m-by-n) and Q_2 ((n-m)-by-n rows).

$$\left(\begin{array}{c} Q_1 \\ \text{---} \\ Q_2 \end{array} \right) \left(\begin{array}{c} y_1 \\ \vdots \\ y_n \end{array} \right) = \left(\begin{array}{c} U \\ \text{---} \\ 0 \end{array} \right) \left(\begin{array}{c} \beta_1 \\ \vdots \\ \beta_m \end{array} \right) \tag{4}$$

The least squares navigation state solution is:

$$\underline{\beta} = U^{-1} Q_1 \underline{y} \tag{5}$$

The parity equation is:

$$Q_2 \underline{y} = 0 \tag{6}$$

The measurement vector $\underline{y}$ contains noise ($\underline{e}$) and measurement biases ($\underline{b}$). If $\underline{y}$ is replaced by ($\underline{y}$ - $\underline{e}$ - $\underline{b}$), the 0 in equation (6) can be replaced by the parity vector $\underline{p}$.

$$\underline{p} = Q_2 \underline{y} - Q_2 \underline{e} - Q_2 \underline{b}$$
$$\underline{p} = -Q_2 \underline{e} - Q_2 \underline{b} \tag{7}$$

Thus, a parity vector will be determined by the noise and bias errors. From the parity vector, it can be determined which instrument is in error and whether or not to raise an alarm.

PARITY SPACE AND DETECTION PROBABILITIES

Consider a situation with one redundant measurement. In this case, the parity vector will be reduced to a scalar, and the detection statistic reduces to the absolute value of the scalar. In the case where no measurement bias exists, figure 2 shows the distribution of the parity scalar. Since there is no bias error, position error is definitely under the alarm threshold and the system is either in normal operation or a false alarm exists. The probability of a false alarm (P_{FA}) is obtained by integrating the areas outside of T_D. For noise having a normal distribution (generally a good assumption), this integral is a standard Gaussian function.

Figure 3 illustrates the case where a large measurement bias exists, making the position error larger than the alarm threshold. In this case the system is either correctly detecting a fault or a missed detection is present. The probability of a missed detection (P_{MD}) is the integral of the area inside T_D. Again, if Gaussian noise is assumed, this is a standard Gaussian function.

PROTECTION RADIUS

The above example uses a detection threshold, measurement noise, and measurement bias error as parameters to find P_{FA} and P_{MD}. Accuracy requirements are stated in a form like "the probability of exceeding 100 meters accuracy is no greater than 0.05". In order to compare FDI results with such specifications, it helps to rearrange the procedure. This means using the above parameters to determine the protection radius, which is the largest horizontal position error that is guaranteed to be detected with the required probabilities of alarm and missed detection. If all parameters are kept constant, the protection radius will vary only as a function of satellite geometry.

One way of finding the protection radius is to use all satellites in view and go through the full parity space/parity vector derivation. Another possibility is to take only

the best 5 satellites (geometry wise) in view and use only those for the calculation. The second method has the advantage that, since there is only one redundant measurement, the parity vector is reduced to a scalar and the algorithm is simpler. Also, a receiver would only be required to track five satellites.

A comparison of the two methods is shown in figures 4 and 5. These are plots of the protection radius as satellite positions change over one day at a single location. Many different locations were tested (with simulated satellite data) worldwide; figure 4 shows the best case result, figure 5 the worst case. As expected, the all-in-view method always gives a better result than the best-of-five.

FAULT ISOLATION

In the case with one redundant measurement, faults can be detected but not isolated. Isolation requires two or more redundant measurements. Consider the case of two redundant measurements. The parity vector now has two elements, making parity space two dimensional. Each measurement in parity space can be represented by an axis extended radially outward from the origin, in both (positive and negative) directions. The exact orientation of each axis depends on satellite geometry; thus each axis will rotate over time. The basic premise of fault isolation in parity space is that a bias error in measurement i will lie along measurement axis i in parity space. A growing bias error will move outward along a line parallel to the measurement axis.

Figure 6 illustrates a case of flight test data consisting of 4 GPS and 3 Loran-C measurements. There are 5 unknowns, which are the 3 user coordinates, the user clock offset from GPS time, and the user clock offset from Loran-C time. This leaves 2 redundant measurements, so parity space is 2 dimensional and it contains 7 axes, 1 for each measurement. The irregular plots are the traces over time of the end of the parity vector, as a slow growing ramp error is being artificially injected into each of the measurements. Each growing error runs parallel to the parity axis for that measurement. Measurement 2 seems to violate this rule, but what could not be shown on one graph is that axis 2 rotated significantly (as compared to the others) over the course of the data run; thus, the plot for instrument 2 ends up curved as it attempts to stay parallel with the axis as it rotates. Also note that the actual unaltered measurements contained a bias on instrument 4, as shown by the tracks originating a small distance out on axis 4.

The simplest method for isolating the faulty instrument is the "snapshot" batch estimator. This method, upon detecting a fault, looks for the parity axis that is closest to the parity vector, and assumes that the measurement belonging to that axis is faulty. The history method, on the other hand, determines which axis is parallel to the track traced out by the parity vector over time. The history method is more accurate, but requires more computation, has a built in delay and must restart if any measurements are lost.

CONCLUSIONS

A fault detection algorithm for a multisensor navigation system has been presented. A protection radius has been calculated using the all-in-view and the best-of-five methods, with the all-in-view proving significantly superior. Fault isolation methods have also been shown, using both snapshot and continuous tracing methods. Efforts continue on exploring the tradeoffs between algorithms, with the goal of determining whether minimum navigation system requirements can be satisfied using fault detection and isolation.

REFERENCES

1. Kline, P. A. and Van Graas, F.: Fault Detection and Isolation for Multisensor Navigation Systems. NASA Conference Publication 3131, Joint University Program for Air Transportation Research 1990-1991.

2. Daly, K. C., Gai, E. G., and Harisson, J. V.: Generalized Likelihood Test for FDI in Redundant Sensor Configurations. Journal of Guidance and Control, Vol. 2, Jan.-Feb. 1979.

3. Gai, E. G., Adams, M. B., and Walker, B. K.: Determination of Failure Thresholds in Hybrid Navigation. IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-12, Nov. 1976.

4. Morrell, F. R., Bailey, M. L., and Motyka, P. R.: Flight Demonstration of Redundancy Management Algorithms for a Skewed Array of Inertial Sensors. Proceedings of the AIAA/AHS/ASEE Aircraft Design and Operations Meeting, AIAA Paper 88-4434, Atlanta, GA, September 7-9, 1988.

5. Golub, G. H., and Van Loan, C. F.: Matrix Computations, Second Edition. The Johns Hopkins University Press, Baltimore, MD, 1989.

6. Brenner, M.: Implementation of a RAIM Monitor in a GPS Receiver and an Integrated GPS/IRS. Proceedings of the Third International Satellite Division Meeting of the Institute of Navigation, September 1990.

7. Van Graas, F. and Farrell, J. L.: Receiver Autonomous Integrity Monitoring (RAIM): Techniques, Performance and Potential. Proceedings of the 47th Annual Meeting of the Institute of Navigation, Williamsburg, VA, June 10-12, 1991.

Estimation Space

Radial Position Error

Missed
Detection

Correct
Fault
Detection

Case II

Alarm
Threshold

Case I

Normal
Operation

False
Alarm

Detection
Threshold

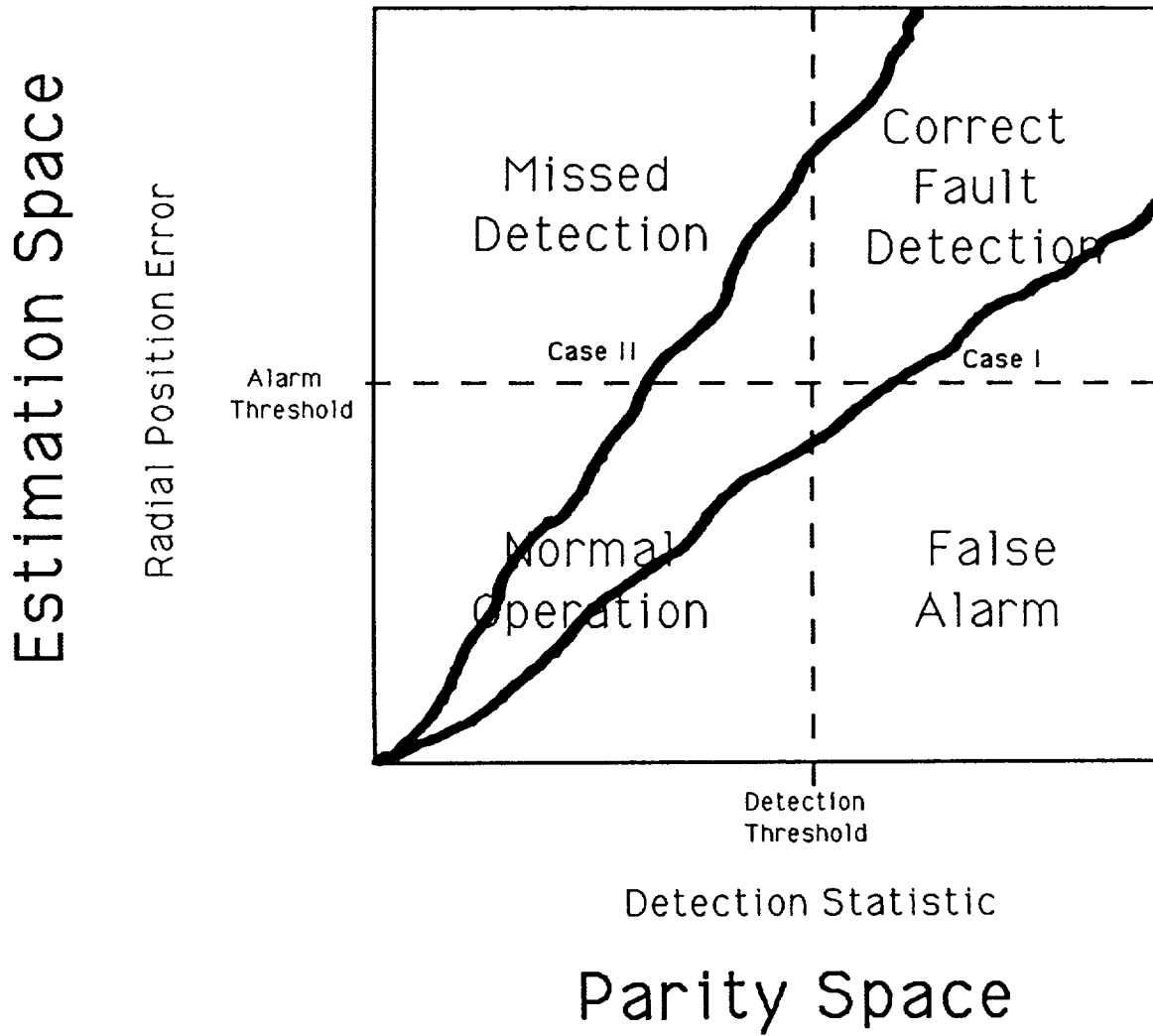Detection Statistic

Parity Space

Figure 1.   Two slowly growing measurement errors plotted in parity space
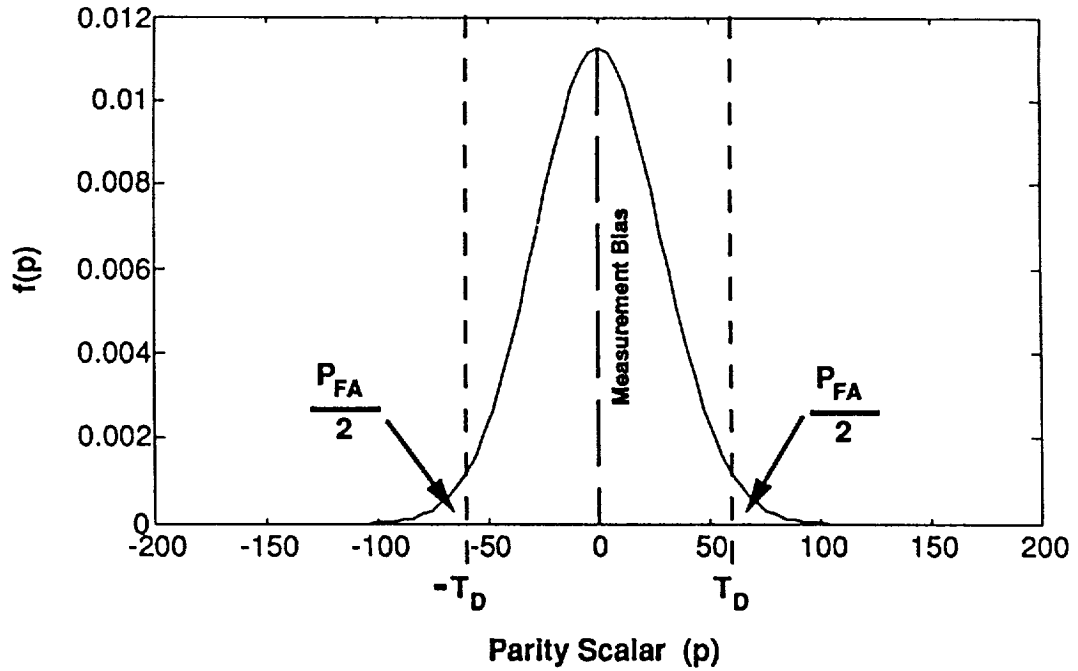versus estimation space.

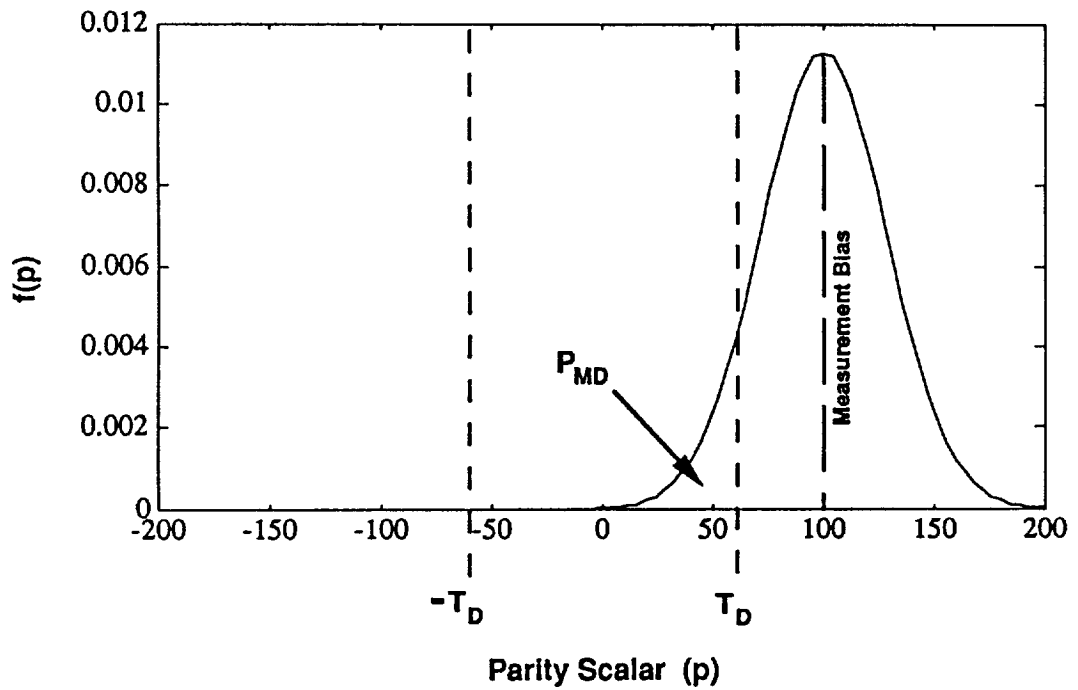Figure 2.   Probability density function for the parity scalar in the absence of a measurement bias error.

Figure 3.   Probability density function for the parity scalar in the presence of a measurement bias error.
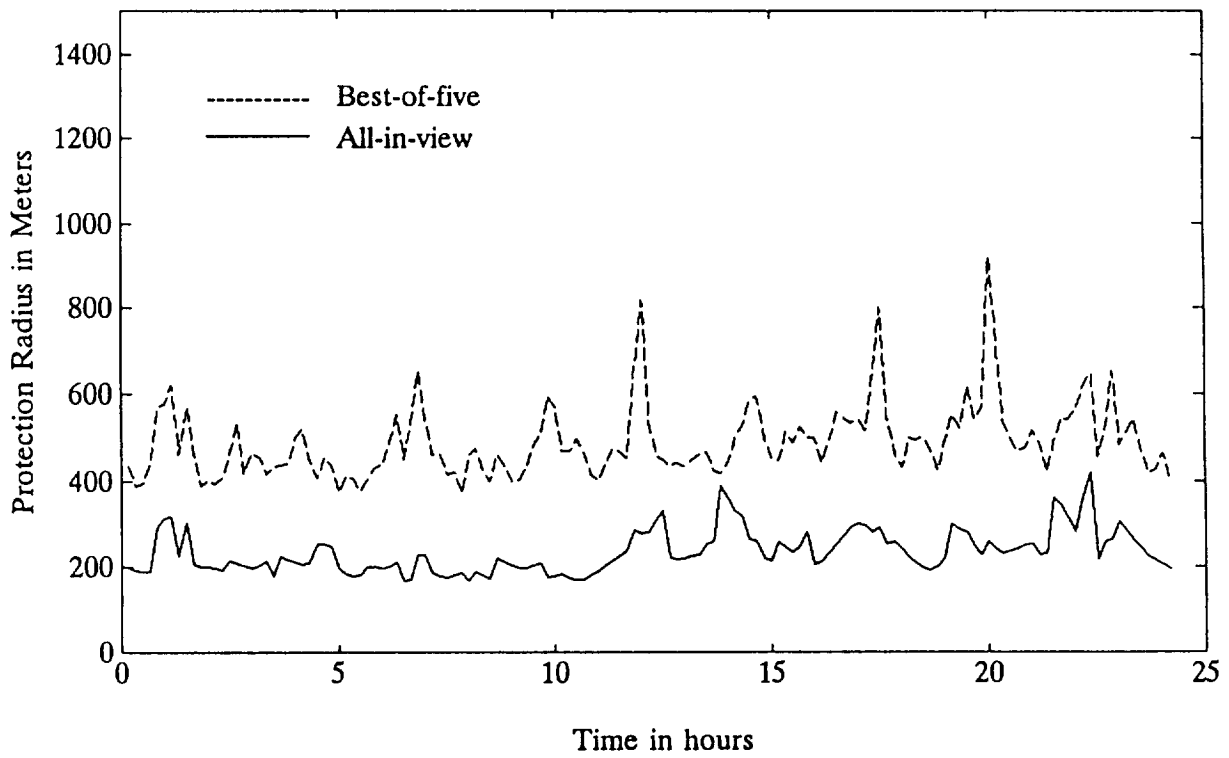
Figure 4.   Best case protection radius (location: S3 W80).
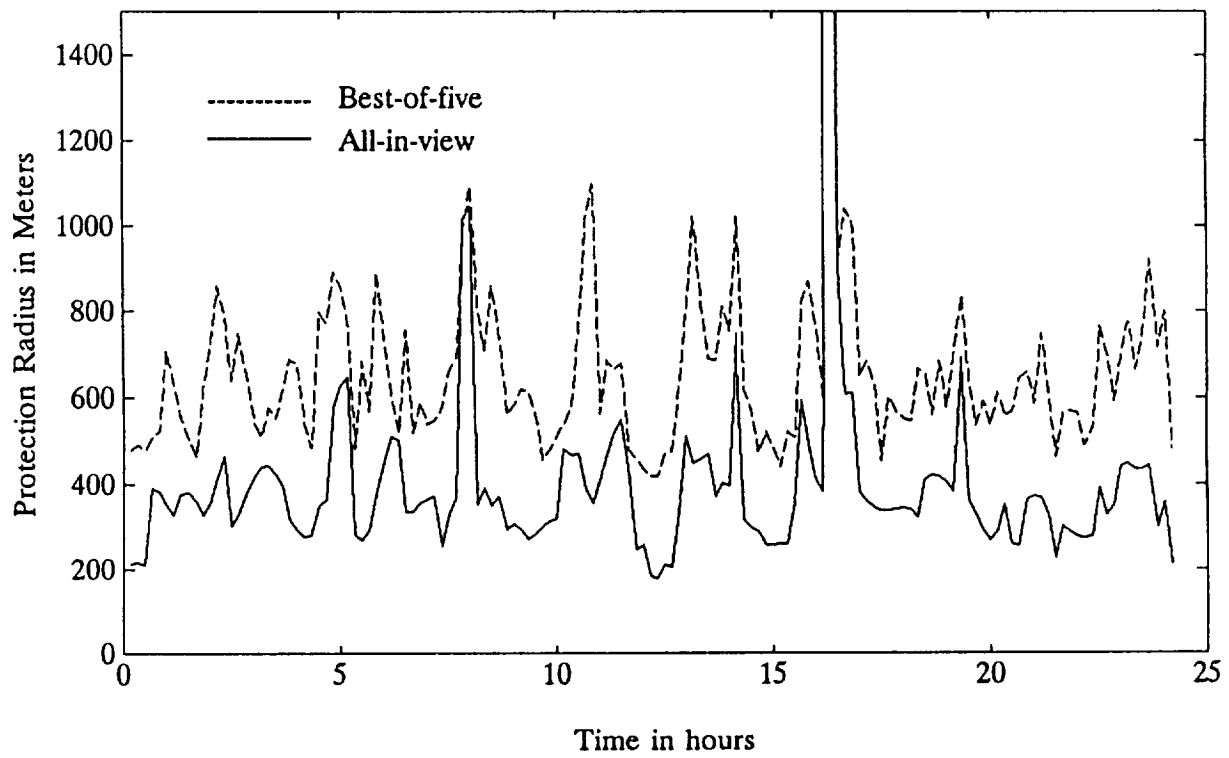

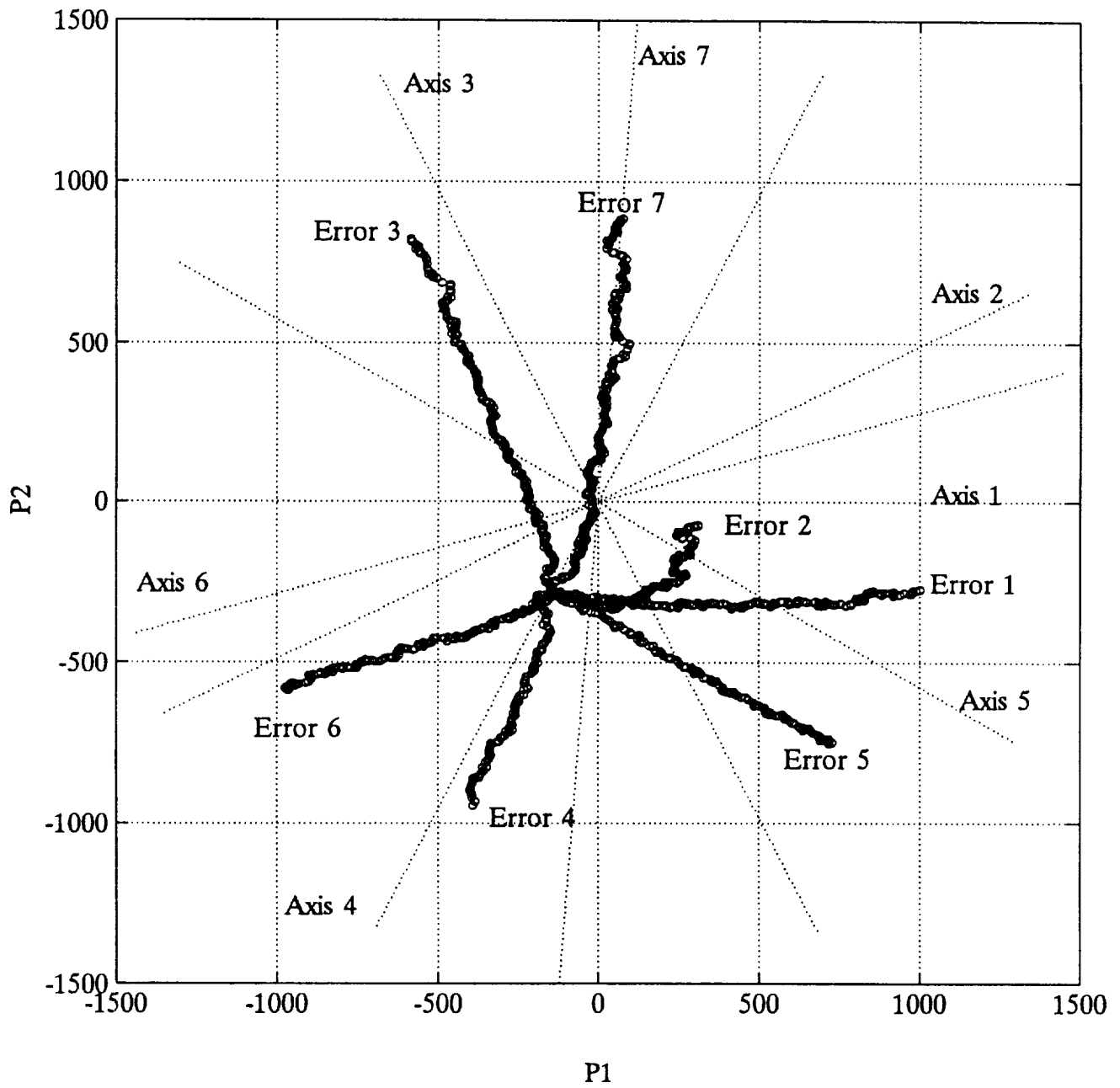Figure 5.   Worst case protection radius (location: N36 E140).

Figure 6.   Parity vector trajectories as a function of time.