DYNAMIC RESPONSE AND CONTROL OF A

JET-TRANSPORT AIRCRAFT

ENCOUNTERING A SINGLE-AXIS VORTEX

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The dynamic responses of a jet-transport aircraft to two types of single-axis wind vortex encounters are studied. Aircraft attitude, flight path angle, and aerodynamic angle excursions are analyzed and dominating dynamic forcing effects are identified for each encounter.

A simple departure-preventing LQR controller is designed to demonstrate the benefits of using automatic control to reduce the wind vortex hazard. A Proportional-Integral-Filter controller structure successfully regulates the critical parameters, roll angle, ϕ, and sideslip angle, β, for the two different vortex encounters considered in this study.

WIND ROTOR FORMATION

A mountain wave wind vortex is formed by strong winds that flow perpendicular to a mountain range. A low pressure system develops on the leeward side of the mountain that tends to force the air into a rotating air-mass. Once this horizontal cyclone of rotating air forms, it tends to be a stable air-mass that moves with axis parallel to the ground in the direction of the prevailing wind. Such wind rotors have been known to travel more that 20 miles from the forming mountains. Once a rotor moves away from the mountain, another one tends to form in its wake. Thus, several rotors formed from the same mountain range may be found moving in lines in the direction of the prevailing wind.

MOUNTAIN
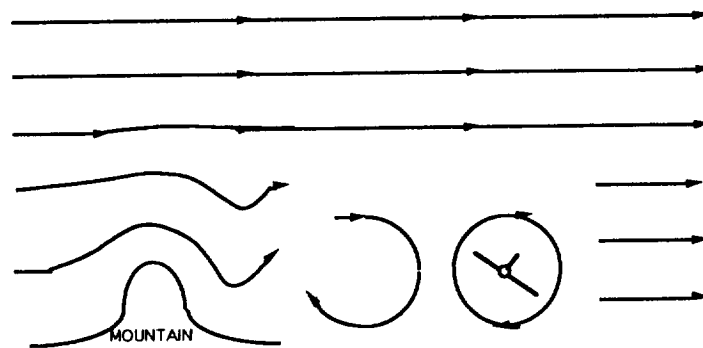
WIND ROTOR MODEL

A single-axis-vortex wind velocity profile was implemented to simulate the velocity field of a wind rotor. The profile was experimentally verified using full-scale Boeing 747 shed-wake-vortex data. This two-dimensional model is defined by the specification of two parameters: the core radius, r_c, and the core velocity, V_c. The core radius is the distance from the core center, where tangential velocity, V_t, is zero, to the location of maximum tangential velocity. The core velocity is equal to the maximum tangential velocity at the core radius.

The flow behaves as an ideal vortex outside the core ($r > r_c$), with tangential velocity proportional to $1/r^2$, where r is the distance from the rotor center . Inside the core ($r < r_c$), viscous damping effects dominate, the tangential velocity departs from the ideal profile, and it increases in proportion to the distance, r. A piecewise-linear model described by three linear equations provides a good approximation to the experimentally verified model.

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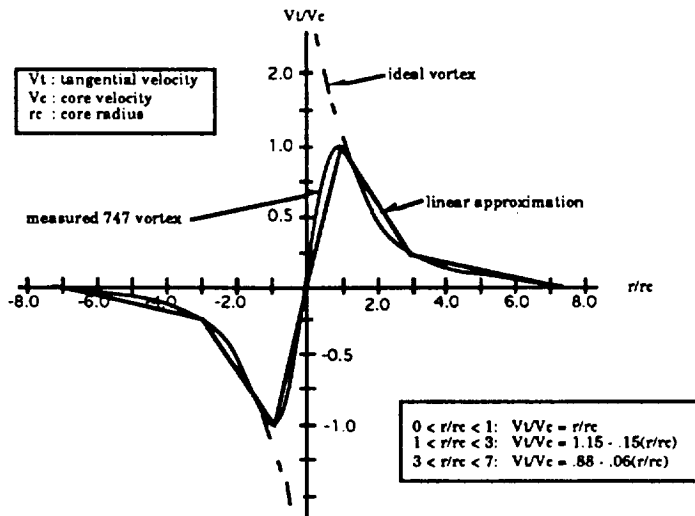
WIND EFFECTS ON AIRCRAFT

Wind terms have been included explicitly in the translational kinematic and dynamic equations of motion. The time derivative of the wind components acts as a forcing term in the aircraft dynamics, inducing both linear and angular rate derivatives. The wind terms do not appear explicitly in the rotational equations.

In addition to the wind time derivatives, there are specific rotational moments due to the spatial velocity gradient acting along the various aerodynamic surfaces of the aircraft. These spatial variations are dependent on the specific aircraft configuration and enter the rotational dynamics through the aerodynamic force and moment calculations.

In the presence of a spatial shear gradient, the roll moment for example, includes not only the damping from the roll-rate-induced velocity field, the control, yaw rate, and sideslip effects, but also the contribution from the velocity field of the spatial shear. Since the shear is a local wind field similar to that induced by the aircraft in roll, the shear gradients become effective roll rates and enter the roll moment coefficient equation through multiplication by elements of the roll-rate derivatives. However, the spatial gradient acts as a roll rate only on the aerodynamic surfaces perpendicular to the gradient flow field. This simple shear-gradient analysis can be extended to each force and moment coefficient calculation that is necessary in the dynamic simulation.

- Equations of motion

  Translational kinematics $\quad \dot{\vec{r}}_E = L_{EB}\dot{\vec{v}}_B + \vec{w}_E$

  Translational dynamics $\quad \dot{\vec{v}}_B = \dfrac{F_B}{m} - H_I^B g - \tilde{w}_B v_B - \dot{\vec{w}}_B$

- Force and moment coefficients
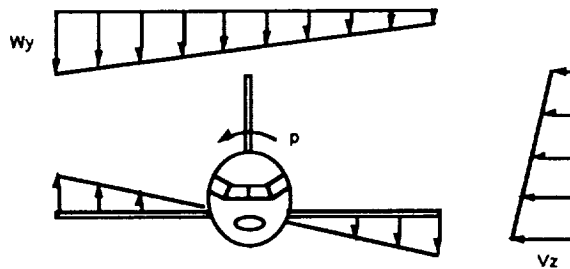

C_l = roll moment coefficient = $(C_l)_{ROLL}$ + $(C_l)_{YAW}$

$(C_l)_{ROLL}$ = $(C_{l_p})p$ + $(C_{l_{p_{wing}}}$ + $C_{l_{p_{horizontal tail}}})w_y$ + $(C_{l_{p_{vertical tail}}})v_z$

AIRCRAFT-VORTEX ENCOUNTER GEOMETRY

Dynamic simulations of a twin-jet transport encountering a single-axis horizontal vortex during landing approach are presented. Two different aircraft-vortex geometrical encounters are simulated: a vortex with axis parallel to the extended runway centerline, referred to as a finite-length rotor, and a vortex with axis at a 60° angle to the extended runway centerline, referred to as an infinite-length rotor. Although the dominating dynamic effects are different in each wind rotor encounter, both simulations produce large bank angles and pitch attitudes that result in drastic deviations from the approach path.

a. finite-length rotor

b. infinite-length rotor
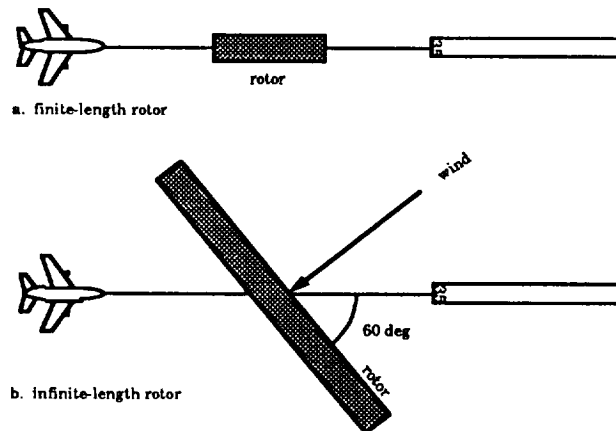
VORTEX ENCOUNTER SIMULATIONS

Results show that both cases can produce severe changes to bank and pitch attitude, velocity, and sideslip angle during an approach. The finite-length wind rotor simulation produces strong shearing moments that dominate the aircraft's response. The 60° encounter angle, or infinite-length simulation has a dynamic response dominated by the time-rate-of-change of the strong vortical winds, with shearing moments having little influence. The effects of these winds can be seen clearly in the sideslip angle and angle of attack plots; much larger aerodynamic angles are induced in the 60° rotor angle simulation than in the co-axial rotor simulation. Therefore, to successfully reduce the initial effects of a wind vortex for any general vortex orientation, shear-induced roll effects and wind-induced sideslip angle effects must be attenuated.

Another significant difference between the two encounters is that for the co-axial case the dynamic response changes relatively little with various initial conditions; however, in the non zero rotor angle case, even small changes in initial conditions result in significant changes in the resulting attitude and flight path angles.
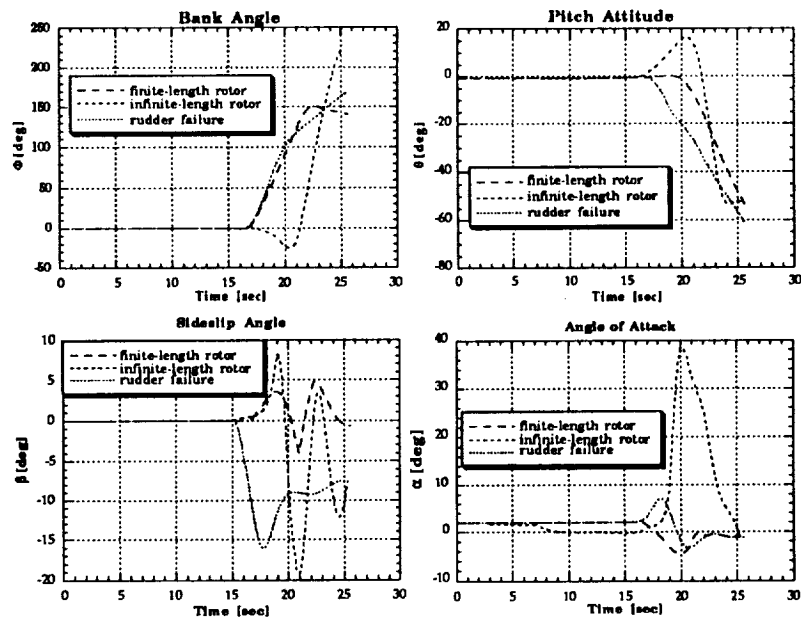
LINEAR QUADRATIC REGULATOR

A simple LQR Proportional-Integral-Filter (PIF) controller is designed based on a linear state-space model that considers only four lateral dynamic states and two controls. LQR control offers a systematic approach to developing constant-gain feedback control laws for multi-input multi-output systems by combining state-space, time-domain, and optimal control concepts. The aircraft and controller is then flown through the two basic wind rotors considered: the co-axial or finite-length encounter and the 60° rotor angle or infinite-length encounter. In both cases the controller's objective is to reduce roll angle and sideslip angle excursions.

$$\Delta \dot{x}(t) = \frac{\delta F}{\delta x} \Delta x(t) + \frac{\delta F}{\delta u} \Delta u(t) \qquad \Delta x = [\Delta v \; \Delta p \; \Delta r \; \Delta \phi]^T$$

$$\Delta u = [\Delta \delta a \; \Delta \delta r]^T$$

objective: minimize cost, J

$$J = \int_0^\infty \left(\Delta \tilde{x}^T(t) Q \Delta \tilde{x}(t) + \Delta \tilde{u}(t)^T R \Delta \tilde{u}(t) \right) dt$$

where: Q = state weighting matrix
         R = control weighting matrix

control law of form:

$$\Delta u(t) = -C \Delta x(t)$$

LQR CONTROL SIMULATIONS

Results for the two aircraft-vortex simulations are presented. During the finite-length encounter the roll angle remains below 8° and the sideslip angle below 3° ; during the infinite-length encounter angle case the roll angle remains below 10° and sideslip angle below 6°. The performance in both cases is superior to the published maximum allowable roll angle, 20°, for a jet-transport on final approach to landing. These simulations demonstrate the value of using automatic control to reduce vortex-induced excursions.
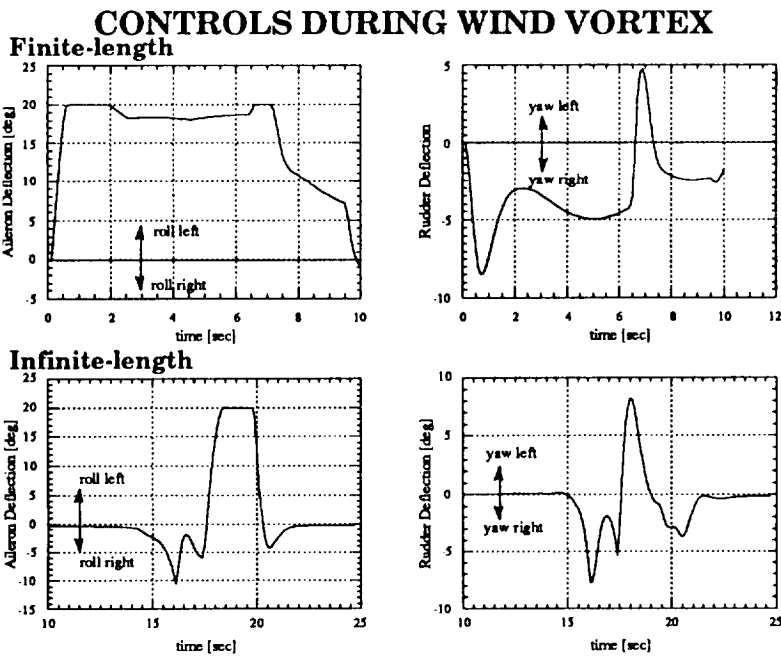
Finite-length

Infinite-length
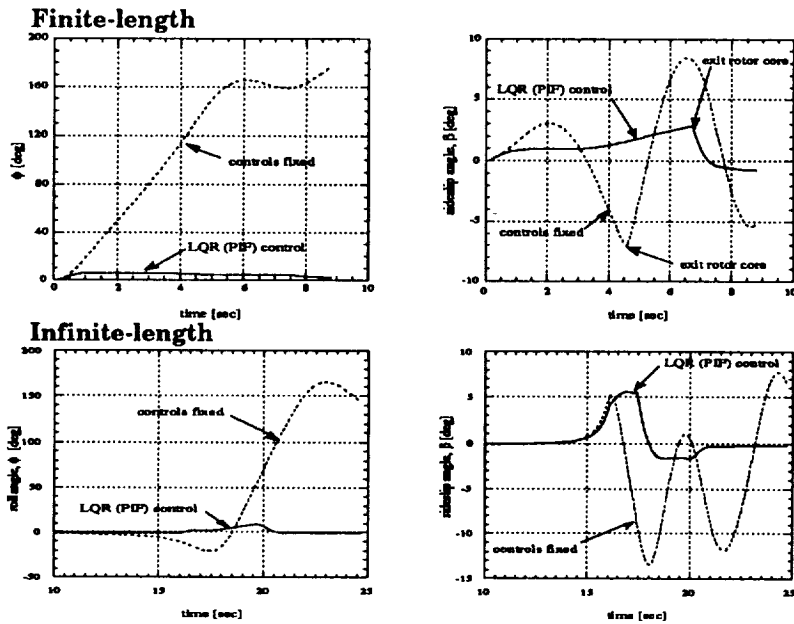
AILERON AND RUDDER CONTROL IN A VORTEX

The control histories for the vortex encounter simulations show that for this controller design, aileron is used almost exclusively to control roll excursions while rudder independently controls sideslip excursions for both the finite-length and infinite-length cases. The control plots also show that aileron deflections are much greater than rudder deflections, even though the same relative weighting was used on both controls in the design. This is expected since large rudder deflections may excite the lightly damped dutch-roll mode and cause severe sideslip angle oscillations.

Increasing the weighting on roll angle forces the rudder to aid in roll excursion reduction at the expense of sideslip angle control. This type of behavior may be preferred in some instances, particularly those in which an unusually large rotor is present. Under these circumstances, rolling to inverted flight may be unavoidable without immediate application of both full aileron and full rudder against the roll.

CONTROLS DURING WIND VORTEX

SUMMARY

The controls-fixed simulations show that the dominating dynamic effects in the finite-length and infinite-length simulations are very different, although both encounter types have the ability to flip a jet-transport configured for final approach. In order to reduce the vortex hazard for all vortex encounters it is necessary to control both roll angle, ϕ, and sideslip angle, β.

The LQR formulation provides insights into optimal control inputs during a wind vortex encounter, given a defined control objective. The simple LQR Proportional-Integral-Filter feedback control system designed in this section demonstrates that proper rudder control of sideslip combined with aileron roll control can help reduce the wind vortex hazard, even for a wide variety of vortex alignments with respect to the flight path. Results from the aircraft-rotor simulations show that rudder is used to a lesser degree than aileron. There is evidence that a very strong vortex or possibly a vortex at an orientation angle not considered in this study may require a greater controller emphasis on roll regulation and less emphasis on sideslip attenuation. Thus, proper gain scheduling appears to be a requirement for a vortex-alleviation control system that is effective over a broad range of vortex encounters.

Controls-fixed simulations:
- **both co-axial and 60° rotor angle can flip a jet-transport**
- **shear effects dominate co-axial case**
- **strong vortical winds dominate 60° rotor angle case**
- **critical parameters are ϕ, β**

LQR (PIF) control:
- **four-state model, two controls**
- **reduces vortex hazard for range of encounters**
- **aileron used to greater degree than rudder**
- **gain scheduling may be necessary**