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P.104

NASA Technical Memorandum

NASA TM - 108399

PRELIMINARY ANALYSIS TECHNIQUES FOR RING AND STRINGER STIFFENED CYLINDRICAL SHELLS

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Preliminary Design Office
Program Development

March 1993

(NASA-TM-108399) PRELIMINARY ANALYSIS TECHNIQUES FOR RING AND STRINGER STIFFENED CYLINDRICAL SHELLS (NASA) 104 p

N93-23104

Unclas

G3/39 0153696

National Aeronautics and Space Administration
George C. Marshall Space Flight Center

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	March 1993	Technical Memorandum

4. TITLE AND SUBTITLE
Preliminary Analysis Techniques for Ring and Stringer Stiffened Cylindrical Shells

5. FUNDING NUMBERS

6. AUTHOR(S)
J. Graham

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Washington, DC 20546

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA TM - 108399

11. SUPPLEMENTARY NOTES
Prepared by Structures and Thermal Analysis Branch, Preliminary Design Office, Program Development.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified — Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This report outlines methods of analysis for the buckling of thin-walled circumferentially and longitudinally stiffened cylindrical shells. Methods of analysis for the various failure modes are presented in one cohesive package. Where applicable, more than one method of analysis for a failure mode is presented along with standard practices. The results of this report are primarily intended for use in launch vehicle design in the elastic range. A Microsoft Excel worksheet with accompanying macros has been developed to automate the analysis procedures. These programs are available by request from the author.

14. SUBJECT TERMS
Skin/Stringer, Buckling, Shells

15. NUMBER OF PAGES
108

16. PRICE CODE
NTIS

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	Unlimited

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89)

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LIST OF SYMBOLS

A	Area (in^2)
b	Characteristic Width (in)
C	Coefficient
D	Diameter (in), Coefficient
d	Frame Spacing (in)
E	Modulus of Elasticity (lb/in^2)
E_s	Secant modulus (lb/in^2)
E_t	Tangent Modulus (lb/in^2)
f	Stress (lb/in^2)
F	Stress (lb/in^2)
h	Characteristic Height (in)
I	Area Moment of Inertia (in^4)
k	Buckling Coefficient
L	Length (in)
m	Integer
M	Bending Moment (in-lb)
n	Integer
N_x	Axial Buckling Line Load (lb/in)
N_y	Hoop Direction Buckling Line Load (lb/in)
p	Pressure (lb/in^2)
P	Axial Load (lb)
R	Radius (in)
t	Thickness (in)
w_e	Effective Skin Width (in)
ε	Strain
h	Plasticity Correction Factor
κ	Curvature
l	Variable
m	Poisson's Ratio (elastic)
u	Poisson's Ratio (elastic)
x	Variable
ρ_s	Stringer Section Radius of Gyration
ρ_f	Frame Section Radius of Gyration

s	Normal Stress (lb/in^2)
t	Shear stress (lb/in^2)
( )	Equation Number

<u>Subscripts</u>:

c - compressive

cr - critical

e - effective or equivalent

f - frame

l - land

sk - skin

sm - smeared

st - stringer

x,y,z - coordinate subscripts

TECHNICAL MEMORANDUM

PRELIMINARY ANALYSIS TECHNIQUES FOR RING AND STRINGER STIFFENED CYLINDRICAL SHELLS

I. INTRODUCTION

Over the years many methods of stiffened panel analysis and design have been proposed and substantiated by test. Most methods deal with only one element of the stiffened structure, such as the skin, the stringer, or the ring. Not many references combine all the elements of skin stringer design and analysis into one cohesive process. This is especially true when the skin of a structure is allowed to buckle prior to application of the ultimate load. This report outlines methods of analysis for the major failure modes for the buckling of thin-walled circumferentially and longitudinally stiffened cylindrical shells. The report is intended particularly to address launch vehicle design issues. Loading on the vehicle will consist of pure bending, axial compression, and shear, all in the elastic range. Generally, any advance of the load beyond the buckling limit is considered a structural failure and must be avoided in launch vehicle design. (The skin, however, may be allowed to buckle at limit loads.) A Microsoft Excel worksheet with accompanying macros has been developed to facilitate application of the various analysis methods. These analysis programs are available by request from the author.

The analysis methods presented are organized according to failure mode. All necessary design curves have been curve fit to allow automated analysis in the spreadsheet program. Sections detailing the calculation of stress in the vehicle, as well as calculation of margins of safety, are also included in the paper. The appendices contain hand calculations, additional analysis information, and the analysis programs. This report will focus on the integrally Tee stiffened shell. The reader should note that, unless otherwise stated, all methods presented in this report are for use in the elastic region.

II. ANALYSIS METHODS

Analysis of the stiffened shell begins with recognition of the various failure modes. The failure modes listed below encompass the most significant failure modes of the shell. Analysis techniques for each of the failure modes listed will be presented.

Buckling failure modes can take one or more of the following forms.

(1) Classical bifurcation buckling

(2) General instability

(3) Stringers

(a) Local buckling

(b) Crippling

(c) Column failure

(4) Skin

(a) Compression buckling

(b) Shear buckling

(c) Pressure.

Frames, of course, may experience cap failures, web failures, and other buckling failures. However, the interest in frames at this level of design is to determine an acceptable moment of inertia, or other overall general characteristic of the frame design, which will stabilize the structure against general instability failure. For these reasons, detailed frame design is excluded from this report. Also, the primary focus of this report is application to pressurized shells. Pressurization precludes penetration of the tank by rivets except in extreme cases. Therefore, inter-rivet failure and face sheet wrinkling have been excluded as failure modes.

A. Bifurcation Buckling

The difference between bifurcation buckling (also commonly referred to as "classical" or "classical bifurcation" buckling) and other types (or modes) of buckling failure are often confusing. The point (load value) at which a column fails due to bifurcation buckling represents the intersection of two equilibrium paths in the structure. The failure modes presented in subsequent sections of this report represent collapse, or failure, at a limit point. Figures 1*a* and 1*b* illustrate the difference between bifurcation buckling and failure at a limit point. The variable P represents the applied load, and Δ represents the displacement.

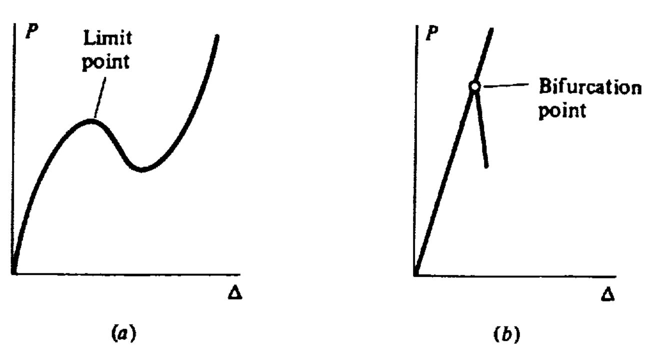


Figure 1. Bifurcation buckling.

Figure 1*a* represents the load-displacement curve for a Bellville spring. The point on the equilibrium path at which load P is a relative maximum is called a limit point. A limit point could be column buckling, skin buckling, stringer crippling, etc. Figure 1*b* represents the load-displacement curve for a thin-walled cylindrical panel under axial compression. In this figure, the "*primary* (or *fundamental)* equilibrium path is intersected by a *secondary* path."[1] The point of intersection is called the bifurcation point.

Failure of a general shell usually occurs through collapse at some limit point rather than through bifurcation. However, bifurcation buckling of the shell must be considered in the design process. It is important that the analyst realize that "the classical (or bifurcation) buckling analysis may give results of little or no value if the shell geometry deteriorates appreciably (Brazier effect) or stresses are redistributed . . . in the subcritical load range."[2] In the examples used in this report, the skin of the cylindrical shell is allowed to buckle at the limit load while the stringers and effective skin are allowed to buckle only at ultimate load. There is considerable redistribution of stress. Therefore, the classical bifurcation solution alone is of little use, but will be calculated as an illustration of the method.

Computation of the linear bifurcation buckling load and application of an empirical knock-down factor provides a conservative method of determining an appropriate allowable load level. The following analysis techniques combine the wide column allowables with the bifurcation buckling allowable reduced by a "knock-down factor" which is a function of $(R/t)_e$.

The bifurcation analysis does yield good results for bending of cylinders with $(R/t)_e$ values large enough so that the Brazier effect is negligible. It has been shown "that the use of wide column load as a design limit for stringer-stiffened cylinders was unduly conservative. It was suggested that a term be added to the wide-column load which corresponds to the curvature effect. This term was obtained as the difference between the classical buckling load and the wide column load multiplied by a reduction factor."[3]

The effect of curvature is introduced by taking the difference between the wide column and classical allowables, multiplying by a "knock-down factor" (φ), and adding this result to the wide column allowable.[3]

$$N_{CR} = N_{WC} + \varphi(N_{CL} - N_{WC}) \ . \tag{1}$$

1. Classical

Presented below are the constitutive relationships for the orthotropic shell.[1] Examples of construction that may be treated as orthotropic include corrugated sheets, fiber reinforced plastic sheets, and plates with closely spaced stiffeners. This method yields adequate results for closely spaced rings. For spacing greater than 30 to 40 in, setting the ring properties equal to zero may yield an adequate solution.[1]

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_{12} \\ M_1 \\ M_2 \\ M_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & C_{14} & C_{15} & 0 \\ C_{21} & C_{22} & 0 & C_{24} & C_{25} & 0 \\ 0 & 0 & C_{33} & 0 & 0 & C_{36} \\ C_{41} & C_{42} & 0 & C_{44} & C_{45} & 0 \\ C_{51} & C_{52} & 0 & C_{54} & C_{55} & 0 \\ 0 & 0 & C_{63} & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \\ \kappa_1 \\ \kappa_2 \\ \kappa_{12} \end{Bmatrix},$$

where $[C_{ij}]$ represents the stiffness matrix for the shell. Definition of the stiffness parameters is given below:[4]

$$C_{11} = C+E*\frac{A_{st}}{b_{st}}, \qquad C_{12} = \mu*C, \qquad C_{14} = E*\frac{A_{st}}{b_{st}}*e_{st},$$

$$C_{21} = C_{12}, \qquad C_{22} = C+E*\frac{A_f}{d_f}, \qquad C_{25} = E*\frac{A_f}{d_f}*(e_f),$$

$$C_{33} = G*t_{sk},$$

$$C_{41} = C_{14}, \qquad C_{44} = D+\frac{E}{b_{st}}[I_{st}+A_{st}*e_{st}^2], \qquad C_{45} = \mu*D,$$

$$C_{52} = C_{25}, \qquad C_{54} = C_{45}, \qquad C_{55} = D*\frac{E}{d_f}[I_f+A_f*e_f^2],$$

$$C_{66} = 2*(1-\mu)*D+G*\left[\frac{J_{st}}{b_{st}}+\frac{J_f}{d_f}\right],$$

where

J = torsional stiffness constant

A = area of stringer or ring

I = area moment of inertia of stringer or ring

e = distance from the skin middle surface to the centroid of the stiffener cross section

b_{sk} = stringer spacing.

The coupling parameters C_{14} and C_{25} are positive for stiffeners outside the skin and negative for stiffeners inside the skin.

$$G = \frac{E}{2*(1+\mu)},$$

$$D = E*\frac{t_{sk}^3}{12*(1-\mu^2)},$$

$$C = E*\frac{t_{sk}}{1-\mu^2}.$$

The buckling coefficients are written in terms of the axial half wavelength number (m), and the full circumferential wave number (n):

$$\lambda(m) = \left[m\frac{\pi}{L}\right]^2 = \lambda_m \quad , \quad \xi(n) = \left[\frac{n}{R}\right]^2 = \xi_n \ .$$

From reference 4, the matrices a_0 and a_1 are defined as,

$$a_0(m,n) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} ,$$

and

$$a_1(m,n) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} ,$$

where,

$$A_{11}(m,n) = C_{11}\lambda_m + C_{33}\xi_n \ ,$$

$$A_{12}(m,n) = [C_{12}+C_{33}]m\left(\frac{\pi}{L}\right)\frac{n}{R} \ ,$$

$$A_{22}(m,n) = C_{33}\lambda_m + C_{22}\xi_n \ ,$$

$$A_{13}(m,n) = \frac{C_{12}}{R}\, m\left(\frac{\pi}{L}\right) + C_{14}\lambda_m m\left(\frac{\pi}{L}\right) + [C_{15}+2C_{36}]m\left(\frac{\pi}{L}\right)\xi_n \ ,$$

$$A_{23}(m,n) = [C_{15}+2C_{36}]\lambda_m\left(\frac{n}{R}\right) + \frac{C_{22}}{R}\left(\frac{n}{R}\right) + C_{25}\xi_n\left(\frac{n}{R}\right) \ ,$$

$$A_{33}(m,n) = C_{44}\lambda_m^2 + [C_{66}+2C_{45}]\lambda_m\xi_n + C_{55}\xi_n^2 + \frac{C_{22}}{R^2} + 2\left(\frac{C_{25}}{R}\right)\xi_n + 2\left(\frac{C_{15}}{R}\right)\lambda_m \ .$$

Also from reference 4, the basic buckling equation is defined as,

$$N_x\lambda_m + N_y\xi_n = \frac{|a_1(m,n)|}{|a_0(m,n)|} \ .$$

Here, the effect of internal pressure is included by calculation of the pressure induced line load in pounds per inch ($N_y = -p*R$), where p is internal gauge pressure. N_x is then determined by the equation

$$N_x = \frac{1}{\lambda_m}\left[\frac{|a_1(m,n)|}{|a_0(m,n)|} - N_y\xi_n\right] .$$

The classical bifurcation buckling load (N_{CL}) is determined by attempting all combinations of m and n. N_{CL} is the minimum of the buckling values obtained.

2. Wide Column

The wide column buckling allowable is obtained from the matrix equations by the following relation:[3]

$$N_{WC} = R^2 \bar{C}_{44} \left[\frac{\pi}{L}\right]^2 ,$$

$$\bar{C}_{44} = C_{44} - \frac{C_{14}^2}{C_{11}} \quad \text{and} \quad \bar{C}_{55} = C_{55} - \frac{C_{25}^2}{C_{22}} .$$

In order to combine the wide column and classical buckling solutions, the "knock-down factor" (φ) is required:

$$\varphi = f(R/t)_e .$$

The "knock-down" factor (φ) as a function of $(R/t)_e$ is obtained from figure 2.[3] A probability level of 99 percent is recommended.

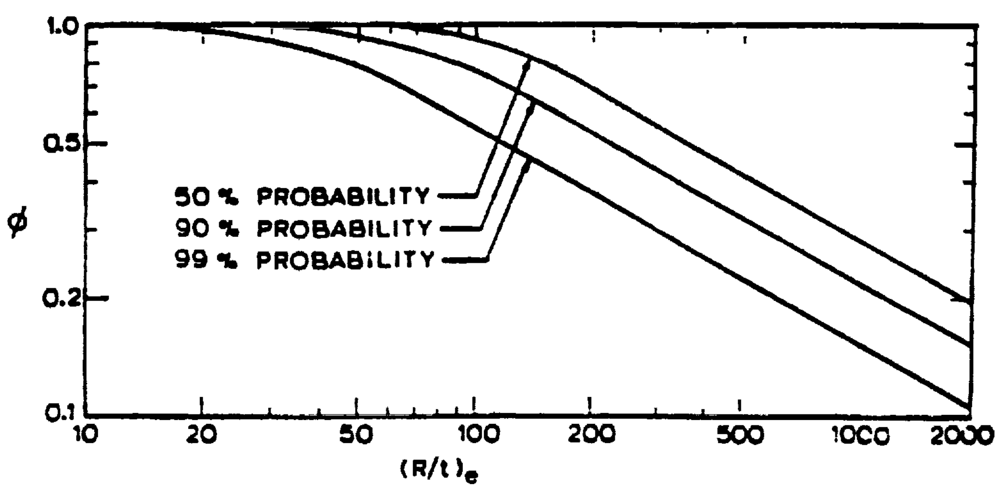


Figure 2. Empirical "knock-down" factors.

$$(R/t)_e = \frac{1}{\sqrt{\dfrac{5.46\left(\bar{C}_{44}+\bar{C}_{55}\right) C_{22}}{\left(C_{11}C_{22}-C_{12}^2\right)}}} . \quad \text{(ref. 3)} \tag{2}$$

All quantities are now known and can be applied to the relation:

$$N_{CR} = N_{WC} + \varphi(N_{CL} - N_{WC}) .$$

B. General Instability

The purpose of general instability calculations is to avoid general instability failure, as illustrated in figure 3.[5] Frames are designed to preclude general instability failure, or rather to ensure panel failure as illustrated in figure 4.[5]

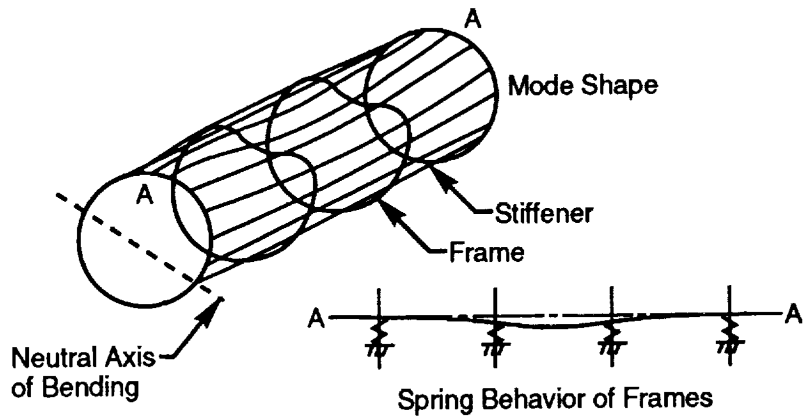


Figure 3. General instability buckling.

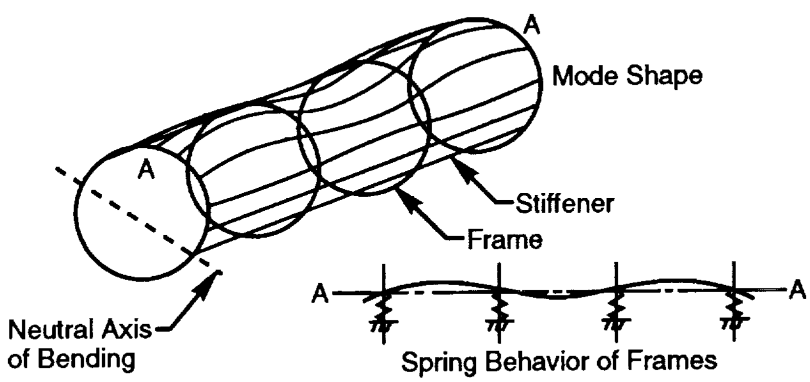


Figure 4. Panel instability buckling.

1. Shanley Criteria for Cylindrical Shells in Bending

To prevent general instability, Shanley has determined an expression—equation (3)—for the required product of frame modulus of elasticity and moment of inertia for pure bending of a stiffened shell:[5]

$$(EI)_f = C_f \, MD^2/L \ . \tag{3}$$

The coefficient C_f has been determined through experimentation to be 62.5×10^{-6} (or 1/16,000). Figure 5 shows the data from which this coefficient comes. One can observe the crossover point from general instability failures to panel failures when the value of C_f is approximately equal to 62.5×10^{-6}.

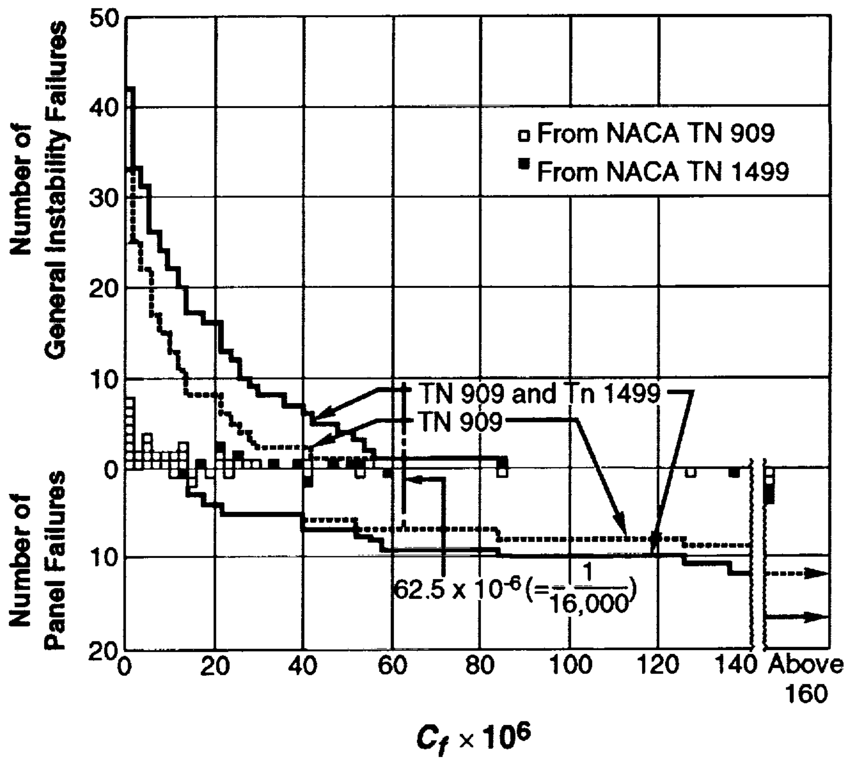


Figure 5. Frame buckling coefficient.

Of course, not all loading in launch vehicles is pure bending. This problem may be remedied by calculation of an equivalent moment where,[6]

$$M_{eq} = PR/2 \ , \tag{4}$$

where P represents the axial load on the cylindrical shell. The equivalent moment is combined with the pure moment to get total effective moment which is then returned to equation (3) for calculation of the required frame (EI).

It should be pointed out that the Shanley method of frame sizing may give either conservative or unconservative results. The results depend on the configuration under analysis. The Shanley method should only be used as an initial sizing measure. Correlation of results with other methods would be advisable.

2. Becker Method

Other methods for calculation of general instability failure levels include those developed by Becker.[7]

$$F_c = gE(I_f t)^{0.5}/Rt_s \ , \qquad \text{(ref. 5)} \tag{5}$$

$$g = 4.80[(b/d)(\rho_s/\rho_f)(t_s/t_f)^2(\rho_s/b)^2]^{0.25} \ , \tag{6}$$

where

b = stringer spacing

d = frame spacing

R = cylinder radius

t = skin thickness

A_{st} = stringer area

A_f = frame section area

t_s = distributed stringer area = A_{st}/b

t_f = distributed frame area = A_f/d

I_f = bending moment of inertia of frame section

I_f = distributed frame moment of inertia = I_f/d

ρ_s = stringer section radius of gyration

ρ_f = frame section radius of gyration

L = length of cylinder

E = modulus of elasticity.

If the frames are not attached to the skin, the coefficient 4.8 in equation (6) should be replaced by 3.25. The effective skin width for frames should be taken as the total frame spacing.[5] The effective width for stringers for use in frame calculations is given by the following equation.[5] Effective skin is that skin which is assumed to act with an adjoining element and carries the same stress as that element.

$$\frac{w_e}{b_{st}} = 0.5\left(\frac{Fc_{cr}}{F_c}\right)^{1/2} . \qquad \text{(ref. 5)} \qquad (7)$$

Fc_{cr} is the critical buckling stress for a curved skin panel, and F_c is the compressive stress at bending general instability—applied ultimate stress.

A distinct advantage of using the Shanley method is that design parameters such as stringer geometry and skin thickness do not have to be known. Use of the Becker method, however, requires that a preliminary design exist for evaluation including stringer and skin definition. Also, there are several ambiguities in the Becker equation that Dr. Bruhn does not clarify. These ambiguities stem from the definition of sectional properties, and whether or not to include effective areas and the like. After review of the original reference, an equivalent equation for critical stress can be obtained which more clearly defines the use of effective skin for the stringers and frame.

$$\sigma_{cr} = CE_cQ_b , \qquad \text{(ref. 7)} \qquad (8)$$

where

C = 4.800 for frames attached to the skin

C = 3.25 for frames not attached to the skin

$$Q_b = \frac{(\rho_{st}\rho_f)^{3/4}(b_{st}d)^{-1/4}}{R} .$$

In determining the radius of gyration for the stringers and frames, the effective widths are used. The effective width for the frames is defined as the frame spacing itself, while effective width to be used with the stringers is calculated by equation (7). According to Becker, these equations are valid only in the range for:

$$\frac{L^2}{RT} \geq 100 .$$

Good agreement can be had between the Becker and Shanley methods even for values of L^2/RT below 100. The frame section properties, however, should not be increased by the standoff distance of the frame from the skin. The properties should be calculated as if the frame were adjacent to the effective skin. For critical values of stress in the plasticity region, the secant modulus can be substituted for Young's modulus with credible results.

C. Stringers

1. Local Elastic Buckling

"Thin flat sheet is inefficient for carrying compressive loads because the buckling stresses are relatively low. However, this weakness, or fault, can be greatly improved by forming the flat sheet into composite shapes such as angles, channels, zees, etc."[5] Calculation of the composite buckling strength is thus necessary to prevent failure of the stringer column. However, since the stringer will continue to carry load after local buckling has occurred, local buckling may be allowed in some instances. Local buckling is more likely to be a design driver where substantial deformation of the stringer flanges causes debonding of insulation or other material. In these instances, it is acceptable to compare the local buckling allowable to the limit load for margin of safety calculations.

Analysis of the local elastic buckling failure mode is easily accomplished by dividing the flanges of the section into individual plate elements with large *a/b* ratios. Jumping ahead to buckling of the skin, and using the buckling equation for a flat plate, equation (26),

$$F_c = \frac{k_c \pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 .$$

Figure 11 is used to determine the buckling coefficient k_c. (Figure 11 and the buckling equation are explained in detail in the skin buckling section of this report.) Since at least two of the flanges are usually of equal size, they buckle at the same stress. Therefore, they cannot be relied upon for edge support. For this reason, a simply supported edge condition is assumed along the longitudinal junction of the flange elements. If the opposite edge is free, the buckling coefficient is 0.43. If the opposite edge is also simply supported, choose k_c equal to 4.0.

The flange width b extends to the centerline of the adjacent leg for formed angles. For extruded angles, the width b extends to the inside edge of the adjacent flange or leg.[5] The smallest buckling stress found in the composite shape, not the average, becomes the critical buckling stress for local elastic buckling.

2. Crippling

"Tests of short lengths of sections composed of flange-plate elements often show that after the section has buckled locally, the unit still has the ability to carry greater loads before failure occurs. . . . For cases where local buckling occurs at low stress, the crippling or failing stress will be higher. When local buckling occurs at high stress, such as 0.7 to 0.8 *Fcy*, buckling and crippling stress are practically the same."[5] For clarification, stringer crippling may be viewed as a material failure where critical values are compared against material ultimate or yield stress. Local elastic buckling is considered a stability failure.

Three methods for computation of the stringer crippling allowable are presented here. The method from the NASA Structures Manual is the simplest of the methods to employ.[8]

a. NASA Structures Manual. The NASA Structures Manual provides a detailed step-by-step procedure for determining the overall strength of a sheet and stiffener combination. The method is very similar to the Gerard method, but with some modifications. No definite reference for this method was given. However, all references listed by the structures manual were dated earlier than the paper delivered by Gerard which details his method of stringer crippling determination.

The stringer crippling stress is determined by the following equation:[8]

$$F_{cs} = \frac{\Sigma b_n t_n f_{ccn}}{\Sigma b_n t_n} \quad . \tag{9}$$

One can easily see that this stress represents the average failing stress of the stringer elements or flanges. The failing stress of individual elements is f_{ccn}. Dimensions of the individual elements are determined consistent with figure 6.[8] Note that two number 1 elements will be needed for analysis of the Tee stringer. The failing stress of each individual element is found from figure 7.[8]

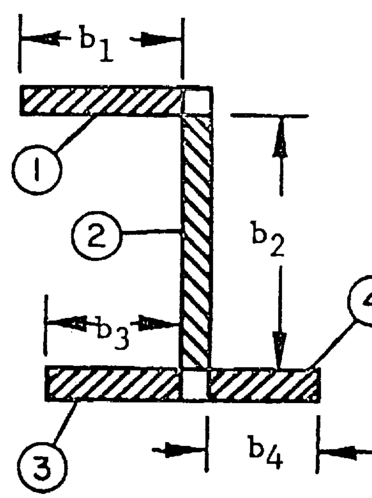


Figure 6. Stringer geometry.

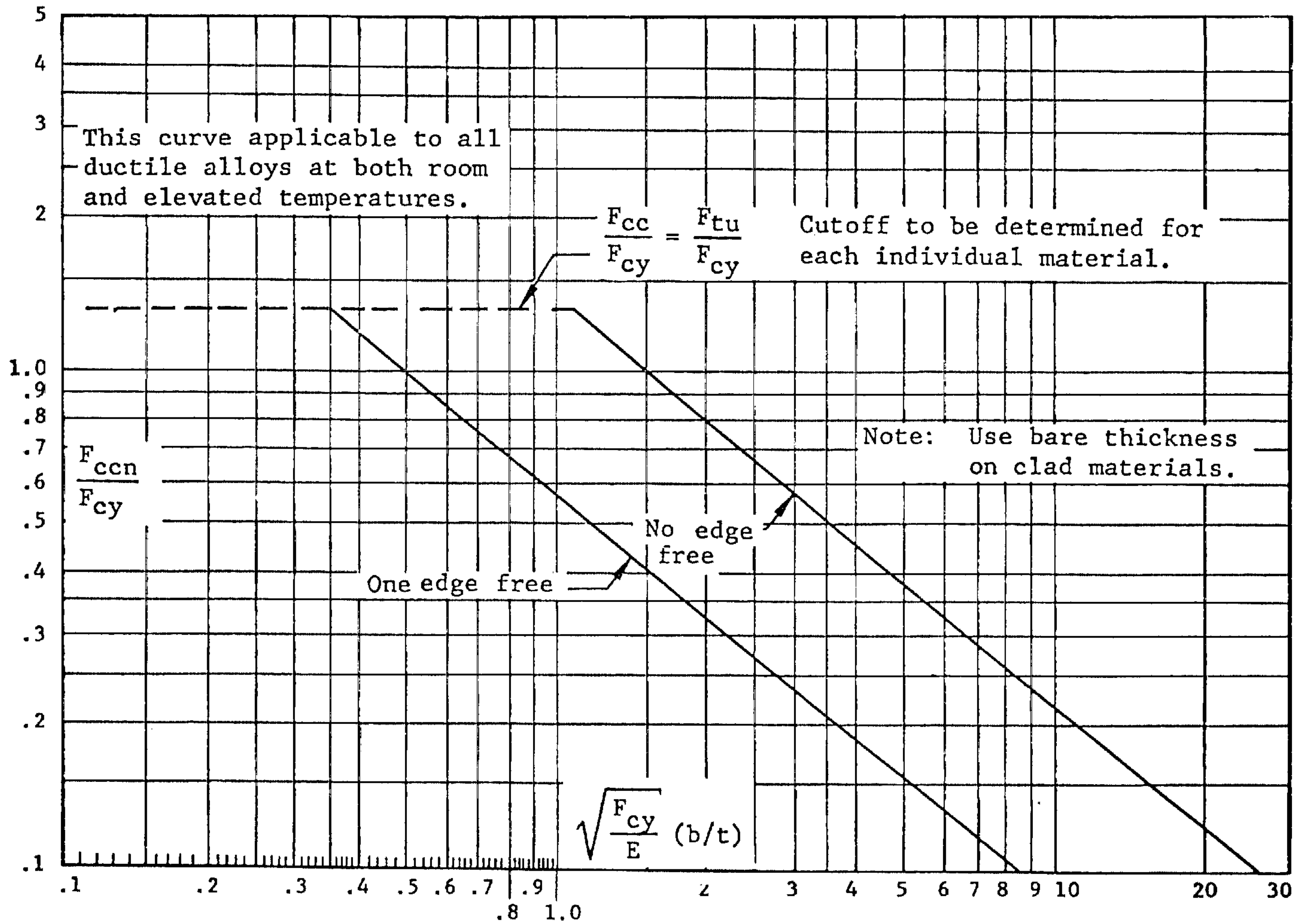


Figure 7. Nondimensional crippling curves.

Following the procedure outlined in the manual, the effective width is not used in determination of the crippling stress of the stringer. This seems contrary to assumptions of the stress distribution in the shell because at this point the effective skin acts with the stringer; experiencing the same stress and load. Common practice is to include the effective width as another flange element of the stiffener when performing crippling stress calculations. The example problem will include the effective skin as part of the stringer in determining crippling stress.

b. Needham. The Needham method is most useful for formed or extruded stringers that are mechanically attached to the skin—such as a hat stringer riveted to the skin. The method consists of dividing a stringer into angle sections. The strength of each of the angle sections is determined, and then the total strength of the stringer is achieved by summing the individual section strengths. Needham has arrived at equation (10) for determination of angled section strengths:[5]

$$\frac{F_{cs}}{\sqrt{(F_{cy}E)}}=\frac{C_e}{\left(\frac{b'}{t_{sk}}\right)^{0.75}} , \tag{10}$$

where

b'/t_{sk} = equivalent b/t of section = $(a+b)/2t_{sk}$
(a and b are the leg elements of the angle)

C_e = coefficient that depends on the edge support
0.316 (two edges free)
0.342 (one edge free)
0.366 (no edge free).

The crippling load of the angle may then be determined as follows:

$$P_{cs}=F_{cs}A \ , \qquad \text{(ref. 5)} \tag{11}$$

where A is the area of the element in question.

The total crippling stress of the stringer representing the average of all stringer angles is then:

$$F_{cs}=\frac{\sum \text{Crippling Loads of Angles}}{\sum \text{Area of Angles}} \ . \qquad \text{(ref. 5)} \tag{12}$$

c. Gerard. The Gerard method can be thought of as a broader application of Needham. The crippling stress equations for various stringer configurations are presented here. Equation (13) is for sections with distorted unloaded edges such as angles, tubes, V groove plates, multicorner sections, and stiffened plates. The accuracy of this equation is said to be ±10 percent, as reported by Gerard.[9]

$$\frac{F_{cs}}{F_{cy}}=0.56\left[\left(gt^2/A\right)\left(E/F_{cy}\right)^{1/2}\right]^{0.85} . \qquad \text{(ref. 5)} \tag{13}$$

Equation (14) is required for sections with straight unloaded edges such as plates, Tee, cruciform, and H sections. Reported accuracy is within ±5 percent.

$$\frac{F_{cs}}{F_{cy}} = 0.67\left[\left(gt^2/A\right)\left(E/F_{cy}\right)^{1/2}\right]^{0.4} . \qquad \text{(ref. 5)} \qquad (14)$$

For two corner sections, J, Z, and channel sections, use equation (15). Accuracy is within ±10 percent.

$$\frac{F_{cs}}{F_{cy}} = 3.2\left[\left(t^2/A\right)\left(E/F_{cy}\right)^{1/3}\right]^{0.75} . \qquad \text{(ref. 5)} \qquad (15)$$

Equations (13) through (15) represent approximations or simplifications of the data presented by Gerard. The general equation for stringer crippling is given by Gerard as:

$$\frac{F_{cs}}{F_{cy}} = \beta_g\left[\left(g\bar{t}_w t_{sk}/A\right)\left(E/F_{cy}\right)^{1/2}\right]^m . \qquad \text{(ref. 9)} \qquad (16)$$

The coefficient β_g is determined experimentally and results are tabulated by Gerard as functions of $\bar{t}_w/t_{sk}$. Using $\beta_g = 0.56$ would be considered an average for stiffened plates. The actual values range from 0.562 to 0.464, as shown in table 1. The constant g is the sum of the number of flanges of the angled elements and the number of cuts required to divide the stringer into angled elements. The exponent m is 0.85.

Table 1. Gerard coefficients.

$\bar{t}_w/t_{sk}$	β_g
1.16	0.562
0.732	0.505
0.464	0.478

$\bar{t}_w$ is the average thickness of the flange sections as determined by equation (17):

$$\bar{t}_w = \frac{\Sigma b_i t_i}{\Sigma b_i} . \qquad \text{(ref. 8)} \qquad (17)$$

The skin thickness is represented by t_{sk}, and the exponent m is determined experimentally. However, it does not change with $\bar{t}_w/t_{sk}$, and is dependent on the type of stringer arrangement.

The data presented above were for Y-stiffened panels. Inspection of the integrally stiffened panel indicates that it closely duplicates the Z-stiffened panel. Coefficients for Z-stiffened panels are: $m = 0.85$, and $\beta_g = 0.558$. β_g data as a function of $\bar{t}_w/t_{sk}$ are the same as Y-stiffened panels. The number of flanges and cuts will change.

Bruhn also presents a series of illustrations in which both methods are used to determine stringer crippling stress. Depending on the stringer configuration, there can be significant differences in the Needham and Gerard methods. Also, the crippling value is subject to upper limits that should not be exceeded unless test data can substantiate such a move. A table of upper limits is presented as table 2.[5]

Table 2. Maximum crippling stress.

Type of Section	Maximum F_{cs}
Angles	$0.7\ F_{cy}$
V Groove Plates	F_{cy}
Multicorner Section, Including Tubes	$0.8\ F_{cy}$
Stiffened Panels	F_{cy}
Tee, Cruciform, and H Sections	$0.8\ F_{cy}$
Two Corner Sections, Zee, J Channels	$0.9\ F_{cy}$

The reader should note that the entire width of skin between stringers is used as a flange in determining the stringer crippling value (using Gerard's method), as opposed to using the effective width only. This convention is maintained since the empirical equations were obtained using that convention.

3. Column Failure

In general, column failure is the limiting failure mode for most longitudinally stiffened vehicle structures. The primary buckling equation for elastic failure is simply stated as equation (18); the Euler buckling equation:[5]

$$F_c = \frac{\pi^2 E}{(L/\rho)^2} \ . \tag{18}$$

Equation (18) can be rewritten involving the tangent modulus for stresses in the inelastic region. Where E_t is approximately E for stresses in the elastic range:

$$F_c = \frac{\pi^2 E_t}{(L/\rho)^2} \ . \tag{19}$$

The radius of gyration (ρ) for the column (or stringer), is calculated by equation (20):[5]

$$\rho = \sqrt{I/A} \ . \tag{20}$$

The buckling strength of a column is also heavily influenced by the end restraint on the column. Adding the end-fixity coefficient c into equation (19) allows incorporation of the end-fixity constraint into the buckling equation. A new effective column length is determined by equation (21):[5]

$$L' = (L/\sqrt{c}) \ . \tag{21}$$

The Euler equation thus becomes:

$$F_c = \frac{\pi^2 E_t}{(L'/\rho)^2} \ . \tag{22}$$

Determination of the tangent modulus is accomplished through use of the basic Ramberg-Osgood relationship.[5] Note that $E \approx E_t$ for stresses in the low elastic range.

$$\frac{E_t}{E} = \frac{1}{1+\frac{3}{7}n\left(\frac{F}{F_{0.7}}\right)^{n-1}} , \tag{23}$$

$$n = 1 + \frac{\ln\left(^{17}/_{7}\right)}{\ln\left(^{\sigma_{0.7}}/_{\sigma_{0.85}}\right)} ,$$

where $\sigma_{0.7}$ is the secant yield stress found by drawing a line on the material stress-strain curve from the origin with a slope of 0.7E and reading the stress at intersection with the stress-strain curve. $\sigma_{0.85}$ is found similarly.

End-fixity coefficients for various end constraints are presented as figure 8. These coefficients are presented for completeness. In practice, $c = 1$ or 1.5 is generally applied to the skin-stringer design problem. A value of 1.0 would be conservative.

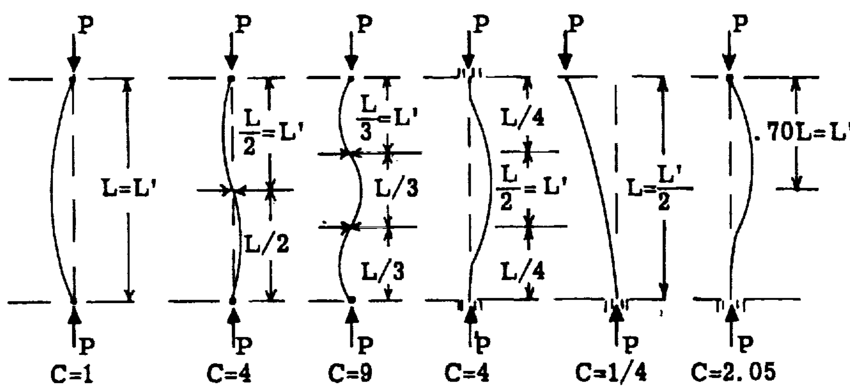


Figure 8. End-fixity coefficients.

The NASA Structures Manual presents design charts (appendix D) for determining the end-fixity coefficients based on the bending stiffness of the end restraint. This data can be used in determining the slenderness ratio (L'/ρ) of the column. However, the analyst may be inclined to determine his or her own coefficient depending on the fidelity and conservatism desired of the analysis.

Figure 9 shows a typical curve of F_{cr} (critical buckling stress) as a function of L'/ρ. Buckling of columns with stable cross sections such as tubes and other closed sections follows the curve *ABFC*. Equation (22) is applied to determine the critical buckling stress.

However, for columns of unstable cross section such as channels, Tees, and Tee stringers with effective skin width, curve *DEFC* must be employed. Critical stresses in the *FC* region are found through the Euler equation—equation (22). If the slenderness ratio of the column shows it to be in the transition region of the curve of figure 9, the buckling limit will be below that predicted by simple Euler buckling.

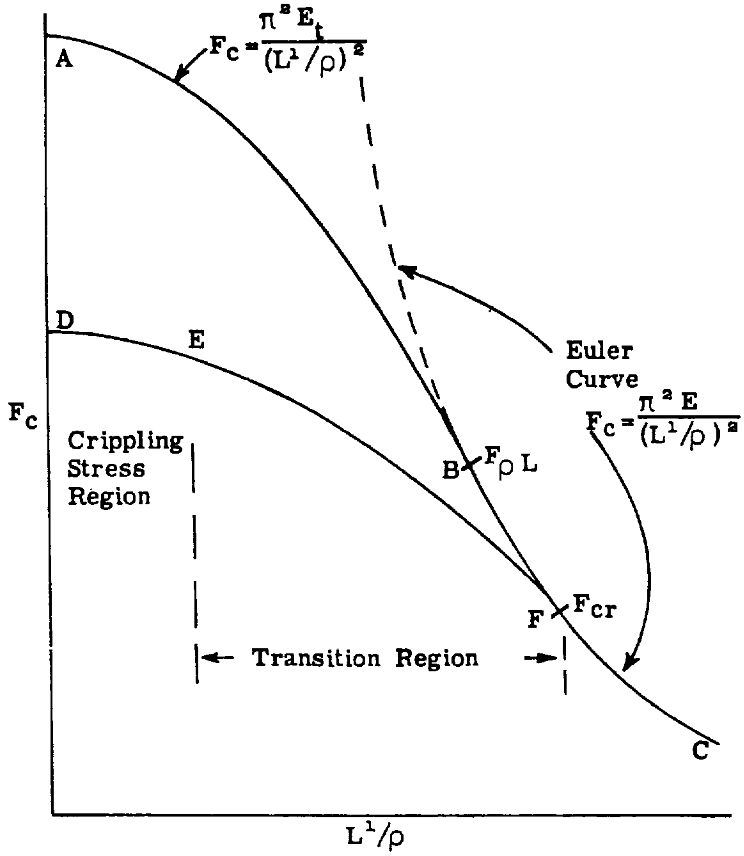


Figure 9. Typical column buckling curve.

Stresses in the transition region are determined using the Johnson-Euler equation (24), where F_{cs} is the crippling strength of the stringer:[5]

$$F_c = F_{cs} = \frac{F_{cs}^2}{4\pi^2 E}\left(\frac{L'}{\rho}\right)^2 . \tag{24}$$

To find out in which section of the buckling curve a column lies, it is necessary to compute the intersection point of the Euler and Johnson-Euler curves. Setting the buckling stresses from Johnson-Euler and Euler equations equal to one another and solving for L'/ρ, the intersection point is shown to lie at:

$$(L'/\rho) = \pi\sqrt{\frac{2E_c}{F_{cs}}} . \qquad \text{(ref. 5)} \tag{25}$$

For values of L'/ρ greater than the intersection value, use the Euler equation. For values of L'/ρ less than or equal to the intersection value, use the Johnson-Euler equation.

Now that methods for calculating the buckling strength of a column have been presented, it is necessary to apply these methods to the skin-stringer design problem. "As load is increased, the sheet buckles between the stiffeners and does not carry greater stress than the buckling stress for the skin. However, as the stiffeners are approached, the skin being stabilized by the stiffener to which it is attached can take a higher stress, and immediately over the stiffener the skin carries the same stress as the ultimate strength of the stiffener, assuming the sheet has a continuous connection to the stiffener."[5]

In general, attempts to determine the amount of skin acting with the stringer have resulted in long and complex equations. To simplify the determination of the effective skin width, equations have been developed to determine the skin width that would be considered as taking a uniform stress equivalent to the stress in the effective stringer under actual nonuniform conditions. Figure 10 illustrates the progression of panel buckling and the assumption of uniform stress made over the effective skin.[5]

A method for effective sheet width determination is presented here. The procedure follows the analysis of a flat sheet in compression with long edges simply supported. The critical buckling stress for the flat sheet is found from equation (26):[5]

$$F_c = \frac{k_c \pi^2 E}{12(1-\upsilon^2)} \left(\frac{t}{b}\right)^2 . \tag{26}$$

The k_c value determined from experiment approaches 4.0 if the long edges of the sheet are assumed to be simply supported.

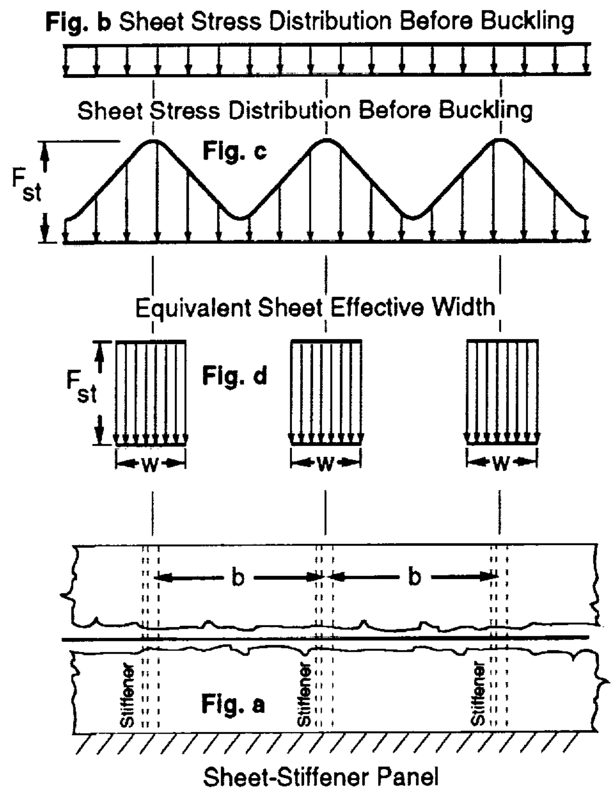


Figure 10. Progression of panel buckling.

The problem of determining effective width has been simplified somewhat by Von-Karman and Sechler.[5] Their method consists of solving for an effective width (w_e) in place of b in equation (26) with the critical stress equal to the yield stress of the material. However, since the buckling stress can be greater than the yield stress, the yield stress has subsequently been replaced by the stress in the stringer itself (F_{st}). Substituting 4.0 for k_c and 0.3 for Poisson's ratio, equation (26) reduces to:

$$F_c = 3.60E(t/w_e)^2$$

or (ref. 5) (27)

$$w_e = 1.90t(E/F_{st})^{0.5} \cdot$$

The "t" of equation (27) is defined as the thickness of the skin plus the skin and stringer land divided by two (see appendix A).[5] However, experiments by Newell indicate the constant 1.9 is too high, and 1.7 would be more appropriate,[5] possibly due to the conservatism in the buckling coefficient. Although the preceding derivation was accomplished using a buckling coefficient of 4.0, a simply supported flat plate, the determination of effective width for curved plates does not differ significantly from flat plates for values of $Z < 30$.[10]

$$Z = \frac{b^2}{Rt}\left(1-\upsilon^2\right)^{1/2} .$$

The stringer should now be treated as a wide column made up of the stringer and its effective skin, and the appropriate Johnson-Euler or Euler equation applied to determine buckling allowables.

D. Skin

1. Compression Buckling

The first evaluation of the skin strength comes in the form of simple flat sheet buckling. By using equation (26) and setting b to the stringer spacing and t equal to the skin thickness, the buckling strength of the skin between stringers can be determined:

$$F_c = \frac{k_c \pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 .$$

Just as in determination of the effective width, the buckling strength of the skin is very much dependent on the buckling coefficient. A value of 4.0 is commonly used and is considered conservative because of its assumption that the long edges of the sheet are simply supported. Figure 11 illustrates the effect of increasing the a/b ratio on the k_c values.[12] For long simply supported sheets (constraint C), the buckling coefficient value approaches 4.0. Figure 12 offers an alternative to this conservatism by allowing the engineer to choose a buckling coefficient that lies somewhere between the conservative simply supported case and the nonconservative clamped edge case.[13]

One should immediately notice that the buckling coefficient curves discussed thus far are for flat sheets, and the problem being investigated is one of stiffened cylindrical structures or curved sheets. Knowing this, however, many analysts use the flat sheet data instead of that for curved sheets, which are generally stiffer than their flat counterparts. For curved sheet panels, the buckling equation remains the same, however, k_c is determined from figure 13. Curved sheets of large radius ($b^2/Rt < 1$) can be analyzed as flat plates.[14] The data used in obtaining figure 13 are for a simply supported edge condition.[5] When curved sheet k_c values are compared to those from figure 12, the flat sheet may be larger. This is caused by the simply supported edge restraint used to obtain figure 13 data. It is recommended that the maximum k_c resulting from figures 11, 12, and 13 be used in determining skin buckling coefficients. Data from figure 11 or $k_c = 4.0$ may be used for conservatism.

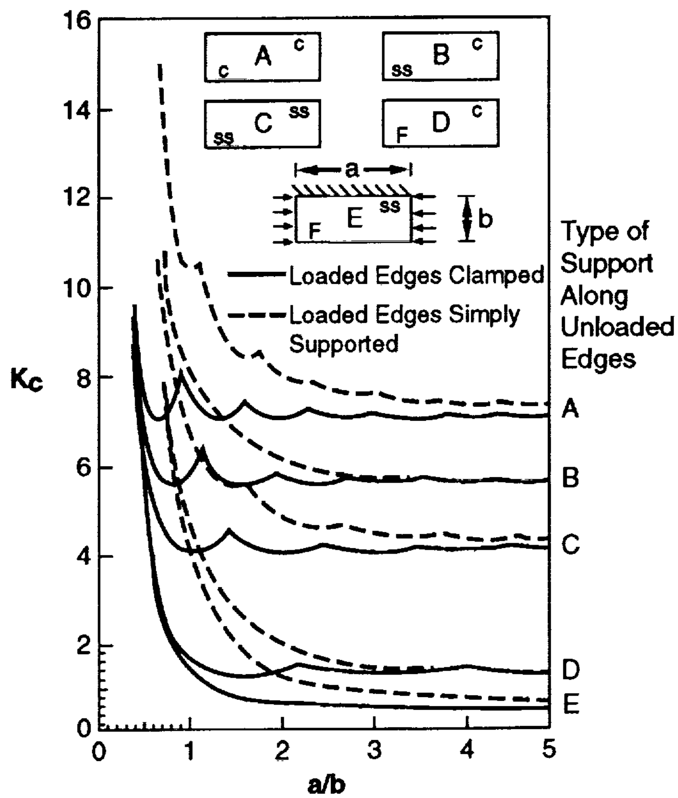


Figure 11. Compression buckling coefficient.

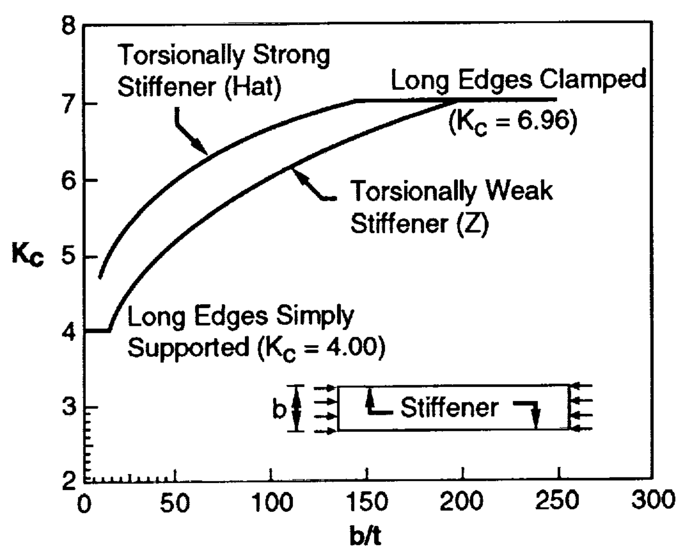


Figure 12. Compression buckling coefficient.

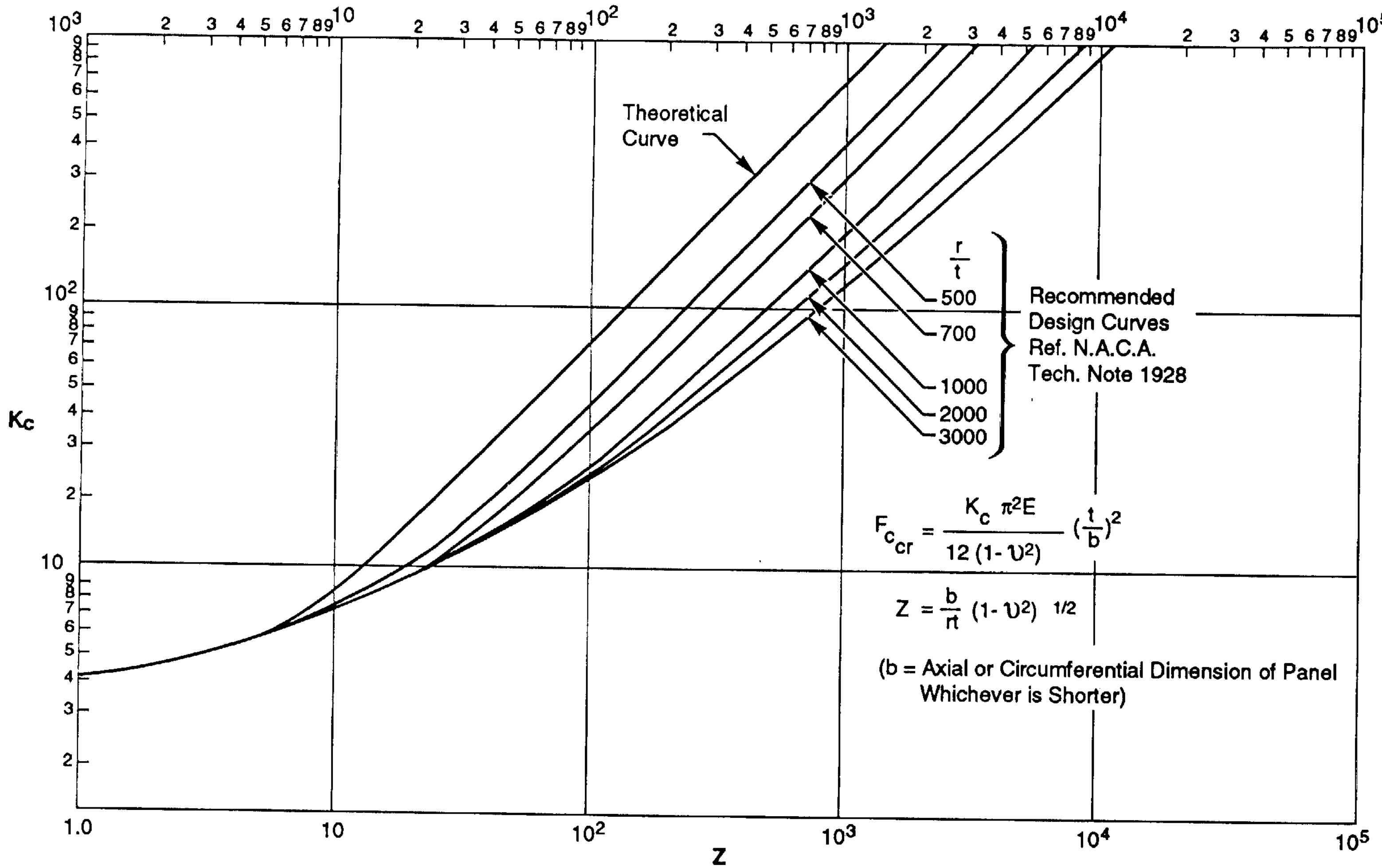


Figure 13. Compression buckling coefficient for curved sheets.

A major contribution of the NASA Structures Manual to the analysis of the skin and stringer structure is the determination of critical load that includes the load carrying capability of the post-buckled skin. From reference 8, the equation for critical buckling load (modified for integral stiffeners) is written:

$$P_{cr} = (F_{cr_{column}})(A_{st}+t_{sk}w_e)+(F_{cr_{skin}})(b_{st}-w_e) \ . \tag{28}$$

One can see the critical load in the column (P_{cr}) is increased by the buckling stress of the skin multiplied by the area of skin not counted as effective skin acting with the stringer. However, from an analysis procedure standpoint, it is easier to use the load carrying capability of the buckled skin to reduce the ultimate stress in the column, rather than increase the column capability.

Equation (28) is further modified by the observation that the skin does not carry the full buckling stress after failing. Bruhn[5] suggests that the maximum stress assumed in the buckled skin should be no greater than

$$\sigma_{cr} = 0.3*E*t/R \ .$$

In practice, the buckling stress value for the skin may be reduced 10 percent so that equation (28) becomes:

$$P_{cr} = (F_{cr_{column}})(A_{st}+t_{sk}w_e)+0.9(F_{cr_{skin}})(b_{st}-w_e) \ . \qquad \text{(ref. 4)} \tag{29}$$

The reader should note that the load carrying capacity of the buckled skin can be accounted for directly by raising the wide column buckling load or reducing the ultimate stress by reducing the applied load, as has been discussed. Also, the buckled skin capacity for load can be retrieved indirectly by determining an area not included with the stringer or effective skin that will act to carry some load. This area, designated A_e, will be used to increase the moment of inertia and total area calculations of the shell, and thereby, lower the stress level. Use of the ineffective area is covered under the section addressing stress calculations using simplified beam theory. The external tank (ET) stress method uses the P_{cr} approach outlined above.

2. Shear Buckling

It is not often in launch vehicle design that shear buckling of a launch vehicle becomes a driving load condition. In general, consideration of axial and bending loads far outweigh any consideration of shear effects. For completeness however, determination of the shear buckling capability of the skin will be presented here.

The critical elastic shear buckling stress is given by the following equation:

$$\tau_{cr} = \frac{\pi^2 k_s E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 \ . \qquad \text{(ref. 5)} \tag{30}$$

If buckling occurs above the proportional limit, equation (31) must be employed:

$$\tau_{cr} = \frac{\eta_s \pi^2 k_s E}{12(1-\upsilon^2)} \left(\frac{t}{b}\right)^2 , \qquad \text{(ref. 5)} \qquad (31)$$

where η_s represents the plasticity correction factor. Correlation with test results indicates that an $\eta_s = G_s/G$ yields best results. G_s being the shear secant modulus, and G being the shear modulus. The coefficient k_s is chosen from figure 14 using the hinged edge constraint (flat plate).[5]

Returning to the original reference by Gerard and Becker, a curve for shear buckling of a curved panel with simply supported edges can be obtained (fig. 15).[15] It is recommended that the largest of these buckling coefficients be used in determination of the shear buckling stress, keeping in mind that b is always the shorter of the panel dimensions.

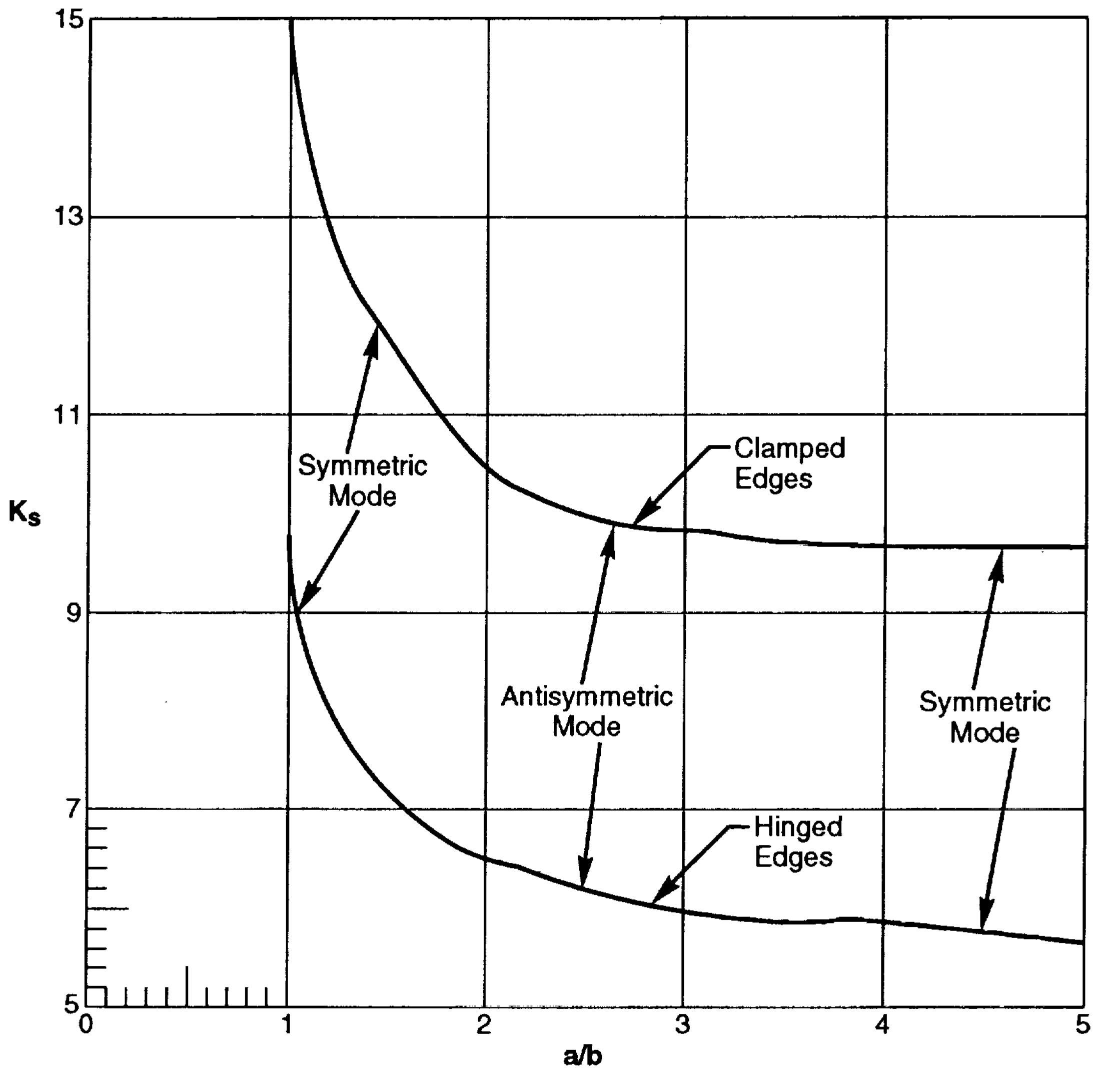


Figure 14. Shear buckling coefficients.

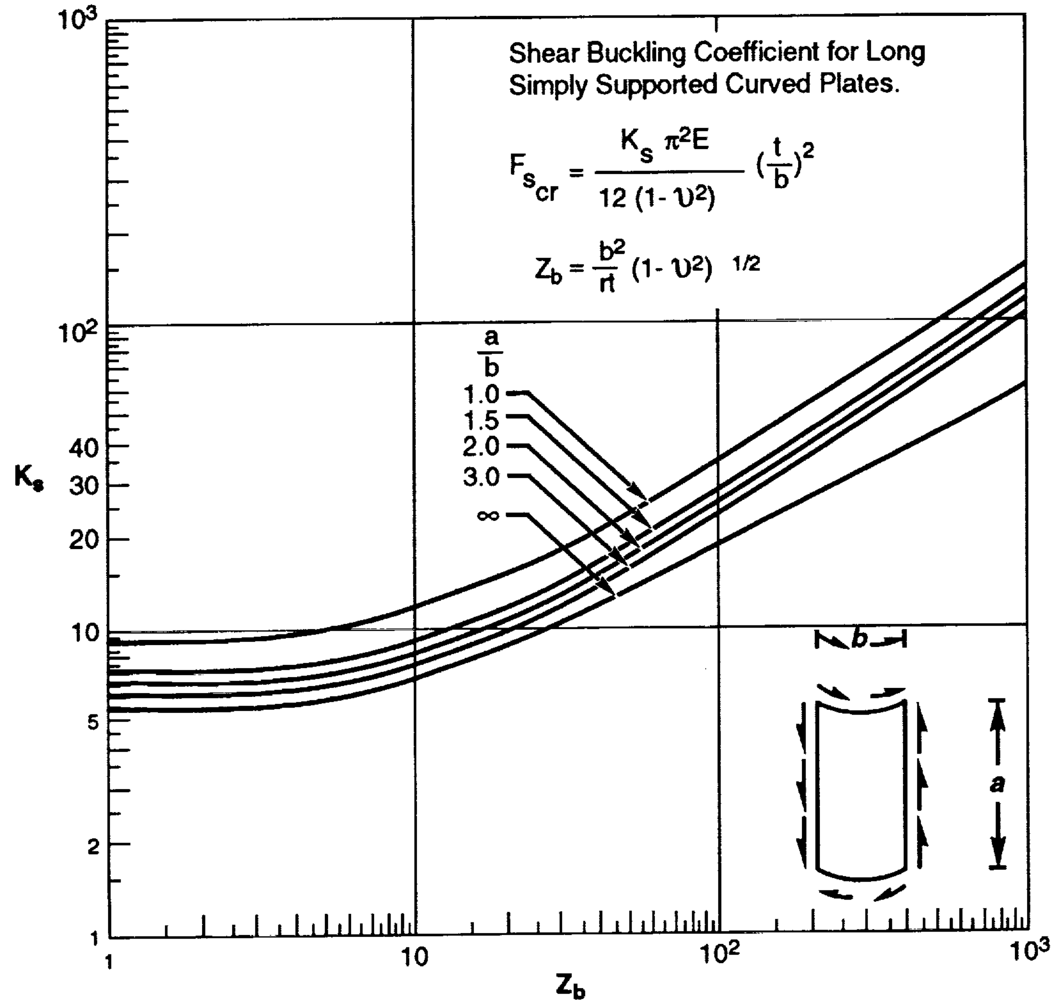


Figure 15. Shear buckling coefficients.

3. Internal Pressure

Internal pressure in a tank creates a biaxial tensile stress state in the skin and thereby increases its resistance to buckling. The addition of internal pressure increases the buckling strength of the curved sheet by the following interaction relationship:[6]

$$R_c^2 + R_p = 1 \ , \tag{32}$$

where R_c is the ratio of compressive buckling stress to critical compressive buckling stress, and R_p is the ratio of applied internal pressure over the external pressure that would buckle the cylinder for which the curved panel is a section. Buckling due to radial pressure is found by use of figure 16. For internal pressure, R_p is negative. The buckling equation is the same as for flat plate, substituting k_y for k_c.[5]

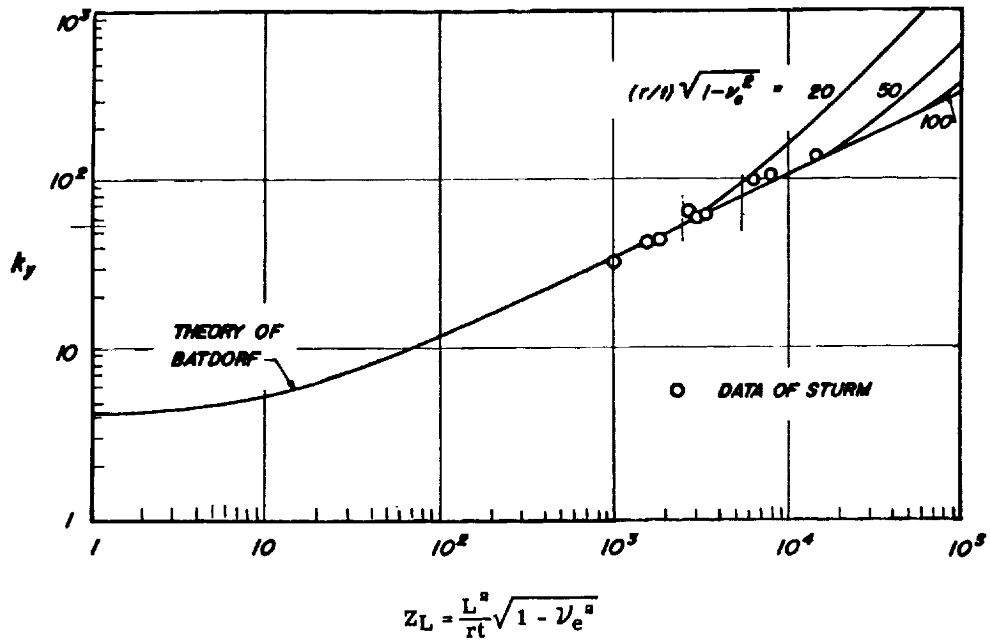


Figure 16. Pressure buckling.

A more sophisticated method of determining the increase in buckling strength due to the effects of internal pressure is found in reference 16. Assuming all edges are simply supported, the relation between critical meridional and hoop stresses is given by the following equation:

$$\sigma_x' \frac{m^2}{a^2} + \sigma_y' \frac{n^2}{b^2} = 0.823 \frac{E}{1-\upsilon^2} t^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right) . \tag{33}$$

Here m and n signify the number of half waves in the x and y directions, respectively. To find σ_y' for a given σ_x', take $m = 1, n = 1$ if,

$$C\left(1-\frac{4a^4}{b^4}\right) < \sigma_x < C\left(5+\frac{2a^2}{b^2}\right) ,$$

where

$$C = \frac{0.823Et^2}{(1-\upsilon^2)a^2} .$$

If σ_x is too large to satisfy the inequality, take $n = 1$ and m to satisfy:

$$C\left(2m^2-2m+1+2a^2/b^2\right) < \sigma_x < C\left(2m^2+2m+1+2a^2/b^2\right) .$$

If σ_x is too small to satisfy the first inequality, take $m = 1$ and n to satisfy:

$$C\left[1-n^2(n-1)^2 \frac{a^4}{b^4}\right] < \sigma_x < C\left[1-n^2(n+1)^2 \frac{a^4}{b^4}\right] .$$

Internal pressure also increases the shear allowable by the following relation:[5]

$$R_s^2 + R_p = 1 \ . \tag{34}$$

This equation is employed in the same manner as equation (32). R_s is the ratio of applied shear stress to the critical allowable shear stress for buckling.

The alternative method presented by reference 16 maintains that the unit shear stress for buckling with all edges simply supported is given by equation (35):

$$\tau' = \sqrt{C^2\left(2\sqrt{1-\frac{\sigma_y}{C}} + 2 - \frac{\sigma_x}{C}\right)\left(2\sqrt{1-\frac{\sigma_y}{C}} + 6 - \frac{\sigma_x}{C}\right)} \ , \tag{35}$$

$$C = \frac{0.823}{(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 E \ ,$$

(σ_x and σ_y are negative when tensile).

III. EXAMPLE PROBLEM

The launch vehicle shown in figure 17 will serve to illustrate the methods presented in this report. The hydrogen tank in particular will be examined. The diameter of the vehicle is 331 in. A Zee-shaped intermediate ring frame is shown in figure 18. The internal stringer geometry is shown in figure 19.[17]

The forces acting on the vehicle are due to ground winds acting against the vehicle prior to launch and the vehicle's own weight. It is assumed for analysis purposes that no pressure is present in the tank at this time. At the hydrogen tank barrel section to be examined, the shear load is 53,678 lb, the bending moment is 4.924×10^7 in-lb, and the axial compressive load is 1,571,825 lb. Recall that the skin is allowed to buckle at limit load.

The skin and stringers are machined from aluminum 2219-T87 and the rings are extruded 2024. A safety factor of 1.4 will be used.

A. Bifurcation Buckling

A Fortran program written using MacTran—a Fortran development program for the Macintosh—was developed to facilitate the matrix method buckling analysis procedure. The code itself is found in appendix E along with a sample output. The code is not as autonomous as the worksheet developed for use with the other methods. Sectional properties for the shell configuration must be hard-coded into the Fortran program for execution.

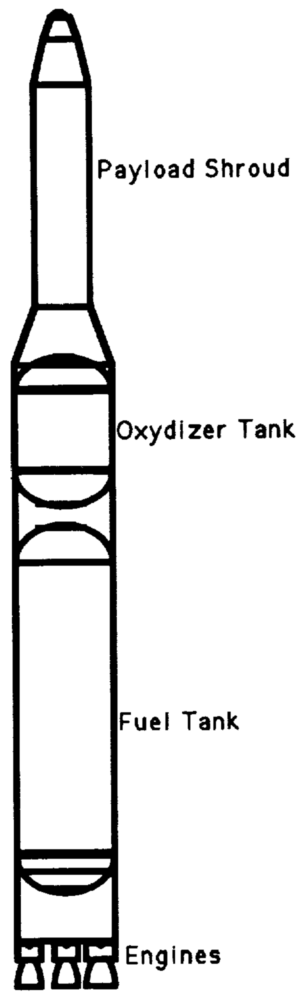


Figure 17. Vehicle configuration.

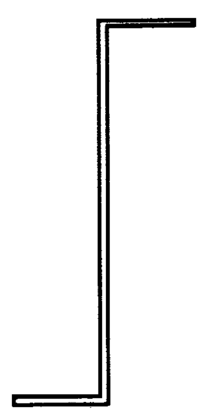


Figure 18. Intermediate ring frame.

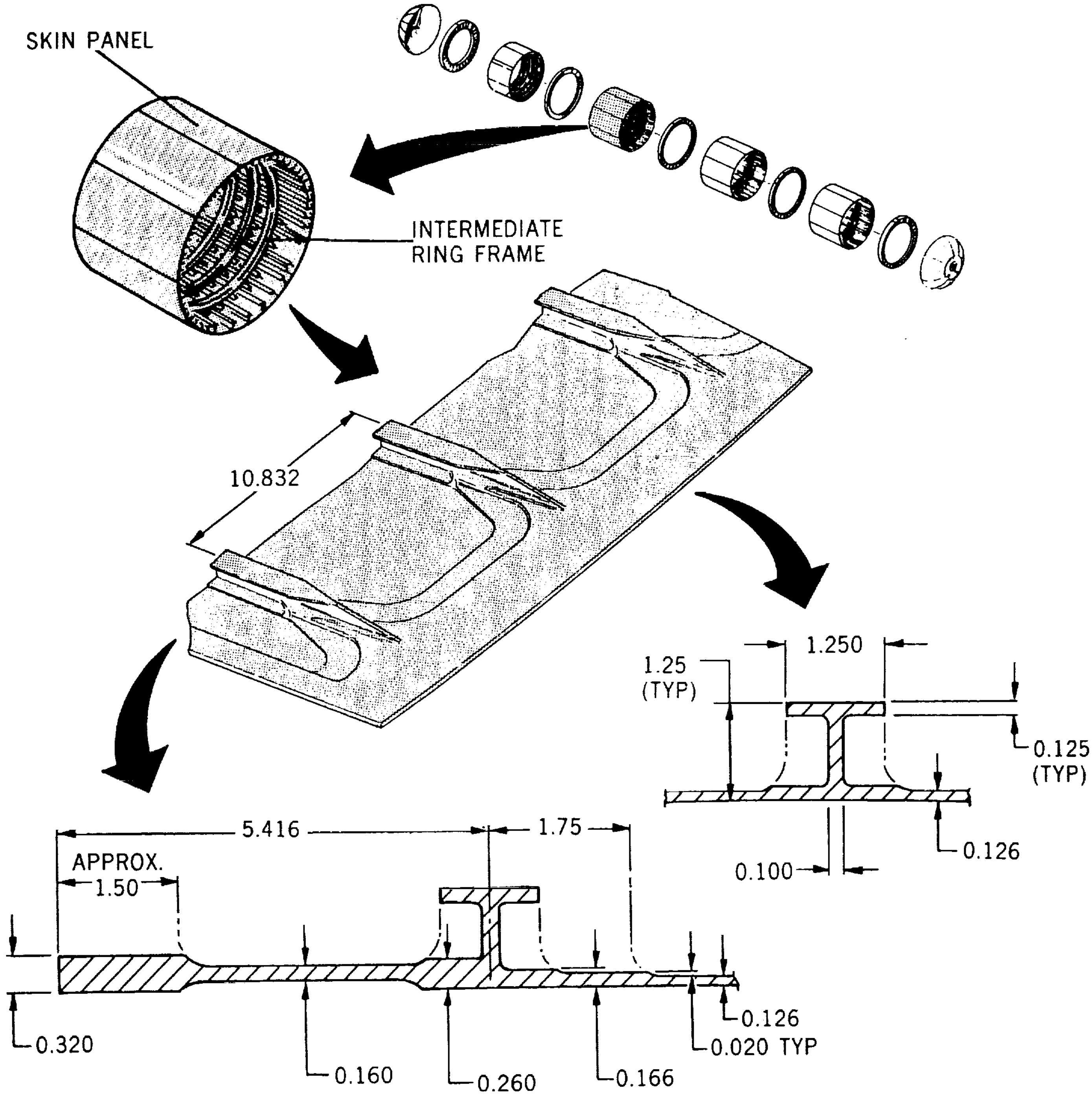


Figure 19. Stiffened panel configuration.

Once again, recall that the bifurcation buckling methods may not be used in instances where significant redistribution of stress occurs in the subcritical load range—as occurs in this example problem. Determination of the bifurcation buckling allowable is presented for demonstration purposes.

Using the same stiffened shell configuration shown previously, the stiffness matrix $[C_{ij}]$ is shown below:

$$\begin{bmatrix} 1{,}805{,}277.3 & 503{,}943.5 & 0.0 & -215{,}235.7 & 0.0 & 0.0 \\ 503{,}943.5 & 1{,}527{,}101.4 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 511{,}578.9 & 0.0 & 0.0 & 0.0 \\ -215{,}235.7 & 0.0 & 0.0 & 210{,}811.6 & 666.7 & 0.0 \\ 0.0 & 0.0 & 0.0 & 666.7 & 2{,}020.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 11{,}584.0 \end{bmatrix} .$$

N_y is zero in this case since there is no pressure acting on the cylinder when the stated loads are applied. The minimum Nx load is found when $m = 1$ and $n = 12$. The classical buckling load is determined to be

$$N_{CL} = 5{,}707 \text{ lb/in.}$$

The wide column allowable is calculated as

$$N_{WC} = 1{,}137 \text{ lb/in.}$$

The "knock-down" factor resulting from an $(R/t)_e$ value of 130.6 is 0.458. Therefore,

$$N_{CR} = 3{,}231 \text{ lb/in.}$$

In this particular case, the classical bifurcation load is of little use since there has been "considerable redistribution" of stress (i.e., the skin has failed).

B. General Instability

The ring shown in figure 18 will be used as the intermediate stiffening ring in this example problem. The ring spacing is 40.08 in. Aluminum 2024-T42 extrusion has a compressive modulus of 11.0×10^6 lb/in^2 and a compressive yield strength of 38,000 lb/in^2.[11]

Using $C_f = 1/16{,}000$, the critical equivalent bending moment can be solved for using equation (3).

$$(EI)_f = C_f \, MD^2/L \ ,$$

$$M_{eq} = 2.7279\times10^8 \text{ in-lb} .$$

This equivalent moment is converted to a line load or stress for comparison to applied stresses in the structure. Comparison of the frame capability and the applied stress yields a margin of safety of 0.428 using the Shanley method. See the section on margin of safety calculations for more details.

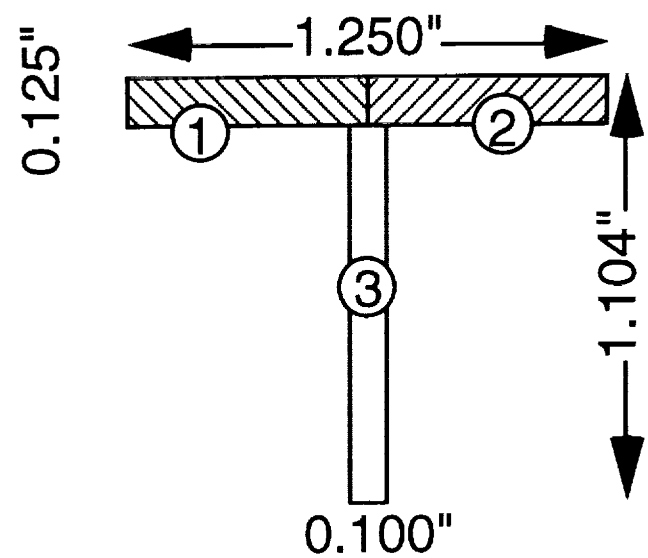
Evaluating this frame using the Becker method the critical stress is 32,087 lb/in². This allowable stress determined by the Becker method equates to a margin of safety of 0.019. Evaluation of the d^2/Rt term reveals it to be less than 100; not in the range specified by Becker. As a test, a configuration was set up so that d^2/Rt was greater than 100. For the test case, the two methods produced identical results.

The reader should take special note that the capability of the frames is compared to the applied stresses rather than to the equivalent moments themselves. The reason for this is that the stress in the shell is reduced by the load carrying capability of the buckled skin. This reduction in load is not reflected in the applied equivalent moment calculations, but is included in the Nx' value that is introduced as part of the ET stress method in appendix B, and in the final stress value produced using the simplified beam theory.

C. Stringers

1. Local Elastic Buckling

The skin stringer panels under investigation are machined rather than formed or extruded. Divide the Tee stringer into three flange sections. Taking the conservitive definition of flange elements, sections one and two represent the halves of the stringer cap. Section three is the web section, which actually extends to the centerline of the cap.



The local buckling strength of flanges one and two, and that of flange three (the web) are determined in the following:

$$F_{c_1} = \frac{0.43\pi^2 10.8E6}{12(1-0.33^2)}\left(\frac{0.125}{0.625}\right)^2 = 171{,}452 \text{ lb/in}^2 ,$$

$$F_{c_3} = \frac{4.0\pi^2 10.8E6}{12(1-0.33^2)}\left(\frac{0.1}{1.0415}\right)^2 = 367{,}584 \text{ lb/in}^2 \ .$$

Obviously, the minimum buckling strength is above the material yield strength. Therefore, the local buckling limit is set equal to the yield strength of the material (51,000 lb/in^2). As the reader may have already discerned, calculation of the local buckling strength for this particular example problem is unnecessary. The stringers are internal to the tank structure and have no insulation or other bonded materials to be concerned with. The calculation is carried out for completeness and added confidence in the design.

2. Crippling

a. NASA Structures Manual. The following illustrates the procedures found in the NASA Structures Manual for determination of stringer crippling stress.

Step (1) Maintaining an end-fixity coefficient of 1.5, L'/ρ equals 80.15.

Step (2) Writing equations for the no edge free and one edge free curves from figure 7.[4]

$$\left(\frac{f_{ccn}}{Fcy}\right)_{NEF} = \frac{1.387194}{\left\{\sqrt{\frac{F_{cy}}{E_c}}\left(\frac{b_n}{t_n}\right)\right\}^{0.8071793}} \ ,$$

$$\left(\frac{f_{ccn}}{Fcy}\right)_{OEF} = \frac{0.5693108}{\left\{\sqrt{\frac{F_{cy}}{E_c}}\left(\frac{b_n}{t_n}\right)\right\}^{0.8127115}} \ .$$

The maximum cutoff value for each material is given by:

$$\frac{f_{ccn}}{Fcy} = \frac{F_{tu}}{F_{cy}} \ ,$$

or simply F_{tu}. The procedures outlined make no mention of stresses in the inelastic region for stringer crippling. Obviously, if f_{ccn} were to reach the cutoff limit it would be past the proportional limit stress and some correction factor would be in order. In this report, the cutoff stress will be limited to the yield stress.

$$\left(\frac{f_{ccn}}{F_{cy}}\right)_{\text{flange}} = \frac{0.5693108}{\left\{\sqrt{\frac{51{,}000}{10.8\times10^6}}\left(\frac{0.575}{0.125}\right)\right\}^{0.8127115}} = 1.45159 \ .$$

The limit value is $f_{ccn}/F_{cy} = 1.0$; therefore, the crippling stress for the flange element is 51,000 lb/in^2. Notice from figure 6 that the flange width is not simply half the stringer cap width.

Crippling strength for the web is determined using the no-edge-free equation as follows.

$$\left(\frac{f_{ccn}}{Fcy}\right)_{web} = \frac{1.387194}{\left\{\sqrt{\frac{51{,}000}{10.8\times10^6}}\left(\frac{0.979}{0.10}\right)\right\}^{0.8071793}} = 1.91 \ .$$

Again the limit ratio of $f_{ccn}/F_{cy} = 1.0$ is employed and the crippling stress for the web becomes 51,000 lb/in^2.

The final effective width after iteration using the ET stress method is 4.28 in, so that:

$$\left(\frac{f_{ccn}}{Fcy}\right)_{we} = \frac{1.387194}{\left\{\sqrt{\frac{51{,}000}{10.8\times10^6}}\left(\frac{2.09}{0.1318}\right)\right\}^{0.8071793}} = 1.29 \ .$$

The resulting crippling stress for the effective skin is 51,000 lb/in^2, and the weighted average of the three crippling stresses is also 51,000 lb/in^2. Based on literature review and experience, it is recommended that the designer use the NASA Structures Manual method for determining stringer crippling first. The structures manual method is easier to program and more straight forward in its application. The analyst attempting to employ the Gerard method should review the papers authored by Gerard. The method of determining the number of cuts and flanges is somewhat confusing.

b. Gerard. The shell geometry to be evaluated in this example is shown in figure 18. The Gerard method of stringer crippling analysis is obviously more applicable to the example than the Needham method. Dividing the Tee into angled sections, as called for by the Needham method, would be difficult without splitting the web. Therefore, the Gerard method is illustrated here. The Needham method will not be used in the example problem.

Since the web and stringer cap are of different thicknesses, an equivalent thickness must be used. This $\bar{t}_w$ is obtained from equation (17).

$$\bar{t}_w = \frac{\Sigma b_i t_i}{\Sigma b_i} \ .$$

To include the stringer land thickness with the rest of the effective skin, an average thickness is used. Figuring this average thickness in much the same way as $\bar{t}_w$; $t_{av} = 0.1318$ in.

$$\bar{t}_w = \frac{2(0.625)(0.125)+2(0.4895)(0.1)+2(2.14)(0.1318)}{(2(0.625)+2(0.4895)+2(2.14))} = 0.1257 \text{ in} \ .$$

Then from equation (16), the stringer crippling stress with g equal to (7) (six flanges, one cut) is:

$$F_{cs} = 51{,}000(0.5346)\left[\frac{7(0.1257)(0.1318)}{0.8183}\sqrt{(10.8\times10^6/51{,}000)}\right]^{0.85} = 50{,}464 \text{ lb/in}^2 \ .$$

β_g is determined by interpolation from table 1. Table 2 shows the stringer crippling stress of a stiffened panel to be less than or equal to the proportional limit stress. Therefore,

$$F_{cs} = 50{,}464 \text{ lb/in}^2 \ .$$

3. Column Failure

Calculation of the column buckling stress involves the stringer properties as well as the effective skin width. Using equation (27) with the stress in the stringer equal to 31,503.6 lb/in^2, the effective width is:

$$w = 1.7t(E/F_{st})^{0.5} = 4.28 \text{ in} .$$

The properties of the stringer and the effective skin width are then combined to form a wide column. From appendix A, the stringer meets the requirements for a Tee section, and the effective width is simply the 4.28 in centered under the Tee web. The moment of inertia of the stringer and its effective skin is 0.131 in^4. The end-fixity coefficient used in calculation of L' was 1.5, yielding an L' of 32.7 in. Inspection of the L'/ρ value for this panel reveals that it lies in the Euler buckling region. The resulting critical wide column buckling stress is:

$$F_c = \frac{\pi^2 10.8\times10^6}{(32.7/0.3997)^2} = 15{,}926 \text{ lb/in}^2 .$$

Note that many values used in hand calculations are taken from the worksheet. Small differences may arise between those numbers and the ones shown here due to round-off errors not as prevalent in the worksheet.

Had the L'/ρ value fallen in the transition region, the Johnson-Euler equation would have been applied as shown below:

$$F_c = 51{,}000 - \frac{(51{,}000)^2}{4\pi^2(10.8\times10^6)}\left(\frac{32.7}{0.3997}\right)^2 = 10{,}169 \text{ lb/in}^2 .$$

It is useful here to review the transition from Johnson-Euler to Euler equations. This transition occurs at the intersection point of the two curves when plotted as functions of L'/ρ. This critical L'/ρ value is found by setting the two equations equal to one another. If the L'/ρ value is larger than the critical value, use Euler. If the L'/ρ value is smaller than the critical value, use Johnson-Euler.

$$(L'/\rho)_{\text{critical}} = \pi\sqrt{\frac{2E_c}{F_{cs}}} .$$

In this case, the critical value is L'/ρ = 64.65 in.

D. Skin

1. Compression Buckling

For compression buckling of the skin, equation (26) is applied with b equal to the stringer spacing and t the skin thickness. The buckling coefficient value most used by Bruhn and others is the conservative 4.0. The compression modulus for Al 2219 is 10.8×10^6 lb/in^2, and u is 0.33.[11]

$$F_c = \frac{k_c \pi^2 E}{12(1-\upsilon^2)} \left(\frac{t}{b}\right)^2 .$$

Using 4.0 for k_c yields a critical stress of 5,395 lb/in^2, a very low number indeed. Buckling stresses are typically well below the material limits ($F_{tu\ 2219\text{-}T87}$ = 63,000 lb/in^2). If a less conservative approach is taken and k_c is chosen from figure 12 (k_c = 5.80), the critical stress increases to 7,823 lb/in^2—a considerable increase.

Using data for curved simply supported sheets, and reading k_c from figure 13 with Z = 5.311, the buckling coefficient is 5.09. The buckling stress is then computed to be 6,861 lb/in^2. Note here that the buckling coefficient, and therefore, the buckling stress for the curved panel, is lower than that of the flat sheet. The reason for this apparent contradiction is that the curved panel data is for simply supported edges, while that for the flat sheet is for an edge condition between simply supported and clamped. It is recommended that the maximum of the flat sheet and curved panel buckling coefficients be used. Therefore, the critical compression buckling stress for the skin between the stringers is 7,823 lb/in^2.

2. Shear Buckling

Shear buckling is handled much the same way as compression buckling. The buckling coefficients are read from figures 14 (flat sheet) and 15 (curved panels). The buckling coefficient for flat sheet is 5.8, while for curved panels it is read as 6.05 (simply supported curved panels). The resulting shear buckling stress is 8,160 lb/in^2. Again, the maximum is used and the critical shear buckling stress is reported as 8,160 lb/in^2.

E. Example Summary

Presented, thus far, in this report have been the basic methods of stringer stiffened panel design as presented by Dr. Bruhn, Mr. Almroth, and others. The methods used have been gleaned from many portions of Bruhn's books, books and papers by Almroth, and various other government and journal publications. It should be noted that the methods presented by Bruhn were devised before the advent of modern computational devices. Therefore, most of the methods rely on design curves to lessen the computational intensity of the problem. However, curve fitting of the appropriate design curves can make the methods acceptable to modern programming techniques. All necessary design curves have been curve fit and programmed as macro routines in the Excel programs contained in the appendices. Despite their age, many of the methods compiled by Bruhn and presented in this section are still used extensively in the aerospace structural design field. An engineer must understand and know how to apply these methods before exploring the more recent works. A summary of analysis results is shown in table 3.

Table 3. Summary of critical stresses.

Failure Mode		Critical Stress (lb/in^2)
General Instability		
	Shanley	45,000
	Becker	32,087
Local Crippling		51,000
Column Failure		15,926
Skin Compression		7,823
Skin Shear		8,160

The following section of this report details the determination of applied stress and margins of safety. Obviously, the applied stress cannot be determined independently of certain failure modes already covered in detail. Discussion of applied stresses and margins of safety is placed in a separate section for organizational purposes only—to separate failure analysis techniques from applied stress calculations.

IV. APPLIED STRESSES AND MARGINS OF SAFETY

This section of the report provides a summary and explanation of loads and stresses applied to the example configuration. Determination of the compressive stress level at limit load is fairly simple—all the skin is effective and moment of inertia and area calculations are quite simple since they involve the stringers and the entire skin rather than portions that are "effective" and portions that are "ineffective." Equations (36) and (37) are used to determine the maximum compressive stress level.

$$\sigma_b = \frac{MR}{I} , \tag{36}$$

$$\sigma_{ax} = \frac{P}{A} . \tag{37}$$

Determination of compressive stress at ultimate load can be considerably more involved. If the skin has not failed at ultimate load, the stress calculation procedure is identical to that for limit stresses. If the skin has failed however, the process becomes a bit more cumbersome. Ultimate stresses in the shell can be calculated through several methods with varying degrees of accuracy. The most accurate calculation, of course, requires the most rigorous analysis of the shell configuration. If the skin buckles after limit load, the effective skin provides some stress relief to the stringer columns. This stress relief depends upon the effective width of skin acting with the stringer. The effective width is dependent on the stress. One can easily visualize the iterative process necessary for determining stress and effective width. Shear stress calculations are performed at the limit load level and the procedure is quite well known. Calculation of the shear stress is shown in detail in appendix B.

Figure 18 gives details of the configuration to be evaluated.[17] The worst case mechanical loads (limit loads) on the shell are shown in the following and are typical of launch vehicle ground wind induced loads.[18]

Shear =	53,678 lb
Moment =	4.924×10^{7} in-lb
Axial =	1,571,825 lb .

This particular load case occurs during the prelaunch phase, and the vehicle at this point is considered as being fully fueled and unpressurized.

A. Simplified Beam Theory

The most accurate determination of stress involves calculation of an initial stress estimate using the entire skin as effective. Since the bending moment acts in compression on one side of the neutral axis and in tension on the other, the neutral axis of the shell will be shifted toward the tensile moment side of the shell. The neutral axis shift is a direct result of increased effective width at the lower compressive stress.

Figure 20 illustrates the iterative shift in neutral axis caused by the application of bending moment.[5]

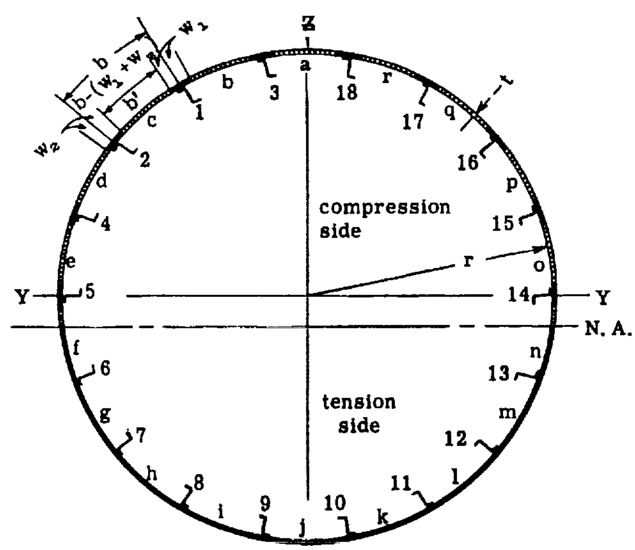


Figure 20. Illustration of neutral axis.

The procedure begins with an estimate of stress using the moment of inertia and area including the entire skin. At this point the stress in each bay must be calculated individually along with an accompanying effective width. When this is done, a new area, moment of inertia, and neutral axis is computed using only the stringer, its effective skin, and A_e as defined below. Including this area (A_e) can be viewed as having the same effect as the reduction in load presented by the NASA Structures Manual. Stress in each bay is then computed with the new cross-sectional properties. This procedure is repeated until the neutral axis location converges.

$$A_{e_i} = (b_{st} - \tfrac{1}{2}(w_{e_{i+1}} - w_{e_i}))\left[\frac{\sigma_{CR_i}}{\sigma_{st_i}}\right] . \qquad \text{(ref. 5)} \qquad (38)$$

The symbols w_{ei} and w_{ei+1} refer to the effective widths on either side of the stringer location where A_e is desired. Since the effective width changes very little between adjacent stringers on a large diameter cylinder, equation (38) can be rewritten as equation (39):

$$A_{e_i} = (b_{st} - w_{e_i}) \left[\frac{\sigma_{CR_i}}{\sigma_{st_i}} \right] . \tag{39}$$

The stress in each bay may be calculated on the stringer location by:

$$\sigma_{b_i} = \frac{My_i}{I} \quad \text{and} \quad \sigma_{ax_i} = \frac{P}{A_i} .$$

The y_i in this equation refers to the stringer distance from the centroid.

B. ET Stress Report

This method is referenced from the ET stress report produced for NASA by the Martin Marietta Corporation. The method in the stress report lacks a great deal of referencing, but has been used successfully in the analysis of the Space Transportation System ET. The main advantage of this method is that it is much simpler, requiring iterations only with the maximum stress in the shell. That is, stresses at each station about the circumference of the shell do not have to be calculated. This greatly reduces the computational intensity of the process. Also, this method allows inclusion of hoop stresses in determining the longitudinal stress. This feature could be included in the simplified beam theory method with a bit of derivation. The disadvantage of the method is its conservatism. Choosing the proper method depends on the analyst's expectations of fidelity in the analysis, confidence in the given load set, and ultimately, the cost of failure.

Using this method, a more detailed stress breakdown can be obtained for the skin, stringer, and stringer land. The equations for normal stresses in the skin, land, and stringer are presented along with figure 21 for explanation.[4]

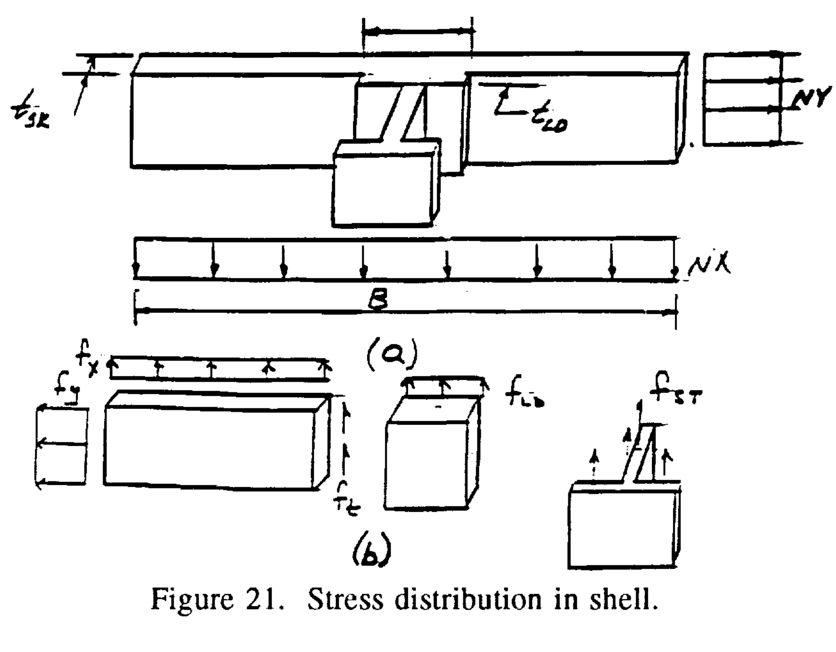


Figure 21. Stress distribution in shell.

$$f_{sk} = \frac{1}{t_{sm}}\left[N_x + \upsilon N_y\left(\frac{t_{sm}-t_{sk}}{t_{sk}}\right)\right] ,$$

$$f_{ld} = \frac{1}{t_{sm}}\left[N_x + \upsilon N_y\left(\frac{t_{sm}-t_{ld}}{t_{ld}}\right)\right] ,$$

$$f_{st} = \frac{1}{t_{sm}}\left[N_x - \upsilon N_y\right] ,$$

$$f_y = \frac{N_y}{t_{sk}} .$$

The line loads (N_y and N_x), in pounds per inch, are determined based on stresses developed in a thin cylindrical shell by internal pressure and axial and bending loads, respectively:

$$N_y = pR \quad \text{and} \quad N_x = \frac{\text{Axial}}{2\pi R} \pm \frac{\text{Moment}}{\pi R^2} ,$$

and t_{sm} is the smeared thickness of the stringer, land, and skin thickness so that,

$$t_{sm} = \frac{A_{st}+A_{ld}+A_{sk}}{b_{st}} .$$

Stress in the stringer is now determined based on the applied line loads and the shell geometry. Now, providing the skin buckles at F_{cr}, the longitudinal load capability of the skin panel is $0.9*F_{cr}*t_{sk}$, or 90 percent of its buckling load. This capacity of the buckled skin to carry load reduces the compressive load that the stringer must support, just as A_e adds to the load carrying capacity of the shell. The effective width acting with a stringer is calculated by equation (27). This varies somewhat from the ET stress report method for determining effective width, but should give comparable results. Once the effective width is known, the load carried by the ineffective skin is determined and subtracted from the total load applied. Line loads are converted to forces in each panel by multiplying by a characteristic length—either stringer spacing or effective width depending on whether or not the skin has failed. The new load that must be carried by the stringer is now known and the process repeats itself until convergence is obtained on the effective width or the stress.

C. Margins of Safety

Table 4 is a summary of stresses produced using the ET stress method and simplified beam theory. As shown in the stress table, the shear stress is very low compared to the compressive stresses resulting from bending and axial load. Shear stress is often ignored for preliminary analyses.

Table 4. Applied stresses.

	Limit (lb/in^2)	Ultimate (lb/in^2)
Shear	819.36	
Moment	3,770.4	
Axial	9,959.5	
Total Compressive	13,729.9	31,503.6 (ET) 24,645.7 (Beam)

The margin of safety (MS) is a numerical evaluation of a structure's load carrying capacity compared to the applied load. In general, when there is only one type of loading, the MS calculation takes the following form:[5]

$$MS = \frac{\text{Allowable Stress or Load}}{\text{Applied Stress or Load}} - 1.0 \ . \tag{40}$$

Evaluating the general instability equations first, the critical stress determined using the Shanley method is 45,000 lb/in^2. However, 45,000 is greater than the yield stress for the ring of 38,000 lb/in^2. Therefore, the capability of the frame is limited to the yield stress. The applied ultimate stress is 31,503.6 lb/in^2. The following MS calculation results:

$$MS = \frac{38{,}000}{31{,}504} - 1.0 = 0.2062 \ .$$

For the stringer and its effective skin, the wide column buckling allowable is lower than the stringer crippling value. Therefore, the wide column value is used to determine the minimum MS. The applied compressive stress is 31,503.6 lb/in^2, and the critical buckling stress is 15,917 lb/in^2. The resulting margin of safety is –0.495. (The shear load is ignored for the wide column margin of safety because of its relative insignificance.) Obviously the column is inadequate for the applied loads and must be redesigned.

The skin is under combined shear and compressive load. The margin of safety under combined shear and compression loading from reference 5 is,

$$MS = \frac{2}{R_c + \sqrt{R_c^2 + 4 * R_s^2}} - 1 \ , \tag{41}$$

where R_c is the applied compressive stress divided by the critical buckling or allowable stress. R_s is the applied shear stress divided by the allowable shear stress in the skin. The MS for the skin is determined using limit loads. The resulting MS for the skin is –0.4316, with $R_c = 1.7551$ and $R_s = 0.0874$. The shear stress contribution to MS is practically negligible. This is typical in launch vehicle design. As with the column buckling MS, the skin is inadequate for the loads applied and must be redesigned.

V. CONCLUSIONS

In this report, many of the most popular methods for determining buckling capability in a ring and stringer-stiffened cylindrical shell have been presented. Methods for determining the skin buckling load, the stringer failure allowable, and the wide column allowable have been presented. Two methods for determining the necessary ring geometry to preclude general instability have also been presented. Where applicable, the conservative approach taken by most designers has been pointed out, along with methods for reducing unnecessary conservatism.

Microsoft Excel spreadsheets have been developed in conjunction with this report to facilitate the use of the methods presented. All design curves necessary for calculation of critical buckling allowables have been curve fit and included in the worksheet as macros which act as subroutines for calculation of buckling coefficients from the curves. Calculation sheets done by hand have also been included in the appendices. The most common problem facing the analyst in this area is the proper designation of sectional properties, i.e., when to include effective widths and when not to. Considerable effort has been made to track down any discrepancies of various reports on the use of sectional properties. Where clear instructions were not given by the author, the most logical option regarding the use of sectional properties was chosen.

Although limited in its scope of application, a method for determination of the bifurcation (classical) buckling load has been included. This method cannot be used in cases where considerable redistribution of stress occurs in the subcritical load range—as happens with the example problem presented. A Fortran program has been written to facilitate determination of the buckling allowable using the methods devised by Almroth.

The report also addresses determination of the applied stress and MS. Neither the applied stress nor the buckling capability of the shell can be determined independent of one another. Recognize that the MS is the measure of structural margin most often used in conjunction with the safety factor.

This report has been designed to serve as a reference for the analyst in need of determining the buckling capability of stiffened cylindrical shells. The methods are easily modified for use with stiffeners other than the integral Tee's used for the example analysis. However alterations to the programs would be necessary. Critical stress calculations for the various methods are given in the following chart:

Summary of Critical Stresses

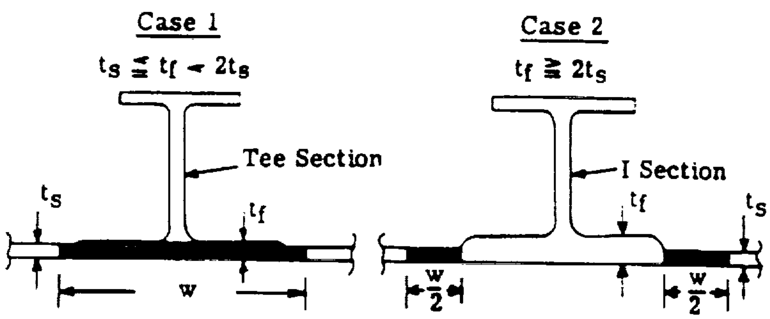
Failure Mode		Critical Stress (lb/in^2)
General Instability		
	Shanley	38,000
	Becker	30,930
Local Buckling		51,000
Crippling (NSN)		51,000
Crippling (Gerard)		50,464
Column Failure (Euler)		15,917
Skin Compression		7,823
Skin Shear		8,160

The methods presented in this report are not meant to exclude or replace the use of analysis codes such as BOSOR or PANDA. Neither have all possible failure modes for tank design been addressed. The report has addressed the major failure modes associated with stiffened circular cylinders and provides the methods necessary for assessing vehicle design. Once the major geometric properties of a design have been established, more rigorous analysis using further refined hand techniques and computer algorithms would be in order.

Also not covered in this report are the optimization techniques. Considerable work has been done in the optimization of stiffened shells area. Most of the methods are mathematically derived, rather than being based on empirical data as most of Bruhn's methods are. It is suggested that the engineer first become familiar with the methods presented in this report, and then investigate the various optimization techniques. An optimized design will likely need to be checked against the methods presented in this paper to ensure the capability of the design.

APPENDIX A

Tee Stringer Criteria



$$w_e = 1.7t(E/F_{st})^{1/2}$$

$$t = (t_s+t_f)/2$$

APPENDIX B

Hand Calculations

General Instability

Shanley

$$(EI)_f = C_f \frac{MD^2}{L} \qquad \text{[Bruhn, "Aircraft" equation (9.7)]}$$

$$C_f = 1/16{,}000$$

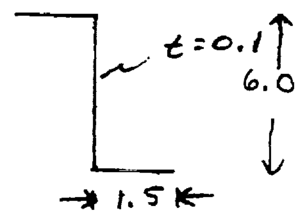
Solving for $M \Rightarrow M_{CR} = \dfrac{(EI)_f L}{C_f D^2}$;

FRAME = 2024 Al

$E_f = 11.0E6$ in-lb
$I_f = 4.2369\ \text{in}^4$

$$M_{CR} = \frac{(11.0E6)(4.2369)(40.08)}{(1/16{,}000)(331)^2}$$

$$M_{CR} = 2.7279\times10^8 \text{ in–lb}$$



Becker

$F_{CR} = CE_cQ_b$ — $C = 3.25$ Frame not attached to skin

[NACA TN 3786: pg. 33] — $C = 4.80$ Frame attached to skin.

$$Q_b = \frac{(\rho_s \rho_f)^{3/4}(bd)^{-0.25}}{R}$$

ρ_s = radius of gyration of stringer and effective skin determined by: $\dfrac{w_{e_{st}}}{b} = 0.5\ SQRT\left(\dfrac{F_{CR}}{F_{ST}}\right)$

$$w_{e_{st}} = 0.5\ SQRT\left(\frac{19{,}904}{29{,}979}\right)^{10.832}$$

F_{CR} = Buckling stress of column

$$w_{e_{st}} = 4.41 \text{ in}$$

F_{st} = Stress in stringer

$A_{eff_{st}} = 0.810\ \text{in}^2 \quad I_{eff_{st}} = 0.171\ \text{in}^4$

ρ_f = radius of gyration of frame and effective skin.

$$\rho_{st} = \sqrt{\frac{I}{A}} = 0.4595\ in$$

$w_{ef} = d$

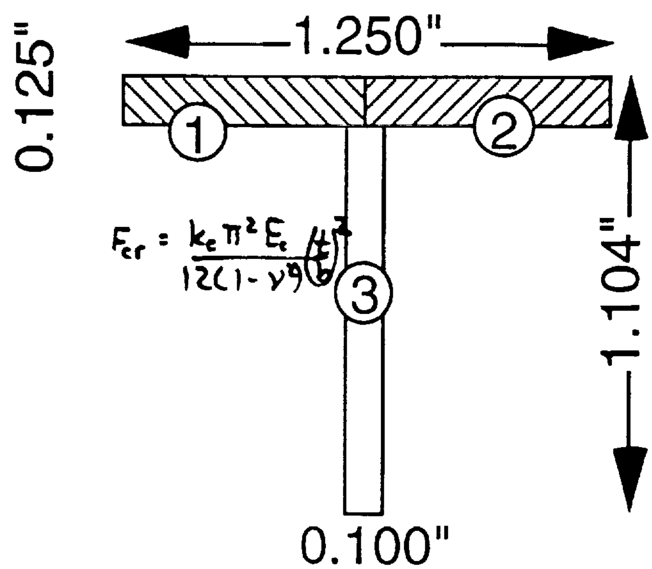
$$Q_b = \frac{[(0.4595)(1.3789)]^{3/4}[(10.832)(40.08)]^{-1/4}}{165.5} = 0.000940$$

$$F_{CR_{FRAME}} = (3.25)(11.0E6)(0.00094) = 33{,}608.12\ \text{lb/in}^2\ .$$

*Note that moment of inertia of frame is computed without the large stand-off caused by mounting on top of the intergral stringers.

Stringer Local Elastic Buckling

$$F_{CR_{1\&2}} = \frac{k_{c_1}\pi^2 E_c}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2$$



$k_c = 0.43$

$t = 0.125$ in

$b = \frac{1.25}{2} = 0.625$ in

$$F_{CR} = \frac{0.43\,\pi^2\,(10.8E6)}{12(1-0.33^2)}\left(\frac{0.125}{0.625}\right)^2$$

$= 171{,}452$ lb/in^2

$$F_{CR_3} = \frac{4.0\,\pi^2\,E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \qquad k_c = 4.0$$

$t = 0.1$ in

$b = 1.104 - \frac{0.125}{2} = 1.0415$ in

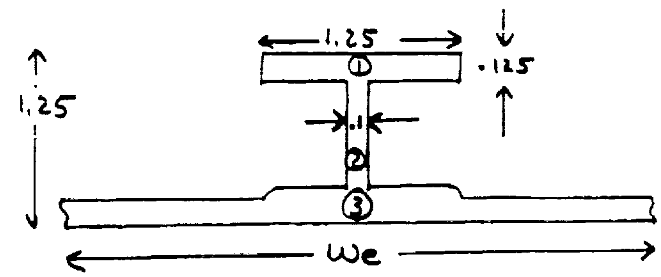
$$F_{CR_3} = \frac{4.0\,\pi^2\,(10.8E6)}{12(1-0.33^2)}\left(\frac{0.1}{1.0415}\right)^2 = 367{,}584 \text{ lb/in}^2$$

Minimum $F_{CR} > F_{CY}$ $\therefore$ $F_{CR} = F_{CY}$

Local Buckling Limit = 51,000 lb/in^2

Stringer Crippling

*NASA Structures Manual



$$\left(\frac{f_{ccn}}{F_{cy}}\right)_{\text{NO EDGE FREE}} = \frac{1.387194}{\left\{\left[\frac{F_{cy}}{E_c}\right]^{1/2}\left(\frac{b_n}{t_n}\right)\right\}^{0.8071793}}$$

Curve fits from NASA Structures Manual C1.3.1–13

$$\left(\frac{f_{ccn}}{F_{cy}}\right)_{\text{ONE EDGE FREE}} = \frac{0.5693108}{\left\{\left[\frac{F_{cy}}{E_c}\right]^{1/2}\left(\frac{b_n}{t_n}\right)\right\}^{0.8127115}}$$

Corresponding to those in ET stress report.

$$\left(\frac{f_{cc_1}}{F_{cy}}\right) = \frac{0.5693108}{\left\{\left[\frac{51{,}000}{10.8E6}\right]^{1/2}\left(\frac{0.575}{0.125}\right)\right\}^{0.8127115}} = 1.4516$$

$$f_{cc_1} = (1.4516)(51{,}000) = 74{,}030.83 > 51{,}000 \quad \therefore \quad f_{cc_1} = 51{,}000 \text{ lb/in}^2$$

$$\left(\frac{f_{cc_2}}{F_{cy}}\right) = \frac{1.387194}{\left\{\left[\frac{F_{cy}}{E_c}\right]^{1/2}\left(\frac{0.979}{0.1}\right)\right\}^{0.8071793}} = 1.91 > 1.0 \qquad \therefore f_{cc_2} = 51{,}000 \text{ lb/in}^2$$

$$\left(\frac{f_{cc_3}}{F_{cy}}\right) = \frac{1.387194}{\left\{\left[\frac{F_{cy}}{E_c}\right]^{1/2}\left(\frac{2.09}{0.1318}\right)\right\}^{0.8071793}} = 1.29 > 1.0$$

$$t_{av} = \frac{4.28(0.126)+1.25(0.02)}{4.28}$$

$$t_{av} = 0.1318$$

$$f_{cc_3} = 51{,}000$$

Stringer Crippling

$$F_{cc} = \frac{\sum b_n t_n f_{cc_n}}{\sum b_n t_n}$$ [NASA Stress Manual, Section C1, page 11, equation (1)]

$$F_{cc} = 51{,}000 \; \{(0.575)(0.125)(2)+(0.979)(0.1)+2^*(2.14)(0.1318)\}+$$
$$\{(0.575)(0.125)(2)+(0.979)(0.1)+(2.14)(0.1318)(20)\}$$

$$F_{cc} = F_{cs} = 51{,}000 \text{ lb/in}^2$$

<u>Stringer Crippling</u>

<u>Gerard</u>:

$$w_e = 4.28 \ , \quad b_f = 1.25 \ , \quad b_w = 0.979$$

Using an average thickness to evaluate the effective skin with the land:

$$t_{av} = \frac{1.25(0.02)+4.28(0.126)}{4.28} = 0.1318$$

$$\bar{t}_w = \frac{\sum b_i t_i}{\sum b_i} = \frac{2(0.625)(0.125)+0.979(0.1)+4.28(0.131)}{2(0.625)+0.979+4.28}$$

$$\bar{t}_w = 0.1257 \text{ in}$$

$$A = 1.25\,(0.125)+0.979(0.1)+4.28(0.1318) = 0.8183 \text{ in}^2$$

$$\frac{\bar{t}_w}{t_{sk}} \Rightarrow \text{Interpolate}$$

$$\frac{\bar{t}_w}{t_{sk}} = \frac{0.1257}{0.1318} = 0.954$$

$$\frac{1.16-0.732}{0.954-0.732} = \frac{0.562-0.505}{\beta-0.505}$$

$$1.9279 = \frac{0.0570}{\beta-0.505} \Rightarrow \beta-0.505 = 0.0296$$

$$\underline{\beta = 0.5346}$$

$$\frac{F_{cs}}{F_{cy}} = \beta\left[(7)\,\frac{(\bar{t}_w)(t_{av})}{A}\left(\frac{E}{F_{cy}}\right)^{1/2}\right]^{0.85}$$

$$F_{cs} = 0.99\,F_{cy}$$

$$F_{cs} = 50{,}463.89 \text{ lb/in}^2 = 50{,}464 \text{ lb/in}^2$$

Column Buckling

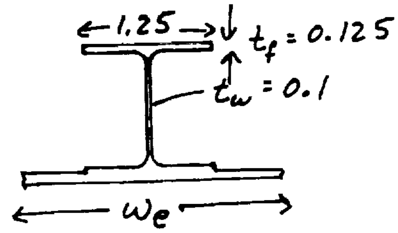
Determine Johnson-Euler Intersection

$$\left(\frac{L'}{\rho}\right)_{\text{INTERSECTION}} = \pi\left[\frac{2E_c}{F_{cs}}\right]^{1/2} = \pi\left\{\frac{(2)(10.8E6)}{51{,}000}\right\}^{1/2} = 64.65 \text{ in}$$

$$\rho_{\text{COLUMN}} = \left\{\frac{I}{A}\right\}^{1/2}$$

I and A = Sum of stringer and effective width.

$w_e = 4.28$ in



$$A_{\text{COLUMN}} = 0.254+1.25(0.02)+t_{sk}w_e$$

$$A_{\text{COLUMN}} = 0.82 \text{ in}^2$$

$$\text{Centroid} = \frac{\Sigma A_i d_i}{\Sigma A_i}$$

$$\text{Centroid} = [(1.25)(0.125)(1.1875)+(0.1)(0.979)(0.6355)+(1.25)(0.02)(0.136)$$

$$+ w_e t_{sk}(t_{sk}/2)] \div A_{\text{COLUMN}} = \qquad = 0.35 \text{ in}$$

$$I_{\text{COLUMN}} = \frac{1}{12}(1.25)(0.125)^3+1.25(0.125)(1.1875-0.33)^2+\frac{1}{12}(0.1)(0.979)^3+(0.1)(0.979)(0.6355-0.33)^2$$

$$+\frac{1}{12}(1.25)(0.02)^3+(1.25)(0.02)(0.136-0.33)^2+\frac{1}{12}w_e t_{sk}^3+w_e t_{sk}\left({}^{t_{sk}}/_2-0.33\right)^2$$

$$I_{\text{COLUMN}} = 0.131 \text{ in}^4$$

$$\rho = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.131}{0.82}} = 0.3997 \text{ in}$$

$$\frac{L'}{\rho} = \frac{40.08/\sqrt{1.5}}{0.3997} = 81.87$$

Column Buckling

Using an end-fixity coefficient of 1.5 and a ring spacing of 40.08 in, the effective length is 32.72 in. When divided by ρ, this length becomes the slenderness ratio.

$$\frac{L'}{\rho} = 81.83 > \left(\frac{L'}{\rho}\right)_{\text{INTERSECT}} \quad \therefore \text{ Column is in the Euler regime.}$$

Critical buckling load for the column is:

$$F_{CR} = \frac{\pi^2 E}{\left(\frac{L'}{\rho}\right)^2} = \frac{\pi^2(10.8E6)}{(81.83)^2} = 15{,}918.3 \text{ lb/in}^2 .$$

Skin Buckling

Compression Buckling of Flat Sheet

$$F_{CR} = \frac{k_c \pi^2 E_c}{12(1-\nu^2)} \left(\frac{t_{sk}}{b_{st}}\right)^2 \qquad \text{[Bruhn, "Aircraft," equation (C5.1)]}$$

From figure C5.6 with $\frac{b}{t} = \frac{10.832}{0.126} = 85.968$

$k_c = 5.802$. Could use 4.0 for conservatism.

$$F_{CR} = 5.802 \frac{\pi^2(10.8E6)}{12(1-0.33^2)} \left(\frac{0.126}{10.832}\right)^2 = 7{,}825.7 \text{ lb/in}^2$$

Compression Buckling of Curved Panel

$$F_{CR} = \frac{k_c \pi^2 E_c}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

From Bruhn, "Aircraft," figure C9.1 $\qquad Z = \frac{b^2}{Rt}\left(1-\nu^2\right)^{1/2}$

$$Z = \frac{10.832^2}{(165.5)(0.126)}\left(1-0.33^2\right)^{1/2} = 5.311 \quad \text{with} \quad R/Z = 1{,}313.49$$

From figure C9.1 $\qquad k_c = 5.09$

$$F_{CR} = \frac{5.09\,\pi^2\,10.8E6}{12(1-\nu^2)} \left(\frac{0.126}{10.832}\right)^2 = 6{,}860.5 \text{ lb/in}^2$$

Taking the larger — $F_{CR} = 7{,}825.7$ lb/in^2.

Skin Buckling

Shear Buckling of Flat Panels

$$\tau_{CR} = \frac{k_s \pi^2 E_c}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \qquad \text{[Bruhn, "Aircraft,"" equation (C5)]}$$

From figure C5.11 k_s for

$$\frac{a}{b} = \frac{40.08}{10.832} = 3.7 = 5.8$$

$$k_s = 5.8 \text{ (hinged edges)}$$

$$\tau_{CR} = \frac{5.8\ \pi^2 10.8E6}{12(1-0.33^2)} \left(\frac{0.126}{10.832}\right)^2 = 7{,}822.89 \text{ lb/in}^2$$

For simply supported curved panels,

$$k_s = 6.49$$

$$\tau_{CR} = 8{,}753 \text{ lb/in}^2 \ .$$

Applied Compressive Stress

- Stress at Limit Load: $S.F. = 1.0$

Limit loads are: moment = 4.924×10^7 in-lb

axial = 1,571,825 lb

$$\sigma_b = \frac{mz}{I} \qquad \sigma_A = \frac{P}{A} \;,$$

σ_b = bending stress
σ_A = axial stress
m = moment
z = distance from centroid
I = moment of inertia
P = axial load
A = area .

*At limit load, all skin is effective. Therefore,

$$I = (\text{number of stringers})\, I_{ST} + \Sigma A_{ST} Z^2 + \pi R^3 t_{sk} \;;\; \text{let } t_{sk} = \bar{t} \text{ to include land}$$

$$I = 96*0.0277 + 0.254\, \Sigma Z^2 + \pi(165.5)^3 \bar{t}$$

$$\bar{t} = \frac{(0.02)(1.25)+(0.126)(10.832)}{10.832} = 0.1283 \text{ in}$$

$$I = 2{,}161{,}079.1 \text{ in}^4$$

$$\text{Area} = 2\pi R\bar{t} + \Sigma A_{ST} = 2\pi(165.5)(0.1283)+(96)(0.254)$$

$$A = 157.79 \text{ in}^2$$

$$\text{Maximum bending stress} = \sigma_{b_{max}} = \frac{(4.924\times10^7)(165.5)}{2{,}161{,}079}$$

$$\sigma_{b_{max}} = 3{,}770.9 \text{ lb/in}^2$$

$$\text{Axial stress} = \sigma_A = \frac{1{,}571{,}825}{157.79} = \sigma_A = 9{,}960.93$$

$$\text{Total compressive stress} = \sigma_{b_{max}} + \sigma_A$$

$$\sigma_T = 13{,}731.83 \text{ lb/in}^2$$

Applied Compressive Stress

• Stress at Ultimate Load: $S.F. = 1.4$

Ultimate loads are: moment = $(4.924\times10^7)*1.4$ in-lb

axial = $(1{,}571{,}825)*1.4$ lb

Check Skin Buckling:

$$F_{CR_{sk}} = \frac{k_c \pi^2 E_c}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$ [ref. Bruhn, "Aircraft," (C5.1)]

k_c = max from curves C5.6 or C9.1

$$k_c = 5.8$$

$$F_{CR_{sk}} = \frac{5.8\pi^2(10.8E6)}{12(1-0.33^2)}\left(\frac{0.126}{10.832}\right)^2 = 7{,}822.89 \text{ lb/in}^2$$

If skin does not buckle, I and A remain the same as limit.

$$\sigma_b = \frac{(4.924\times10^6)(1.4)(165.5)}{2{,}161{,}079} = 5{,}279.26 \text{ lb/in}^2$$

$$\sigma_A = 13{,}945.3 \text{ lb/in}^2 = \frac{(1{,}571{,}825)1.4}{157.79}$$

$$\sigma_T = 19{,}224.56 \text{ lb/in}^2$$

13,945.3 > 7,822.89: • Skin buckles prior to ultimate load causing redistribution of stress.

* Note that $\sigma_{A_{\text{Limit}}}$ also > 7,822. This indicates a negative margin for skin buckling. The skin must be redesigned to prevent buckling at limit load.

<u>Applied Compressive Stress</u>

- The skin fails prior to ultimate load causing a redistribution of stress. The stresses at ultimate load must now be determined based on the effective widths acting with each stringer.

* <u>Simplified Beam Theory</u>

Step 1: An initial estimate of stress in each stringer bay is made based on the entire skin as effective.

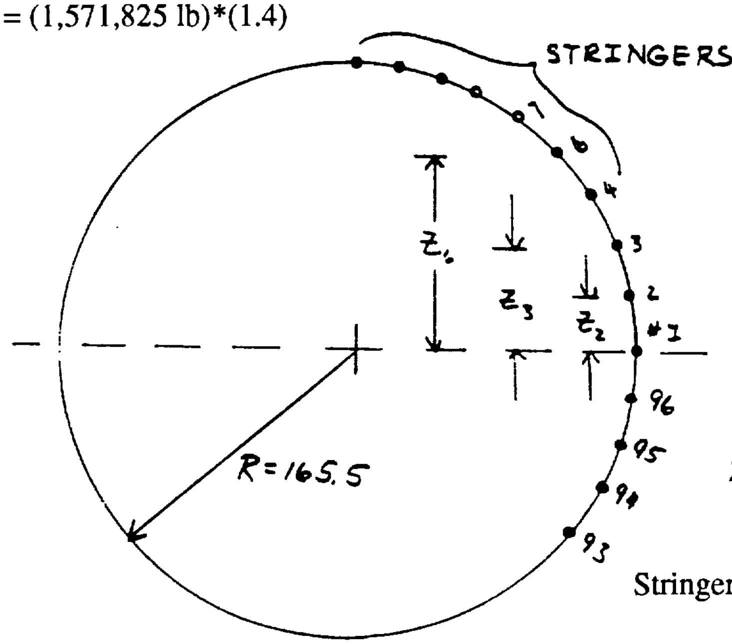
$I = 2{,}161{,}079 \text{ in}^4$

$A = 157.79 \text{ in}^2$

$m = (4.924\times10^7 \text{ in-lb})*(1.4)$

$P = (1{,}571{,}825 \text{ lb})*(1.4)$

Shell Geometry



$A_{ST} = 0.254 \text{ in}^2$
$I_{ST} = 0.0277 \text{ in}^4$

$Z = 0$, original neutral axis

Stringer Pitch Angle $= \frac{2\pi}{96} = 3.75°$

*Recall that 96 = number of stringers.

$z_1 = 0$
$z_2 = \sin(3.75)165.5 = 10.8242$ in
$z_3 = \sin(7.5)165.5 = 21.60$ in
$z_{96} = -10.8242$

Applied Compressive Stress

$$\sigma_A = \frac{1{,}571{,}825*1.4}{157.79} = 13{,}946.09 \text{ lb/in}^2$$

$$\sigma_{b_1} = 0 \;, \quad \sigma_{b_2} = \frac{(4.924\times10^7)(1.4)(10.8242)}{2{,}161{,}079} = 345.28 \text{ lb/in}^2$$

$$\sigma_{b_3} = \frac{(4.924\times10^7)(1.4)(21.6)}{2{,}161{,}079} = 689.02 \text{ lb/in}^2$$

$$\sigma_{b_{96}} = \frac{(4.924\times10^7)(1.4)(-10.8242)}{2{,}161{,}079} = -345.28 \text{ lb/in}^2$$

$$\sigma_1 = 13{,}946 \text{ lb/in}^2 \;; \quad w_{e_1} = 1.7\,(t_s+t_f)/2\sqrt{\frac{E_c}{\sigma_1}}$$

[ref. Bruhn, "Aircraft," equation C7.16]

$$w_{e_1} = 1.7(0.1360)\left\{\frac{10.8\times10^6}{13{,}946}\right\}^{1/2} = 5.9608 \text{ in}$$

$$A_{e_i} = (10.832-5.9608)(0.126)\left(\frac{7{,}822.89}{13{,}946}\right)$$

$$A_{e_i} = 0.3443 \text{ in}^2$$

$$\sigma_2 = 13{,}946+345.28 = 14{,}291.28 \text{ lb/in}^2$$

$$w_{e_2} = 5.8884 \text{ in} \;, \; A_{e_2} = 0.3410 \text{ in}^2$$

$$\sigma_3 = 13{,}946+689.02 = 14{,}635.0$$

$$w_{e_3} = 5.8188 \text{ in} \;, \; A_{e_3} = 0.3376 \text{ in}^2$$

$$\Downarrow$$

$$\sigma_{96} = 13{,}946-345.28 = 13{,}600.72$$

$$w_{e_{96}} = 6.036 \text{ in} \;, \; A_{e_{96}} = 0.3476 \text{ in}^2$$

* Note: $A_e = b't(F_{cr}/\sigma_i)$ (ref. Bruhn, "Aircraft," equation (2), page A20]

$b' = b_{ST} - w_e$

* Note: A_e is not $w_e * t_{sk}$!

Applied Compressive Stress

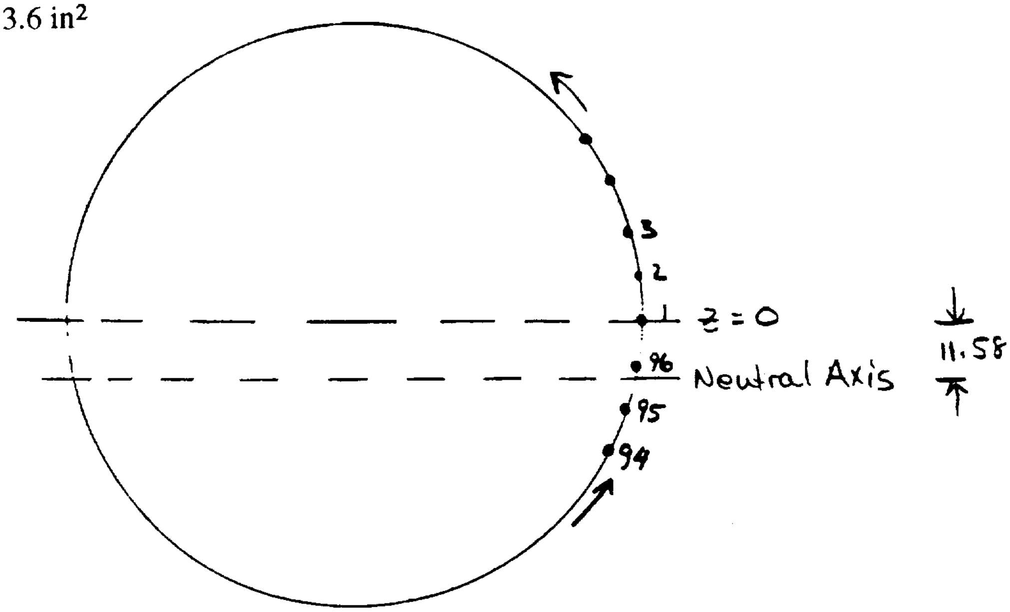
Step 2: Determine new centroid using A_e, $w_{e_i}*t_{av_i}$, and A_{ST}.

$$t_{av_i} \text{ is new } \bar{t} = \frac{(0.02)(1.25)+(0.126)(w_{e_i})}{w_{e_i}}$$

New centroid = −11.58 in

New moment of inertia = 1,859,548 in^4

New area = 133.6 in^2



Step 3: Recalculate stresses, A_e and w_e based on new sectional properties.

$$\sigma_1 = \frac{(4.924\times10^7)(1.4)(0+11.58)}{1,859,548} + \frac{(1,571,825)(1.4)}{133.6}$$

$$\sigma_1 = 429.28+16,471.2 = 16,900.5 \text{ lb/in}^2$$

$$\sigma_2 = \frac{(4.924\times10^7)(1.4)(10.8242+11.58)}{1,859,548} + 16,471.2$$

$$\sigma_2 = 17,301.7 \text{ lb/in}^2$$

$$\Downarrow$$

$$\sigma_{96} = \frac{(4.924\times10^7)(1.4)(-10.8242+11.58)}{1,859,548} + 16,471.2$$

$$\sigma_{96} = 16,474.0 \text{ lb/in}^2$$

Applied Compressive Stress

Step 4: With new stresses, A_e and w_e, recalculate neutral axis location, moment of inertia, and area.

Step 5: Repeat until convergence on neutral axis location is obtained.

Final Results:

Moment of inertia = 1,713,065.85 in^4

Area = 124.185 in^2

Neutral axis or centroid = −11.654 in

$$\sigma_{max} = \sigma_{25} = 24{,}645 \text{ lb/in}^2$$

$$w_{e_{25}} = 4.84 \text{ in.}$$

Applied Compressive Stress

External Tank Stress Method

Step 1: Determine initial line loads.

$$N_x = \frac{1{,}571{,}825*1.4}{2\pi R} + \frac{(4.924\times10^7)(1.4)}{\pi R^2}$$

$$N_x = 2{,}917.3 \text{ lb/in}$$

$$N_y = \frac{(\text{Pressure})(R)}{2} \;\; ; \text{ Pressure} = 0,\ N_y = 0$$

Step 2: Determine initial stress based on line load and smeared thickness.

$$f_{sk} = \frac{1}{t_{sm}}\left[N_x + \nu N_y\left(\frac{t_{sm}-t_{sk}}{t_{sk}}\right)\right]$$

$$f_{sk} = \frac{2{,}917.3}{t_{sm}}$$

$$t_{sm} = \frac{A_{st}+A_{ld}+A_{sk}}{b_{st}} = \frac{0.254+1.25(0.02)+b_{st}t_{sk}}{b_{st}}$$

$$t_{sm} = 0.1518 \text{ in}$$

$$f_{sk} = \frac{2{,}917.3}{0.1518} = 19{,}223.49 \text{ lb/in}^2$$

Step 3: Calculate effective width.

$$w_e = 1.7t\sqrt{\frac{E_c}{f_{sk}}} = 1.7(0.136)\left\{\frac{10.8E6}{19{,}223.49}\right\}^{1/2}$$

$$w_e = 5.48 \text{ in}.$$

Applied Compressive Stress

Step 4: Determine new loads in the column.

The total equivalent axial load carried by the panel is,

$$p_{eq} = 2{,}917.3*10.832 = 31{,}600.2 \text{ lb}$$

The load carried by the buckled skin is,

$$P_{sk} = 0.9\, F_{cr}\, t_{sk}\, (10.832 - w_e)\ ;\ w_e = 5.48$$

$$P_{sk} = 4{,}764.9 \text{ lb}\ .$$

Load which must be supported by the column is

$$31{,}600 - 4{,}764.9 = 26{,}835 \text{ lb}\ .$$

The resulting line load, t_{sm}, and stress are:

$$N_x' = \frac{26{,}835}{w_e} = 4{,}896.9 \text{ lb/in}$$

$$t_{sm}' = \frac{0.254 + 1.25(0.02) + t_{sk} w_e}{w_e}$$

$$t_{sm}' = 0.1769 \text{ in}$$

$$f_{sk}' = \frac{4{,}897}{0.1769} = 27{,}680.36 \text{ lb/in}^2\ .$$

Step 5: Continue iteration until convergence on stress is reached.

The final results are:

$$w_e = 4.28 \text{ in}$$

$$t_{sm}' = 0.1912 \text{ in}$$

$$N_x' = 6{,}020.26 \text{ lb/in}$$

$$f_{sk} = 31{,}488 \text{ lb/in}^2 \approx 31{,}503 \text{ lb/in}^2 \text{ from spreadsheet.}$$

APPENDIX C

Spreadsheet Output

Skin-Stringer Spreadsheet

T Stringers

No st =	96.0
Total Height =	1.25 in
b f =	1.25 in
t f =	0.13 in
t w =	0.10 in
Stringer Land =	0.02 in
bw =	0.98 in
b st =	10.83 in
Height of T =	1.12 in
A st =	0.25 in^2
Centroid =	0.83 in
I st =	0.03 in^4
ρ =	0.32 in

Skin

t sk =	0.126 in
Skin Radius =	165.50 in

Z Intermediate Frames

Barrel Length =	240.45 in
No Frames =	5.00 in
Overall Height =	6.00 in
b f =	1.50 in
t w =	0.10 in
d =	40.08 in
A f =	0.88 in^2
Cf =	3.13 in
I f =	4.24 in^4
Aeff =	5.93 in^2
Ceff =	0.52 in
Ieff =	11.3 in^4

Load Type	Value	
Shear (lbs)	53,678 lbs	
Moment (in-lbs)	49,240,000 in-lbs	
Axial (lbs)	1,571,825 lbs	
Shear Stress =	819.4 psi	
Comp Stress =	13,729.9 psi	31,503.6 psi
Stress Method =	1	

Margin of Safety Summary	
General Instability	
Shanley	0.206
Becker	(0.019)
Stinger Crippling	0.619
Local Buckling	2.715
Wide Column	(0.495)
Sheet Buckling	(0.432)

S.F.Yield =	1.00
S.F. Ultimate =	1.40

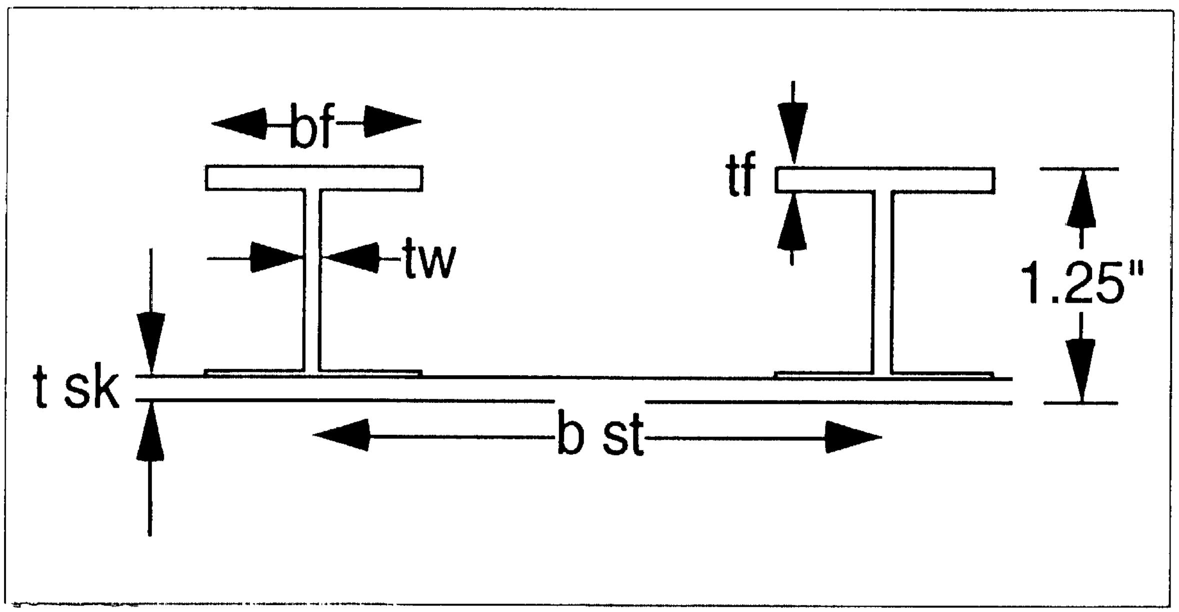
Skin-Stringer Spreadsheet

Stringer Crippling		Wide Column Buckling		Sheet Buckling		General Instability	
				Flat Sheet		Shanley	
tav =	0.132 in	we =	4.28 in	kc =	5.80	Mcr =	2.73E+08
fccf =	51000 psi	Aeff =	0.82 in^2	Fc =	7826 psi	Fccr =	38,000 psi
fccw =	51000 psi	Centroid =	0.35 in	ks =	5.82		
fccwe =	51000 psi	Ieff =	0.131 in^4	Ts =	7851 psi	Becker	d^2/Rt < 100. * Invalid *
Fcc =	51000 psi	End Fixity Coeff =	1.50			C =	3.25
		Slenderness Ratio =	81.83	Curved Panel		Qb =	0.00086454
Elastic Buckling		Critical Slend. Ratio =	64.65	Z =	5.31	Fccr =	30907 psi
				kc =	5.09	Stringer Properties (For Becker Only)	
fccf =	51000psi	Fcr =	15917 psi	Fc =	6860 psi	we =	3.85 in
fccw =	51000psi		(Euler)	ks =	6.05	Aeff =	0.74 in^2
				Ts =	8167 psi	Cntrd =	0.326 in
Fcc =	51,000 psi					Ieff =	0.125 in^4
				Rc =	1.754		
				Rs =	0.104		

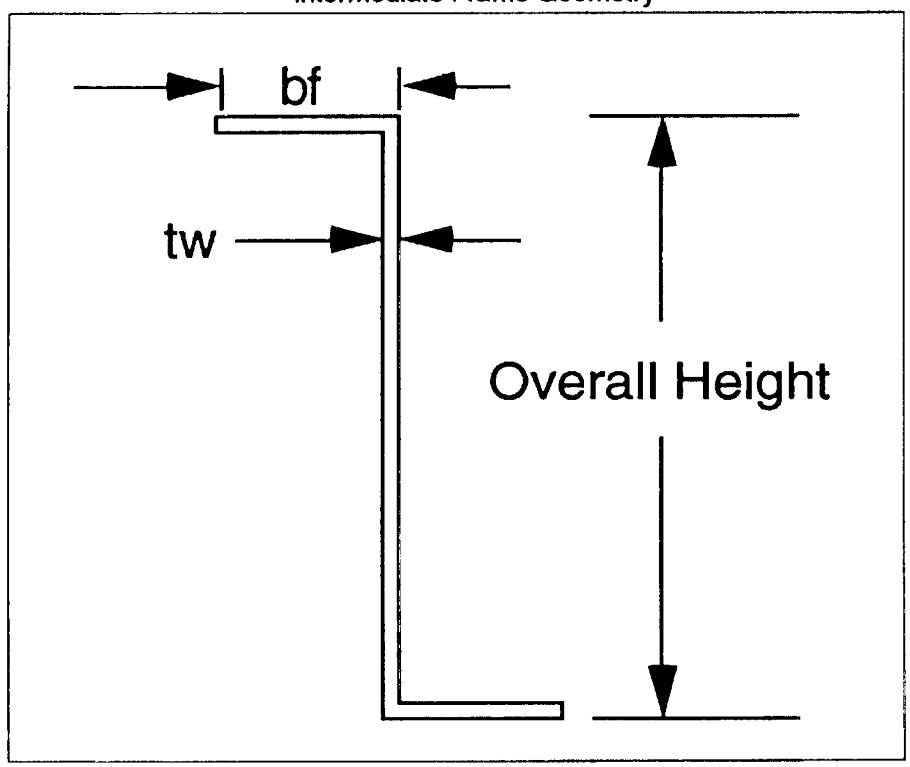
Material Properties							
Stringers & Skin				Frames			
Ftu =	63000psi	E =	10.5E+6 psi	Ftu =	57000 psi	E =	10.8E+6 psi
Fty =	51000psi	Ec =	10.8E+6 psi	Fty =	38000 psi	Ec =	11.0E+6 psi
Fcy =	51000psi			Fcy =	38000 psi		
density =	.10 lb/in^3			density =	.100 lb/in^3		
Poisson=	0.33						

Skin-Stringer Spreadsheet

Stringer Geometry



Intermediate Frame Geometry



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=OPEN("CLOSER")
=HIDE()
=OPEN("BruhnFig")
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=OPEN("Interpolate",1)
=HIDE()
=OPEN("Stress",1)
=HIDE()
=OPEN("Skin-Stringer")
=ACTIVATE("Control Macro")
=HIDE()
=RETURN()
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```
Moi1
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=ARGUMENT("Nost")
=ARGUMENT("Ast")
=ARGUMENT("Ist")
=ARGUMENT("tsk")
=ARGUMENT("bf")
=ARGUMENT("tl")
bst=2*PI()*Rad/Nost
DA=2*PI()/Nost
Ald= (tl+tsk)*bf
tav= (tsk*(bst-bf)+Ald)/bst
Itotal=0
moist=0
moisk= PI()*Rad^3*tav
=FOR("Count",1,Nost)
d=Rad*SIN((Count-1)*DA)
=SET.NAME("moist",moist + Ast*d^2)
=NEXT()

=SET.NAME("Itotal",moisk+moist+Nost*Ist)
=RETURN(Itotal)

Moi2
=ARGUMENT("Rad")
=ARGUMENT("Nost")
=ARGUMENT("Aeff")
=ARGUMENT("Ieff")
=ARGUMENT("tsk")
=ARGUMENT("bf")
=ARGUMENT("tl")

DA=2*PI()/Nost

Itotal=0
moist=0

=FOR("Count",1,Nost)
d=Rad*SIN((Count-1)*DA)
=SET.NAME("moist",moist + Aeff*d^2+Ieff)
=NEXT()

=SET.NAME("Itotal",moist)
=RETURN(Itotal)
```

```
Closer (c)
=SAVE()
=CLOSE()
=UNHIDE("BruhnFig")
=SAVE()
=CLOSE()
=UNHIDE("Interpolate")
=SAVE()
=CLOSE()
=UNHIDE("Control Macro")
=SAVE()
=CLOSE()
=UNHIDE("MOI")
=SAVE()
=CLOSE()
=UNHIDE("Stress")
=SAVE()
=CLOSE()
=UNHIDE("Closer")
=SAVE()
=CLOSE()
=RETURN()
```

```
FigC5.6
=ARGUMENT("BOT")
= ARGUMENT("Zappa")
=2.9384+0.088965*BOT-0.0013486*BOT^2+0.000012063*BOT^3-0.000000053153*BOT^4+0.000000000089722*BOT^5   Torsionally Weak
=4.1134+0.074296*BOT-0.001177*BOT^2+0.00001118*BOT^3-0.000000052865*BOT^4+0.000000000095262*BOT^5   Torsionally Strong

=IF(AND(AND(BOT>=15,BOT<=200),Zappa=1),RETURN(A4))
=IF(AND(AND(BOT>=10,BOT<=145),Zappa=2),RETURN(A5))
=IF(BOT<0,RETURN("b/t Out of Range."))
=IF(AND(BOT<15,Zappa=1),RETURN(4))
=IF(AND(BOT>200,Zappa=1),RETURN(6.96))
=IF(AND(BOT<10,Zappa=2),RETURN("b/t Out of Range"))
=IF(AND(BOT>145,Zappa=2),RETURN(6.96))

=RETURN("Macro Err")


FigC5.11
=ARGUMENT("AOB")
=121.37-314.43*AOB+350.03*AOB^2-195.9*AOB^3+54.702*AOB^4-6.0732*AOB^5   1<a/b<2.2
=-198.31+368.82*AOB-261.64*AOB^2+91.472*AOB^3-15.8*AOB^4+1.0804*AOB^5   2.2<a/b<3.68
=6.5958-0.20958*AOB   3.68<a/b<5.0

=IF(AND(AOB>=1,AOB<2.2),RETURN(A20))
=IF(AND(AOB>=2.2,AOB<3.68),RETURN(A21))
=IF(AND(AOB>=3.68,AOB<=5),RETURN(A22))
=IF(AOB<1,RETURN("a/b Out of Range"))
=IF(AOB>5,RETURN("a/b Out of Range"))
=RETURN("Macro Err")

FigC9.1
=ARGUMENT("Z")
=ARGUMENT("ROT")
=3.8337+0.25748*Z-0.0015272*Z^2+0.0000048691*Z^3   r/t=3000
=3.8337+0.25748*Z-0.0015272*Z^2+0.0000048691*Z^3   r/t=2000
=3.4625+0.3351*Z-0.0061366*Z^2+0.000084875*Z^3   r/t=1000
=5.5977-0.087272*Z+0.015305*Z^2-0.00014876*Z^3   r/t=700
=4.121+0.078764*Z+0.027337*Z^2-0.001383*Z^3+0.000031261*Z^4-0.00000024929*Z^5   r/t=500
=5.0601-1.7512*Z+1.0254*Z^2-0.24468*Z^3+0.027988*Z^4-0.0012329*Z^5   Z>1.4 AND Z<=7.0
=4   Z<=1.4

=IF(AND(Z>=1,Z<=1.4),RETURN(A40))
=IF(AND(Z>1.4,Z<=7),RETURN(A39))
=IF(AND(AND(Z>7,Z<=50),AND(ROT<=3000,ROT>2000)),RETURN(Interpolate!INTERPOLATE(3000,ROT,2000,A34,A35
=IF(AND(AND(Z>7,Z<=50),AND(ROT<=2000,ROT>1000)),RETURN(Interpolate!INTERPOLATE(2000,ROT,1000,A35,A36
=IF(AND(AND(Z>7,Z<=50),AND(ROT<=1000,ROT>700)),RETURN(Interpolate!INTERPOLATE(1000,ROT,700,A36,A37)))
=IF(AND(AND(Z>7,Z<=50),AND(ROT<=700,ROT>500)),RETURN(Interpolate!INTERPOLATE(700,ROT,500,A37,A38)))
=IF(AND(AND(Z>7,Z<=50),AND(ROT<=500,ROT>0)),RETURN("r/t Out of Range"))
=IF(Z>50,RETURN("Z Out of Range"))
=IF(Z<1,RETURN("Z Out of Range"))
=IF(ROT>3000,RETURN("r/t Out of Range"))
=IF(ROT<0,RETURN("r/t Out of Range"))
=RETURN("Macro Err")


FigC9.2
=ARGUMENT("Zb")
=ARGUMENT("AOB92")
=9.2456+0.32745*Zb-0.0013434*Zb^2+0.0000042412*Zb^3   a/b = Infinity
=10.128+0.31493*Zb-0.00031213*Zb^2-0.0000019266*Zb^3   a/b = 3
=10.762+0.37273*Zb-0.00054319*Zb^2-0.000002604*Zb^3   a/b = 2
=11.72+0.41376*Zb-0.0012646*Zb^2+0.0000025354*Zb^3   a/b = 1.5
=14.841+0.34501*Zb+0.0004971*Zb^2-0.0000073339*Zb^3   a/b = 1
=IF(Zb<1,RETURN("Zb Out of Range."))
=IF(AND(AND(Zb>=1,Zb<=100),AOB92>3),RETURN(Interpolate!INTERPOLATE(1000000,AOB92,3,A59,A60)))
=IF(AND(AND(Zb>=1,Zb<=100),AND(AOB92<=3,AOB92>2)),RETURN(Interpolate!INTERPOLATE(3,AOB92,2,A60,A61)))
=IF(AND(AND(Zb>=1,Zb<=100),AND(AOB92<=2,AOB92>1.5)),RETURN(Interpolate!INTERPOLATE(2,AOB92,1.5,A61,A6
=IF(AND(AND(Zb>=1,Zb<=100),AND(AOB92<=1.5,AOB92>=1)),RETURN(Interpolate!INTERPOLATE(1.5,AOB92,1,A62,A
```

```
=IF(Zb>100,RETURN("Zb Out of Range."))
=IF(AOB92<1,RETURN("a/b Out of Range."))
=RETURN("Macro Err")


FigC9.4
=ARGUMENT("Zb94")
=ARGUMENT("AOB94")
=5.1455+0.17118*Zb94                                  a/b = Infinity i
=2.5003*Zb94^(0.4162)                                 a/b = Infinity ii
=5.7438+0.16169*Zb94                                  a/b = 3
=6.2273+0.18037*Zb94                                  a/b = 2
=6.9795+0.19377*Zb94                                  a/b = 1.5
=9.3546+0.25446*Zb94                                  a/b = 1
=IF(Zb94<1,RETURN("Zb Out of Range."),IF(Zb94>100,RETURN("Zb Out of Range.")))
=IF(AND(AND(Zb94>=20,Zb94<=100),AOB94>3),RETURN(Interpolate!INTERPOLATE(1000000,AOB94,3,A78,A79)))
=IF(AND(AND(Zb94>=1,Zb94<=20),AOB94>3),RETURN(Interpolate!INTERPOLATE(1000000,AOB94,3,A77,A79)))
=IF(AND(AND(Zb94>=1,Zb94<=100),AND(AOB94<=3,AOB94>2)),RETURN(Interpolate!INTERPOLATE(3,AOB94,2,A79,A
=IF(AND(AND(Zb94>=1,Zb94<=100),AND(AOB94<=2,AOB94>1.5)),RETURN(Interpolate!INTERPOLATE(2,AOB94,1.5,A8
=IF(AND(AND(Zb94>=1,Zb94<=100),AND(AOB94<=1.5,AOB94>=1)),RETURN(Interpolate!INTERPOLATE(1.5,AOB94,1,A

=RETURN("Macro Err")
```

```
INTERPOLATE
=ARGUMENT("ROT1")
=ARGUMENT("ROT2")
=ARGUMENT("ROT3")
=ARGUMENT("VAL1")
=ARGUMENT("VAL3")
=RETURN((VAL1-VAL3)*(ROT2-ROT3)/(ROT1-ROT3)+VAL3)
```

Stress Calculation Macros

Equivalents -- Determines Pequivalent and Mequivalent.

```
=ARGUMENT("Select")
=ARGUMENT("Rad1")
=ARGUMENT("Mlim")
=ARGUMENT("Mult")
=ARGUMENT("Axlim")
=ARGUMENT("Axult")
Meqlim= Mlim+Axlim*Rad1/2
MeqUlt= MUlt+AxUlt*Rad1/2
Peqlim= Axlim+Mlim*2/Rad1
Peqult= AxUlt+MUlt*2/Rad1
=IF(Select1=1,RETURN(Meqlim))
=IF(Select1=2,RETURN(MeqUlt))
=IF(Select1=3,RETURN(Peqlim))
=IF(Select1=4,RETURN(Peqult))
=RETURN("Select not valid.")
```

Stress_C -- Limit Stress.

```
=ARGUMENT("Mlim")
=ARGUMENT("Axlim")
=ARGUMENT("Rad")
=ARGUMENT("Nost")
=ARGUMENT("Ast")
=ARGUMENT("Ist")
=ARGUMENT("bst")
=ARGUMENT("tsk")
=ARGUMENT("tl")
=ARGUMENT("bf")
=(Mlim*Rad)/MOI!Moi1(Rad,Nost,Ast,Ist,tsk,bf,tl)
=Axlim/(2*PI()*Rad*(bst*tsk+tl*bf)/bst+Ast*Nost)
=RETURN(A31+A32)
```

ET_Stress -- Ultimate Stress.

```
=ARGUMENT("select")
=ARGUMENT("MUlt")
=ARGUMENT("AxUlt")
=ARGUMENT("Rad")
=ARGUMENT("Ec")
=ARGUMENT("tsk")
=ARGUMENT("bst")
=ARGUMENT("Ast")
=ARGUMENT("bf")
=ARGUMENT("tl")
=ARGUMENT("d")
=ARGUMENT("nu")
```

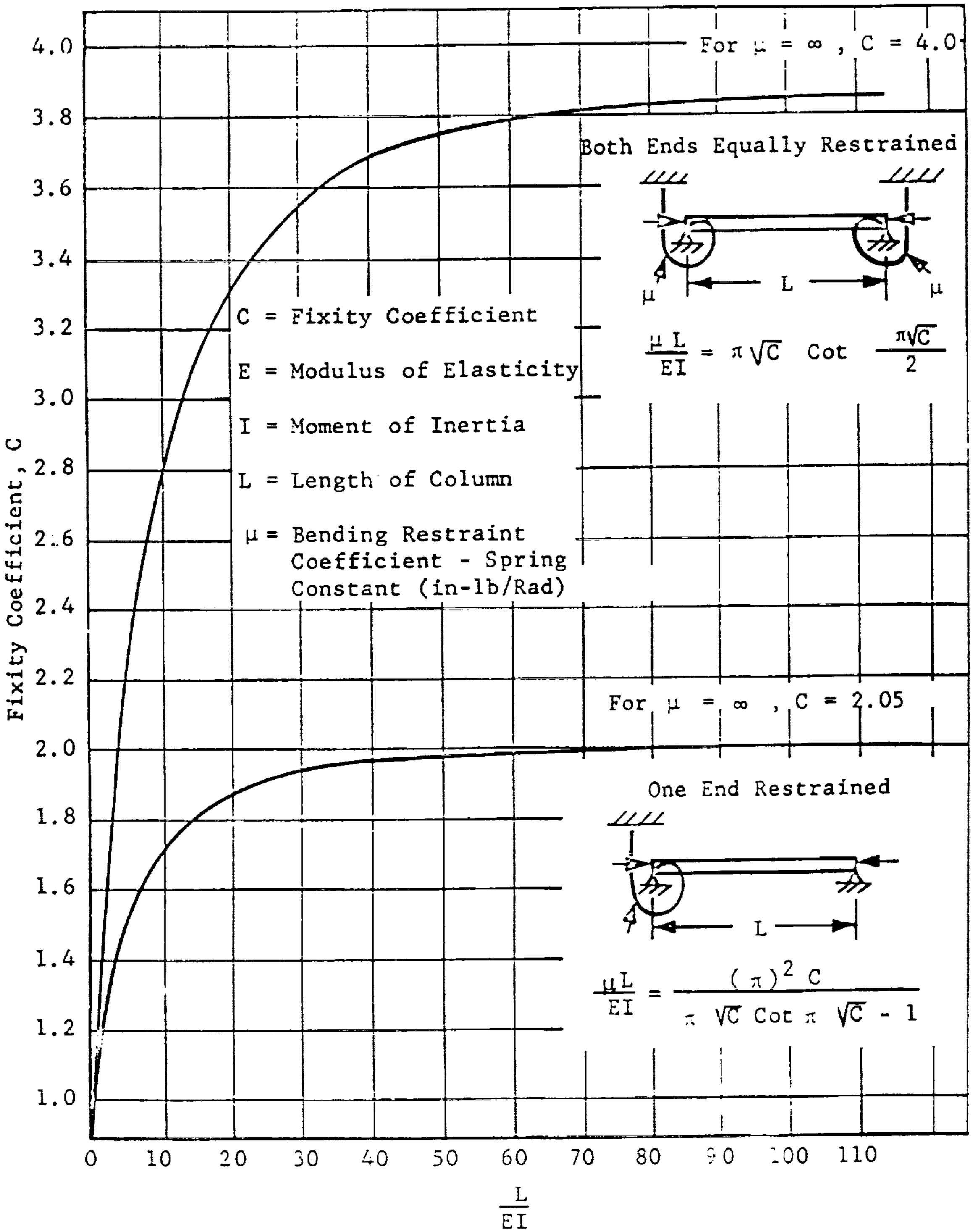
```
Nxult= -(MUlt/PI()/Rad^2+AxUlt/2/PI()/Rad)

Nyult=0

kc= MAX(BruhnFig!FigC5.6(bst/tsk,1),BruhnFig!FigC9.1(MIN(bst,d)^2/Rad/tsk*(

Fcr= -kc*PI()^2*Ec/(12*(1-nu^2))*(tsk/bst)^2
Ald=bf*(tsk+tl)
tsm=(Ast+Ald+(bst-bf)*tsk)/bst
tb=(tsk+(tsk+tl))/2
Fskp= 1/tsm*(Nxult+nu*Nyult*(tsm-tsk)/tsk)

=SET.VALUE(B63,Fskp)                                   -31503.5836277€
Ald=bf*(tsk+tl)
we= 1.7*tb*SQRT(ABS(Ec/Fskp))
=IF((Fskp)>=Fcr,RETURN("No Skin Failure"))
Pb= 0.9*Fcr*tsk*(bst-we)
Ptot=Nxult*bst
Pwe= Ptot-Pb

Nxp= Pwe/we

tsm=(Ast+Ald+(we-bf)*tsk)/we

Fskp= 1/tsm*(Nxp+0.33*Nyult*(tsm-tsk)/tsk)

we=1.7*tb*SQRT(ABS(Ec/Fskp))
=IF(we>bst,SET.NAME("we",bst))
=IF(we<bf,SET.NAME("Ald",we*(tsk+tl)))

=IF(ABS(Fskp)-ABS(B63)>0.01,GOTO(A63))


Fst= 1/tsm*(Nxp-0.33*Nyult)
Fskin= 1/tsm*(Nxp+0.33*Nyult*(tsm-tsk)/tsk)
=IF(select=1,RETURN(Fskin),RETURN(we))
=RETURN(Fskp)


Beam_Theory -- Ultimate Beam Theory.
=ARGUMENT("select")
=ARGUMENT("Mult")
=ARGUMENT("AxUlt")
=ARGUMENT("Rad")
=ARGUMENT("Ec")
=ARGUMENT("tsk")
=ARGUMENT("Nost")
=ARGUMENT("bst")
```

```
=ARGUMENT("Ast")
=ARGUMENT("bf")
=ARGUMENT("tl")
=ARGUMENT("d")
=ARGUMENT("nu")
DA=2*PI()/Nost

Ald= (tl+tsk)*bf
tav= (tsk*(bst-bf)+Ald)/bst
tb=(tsk+(tsk+tl))/2
moist=0
moisk= PI()*Rad^3*tav

=FOR("Count",1,Nost)
d=Rad*SIN((Count-1)*DA)

=SET.NAME("moist",moist + Ast*d^2)
=NEXT()

moi= moist+moisk

Cent=0
Area=2*PI()*Rad*tav+Nost*Ast
Aeff=0
Stressmax= 0
wemax=0
Atrack= 0
DC=0
Ieff= 0

StressA= AxUlt/Area

kc= MAX(BruhnFig!FigC5.6(bst/tsk,1),BruhnFig!FigC9.1(MIN(bst,d)^2/Rad/tsk*(
Fcr= -kc*PI()^2*Ec/(12*(1-nu^2))*(tsk/bst)^2

=FOR("Count",1,Nost)
d=Rad*SIN((Count-1)*DA)
Stress=- MUlt*(d-Cent)/moi-StressA

we=1.7*tb*SQRT(ABS(Ec/Stress))
=IF(we>bst,SET.NAME("we",bst))

=IF(Stress<Stressmax)
=SET.NAME("Stressmax",Stress)
=SET.NAME("wemax",we)
=END.IF()

Ae=ABS((bst-we)*tsk*Fcr/Stress)
Ald=bf*(tl+tsk)
```

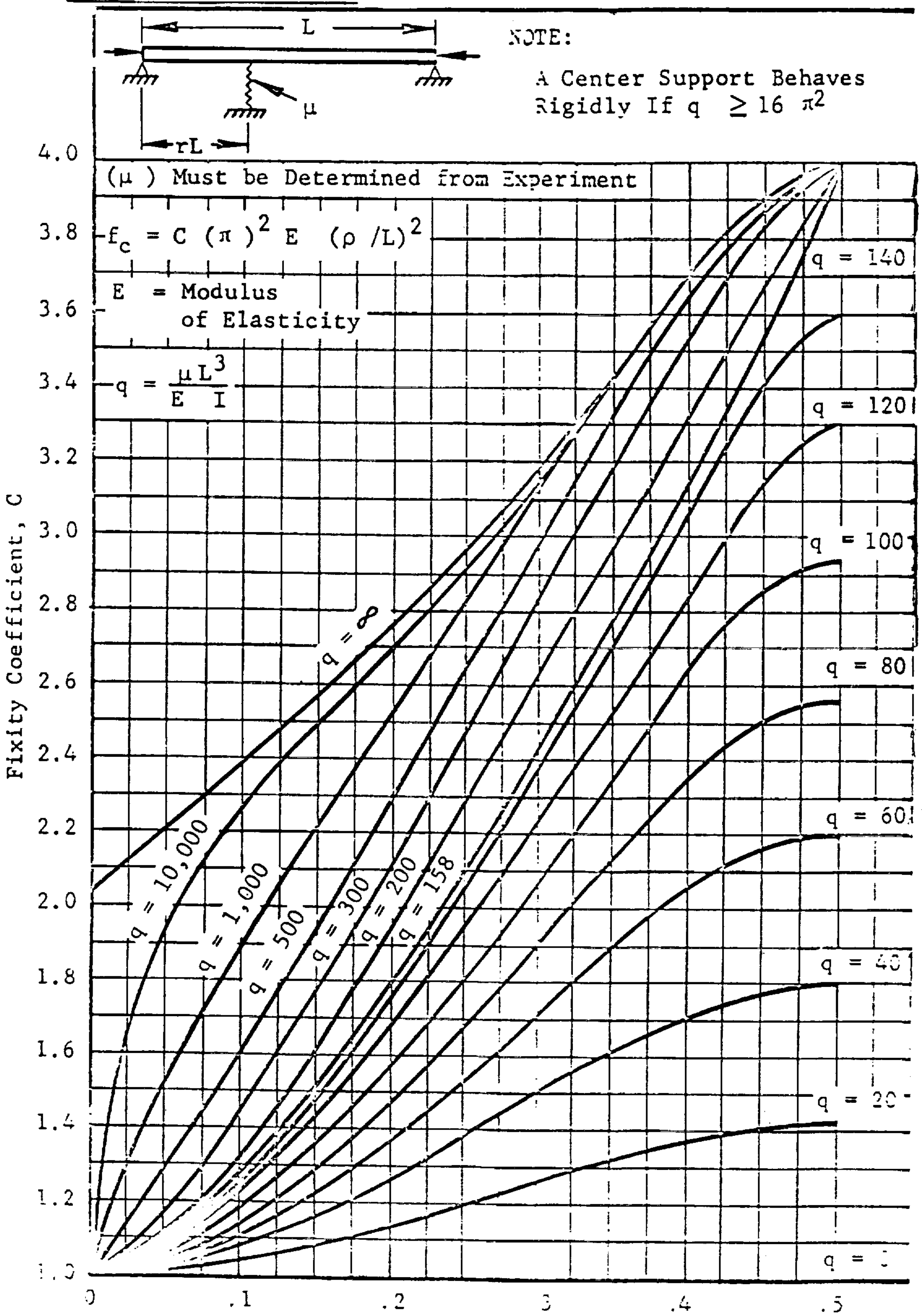
```
=IF(we<bf,SET.NAME("Ald",we*(tl+tsk)))
tav=((we-bf)*tsk+Ald)/we
=IF(we<bf,SET.NAME("tav",Ald/we))

=SET.NAME("Aeff",Aeff+(we*tav+Ast+Ae))
=SET.NAME("Atrack",Atrack+(we*tav+Ast+Ae)*d)
=SET.NAME("Ieff",Ieff+(we*tav+Ast+Ae)*d^2)

=NEXT()

=SET.NAME("DC",Cent-Atrack/Aeff)
=SET.NAME("Cent",Atrack/Aeff)


=SET.NAME("Area",Aeff)

=SET.NAME("moi",Ieff+Aeff*DC^2)

=IF(ABS(DC)>0.1,GOTO(A122))
=IF(select=1,RETURN(Stressmax),RETURN(wemax))
=RETURN("Macro Error")
```

APPENDIX D

End-Fixity Coefficients

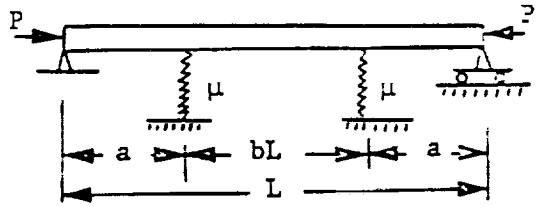


Fixity Coefficient for a Column with End Supports Having a Known Bending Restraint



Fixity Coefficient for a Column with Simply Supported Ends and an Intermediate Support of Spring Constant. -

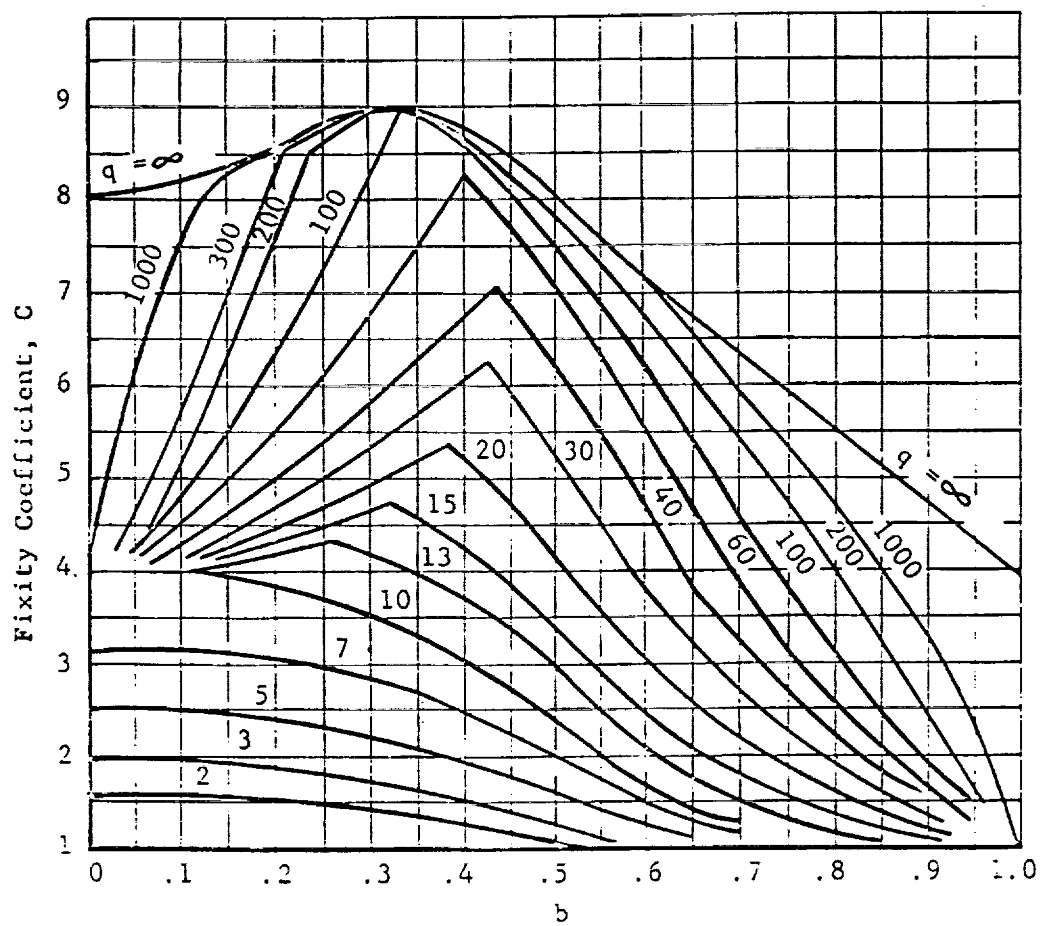
Long Columns (Cont'd)



$$P_c = \frac{C\,\pi^2 EI}{L^2} \text{ or } f_c = \frac{C\,\pi^2 E}{(L/\rho)^2}$$

$$q = \frac{\mu L^3}{8EI}$$

Where (μ) = Spring Constant Which Is Equal To The Number of Pounds Necessary To Deflect The Spring One Inch Extrapolated to Zero Deflection.



Fixity of a Column with Two Elastic, Symmetrically Placed Supports Having Spring Constants, μ

APPENDIX E

Bifurcation Buckling

Source Code

```
C ***  This program utilizes the methods put forth in "Design
C      Criteria for Axially Loaded Cylindrical Shells" and
C      "Buckling of Bars, Plates, and Shells", by
C      B.O. Almroth


       Real Ist,Ir,Jst,Jr,nu,L,C(6,6),A(3,3),AO(2,2),A1(3,3),Ny,p
       Real Lam(15),Nx(15,18),Nxp(15,18),Ncl,Eta(15),Nwc,Ncr,mb
       Real Pcr,Pcl(15,15),Pcla(15)
       Integer m,n

       CHARACTER TAB
       CHARACTER CR
       CHARACTER LF
       TAB = CHAR(9)
       CR = CHAR(13)
       LF = CHAR(14)



       pi = 3.14159
       p=17.3
       Stress = 19177.1
       E = 10.8E6
       nu = 0.33
       G = E/2/(1+nu)
       bs = 10.832
       L = 40.08

       Open(unit=3,file='Bifurcation.Dat',status='unknown')

       Write(3,'(/)')

       Fpl = 5.8*pi**2*E/12/(1-nu**2)*(tsk/bs)**2

       tsk = 0.126
       Ast = 0.279
       Ist = 0.04238
       Jst = 0.023682
       es = -0.77374
       R = 165.5-tsk/2

       twbar = tsk+Ast/bs
       tebar = (Ast+tsk*bsk)/bs
       Write(*,*)tebar

       Ar = 0.0
       Ir = 0.0
       dr = 40.08
       Jr = 0.0
       er = 0.0


       Cp = E*tsk/(1-nu**2)
       D = E*tsk**3/12/(1-nu**2)

C
C   ****  Classical Bifuraction Buckling Analysis   ***
C
       Do 10 I=1,6
       Do 11 J=1,6
       C(i,j) = 0.000
11     Continue
10     Continue

       C(1,1) = Cp+E*Ast/bs
```

```fortran
      C(1,2) = nu*Cp
      C(1,4) = E*Ast*es/bs

      C(2,1) = nu*Cp
      C(2,2) = Cp + E*Ar/dr
      C(2,5) = E*Ar*er/dr

      C(3,3) = G*tsk

      C(4,1) = E*Ast*es/bs
      C(4,4) = D+E/bs*(Ist+Ast*es**2)
      C(4,5) = nu*D

      C(5,2) = E*Ar*er/dr
      C(5,4) = nu*D
      C(5,5) = D + E/dr*(Ir+Ar*er**2)

      C(6,6) = 2*(1-nu)*D + G*(Jst/bs+Jr/dr)

      Write(3,15) ((C(i,j), i=1,6),j=1,6)
15    Format(6f12.1)

      Write(3,315)

      Do 20 i=1,3
      Do 21 j=1,3
      A(i,j) = 0.0
      A1(i,j) = 0.0
21    Continue
20    Continue

      Do 30 i=1,2
      Do 31 j=1,2
      Ao(i,j) = 0.0
31    Continue
30    Continue


      Do 101 m=1,15
      Do 100 n=1,15

      nn = n+3
      Lam(m) = (m*pi/L)**2
      Eta(n) = (n/R)**2


      A(1,1) = C(1,1)*Lam(m) + C(3,3)*Eta(n)
      A(1,2) = (C(1,2) + C(3,3))*m*pi*n/L/R

      A(1,3) = C(1,2)*m*pi/R/L+C(1,4)*Lam(m)*m*pi/L+
     @(C(1,5)+2*C(3,6))*m*pi*Eta(n)/L




      A(2,2) = C(2,2)*Eta(n) + C(3,3)*Lam(m)

      A(2,3) = (C(1,5)+2*C(3,6))*Lam(m)*n/R+C(2,2)*n/R**2+
     @C(2,5)*Eta(n)*n/R

      A(3,3) = C(4,4)*Lam(m)**2+(C(6,6)-2*C(4,5))*Lam(m)*Eta(n)+
     @C(5,5)*Eta(n)**2+C(2,2)/R**2+2*C(2,5)/R*Eta(n)-
     @2*C(1,5)*Lam(m)/R
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
      Ao(1,1) = A(1,1)
      Ao(1,2) = A(1,2)
      Ao(2,1) = A(1,2)
      Ao(2,2) = A(2,2)

      A1(1,1) = A(1,1)
      A1(1,2) = A(1,2)
      A1(1,3) = A(1,3)
      A1(2,1) = A(1,2)
      A1(2,2) = A(2,2)
      A1(2,3) = A(2,3)
      A1(3,1) = A(1,3)
      A1(3,2) = A(2,3)
      A1(3,3) = A(3,3)

      DETAo = Ao(1,1)*Ao(2,2)-Ao(2,1)*Ao(1,2)
      DETA1 = A1(1,1)*(A1(2,2)*A1(3,3)-A1(3,2)*A1(2,3))
     @-A1(1,2)*(A1(2,1)*A1(3,3)-A1(3,1)*A1(2,3))
     @+A1(1,3)*(A1(2,1)*A1(3,2)-A1(3,1)*A1(2,2))

      Ny = -p*R

      Nxp(m,n) = (1/Lam(m))*(DetA1/DetAo-Ny*Eta(n))
      Nx(m,nn)=Nxp(m,nn-3)


      If(m .EQ. 1 .AND. n .EQ.1) Ncl=Nxp(m,n)
      If(Nxp(m,n) .LT. Ncl) Ncl = Nxp(m,n)


C     Write(3,211) m,n
C211  Format(2x,I2,3x,I2,3x,I2)

C     Write(3,215) ((Ao(i,j),i=1,2),j=1,2)
C215  Format(2f12.2)

C     Write(3,315)
315   Format(/)

C     Write(3,415) ((A1(i,j),i=1,3),j=1,3)
C415  Format(3f20.2)
      Write(3,'(2x,''Nxp(''I2,1h,,I2,'')= '',F20.4)') m,n,Nxp(m,n)

100   Continue
101   Continue




       Write(*,'' //''Classical Bifurcation Buckling Allowable = '',
     @f10.2)') Ncl
       Write(3,'' //''Classical Bifurcation Buckling Allowable = '',
     @f10.2)') Ncl


C
```

```
C    ****  Wide Column Buckling Analysis  ***
C

       Cb44 = C(4,4)-C(1,4)**2/C(1,1)
       Cb55 = C(5,5)-C(2,5)**2/C(2,2)
       Nwc = Cb44*(pi/L)**2
       Write(*,'(''Wide Column Buckling Allowable = '',
     @f10.2)') Nwc
       Write(3,'(''Wide Column Buckling Allowable = '',
     @f10.2,)') Nwc


       Det = C(1,1)*C(2,2)-C(1,2)**2
       te = 1/sqrt(5.46*(Cb44+Cb55)*C(2,2)/Det)
       ROte = R/te
       Phi = 6.48/ROTe**(0.54371769)
       Write(*,*)ROTe,Phi

       Ncr = Nwc+phi*(Ncl-Nwc)

       Write(*,'(''Critical Buckling Allowable = '',
     @f10.2,)') Ncr
       Write(3,'(''Critical Buckling Allowable = '',
     @f10.2,)') Ncr


       Close(unit=3)

       Stop
       End
```

Output

$$\begin{bmatrix} 1805277.3 & 503943.5 & .0 & -215235.7 & .0 & .0 \\ 503943.5 & 1527101.4 & .0 & .0 & .0 & .0 \\ .0 & .0 & 511578.9 & .0 & .0 & .0 \\ -215235.7 & .0 & .0 & 210811.6 & 666.7 & .0 \\ .0 & .0 & .0 & 666.7 & 2020.4 & .0 \\ .0 & .0 & .0 & .0 & .0 & 11584.0 \end{bmatrix} = C_{ij}$$

Nxp(1, 1)=	9999.0313
Nxp(1, 2)=	9685.4014
Nxp(1, 3)=	9210.9961
Nxp(1, 4)=	8635.0781
Nxp(1, 5)=	8020.2241
Nxp(1, 6)=	7421.9658
Nxp(1, 7)=	6883.0952
Nxp(1, 8)=	6432.3403
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Classical Bifurcation Buckling Allowable =    5707.59
Wide Column Buckling Allowable =    1137.54
Critical Buckling Allowable =    3231.44
```

APPENDIX F

Margin of Safety Calculations

Margins of Safety

General Instability

Shanley

$$M_{CR} = 272{,}794{,}759 \text{ in-lb}$$

$$P_{eq} = \frac{2M}{R} \qquad \text{[Bruhn, "Missiles," page E1.98]}$$

$$P_{eq} = 3{,}296{,}613 \text{ lb}$$

$$\text{Load per stringer column} = \frac{P_{eq}}{96} = 34{,}339.72 \text{ lb}$$

$$\text{Area per column} = 0.8183 \text{ in}^2$$

$$\text{Stress (allowable)} = \frac{P_{eq}}{A} = \frac{34{,}339.72}{0.8183} = 41{,}964.7 \text{ lb/in}^2$$

$$\sigma_{\text{ALLOWABLE}} > \sigma_{\text{YIELD}} \ \therefore\ F_{CR} = F_{cy} = 38{,}000 \text{ lb/in}^2$$

$$\text{MS} = \frac{38{,}000}{31{,}504} - 1 = 0.2062\,.$$

Margins of Safety

Local Stringer Buckling

$$F_{CR} = 51{,}000 \text{ lb/in}^2$$

$$\text{Limit Stress} = 13{,}730 \text{ lb/in}^2$$

$$\text{MS} = \frac{51{,}000}{13{,}730} - 1 = 2.715\,.$$

Column Buckling and Stringer Crippling

Lowest critical stress in the column results from Euler buckling.

$$F_{CR_{\text{Euler}}} = 15{,}917 \text{ lb/in}^2$$

$$\text{MS} = \frac{15{,}917}{31{,}503} - 1 = -0.4947\,.$$

Margin of Safety

Skin Buckling: Combined Compression and Shear

$$MS = \frac{2}{R_C + \left[R_C^2 + 4R_S^2\right]^{1/2}} \qquad \text{[Bruhn, "Aircraft," page C5.11]}$$

$$R_C = \frac{f_C}{F_{C_{CR}}} \;, \quad R_S = \frac{f_S}{F_{S_{CR}}}$$

$$R_C = \frac{13{,}729.9}{7{,}823} = 1.7551$$

$$R_S = \frac{713.1}{8{,}160} = 0.0874$$

$$MS = \frac{2}{1.7551 + [(1.7551)^2 + 4(0.0874)^2]^{1/2}} - 1$$

$$MS = -0.4316$$

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APPROVAL

PRELIMINARY ANALYSIS TECHNIQUES FOR RING AND STRINGER STIFFENED CYLINDRICAL SHELLS

BY J. GRAHAM

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

I have personally reviewed this report, in its entirety, for technical content and have determined that this report is unclassified.

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Billy W. Shelton, Chief
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Chief, Security Division

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