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USE OF SYSTEM IDENTIFICATION TECHNIQUES FOR IMPROVING AIRFRAME FINITE ELEMENT MODELS USING TEST DATA

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(NASA-CR-192699) USE OF SYSTEM IDENTIFICATION TECHNIQUES FOR IMPROVING AIRFRAME FINITE ELEMENT MODELS USING TEST DATA Final Report (Georgia Inst. of Tech.) 112 p

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SUMMARY

A method for using system identification techniques to improve airframe finite element models using test data has been developed and demonstrated. The method uses linear sensitivity matrices to relate changes in selected physical parameters to changes in the total system matrices. The values for these physical parameters were determined using constrained optimization with singular value decomposition. The method was confirmed using both simple and complex finite element models for which pseudo-experimental data was synthesized directly from the finite element model. The method was then applied to a real airframe model which incorporated all of the complexities and details of a large finite element model and for which extensive test data was available. The method was shown to work, and the differences between the identified model and the measured results were considered satisfactory.

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NOMENCLATURE

$\mathbf{c}$	: coefficient matrix
$\mathbf{C}$	: damping matrix
$\mathbf{C}_A$	: damping matrix for analytical model
$\mathbf{c}_i$	: grouped element damping matrix
$\mathbf{f}(\lambda)$	: lambda matrix
$\mathbf{K}$	: stiffness matrix
$\mathbf{K}_A$	: stiffness matrix for analytical model
$\mathbf{k}_i$	: grouped element stiffness matrix
$\mathbf{M}$	: mass matrix
$\mathbf{M}_A$	: mass matrix for analytical model
$\mathbf{m}_i$	: grouped element mass matrix
$\mathbf{q}(t)$	: displacements at the n degrees of freedom
$\mathbf{Q}(t)$	: n independent forces applied at each DOF
$\mathbf{U}$	: $\mathbf{Z}_1\Lambda$
$\mathbf{U}_I$	: imaginary component of **U**
$\mathbf{U}_R$	: real component of **U**
$\mathbf{V}$	: $\mathbf{Z}_2\Lambda$
$\mathbf{V}_I$	: imaginary component of **V**
$\mathbf{V}_R$	: real component of **V**
$\mathbf{W}$	: $\mathbf{Z}_2$
$\mathbf{W}_I$	: imaginary component of **W**
$\mathbf{W}_R$	: real component of **W**
$y(t)$	: redefined displacement vector
$Y(t)$	: redefined applied force vector
$\mathbf{Y}_1$	: $-(d\mathbf{U}_R{}^T\mathbf{M}_A+d\mathbf{V}_R{}^T\mathbf{C}_A+d\mathbf{W}_R{}^T\mathbf{K}_A)$
$\mathbf{Y}_2$	: $-(d\mathbf{U}_I{}^T\mathbf{M}_A+d\mathbf{V}_I{}^T\mathbf{C}_A+d\mathbf{W}_I{}^T\mathbf{K}_A)$
$\mathbf{Z}_1,\mathbf{Z}_2$	: special modal matrices defined in this paper
α	: eigenvalue
α_i	: adjustable physical mass quantities
β_i	: adjustable physical damping quantities
γ_i	: adjustable physical stiffness quantities
δ	: real component of the complex eigenvalue
λ	: eigenvalue
φ	: modal matrix
Ω	: imaginary component of the complex eigenvalue
$\varphi^{(r)}$	: *r*th modal column
Ψ	: n x 2n rectangular modal matrix
η	: modal coordinates matrix
Φ	: modal matrix
d( )	: differences between experimental data and analytical data

INTRODUCTION

Background

The vast bulk of the work reported to date on identification of structural dynamic systems has focused on identifying mathematical models that reproduce test results, but little consideration has been given to the physical basis for the identified system equations. Typically, the identification procedures make systematic adjustments to the system equation, commonly to the stiffness and/or mass matrices but also to the damping matrix, so that the identified eigenvalues and eigenvectors reproduce as closely as possible the results measured in tests. The result of this process is almost inevitably identified mass, stiffness and damping matrices that are fully populated, that is, which have nonzero values for almost all elements. Such matrices, while capable of producing plausible eigenvalues and eigenvectors, can nonetheless be physically implausible in the sense that the large numbers of nonzero elements throughout the system matrices implies direct connectivity among the degrees of freedom that does not exist physically.

Identified mathematical models that are based on physically implausible system matrices may be quite acceptable if the objective of the study is to develop a *simulation* model. However, such results for analysis purposes are generally unsatisfactory because it is difficult or impossible to relate specific features of the physical system to the analysis results. This problem is particularly troublesome when the objective of the identification of a system model from experimental measurements is an accurate system model that, in turn, will be used to make modifications to or improvements in the original physical system. Such an example might be the modification of an existing aircraft structure to accommodate a new mission. In this case it would be desirable to formulate a structural model for the present structure, verify its accuracy against experimental measurements, and then use it as the basis for the modifications. When the verification process yields identified system matrices that are mathematically acceptable but physically implausible, the resulting model may be useless as the basis for future structural modifications.

The objective of the present work was to develop a method for identifying physically plausible finite element system models of airframe structures from test data. The assumed models were based on linear elastic behavior with general (nonproportional) damping. Physical plausibility of the identified system matrices was insured by restricting the identification process to designated physical parameters only and not simply to the elements of the system matrices themselves. For example, in a large finite element model the identified parameters might be restricted to the moduli for each of the different materials used in the structure. In the case of damping, a restricted set of damping values might be assigned to finite elements based on the material type and on the fabrication processes used. In this case, different damping values might be associated with riveted, bolted and bonded elements.

The method itself is developed first, and several approaches are outlined for computing the identified parameter values. The method is applied first to a simple structure for which the "measured" response is actually synthesized from an assumed model. Both stiffness and damping parameter values are accurately identified. The true test, however, is the application to a full-scale airframe structure. In this case, a NASTRAN model and actual measured modal parameters formed the basis for the identification of a restricted set of physically plausible stiffness and damping parameters.

Review of Previous Pertinent Work

Airframes are generally modelled using powerful finite element analysis packages such as *NASTRAN* that are capable of representing quite detailed aspects of the structural system. The accuracy of such models is determined by comparing the analytical results with flight or ground vibration test results. In the case of helicopter airframes, several recent efforts have focused on the correlation of *NASTRAN* model data with ground vibration test data[1-3]. The conclusions reached in these studies suggest that in cases where there is some degree of correlation, the model frequencies compare favorably with test frequencies, but generally only in the low frequency range below about 15 Hz [1-2]. The frequency response functions at selected locations also compare reasonably well in this range. Outside this range the comparisons are generally unsatisfactory, and the eigenvectors do not usually compare favorably in either range.

Although there have been numerous contributions to the literature in the area of the identification of structural dynamic systems[4-25], the majority of reported methods are based on simply adjusting the elements of one or more of the **K**, **M**, and **C** matrices. While this approach is capable of yielding a system matrix whose eigenvalues and eigenvectors suitably match measured results, the methods generally lose all physical interpretability inherent in the original **K**, **M** and **C** matrices by not maintaining relationships among elements dictated by the model topology. These difficulties are compounded for large-scale models with thousands of degrees of freedom.

The reported papers, that address this problem are by Meirovitch and Norris[25-27], Lim [28-30], Hajela [31], Zimmerman [32], Hickman[33] et.al. and Chen and Garba [33]. In reference 25, the problem of parameter identification in distributed parameter system has been addressed. It is assumed that the response of a structure is measured and the physical parameters of the system are identified by using least square principle. In reference 26, a perturbation technique has been discussed for parameter identification. As a basis for the perturbation procedure, it has been assumed that there is a prior knowledge of approximate values of system parameters. The developed perturbation technique depends on measured system response to a harmonic excitation in a frequency domain. Again, the identification is based on least square principle. In reference 27, the method has been extended to a Raleigh-Ritz type of method where the parameters have been assumed to be known functions with undetermined coefficients. The developed theory considers linear viscous damping. However, the numerical examples do not include damping and are based on simulated measurements that have been obtained by numerical methods. Lim has used a submatrix approach to update stiffness matrices in reference 27. In reference 28, both stiffness and mass matrices have been updated simultaneously. In reference 27 and 28, Lim has attempted to retain the physical significance of the stiffness and mass matrices by use of sub-matrices that represent the stiffness or mass matrices of individual or group of elements. In reference 29, Lim has used a method that is similar to that of references 27 and 28 to identify damages to structures. In all these works, Lim has considered undamped system matrices and has used numerically simulated eigen values and eigen vectors to validate the theory. In references 31-33, the identified physical parameters are the structural failures. These failures have been related to changes in stiffness matrices. The algorithm is based on partial inverses and optimum least squares expansion. Damages are identified only by observation of global changes in properties. Quantities such as parameter sensitivity or parameter perturbations or methods such as singular value decomposition or constraints to limit parameters to physical space have not been used. Also, these methods have not been applied to large scale system matrices. In reference 32, however, a pattern recognition technique has been used on the basis of measured response of the structure.

In our work, we would like to consider measured frequencies and modes as inputs to identify system parameters that can be related to physical variables. Our objective is to identify large scale systems like helicopters that have been specified by a known finite element model. Because of the sensitivity of errors in experimentally determined modes to system parameters, it is also necessary to impose constraints on identified parameters to occupy specified parameter space. For example, we can not have a negative modulus or a negative damping in a passive structural dynamic system. In using measured data, we found that the identification process needed the use of singular value decomposition methods. Because we are using experimentally generated data, the examples include general linear damping matrices. No restriction of proportional damping has been imposed.

Kuo and Wada[35] used nonlinear sensitivity coefficients (NSC) in the identification procedure. Their sensitivity coefficients are between the system parameters and eigenvalues. In the present work the interest is in the change of system matrices as a function of physical variables of the structure. A different type of sensitivity coefficient between system matrices and physical variables has therefore been developed.

The most significant achievement in the present work[40] is to preserve the physical interpretability of the **M**, **C**, **K** matrices so that the identification can provide evidence of possible sources of erroneous modeling and point to specific regions of the model that are unduly sensitive and need additional consideration in modeling. The identification procedure developed in this paper is capable of adjusting physical quantities such as boundary conditions, moments of inertia, stiffnesses, damping or other selected physical parameters.

Mathematical Model

Basic Equations

Any linearly elastic structural system with n discrete degrees of freedom and with general viscous damping (either proportional or nonproportional) can be represented by n coupled ordinary differential equations that can be written in the following form[37]:

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{Q}(t) \tag{1}$$

where **M**, **C**, and **K** are symmetric n x n inertia, damping, and stiffness matrices, respectively. In this formulation, **q**(t) are the displacements at the n degrees of freedom and **Q**(t) are the n independent forces applied at each degree of freedom.

In the case of undamped or proportionally damped systems, there are n complex conjugate pairs of eigenvalues and n distinct modes which are orthogonal with respect to **M** and **K**. Using a transformation matrix of the form:

$$\mathbf{q}(t)=\Psi\eta(t)$$

will allow decomposition of the original system equations (Eq. 1) into n decoupled equations that are straightforward to solve.

This transformation cannot be applied to the general nonproportionally damped problem in the same manner because for this case there are 2n complex modes, $\varphi^{(r)}$, and consequently 2n modal coordinates, $\eta_r(t)$, but there are only n physical coordinates, $q_i(t)$.

One can overcome this difficulty by writing Eq. (1) as a set of 2n ordinary differential equations in the form:

$$\begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix}\begin{Bmatrix} \ddot{q}(t) \\ \dot{q}(t) \end{Bmatrix}+\begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix}\begin{Bmatrix} \dot{q}(t) \\ q(t) \end{Bmatrix}=\begin{Bmatrix} 0 \\ Q(t) \end{Bmatrix}. \tag{2}$$

If one then defines: $\mathbf{y}(t)=\begin{Bmatrix} \ddot{q}(t) \\ \dot{q}(t) \end{Bmatrix}$ and $\mathbf{Y}(t)=\begin{Bmatrix} \mathbf{0} \\ Q(t) \end{Bmatrix}$, the above equations can be written as a set of 2n first order ordinary differential equations:

$$\begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix}\dot{\mathbf{y}}(t)+\begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix}\mathbf{y}(t)=\mathbf{Y}(t). \tag{3}$$

This formulation has the advantage that the modes obtained from the solution of the homogeneous equations, obtained by letting **Y**(t)=**0** in Eqs. (3), are orthogonal, and hence can be used in conjunction with the expansion theorem to obtain the solution of the nonhomogeneous problem. The solution of the homogeneous equations is obtained by assuming as before a solution in the form:

$$\mathbf{y}(t)=\Phi e^{\alpha t} \tag{4}$$

where Φ represents the spatial component of the solution and is a vector consisting of 2n constant elements. The corresponding eigenvalue problem can be written as:

$$\alpha\begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix}\Phi+\begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix}\Phi=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{5}$$

The solution of the eigenvalue problem yields 2n eigenvalues, α_r, and 2n eigenvectors

$$\Phi_r^T=\begin{Bmatrix} \alpha_r\phi_r^T \\ \phi_r^T \end{Bmatrix}, \quad r=1,2,...2n. \tag{6}$$

Equations (5) and (6) provide the solution to Eq. (1), but in order to simplify the computational work, it is convenient to formally separate these complex equations into real and imaginary pairs. Following the approach introduced by Cheng[10], the real and imaginary components of the eigenvalues and eigenvectors are defined, respectively, as:

$$\lambda_r=\delta_r+j\Omega_r$$

$$\Phi_r=\Phi_{R_r}+j\Phi_{I_r} \tag{7}$$

and in addition the following modal matrices are defined:

$$Z_1 = (Re(\alpha_1\phi_1), Im(\alpha_1\phi_1), Re(\alpha_2\phi_2), Im(\alpha_2\phi_2), .. \quad Re(\alpha_n\phi_n), Im(\alpha_n\phi_n)) \tag{8}$$

$$Z_2 = (Re(\phi_1), Im(\phi_1), Re(\phi_2), Im(\phi_2), ... \quad Re(\phi_n), Im(\phi_n))$$

Then the new system equation, Eq. (5), can now be rewritten with purely real terms in the form:

$$\Lambda\begin{bmatrix} 0 & M \\ M & C \end{bmatrix}\begin{Bmatrix} Z_1 \\ Z_2 \end{Bmatrix} + \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix}\begin{Bmatrix} Z_1 \\ Z_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{9}$$

where the eigenvector matrix, Λ, is a block diagonal matrix with blocks

$$\Lambda_r = \begin{bmatrix} \delta_r & \Omega_r \\ -\Omega_r & \delta_r \end{bmatrix} \tag{10}$$

along the diagonal and zeros elsewhere. Equation (9) can be further simplified by the introduction of **U**, **V**, and **W** as follows:

$$MU + CV + KW = 0 \tag{11}$$

where $U = Z_1\Lambda$, $V = Z_2\Lambda$ and $W = Z_2$ or explicitly:

$$U = (Re(\alpha_1{}^2\phi_1), Im(\alpha_1{}^2\phi_1), Re(\alpha_2{}^2\phi_2), Im(\alpha_2{}^2\phi_2), \ldots Re(\alpha_n{}^2\phi_n), Im(\alpha_n{}^2\phi_n))$$

$$V = (Re(\alpha_1\phi_1), Im(\alpha_1\phi_1), Re(\alpha_2\phi_2), Im(\alpha_2\phi_2), \ldots Re(\alpha_n\phi_n), Im(\alpha_n\phi_n))$$

$$W = (Re(\phi_1), Im(\phi_1), Re(\phi_2), Im(\phi_2), \ldots Re(\phi_n), Im(\phi_n)) \tag{12}$$

Finally, Eq. (11) can be separated into explicit real and imaginary equations in the form of the following two equations.

$$MU_R + CV_R + KW_R = 0 \tag{13}$$

$$MU_I + CV_I + KW_I = 0 \tag{14}$$

These equations are same as Eqs. (5), but they do not include complex variables. For the identification procedure, it is much easier to use these equations than to use Eqs. (5) directly.

__Identification Procedure__

To begin, suppose that the mass, damping and stiffness matrices for the initial analytical model are given by M_A C_A and K_A, respectively, and the identified mass, damping and stiffness matrices are given by **M**, **C** and **K**. In a similar manner, the eigenvectors and eigenvalues for the analytical model are given by U_A, V_A and W_A, while U_E, V_E and W_E are the eigenvectors and eigenvalues determined from test data. From these definitions it follows that the relationship between the identified model (based on the test data) and the analytical model can be written as:

$$M = M_A + dM, \quad C = C_A + dC, \quad K = K_A + dK \tag{15}$$

$$U_E = U_A + dU, \quad V_E = V_A + dV, \quad W_E = W_A + dW \tag{16}$$

where dM, dC, dK, dU, dV and dW are the changes. The identified model satisfies equations (13) and (14), so substituting equations (15) and (16) into equation (13) and (14), yields:

$$dU_R^TM_A+dV_R^TC_A+dW_R^TK_A=$$
$$-(U_{ER}^T,V_{ER}^T,W_{ER}^T)(dM,dC,dK)^T \tag{17}$$

$$dU_I^TM_A+dV_I^TC_A+dW_I^TK_A=$$
$$-(U_{EI}^T,V_{EI}^T,W_{EI}^T)(dM,dC,dK)^T \tag{18}$$

These equations can be combined into the following form:

$$\begin{bmatrix} U_{ER}^T & V_{ER}^T & W_{ER}^T \\ U_{EI}^T & V_{EI}^T & W_{EI}^T \end{bmatrix} \begin{Bmatrix} dM \\ dC \\ dK \end{Bmatrix} = \begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix}, \tag{19}$$

where

$$\begin{aligned} Y_1 &= -(dU_R^TM_A+dV_R^TC_A+dW_R^TK_A) \\ Y_2 &= -(dU_I^TM_A+dV_I^TC_A+dW_I^TK_A). \end{aligned} \tag{20}$$

The right side of these equations is known, since $\mathbf{M_A}$, $\mathbf{C_A}$, and $\mathbf{K_A}$ are given by the analytical model and dU_R^T, dV_R^T, dW_R^T, dU_I^T, dV_I^T, and dW_I^T, which are the differences of the eigenvalues and eigenvectors between the analytical model and the experimental data, are known. Finally, the matrix

$$\begin{bmatrix} U_{ER}^T & V_{ER}^T & W_{ER}^T \\ U_{EI}^T & V_{EI}^T & W_{EI}^T \end{bmatrix}$$

contains only experiment data.

The solution to these equations are the changes of **dM**, **dC** and **dK**. Because of matrix symmetry, the number of unknowns in Eq. (19) is $3\ n(n+1)/2$. The number of equations depends on the number of known experimental modes. Suppose this number is m, then the number of equations are m x n. If the number of the equations is larger than or equal to the number of unknowns and the rank of this matrix is equal to $3\ n(n+1)/2$, normal least square methods can be used to solve these equations. Otherwise, singular value decomposition, or constrained optimization can be used to solve Eq. (19) for the changes **dM**, **dC** and **dK**, and these results can then be substituted into Eq. (15) to determine the identified **M**, **C** and **K** matrices. It should be noted that this approach is capable of handling nonproportional damping and underdetermined problems in which fewer modes are measured than are computed from the analytical model.

At this stage the usual identification procedure can be performed. The values of **M**, **C** and **K** can be put into the system equation, Eq. (1), and the experimental data can then be reproduced. However the identified **M**, **C** and **K** cannot be related to particular physical quantities in the actual airframe, because the changes occur throughout the entire **M**, **C** and **K** matrices . In order to preserve the physical interpretability of the identified system, it is necessary to develop a relationship between **dM**, **dC** and **dK** and adjustable physical quantities such as boundary conditions, moments of inertia, stiffnesses or other selected physical parameters. To this end, assume that each of the system matrices can be decomposed into the form:

$$M=\sum_{i=1}^{N_m} m_i\alpha_i, \quad C=\sum_{i=1}^{N_c} c_i\beta_i, \text{ and } \quad K=\sum_{i=1}^{N_k} k_i\gamma_i \tag{21}$$

where α_i, β_i and γ_i are adjustable physical quantities and $\mathbf{m}_i$, $\mathbf{c}_i$ and $\mathbf{k}_i$ are grouped element matrices with common physical quantities.

For example, in the finite element model of actual airframe, there is an ej-th element , (see Fig. 1). The portion of the stiffness matrix that describes bending in the xz plane of an element, assumed to be a principal plane (Fig. 2), in NASTRAN, is given by

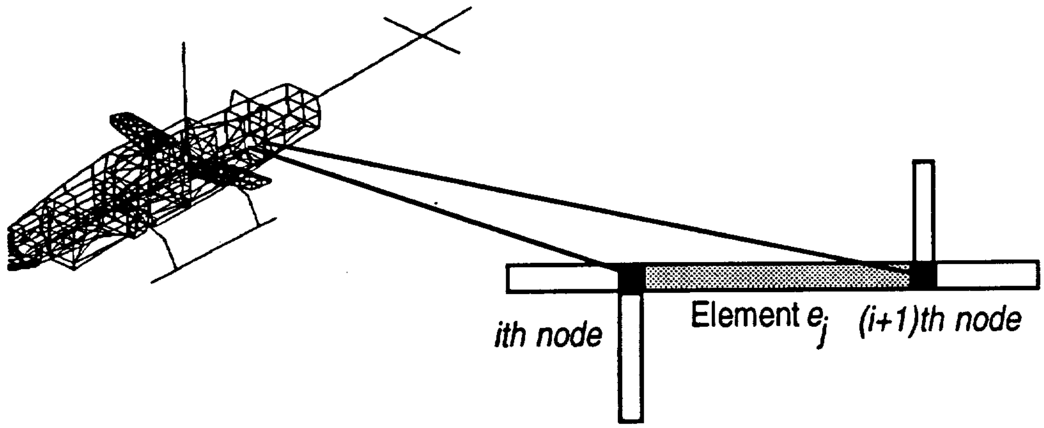

Fig. 1 Typical Bar Element, ej, in Airframe Model

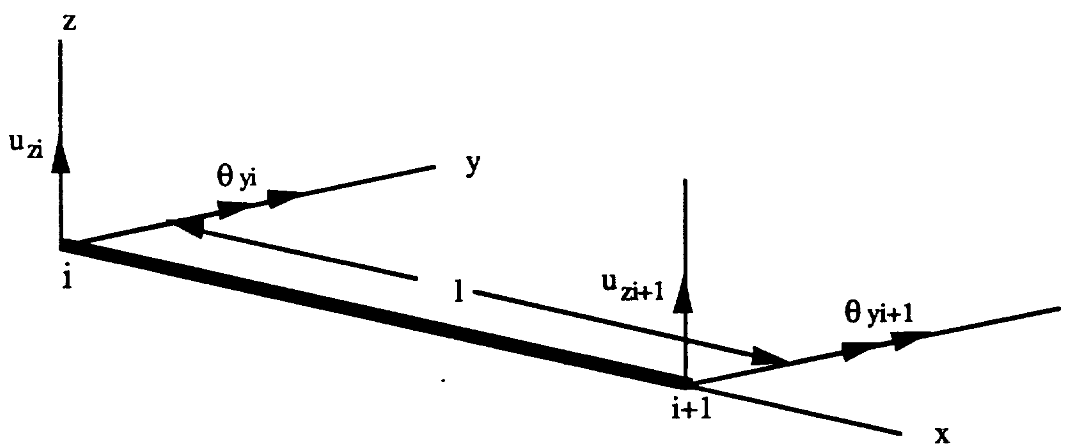

Fig. 2. Degrees of Freedom for Bending in the xz Plane

$$\begin{Bmatrix} F_{zi} \\ M_{zi} \\ F_{zi+1} \\ M_{zi+1} \end{Bmatrix} = \begin{bmatrix} R & -\frac{l}{2}R & -R & -\frac{l}{2}R \\ & \frac{l^2}{4}R+\frac{EI_y}{l} & \frac{l}{2}R & \frac{l^2}{2}R-\frac{EI_y}{l} \\ & \text{sym} & R & \frac{l}{2}R \\ & & & \frac{l^2}{4}R+\frac{EI_y}{l} \end{bmatrix} \begin{Bmatrix} u_{zi} \\ \theta_{zi} \\ u_{zi+1} \\ \theta_{zi+1} \end{Bmatrix}$$

where $R = \left(\frac{1}{k_z AG}+\frac{l^3}{12EI_y}\right)^{-1}$. If the modulus of elasticity, E, is taken here as an adjustable physical quantity, γ_k, then

$$K_{ej} = \begin{bmatrix} R & -\frac{1}{2}R & -R & -\frac{1}{2}R \\ & \frac{l^2}{4}R+\frac{EI_y}{l} & \frac{1}{2}R & \frac{l^2}{2}R-\frac{EI_y}{l} \\ & & R & \frac{1}{2}R \\ sym & & & \frac{l^2}{4}R+\frac{EI_y}{l} \end{bmatrix}$$

$$= \begin{bmatrix} r & -\frac{1}{2}r & -r & -\frac{1}{2}r \\ & \frac{l^2}{4}r+\frac{I_y}{l} & \frac{1}{2}r & \frac{l^2}{2}r-\frac{I_y}{l} \\ & & r & .\frac{1}{2}r \\ sym & & & \frac{l^2}{4}r+\frac{I_y}{l} \end{bmatrix} \gamma_k = \mathbf{k}_{ej}\gamma_k \qquad (23)$$

where γ_k= E and $r = \left(\frac{2(1+\nu)l}{k_z A} + \frac{l^3}{12I_y}\right)^{-1}$. Suppose there are n elements which have the same E so that it is possible to express the stiffness as:

$$\mathbf{k}_k = \sum_{ej=1}^{n} \mathbf{k}_{ej}$$

When the modulus changes from E to E+dE, the corresponding change in γ_k is to γ_k+dγ_k. Considering all different γ_k, **K** changes from **K** to **K**+d**K** where

$$d\mathbf{K}=\sum_{k=1}^{N_k} \mathbf{k}_k d\gamma_k$$

Similar procedures can be generalized to include the damping, other stiffness parameters, and mass.

$$d\mathbf{M}=\sum_{i=1}^{N_m} \frac{\partial \mathbf{M}}{\partial \alpha_i} d\alpha_i=\sum_{i=1}^{N_m} \mathbf{m}_i d\alpha_i$$

$$d\mathbf{C}=\sum_{i=1}^{N_c} \frac{\partial \mathbf{C}}{\partial \beta_i} d\beta_i=\sum_{i=1}^{N_c} \mathbf{c}_i d\beta_i \qquad (24)$$

$$d\mathbf{K}=\sum_{i=1}^{N_k} \frac{\partial \mathbf{K}}{\partial \gamma_i} d\gamma_i=\sum_{i=1}^{N_k} \mathbf{k}_i d\gamma_i$$

Substituting these into Eq. (19) yields a set of linear algebra equations with unknowns $d\alpha_i$, $d\beta_i$ and $d\gamma_i$:

$$\begin{bmatrix} U_R^T\frac{\partial M}{\partial \alpha_1} \dots V_R^T\frac{\partial C}{\partial \beta_1} \dots W_R^T\frac{\partial K}{\partial \gamma_1} \dots \\ U_I^T\frac{\partial M}{\partial \alpha_1} \dots V_I^T\frac{\partial C}{\partial \beta_1} \dots W_I^T\frac{\partial K}{\partial \gamma_1} \dots \end{bmatrix} \begin{Bmatrix} d\alpha_1 \\ \vdots \\ d\alpha_{N_m} \\ d\beta_1 \\ \vdots \\ d\beta_{N_c} \\ d\gamma_1 \\ \vdots \\ d\gamma_{N_k} \end{Bmatrix} = \begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix} \tag{25}$$

The number of unknowns in this equation is much less than the number of unknowns in Eq. (19), and also all the unknowns in this equation have physical meaning in the real structure.

However, neither Eq. (19) or Eq. (25) can be solved directly since the numbers of unknowns and equations are not equal in most of the cases. There exists a number of techniques for dealing with sets of equations that are under or over-determined or with matrices that are either singular or else poorly conditioned. The singular value decomposition, or SVD method[36], is one of the most powerful ways to handle these problems. In the present study it was employed to compute solutions to Eq's. (19) and (25) which are highly under-determined for most practical situations. In this case the SVD method provides a least square type of solution to the problem.

In most cases, the selected physical parameters must also be restricted to positive values in order to make sense physically. However, the identification procedure outlined above cannot guarantee that the identified values will all be positive. This is of particular concern when the parameters are proportional to mass, an elastic modulus or a damping coefficient, all of which must be positive for the systems typically considered. Using a constrained optimization method, this problem can be eliminated. The present problem can be posed as one of minimizing

$$f=d\alpha_1+d\alpha_2+\cdots+d\alpha_{N_m}+d\beta_1+..+d\beta_{N_c}+d\gamma_1+..+d\gamma_{N_k} \tag{26}$$

subject to the constraints

$$\begin{bmatrix} U_R^T\frac{\partial M}{\partial \alpha_1} \dots V_R^T\frac{\partial C}{\partial \beta_1} \dots W_R^T\frac{\partial K}{\partial \gamma_1} \dots \\ U_I^T\frac{\partial M}{\partial \alpha_1} \dots V_I^T\frac{\partial C}{\partial \beta_1} \dots W_I^T\frac{\partial K}{\partial \gamma_1} \dots \end{bmatrix} \begin{Bmatrix} d\alpha_1 \\ \vdots \\ d\alpha_{N_m} \\ d\beta_1 \\ \vdots \\ d\beta_{N_c} \\ d\gamma_1 \\ \vdots \\ d\gamma_{N_k} \end{Bmatrix} = \begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix}$$

and

$$d\alpha_1 \geq 0,\ d\alpha_2 \geq 0,\ ..d\alpha_{N_m} \geq 0,\ d\beta_1 \geq 0, ..d\beta_{N_c} \geq 0,\ d\gamma_1 \geq 0,\ ..d\gamma_{N_k} \geq 0$$

The feasible solution ($d\alpha_1$, $d\alpha_2$, ..., $d\alpha_{N_m}$, $d\beta_1$, ..., $d\beta_{N_c}$, $d\gamma_1$, ..., $d\gamma_{N_k}$) to this problem yields the identified selected physical parameters.

System Identification Procedures

The method described above was applied to several practical examples. For these cases, the analytical finite element model for the structures was assumed correct and was developed using the NASTRAN finite element analysis package[38]. Then, values for selected physical parameters in the model were identified on the basis of measured experimental data (eigenvalues and eigenvectors) so that the analytical model more accurately represented the real structure. The assumption for this procedure was that the identification process could be applied in an iterative fashion by making successive small modifications to the selected physical parameters until satisfactory agreement with experimental results was obtained. For the i-th iteration, there are the following relationships:

$$M^i = M^{i-1} + dM,\ C^i = C^{i-1} + dC,\ K^i = K^{i-1} + dK$$

$$U^i = U^{i-1} + dU,\ V^i = V^{i-1} + dV,\ W^i = W^{i-1} + dW, \quad (27)$$

Substitute these into equation (25), we can obtain

$$\begin{bmatrix} U_R^T\frac{\partial M^{i-1}}{\partial \alpha_1} \ldots V_R^T\frac{\partial C^{i-1}}{\partial \beta_1} \ldots W_R^T\frac{\partial K^{i-1}}{\partial \gamma_1} \ldots \\ U_I^T\frac{\partial M^{i-1}}{\partial \alpha_1} \ldots V_I^T\frac{\partial C^{i-1}}{\partial \beta_1} \ldots W_I^T\frac{\partial K^{i-1}}{\partial \gamma_1} \ldots \end{bmatrix} \begin{Bmatrix} d\alpha_1{}^i \\ \vdots \\ d\alpha_{N_m}{}^i \\ d\beta_1{}^i \\ \vdots \\ d\beta_{N_c}{}^i \\ d\gamma_1{}^i \\ \vdots \\ d\gamma_{N_k}{}^i \end{Bmatrix} = \begin{Bmatrix} Y_1' \\ Y_2' \end{Bmatrix} \quad (28)$$

where

$$Y_1' = -(dU_R^T M^{i-1} + dV_R^T C^{i-1} + dW_R^T K^{i-1})$$

$$Y_2' = -(dU_I^T M^{i-1} + dV_I^T C^{i-1} + dW_I^T K^{i-1}) \quad (29)$$

The convergence criteria was formulated as follows:

(1) Check the physical parameter differences $d\alpha_1, d\alpha_2, \ldots, d\alpha_{N_m}, d\beta_1, \ldots, d\beta_{N_C}, d\gamma_1, \ldots, d\gamma_{N_k}$ either manually or programmatically. If these physical parameter differences are smaller than a tolerance value, the identified physical parameters are obtained.

(2) Check the differences of the experimental eigenvalues and the i-th iteration analytical results which are obtained after running NASTRAN. If the differences are smaller than a tolerance value, the identified system is obtained.

Simple Numerical Example

In order to verify the proposed approach, the identification procedure developed above was applied first to a very simple finite element model with only a few degrees of freedom. It is a simple variable cross section straight rod with fixed ends, and it contains all the desired parameters to be identified such as mass, stiffness and damping. It was modeled using 9 rod elements with lumped masses at each node as shown in Fig. 3, and representative values were assumed for all elements and mass properties. For the purpose of defining the damping, the elements were segregated into 4 groups and a different damping coefficient was specified for each group.

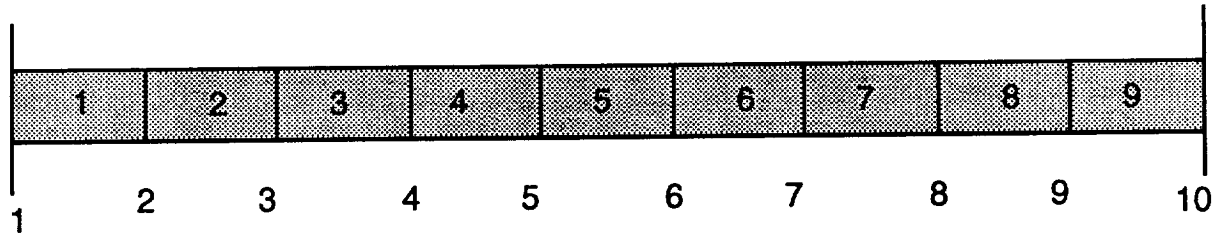

Figure 3. Simple Numerical Example

The assumed physical properties were defined to be typical of an aluminum rod. The length was 228.6 mm (9") and the modulus E =68.9 GPa (10e6 psi). The concentrated masses at each node were those given in Table 1. This model was then employed to generate eigenvalues and eigenvectors which were used to represent "measured" test results. In all the cases presented in this report, the calculations were done in US customary units and the results converted to SI units. As a result, some of the percentage figures may be slightly in error due to numerical roundoff in the conversion of units.

TABLE 1.
ASSUMED PHYSICAL PROPERTIES FOR SIMPLE NUMERICAL EXAMPLE

Index	Mass at Node (kg)	Element Stiffness (MN/m)	Element Damping Coefficient (kN s/m)
1	21.89	367.7	28.34
2	22.76	369.5	28.34
3	23.64	371.3	28.34
4	24.51	373.0	27.58
5	25.39	374.8	27.58
6	26.26	376.5	27.14
7	27.14	378.3	27.14
8	28.01	380.0	26.27
9	28.89	381.8	26.27
10	29.76		

The objective of the identification was to determine the physical parameters such as damping constants (c or ζ) and the cross section area for each rod element. There are three different cases to start to consider with this system. In the first case, the mass and the stiffness matrices were assumed to be accurate, and four damping parameters were identified assuming zero as initial analytical values of the damping matrix. The identified damping parameters are listed in Table 2 .

TABLE 2.
CASE I: IDENTIFYING THE DAMPING PARAMETERS

Damping Parameter	Exact Value	Initial Value	Identified Value	Error (%)
c_1	28.34	0.	28.3420	-0.0029
c_2	28.34	0.	28.3420	-0.0029
c_3	28.34	0.	28.3420	-0.0029
c_4	27.58	0.	27.5805	-0.0022
c_5	27.58	0.	27.5805	-0.0022
c_6	27.14	0.	27.1431	-0.0006
c_7	27.14	0.	27.1431	-0.0006
c_8	26.27	0.	26.2676	-0.0005
c_9	26.27	0.	26.2676	-0.0005

The second case was to identify the stiffness parameters assuming accurate values of mass and damping parameters which were the same for all elements k_i = 376.5 MN/m. The identified stiffness parameters are listed below.

TABLE 3.
CASE II: IDENTIFYING STIFFNESS PARAMETERS

Stiffness Parameter	Exact (MN/m)	Initial (MN/m)	Identified (MN/m)	Error (%)
k_1	367.7	376.5	367.7	0.0000
k_2	369.5	376.5	369.5	-0.0005
k_3	371.3	376.5	371.3	0.0000
k_4	373.0	376.5	373.0	0.0000
k_5	374.8	376.5	374.8	0.0000
k_6	376.5	376.5	376.5	0.0000
k_7	378.3	376.5	378.3	0.0000
k_8	380.0	376.5	380.0	0.0000
k_9	381.8	376.5	381.8	0.0000

In the third case, both the damping and the stiffness parameters were identified under the assumption of accurate mass value alone. The elements of the initial damping matrix were assumed to be zero, and the stiffness parameters were assumed to be the same for all elements (k_i= 376.5 MN/m). The identified damping and stiffness parameters are listed in Tables 4 and 5.

TABLE 4.
CASE III(A): DAMPING PARAMETERS

Damping Parameter	Exact	Initial	Identified	Error (%)
c1	28.34	0.	28.34	0.0000
c2	28.34	0.	28.34	0.0000
c3	28.34	0.	28.34	0.0000
c4	27.58	0.	27.5812	0.0004
c5	27.58	0.	27.5812	0.0004
c6	27.14	0.	27.1437	0.0015
c7	27.14	0.	27.1437	0.0015
c8	26.27	0.	26.2679	0.0008
c9	26.27	0.	26.2679	0.0008

TABLE 5.
CASE III(B): STIFFNESS PARAMETERS

Stiffness Parameter	Exact (MN/m	Initial (MN/m)	Identified (MN/m)	Error (%)
k1	367.7	376.5	367.700	0.0000
k2	369.5	376.5	369.492	-0.0019
k3	371.3	376.5	371.285	0.0094
k4	373.0	376.5	373.044	0.0112
k5	374.8	376.5	374.763	0.0028
k6	376.5	376.5	376.521	0.0047
k7	378.3	376.5	378.290	0.0093
k8	380.0	376.5	379.964	-0.0111
k9	381.8	376.5	381.908	0.0394

All the results were obtained after only one iteration. For these simple cases the method accurately identified the selected physical parameter values (damping and cross section areas).

Application to AH-1G Model

A NASTRAN finite element model (FEM) for the AH-1G helicopter airframe has existed for a long time and was originally developed by Bell Helicopter Textron Inc. It is basically composed of two parts, one is stiffness modeling for idealizing the structures and the other is weight modeling for distributing weights to grid points. There are 4405 different elements with a total of 2764 degrees of freedom in the basic full model. A reduced model, based on Guyan reduction, contains only a total of 63 physical degrees of freedom.

Normally, the input and output data files from NASTRAN dynamic analyses are specially formatted and are quite large for a large finite element model such as the full AH-1G model. For convenience and accuracy, the present system identification programs were designed to automatically read NASTRAN output files and create NASTRAN input data deck files. At each step in the iterative identification procedure, the new modified physical parameters were put into the NASTRAN model bulk data in order to generate the required analytical results, such as eigenvalues, eigenvectors and other parameters, for the next iteration.

The mass, stiffness and damping matrices defined with respect to the internal degrees of freedom are not normal NASTRAN output data. However, such results can be developed by using appropriate Direct Matrix Abstraction Programming (DMAP) utilities so that the identification program can automatically get this NASTRAN output data (see Appendix B).

Results Using Simulated Test Data

The NASTRAN model of an AH-1G airframe includes 4405 different elements with a total of 2764 degrees of freedom. In order to make sure that the identification procedure was appropriate to a such big model, the use simulation has been chosen to begin with. For this identification, the mass and stiffness properties of the analytical model were considered to be accurate, and nonproportional damping properties were identified. The physical damping parameters were associated with 8 distinctly different types of materials and structural fabrication techniques used in the airframe (e.g., aluminum, steel, riveted, welded, bolted, etc.) and one of these damping values was associated with each of the model elements using Eq. (11).

For this case, the test data were synthesized from the original NASTRAN model assuming small values for the extension and rotation viscous damping coefficients (kN-s/m and N-s/rad units):

TABLE 6. ASSUMED INITIAL PHYSICAL DAMPING VALUES

Extension	Rotation
$C_1 = 5.253$	$C_5 = 93.4$
$C_2 = 8.756$	$C_6 = 155.7$
$C_3 = 1.751$	$C_7 = 31.14$
$C_4 = 1.226$	$C_8 = 21.80$

The synthesized data included 24 modes of which 6 were rigid body modes, and the frequency range was from 0.0 to 30.2 Hz. The dimension of the mass, stiffness and damping matrices was 2764 x 2764. The initial values of the physical damping parameters for the analytical NASTRAN model were taken to be zero, and the results for the identified values are shown below:

TABLE 7. IDENTIFIED PHYSICAL DAMPING PARAMETERS

Parameter	Initial	Identified
C_1	5.253	5.429
C_2	8.756	10.490
C_3	1.751	1.746
C_4	1.226	6.069
C_5	93.4	55.91
C_6	155.7	160.35
C_7	31.14	65.96
C_8	21.80	55.91

The error in the identified damping parameters as a function of the number of matrix elements for each of the 8 damping types is shown in Fig. 4. The error for those element types with more than 100 elements present is quite low, but it is much larger for those types with only a few elements present in the complete finite element model. The largest error was associated with what appeared to be elastomeric materials.

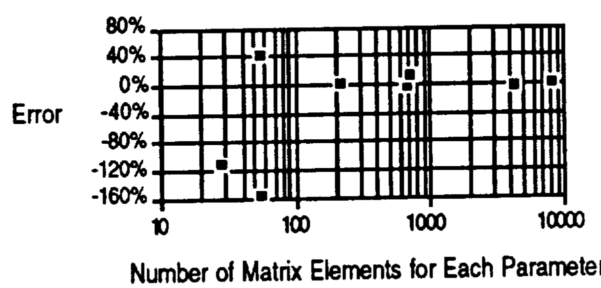

Fig. 4. Error in Damping Estimate as a Function of Number of Matrix Elements in Model

This simulation confirms the identification procedure for a complicated but yet well-defined example. If the assumptions such as nonproportional damping are correct for the airframe and if the experimental data are of high quality, the physical damping parameters can be identified from the test data.

Actual AH-1G Data

Actual test data for an AH-1G airframe were provided by Bell Helicopter Textron Inc. based on ground vibration tests and included both resonance dwell and FRF (frequency response function) data. The experimental data were available for 8 different configurations of the AH-1G that were tested. The principal difference between the tests concerned the degree of complexity of the actual airframe tested. At one extreme, the bare airframe without most attachments was tested while at the other extreme the complete airframe with all attached mechanical components was tested. The test data from the most complex airframe configuration (with all difficult components present) showed the poorest agreement with the corresponding analytical model, while the data from the simplest test airframe showed the best agreement.

For this study, the test data from the most complex airframe configuration was used. Only the FRF data were employed, and the complex eigenvalues and eigenvectors for some 7 triaxial modes were obtained from the FRF data by using the TDAS® curvefitting program[39]. The experimental data were provided as FRF's in TDAS universal file format, and the results generated by TDAS were complex eigenvalues and eigenvectors.

Before use in the identification program, the eigenvectors were normalized. Two options were used to normalize both experimental and analytical eigenvectors. One was to normalize the eigenvectors to the same point, and the other was to normalize based on the minimum deviation between the analytical and experimental eigenvectors.

The full finite element model for AH-1G airframe, as mentioned in the previous section, has a total of 2764 degrees of freedom which is very large for the identification procedure. In order to keep the problem tractable, a Guyan reduction was used in the present application to reduce the analytical model to a total of 63 degrees of freedom, which corresponded to the 23 distinct locations on the airframe at which experimental measurement were made. The error due to the reduction in degrees of freedom from 2764 to 63 is shown in Table 8.

TABLE 8.
EIGENVALUES (FREQUENCY) (WITHOUT ANY DAMPING)

Test	Full Model	Error (%)	Reduced Model	Error (%)
7.2475	7.6734	5.877	7.6932	6.150
8.0458	8.3467	3.740	8.4026	4.435
15.9539	14.6722	-8.034	15.825	-0.810
17.2174	17.3701	0.887	17.784	3.294
23.7396	20.7955	-12.392	22.881	-3.606
24.6675	25.7955	4.573	28.238	14.475
32.6848	31.7526	-2.852	33.786	3.369

Initially, both the full and the reduced models were used as analytical models. Using the actual experimental data, the physical parameters in the analytical models were obtained using the iterative procedure outlined earlier. The initial results for both the full model and the reduced model included several negative identified damping parameters which were obtained using the singular value decomposition method when either zero or positive initial guess values were assumed for the analytical model. Physically of course, the damping parameters should be greater

® TDAS (Test Data Analysis) is a part of I-DEAS which is a computer-aided engineering product of Structural Dynamical Research Corporation (SDRC).

than zero, but mathematically, the identification procedure is oblivious to this constraint. The constrained optimization procedure outlined earlier was therefore used in order to overcome this problem. In addition, the reduced model was used in most of the identification cases, expect when otherwise stated, because of the small error and big savings in computational time.

The complete system identification was carried out in two steps. The first step was to identify the stiffness, and for this process the initial damping values were assumed to be zero. The second step was to use the stiffness values obtained from the first step to identify the damping values. This was done under the assumption that the greatest change in natural frequency can be obtained by changing the stiffness parameter, while changes in the damping parameters will only fine-tune the eigenvalues but will obtain accurate modal damping estimates for the structure.

At the first step, four stiffness parameters associated with elastic moduli for four principal materials used in the airframe were selected to be identified. After two iterations, the differences between the identified and the initial moduli and the analytical and experimental eigenvalues were those shown in the following tables:

TABLE 9.
IDENTIFIED MODULUS VALUES

	Initial (GPa)	After first iteration (GPa)	Change from initial value (%)	After second iteration (GPa)	Change from initial value(%)
mat.-1	22.1	21.7	-1.81	3.937	23.03
mat.-2	72.4	72.5	0.19	9.417	-10.41
mat.-3	200.0	190.5	-4.75	28.435	-1.95
mat.-4	120.7	112.2	-7.01	19.396	10.83

TABLE 10.
IDENTIFIED EIGENVALUES (FREQUENCY)

Test	Original	Error (%)	After first iteration	Error (%)	After second iteration	Error (%)
7.247	7.693	6.15	7.686	6.05	7.426	2.47
8.046	8.403	4.43	8.394	4.33	8.064	0.22
15.95	15.82	-0.81	15.795	-0.99	15.302	-4.09
17.22	17.78	3.29	17.762	3.16	17.180	-0.22
23.74	22.88	-3.61	22.899	-3.53	21.819	-8.08
24.67	28.24	14.5	28.195	14.3	27.544	11.6
32.68	33.79	3.37	33.805	3.43	32.481	-0.62

As the second step, the damping parameters were identified for the previously identified stiffness conditions. Initial estimates for the damping parameters were developed by assuming a nominal damping ratio, $\zeta=5\%$. For the extensional elements, it was therefore assumed that the initial viscous damping values would be $c_E=17.5$ for all extensional viscous damping, and that for the rotational elements (assuming the cross section area to be a circle) it would be $c_R=222$ for all rotational damping.

After one iteration, the results shown in Table 11 were obtained.

TABLE 11.
FINAL RESULTS FOR AH-1G MODEL

Mode	Test		NASTRAN (original)	NASTRAN (final)	
	ω_n (Hz)	ζ (%)	ω_n (Hz)	ω_n (Hz)	ζ (%)
First Lat Bending	7.247	2.19	7.693	7.425	3.00
First Vert Bending	8.046	1.56	8.403	8.057	4.55
Second Lat Bending	15.954	3.05	15.82	15.41	1.70
Second Vert Bending	17.217	1.02	17.78	17.12	7.48
Fuselage Torsion	23.737	1.70	22.88	21.83	0.24
Third Vert Bending	24.667	1.31	28.24	27.702	6.25
Third Lat Bending	32.685	1.95	33.74	32.498	0.97

CONCLUSIONS AND RECOMMENDATIONS

A structural dynamic system identification procedure that is capable of identifying physical parameter changes has been developed. The changes in physical parameters of the system can therefore be related to observed experimental data. In the examples considered, physical parameters, such as the damping constant of a material that will result in a nonproportionally damped system, the modulus of elasticity of a material, and the dynamic stiffness of a beam element have been identified by using the experimentally obtained frequency response functions, modes and eigenvalues.

Following the validation of the developed procedures by using synthesized data on a small model, the method was applied to a large-scale NASTRAN finite element model of a helicopter airframe. Both synthesized data and observed experimentally identified modal data were used. Again, modulus of elasticity, stiffness and damping constants were the parameters considered for the four representative materials used in the airframe. With the exception of one material that had been used to construct a very small number of components, other material constants were identified reasonably accurately where synthesized data were used. When experimental modal data were used, the modal parameters calculated from the identified model did not yield the experimentally observed modes only in cases where the initial *a priori* finite element model output and the experimental model output differed considerably. When experimental output and the *a priori* model output were reasonably close, the results of the identification were satisfactory.

Even though the method was shown to work and the difference between the identified model and the experimental observations were considered satisfactory in some cases, there are some other cases that need improvement to make the procedure applicable to a structural dynamic design process:

(1) While the numerical processes were improved and refined, no similar improvements in the quality of the test data could be realized. One result of this problem was that it was relatively difficult to match measured eigenvalues and eigenvectors with corresponding analytical values. Quite often, the measured and initial eigenvalues matched closely while the eigenvectors differed considerably, and the identified eigenvectors were not significantly closer in agreement. For this reason it is necessary to consider other experimental data, such as the AH-1G dwell data, which have been acquired by other means.

(2) In cases where selected portions of experimental data and *a priori* analytical data differ significantly while a large amount of experimental and analytical data are close together, it is necessary to minimize first the large errors by using H_∞ type of identification before using the least square analysis with singular value decomposition.

(3) It is important that a larger group of identifiable parameters be considered.

(4) It is necessary that we examine the convergence and accuracy of the complete process.

(5) We have used linear sensitivity coefficients. Accuracy and convergence may require nonlinear sensitivity coefficients.

(6) The real damping in a structural dynamic system may not be linear viscous damping with a nonproportional behavior. It is necessary to include other types of damping mechanisms.

(7) As pointed out by Bell's DAMVIBS conclusions[1], nonlinearity is important in considering selected components of the airframe.

(8) We should also examine the experimental parameter estimation processes used to determine modal parameters used as inputs to the identification process.

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APPENDIX A

SYSTEM IDENTIFICATION PROGRAM LISTING

The following pages contain a listing of the current version of the program used to carry out the structural system identification described in this report. The program is written in the CDC version of Fortran 77 and was run on a CDC Cyber 180-990 running under the NOS/VE operating system. The program requires the use of the IMSL Scientific Library (Version 11) in order to carry out the singular value decomposition and the constrained optimization procedures.

The program was used with Version 66C of MSC/NASTRAN which was also run on the same computer system and was used to solve the structural dynamic eigenvalue problems. MSC/NASTRAN was used to run the initial eigenvalue problem and all subsequent iterative solutions. As a result, the program listing also includes the necessary I/O calls needed to operate directly with MSC/NASTRAN input and output data files. The program also requires the experimentally determined eigenvalues and eigenvectors to be present is separate input files for each eigenvalue.

A. Summary of Parameters Used by the Program

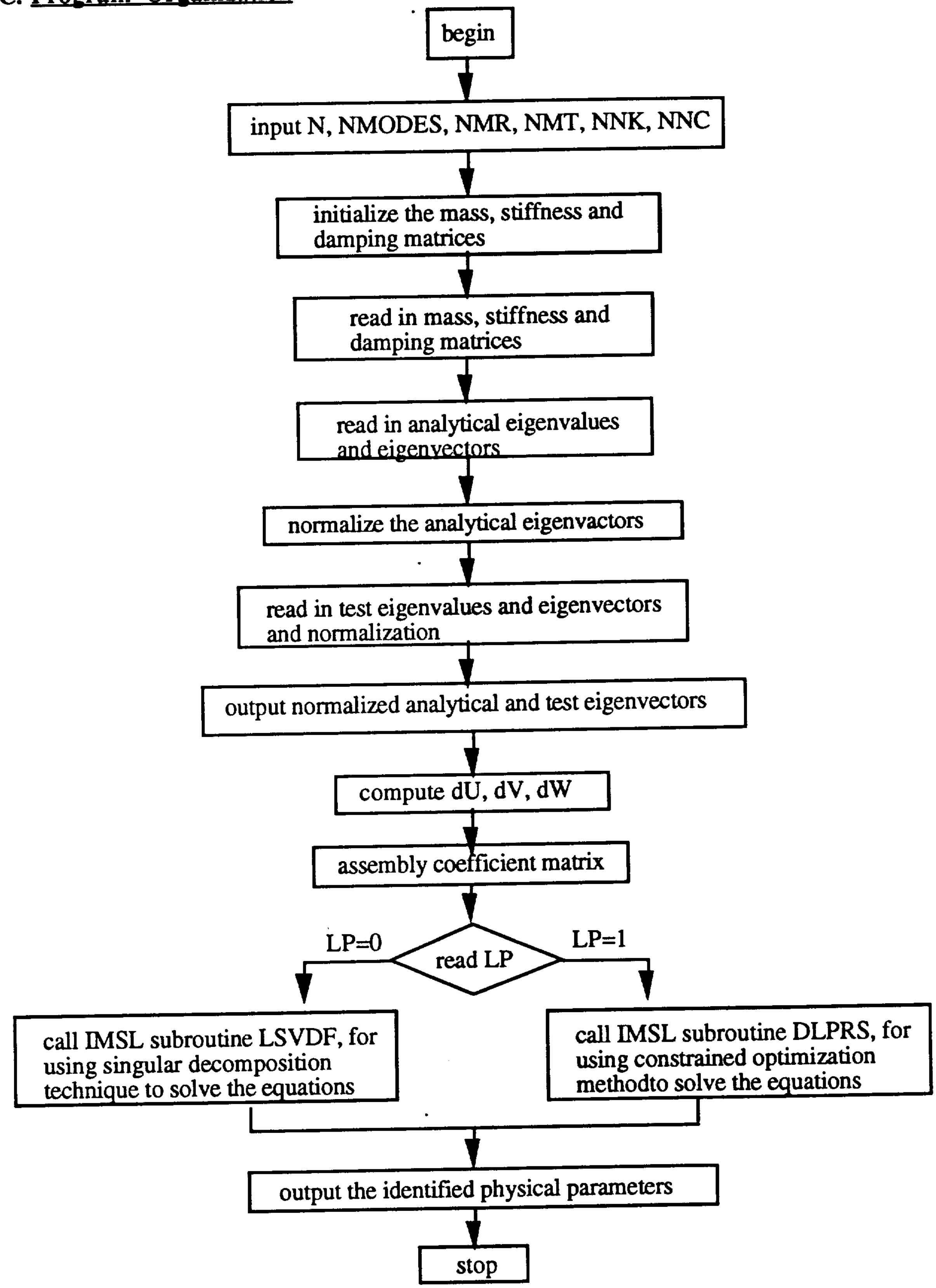
Parameter	Dimension	Definition
LAMBDR, LAMBDI	MD	real and imaginary parts of analytical eigenvalues λ_A
LAMBDTR, LAMBDTI	MD	real and imaginary parts of test eigenvalues λ
WAR, WAI	ND×MD	real and imaginary parts of analytical eigenvectors W_A
WTR, WTI	ND×MD	real and imaginary parts of test eigenvectors W
VTR,VTI	ND×MD	real and imaginary parts of V_A
DUR, DUI	ND×MD	real and imaginary parts of $U-U_A$
DVR, DVI	ND×MD	real and imaginary parts of $V-V_A$
DWR, DWI	ND×MD	real and imaginary parts of $W-W_A$
MASS	NV	mass matrix
STIFF	NV	stiffness matrix
DAMP	NV	damping matrix
DC, D^k	NV	$\mathbf{c}_i$ matrix
DK	NV	$\mathbf{k}_i$ matrix
COEFF	NM×ID	coefficient matrix
Y	NM	vector of the right hand side of the equations
WK	ID2	work vector
BETA	ID	damping parameters
GAMMA	ID	stiffness parameters
MODES	22×2	test eigenvector
NTESTk (k=1,2,5)	20×4	test measurement location definitions
NT	7	numbers of test eigenvectors corresponding to the eigenvectors of NASTRAN model
IRTYPE	12	vector indicating the type of constraints exclusive of simple bounds, where IRTYPE(I)=0,1,2,3 indicates .EQ., .LE., .GE., and range constraints respectively
BL,BU	12	vectors containing the lower and upper limits of the general constraints
A	12×12	matrix containing the coefficients of the constraints

C	12	vector containing the coefficients of the objective function
OBJ		value of the objective function
XLB, XUB	12	vectors containing the lower and upper bounds on the variables
XSOL, DSOL	12	vectors containing the primal and the dual solutions
ND		order of the system, default = 63
N		order of the system, (input)
MD		modes used in the identification, default = 25
NMODES		modes used in the identification, (input)
ID		number of physical parameters, default = 12
NUNK		number of unknowns to be identified (input)
NNK		number of stiffness unknowns
NNC		number of damping unknowns
NMR		number of rigid body motion modes
NMT		number of test modes
LP		choice of solving techniques LP=0, singular value decomposition LP=1, constrained optimization

B. Definition of Input and Output Files

File Name	Definition
TEST1, TEST2, TEST5	files containing 3 different test measurement location definitions
GUYAN_DAMP_F06	NASTRAN output data file including analytical mass, stiffness, damping matrices, eigenvalues and eigenvectors
GUYAN_KCOUT	output file including results
GUYAN_ELECDAT	NASTRAN output data file including the grouped element matrices $\mathbf{c}_i$
GUYAN_ELEKDAT	NASTRAN output data file including the grouped element matrices $\mathbf{k}_i$
TEST_EIGENV1, ..., TEST_EIGENV7	files containing the test eigenvectors
TEST_EIGENVAL	file containing the test eigenvalues

C. **Program Organization**



APPENDIX B

MSC/NASTRAN INPUT FOR FINAL AH-1G SYSTEM IDENTIFICATION RUN

The following pages include the listings of the input file for the final MSC/NASTRAN runs used to compute the structural eigenvalues and eigenvectors for the identified AH-1G structural model.

APPENDIX C

MSC/NASTRAN OUTPUT FOR FINAL AH-1G SYSTEM IDENTIFICATION RUN

The following pages include the listings of the output files from the final MSC/NASTRAN run used to compute the structural eigenvalues and eigenvectors for the identified AH-1G structural model.

Part II

IDENTIFICATION OF DAMPING CONSTANTS OTHER THAN THE VISCOUS DAMPING CONSTANTS

CHAPTER I

Introduction

In this part of the work, damping other than viscous damping has been considered. As a first step, nonlinear Coulomb damping has been studied. This method can also be extended to consider structural damping. This identification procedure uses the Hammerstein Feedback Model (HFM) , which represents the nonlinear dynamic system, and Singular Value Decomposition (SVD) Method for estimating the parameters. The identification of Coulomb damping constant of a Single Degree of Freedom (SDOF) nonlinear dynamic system and the estimating the parameters of Multiple Degree of Freedom (MDOF) nonlinear dynamic system have been illustrated in this report.

The identification of nonlinear dynamical system has received considerably amount of attention. These identification procedures are based on various models of nonlinear dynamical systems. Usually, a nonlinear system is represented by a set of nonlinear differential or integral equations. In many practical applications, an input-output approach of a nonlinear dynamical system is a means of describing a relationship between the input and the output of the system in some straightforward way and is considered to be more useful.

An approach for modeling a nonlinear dynamical system is by the use of Volterra Series [1],[2].

$$\begin{aligned} x(t) &= \int_0^t h_1(\tau)u(t-\tau)d\tau \\ &+ \int_0^t\int_0^t h_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 \\ &+ \int_0^t\int_0^t\int_0^t h_3(\tau_1,\tau_2,\tau_3)u(t-\tau_1)u(t-\tau_2) \\ &\quad u(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \cdots \end{aligned} \tag{1.0.1}$$

The Volterra Series, Eq (1.0.1), is a functional series, It maps past inputs into the present output. This means that many kernel values are required to estimate. Several techniques have been presented [3],[4], [5]. Because we have to decide which terms of Volterra Series are necessary for a given practical problem and to estimate many kernel values, the procedure of identification is usually a difficult procedure.

Several other simple block-oriented input-output models for representing nonlinear dynamical systems are as follows. [7].

- Simple Hammerstein Model.
- Generalized Hammerstein Model.
- Simple Wiener Approach.
- Generalized Wiener Approach.
- Extended Wiener Approach.
- Simple Wiener-Hammerstein Approach.
- Generalized Wiener-Hammerstein Model.
- Extended Wiener-Hammerstein Model.

The block-oriented models have been widely used because of their simplicity.

In 1985, a nonlinear difference equation model NARMAX (Nonlinear Autoregressive Moving Average Models with inputs) was presented by Leontaritis and Billings [9],[10] . The NARMAX model is said to be an unified representation of a finitely realizable nonlinear system. The finitely realizable nonlinear system in essence means that the state space of the system can not be infinite dimensional. This model maps the past inputs and outputs to current output. For the SISO (single input and single output) nonlinear dynamical system with white noise, it can be denoted by [11]

$$x(k) = F[x(k-1), ..., x(k-n_x), u(k-1), ..., u(k-n_u)] \tag{1.0.2}$$

Where $F(*)$ is an unknown nonlinear function. In general, it will be determined for a given real sampled nonlinear system. Leontaritis and Billings proved that a nonlinear discrete time invariant system can always be denoted by Eg.(1.0.2) in a region around an equilibrium point, if the response function of system is finitely realizable and a linearized model exists at the chosen equilibrium.

The NARMAX model is derived assuming zero initial state response, but it can be carried over to the non-zero-initial-state cases. The response functions of a system are different for different initial condition, but the input-output NARMAX model for the system will always be the same within a region around an equilibrium point. Several simple forms of the NARMAX model have been proposed for nonlinear dynamic system identification, such as the Bilinear Model.[11],[12]

$$\begin{aligned} x(k) &= a_0 + \sum_{i=1}^{n} a_i x(k-i) + \sum_{i=1}^{n} b_i u(k-i) \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x(k-i) u(k-j) \end{aligned} \tag{1.0.3}$$

the fractional model.[11], [13],[14]

$$x(k) = \frac{b[x(k-1), \cdots,}{a[x(k-1), \cdots,} \frac{x(k-r), u(k-1), \cdots, u(k-r)]}{x(k-r), u(k-1), \cdots, u(k-r)]} \tag{1.0.4}$$

Haber and Unbehauen [7] prefer the NARMAX model, because the NARMAX model is parametric and has fewer parameters than the Volterra series.

In aerospace engineering applications, a nonlinear structural dynamical system is usually described by a system of nonlinear differential equations. In SISO case , the nonlinear differential equation of a system is of the form

$$\ddot{x} + b\dot{x} + cx + f(\dot{x}, x) = u(t) \tag{1.0.5}$$

where $f(*)$ is a nonlinear function of $\dot{x}$,x. If $f(*)$ is represented by a polynomial extension for simplicity, Eq.(1.0.5) becomes

$$\ddot{x} + b\dot{x} + cx + \alpha_2 x^2 + \alpha_3 x^3 + \ldots + \beta_2 \dot{x}^2 + \beta_3 \dot{x}^3 + \ldots = u(t) \tag{1.0.6}$$

Every term in Eq.(1.0.6) has a distinct physical meaning. Identifying the parameters of Eq.(1.0.6) are useful for dynamic analysis, structural dynamic design, control and design modification. If the nonlinear structural dynamic system is modeled by using Eq.(1.0.6), the problem of the identification of a system is to estimate the parameters : $b, c, \alpha_2, \cdots, \beta_2, \cdots$.

Many techniques for estimating these parameters have been proposed. Hanagud, Meyyappa and Craig (1985) [15] used the method of multiple scales to formulate a procedure for identification of parameters of Eq.(1.0.6). Mook(1988) [16] used a model error method to find the model error $d(t)$ which represents the nonlinear terms of the nonlinear dynamic system and then estimated the nonlinear parameters from $d(t)$ by using a least square method. Yun and Shinozuka [17] proposed an approach that is based on two versions of Kalman filter for identifying the parameters. Ibanez [18] used an approach for estimating parameters in which it is assumed that the system response is dominated by a periodic response at the forcing frequency and an approximate transfer function is constructed. Broersen [19] replaced nonlinear terms in the equation by a series expansion for a system subjected to random excitation. Distefano and Rath, Yun and Shinozuka [20] [21] described several methods of of identification and applied nonlinear Kalman filtering techniques for estimation.

If a structural control is considered, an input- output approach of nonlinear structural dynamic system in time domain and its parameter identification is useful. For this purpose, the Hammerstein Feedback Model (HFM) has been considered here.

CHAPTER II

Background: Hammerstein Operator and Hammerstein Model

In 1924, P. Uryson investigated a nonlinear integral operator [24],[25] of the following type.

$$Ax(t) = \int_{\Omega} k[t, \tau, x(\tau)]d\tau$$
$$t \in \Omega^* \tag{2.0.1}$$

where Ω and Ω^* are two sets of finite Lebesque measures in a finite dimensional space. $t \in \Omega^*$, $\tau \in \Omega$, $-\infty < x(t) < \infty$. $K[t, \tau, x(\tau)]$ is measurable and it satisfies the Caratheodory condition. The Caratheodory condition is that for all $x(\tau)$, it is jointly measurable in the variables $(t, \tau) \in \Omega^* \times \Omega$ and for all $(t, \tau) \in \Omega^* \times \Omega$, it is continuous in $x(\tau)$.

In 1930, A. Hammerstein studied the following integral equation:

$$x(t) = \int_{\Omega} K(t, \tau)f(\tau, x(\tau)]d\tau \tag{2.0.2}$$

This kind of equation is known as Hammerstein equation.
Eq.(2.0.2) is a special form of the Uryson nonlinear integral operator :

$$\int_{\Omega} K_0(t, \tau)f[\tau, x(\tau)]d\tau = Hx(t) \tag{2.0.3}$$

This integral operator H is called Hammerstein integral operator. Hammerstein integral operator H can be denoted by the following form:

$$H = K_0\mathcal{F} \tag{2.0.4}$$

where K_0 represents a linear integral operator with kernel $K_0(t,\tau)$:

$$K_0x(t) = \int_{\Omega} K_0(t,\tau)x(\tau)d\tau \tag{2.0.5}$$

and $\mathcal{F}$ represents the nonlinear superposition operator [25].

$$\mathcal{F}x(\tau) = f[\tau, x(\tau)] \tag{2.0.6}$$

Then Hammerstein equation Eq.(2.0.2) can be expressed by

$$x(t) = K_0\mathcal{F}x(t) \tag{2.0.7}$$

and Hammerstein integral operator can be denoted in following form

$$Hx(t) = K_0\mathcal{F}x(t) \tag{2.0.8}$$

Let $L_\alpha, L_\beta, L\gamma$ express the sets of measurable function $x(\tau)$. They have the norms separately as follows.

$$\| x \|_\alpha = [\int_{\Omega} | x(\tau) |^{-\alpha} d\tau]^\alpha \tag{2.0.9}$$

$$\| x \|_\beta = [\int_{\Omega} | x(\tau) |^{-\beta} d\tau]^\beta \tag{2.0.10}$$

$$\| x \|_\gamma = [\int_{\Omega} | x(\tau) |^{-\gamma} d\tau]^\gamma \tag{2.0.11}$$

and L_0 is the set of $x(\tau)$ with norm

$$\| x \|_0 = sup \mid x(\tau) \mid \tag{2.0.12}$$

There are two theorems about Hammerstein operator H: [25]

Theorem 1: Let $\mathcal{F}$ be an operator acting from L_α to L_γ $(\gamma > 0)$ and let K_0 be a continuous operator acting from L_γ to L_β. Then Hammerstein operator $H = K_0\mathcal{F}$ acts from L_α to L_β and is continuous.

Theorem 2: Let $\mathcal{F}$ act from L_α to L_0 and let K_0 be a regular operator acting from L_0 to L_β $(\beta > 0)$, then the Hammerstein operator $H = K_0\mathcal{F}$ acts from L_α to L_β and is continuous.

The theorems 1,2 are very useful. It permit one to construct the Hammerstein operator if K_0 and $\mathcal{F}$ are known.

In 1966, K.S. Narendra and P.G. Gallman [6] suggested a Hammerstein Model for identification of nonlinear dynamic system. They assumed that the response $x(t)$ of nonlinear dynamical system is

$$\begin{aligned} x(t) &= Hu(t) \\ &= K_0\mathcal{F}u(t) \end{aligned} \tag{2.0.13}$$

where $u(t)$ is an input function. Then the Hammerstein Model suggested by Narendra and Gallman consists of a nonmemory (independent from history) nonlinear gain having a polynomial form followed by a linear discrete system. The nonlinear gain is

$$\begin{aligned} v(t) &= \mathcal{F}u(t) \\ &= c_1 u + c_2 u^2 + \cdots + c_p u^p \end{aligned} \tag{2.0.14}$$

The linear discrete system has a response .

$$x(t) = \int_\Omega K_0(t,\tau)v(\tau)d\tau \tag{2.0.15}$$

Narendra and Gallman used the following form to denote the linear system.

$$\frac{b_1 q^{-1} + \cdots + b_n q^{-n}}{a_0 + a_1 q^{-1} + \cdots + a_n q^{-n}} \tag{2.0.16}$$

where the $q^{-1}, \cdots, q^{-n}$ are the time delay operators. They are defined as

$$\begin{aligned} x(k)q^{-1} &= x(k-1) \\ &\vdots \\ x(k)q^{-n} &= x(k-n) \end{aligned} \tag{2.0.17}$$

This Hammerstein Model suggested by Narendra and Gallman is illustrated in Fig.2.1. Hammerstein model provides a simple input-output model for identification of nonlinear dynamical system. In the past years , Hammerstein model has been widely used

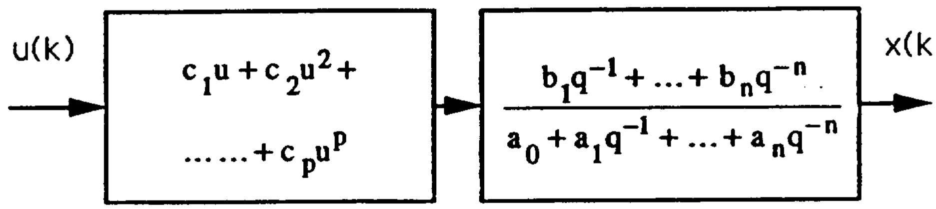


Figure 2.1: The Hammerstein Model

in various fields.

Neil, Francis and Rein [26] presented a simple iterative technique for estimating parameters in a Hammerstein model for a case where noise in the output data is correlated. They suggested a Hammerstein model with noise . The noise is modeled by the following transfer form .

$$H(q) = \frac{1}{1 + c_1q^{-1} + \cdots + c_qq^{-m}} \tag{2.0.18}$$

The expected value of noise $d(j)$ equals to zero. The parameters are estimated by an iterative method.

Greblick and Pawlak [27] used a correlation method for Hammerstein system identification by using non-parametric regression estimation. Shih and Kung [30], [31] used shifted Legendre and Chebyshev expansions for identification of the nonlinear dynamic system described by a Hammerstein model. The input and output are expressed by using a finite number of the shifted Legendre polynomials or of the shifted Chebyshev polynomials. Substituting these into the Hammerstein model in a state variable form, and integrating from 0 to t, then we reduce the nonlinear equations to a linear algebraic equation. The parameters are obtained from the linear algebraic equations. Horng and Chou,[32], used shifted Jacobi series to express the input and output. Substituting the input and output into the Hammerstein model, then integrating the model from 0 to t, a linear algebraic equation is obtained for identification of nonlinear dynamical system. Chung and Sun,[33], used a Taylor's series approximation for Hammerstein model to estimate the parameters. Kung and Shih [34], and Jiang [35], used the Block pulse function for identification of parameters of a nonlinear system with Hammerstein model etc.

Actually, the Hammerstein model suggested by Narendra and Gallman can be considered as a superposition of several linear system models. Their objective was primarilly to consider only nonlinearity in the forcing function $u(t)$, however in a structural dynamic system, nonlinearities are from damping and stiffness terms, (or the plant model). Therefore we will propose a feedback model which is named here as Hammerstein Feedback Model (HFM) for identification of nonlinear feedback system for a nonlinear structural dynamical system and identify the nonlinear plant parameters.

CHAPTER III

HAMMERSTEIN FEEDBACK MODEL OF NONLINEAR DYNAMIC SYSTEM

Consider a linear equation.

$$Lx = f(t) \tag{3.0.1}$$

where f(t) is an almost periodic function (ap-function). The ap-function is defined as: A function $x(t) \in C(R)$ ($C(R)$ denotes a continuous metric function space) , which has the translate

$$x_h(t) = x(t+h) \tag{3.0.2}$$

where $(-\infty < t < \infty)$ and $(-\infty < h < \infty)$, is called as an ap-function, if its translates form a compact set in $C(R)$.
L is a regular ap-operator .

$$L = \frac{d^n}{dt^n} + A_1(t)\frac{d^{n-1}}{dt^{n-1}} + \cdots + A_n(t) \tag{3.0.3}$$

where $A_i(t)$ $(i = 1, \cdots, n)$ are ap-functions. There is at most one function $G(t, \tau)$ $(-\infty < t, \tau < \infty)$, which is called to be the Green's function, such that one can write the solution of Eq.(3.0.1) as:

$$x(t) = \int_\Omega G(t,\tau)\dot{f}(\tau)d\tau \tag{3.0.4}$$

Now consider the nonlinear equation.

$$Lx = f(t, x, \dot{x}, u) \tag{3.0.5}$$

where the function $f(t, x, \dot{x}, u)$ is jointly continuous for t, x and almost periodic in t . It can be easily seen that the ap-function $x(t)$ is a solution if and only if it is a solution of the integral equation, called the Hammerstein integral equation.

$$\begin{aligned} x(t) &= \int_{\Omega} G(t,\tau) f[\tau, x(\tau), \dot{x}(\tau), u(\tau)] d\tau \\ &= H(x, \dot{x}, u, t) \end{aligned} \tag{3.0.6}$$

The right side of Eq.(3.0.6) is known as Hammerstein integral operator, which can be denoted by

$$H = K_0 \mathcal{F} \tag{3.0.7}$$

where K_0 represents a linear integral operator with Green's function G of operator L, and $\mathcal{F}$ represents a nonlinear superposition operator

$$\mathcal{F}(x, \dot{x}, u, t) = f[x(t), \dot{x}(t), u(t), t] \tag{3.0.8}$$

Derivation of Hammerstein integral equation

For linear case, consider a linear dynamic system for instant.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{3.0.9}$$

where A and B are constants. The Laplace transform of Eq.(3.0.9) is

$$s\bar{x} + x(0) = A\bar{x} + B\bar{u} \tag{3.0.10}$$

i.e.

$$\bar{x} = [sI - A]^{-1} x(0) + [sI - A]^{-1} B\bar{u} \tag{3.0.11}$$

The response of system is

$$x(t) = e^{At} x(0) + \int_0^t e^{At} e^{-A\tau} Bu(\tau) d\tau \tag{3.0.12}$$

i.e.

$$x(t) = \int_0^t GBu(\tau) d\tau \tag{3.0.13}$$

If the initial condition is assumed as $x(0) = 0$. For nonlinear case, consider a nonlinear dynamic system

$$\dot{x}(t) = Ax(t) + f[x(t), \dot{x}(t), u(t)] \tag{3.0.14}$$

Initial condition: $x(0) = 0$. Construct a response for the nonlinear dynamic system, which is denoted by Eq.(3.0.14).

$$x(t) = \int_0^t d^{At} e^{-A\tau} f[x(\tau), \dot{x}(\tau), u(\tau)] d\tau \tag{3.0.15}$$

The derivative of Eq.(3.0.15) is

$$\begin{aligned} \dot{x}(t) &= \frac{de^{At}}{dt} \int_0^t e^{-A\tau} f[x(\tau), \dot{x}(\tau), u(\tau)] d\tau \\ &+ e^{At} e^{-At} f[x(t), \dot{x}(t), u(t)] = Ax(t) + f[x, \dot{x}, u] \end{aligned} \tag{3.0.16}$$

Eq.(3.0.15) is the Hammerstein integral equation, which is equivalent to the nonlinear differential equation Eq.(3.0.14). Eq.(3.0.15) can be rewritten as

$$x(t) = H(x, \dot{x}, u) \tag{3.0.17}$$

where H is the Hammerstein integral operator. Since there are two theorems about Hammerstein operator H:

Theorem 1: Let $\mathcal{F}$ be an operator acting from space L_α to space L_γ ($\gamma > 0$) and let K_0 be a continuous operator acting from space L_γ to space L_β. Then Hammerstein operator $H = K_0\mathcal{F}$ acts from space L_α to space L_β and is continuous.

Theorem 2: Let $\mathcal{F}$ act from space L_α to space L_0 and let K_0 be a regular operator acting from space L_0 to space L_β ($\beta > 0$), then the Hammerstein operator $H = K_0\mathcal{F}$ acts from space L_α to space L_β and is continuous.

According to Hammerstein integral equation and theorems 1 and 2 of Hammerstein integral operator, a Hammerstein Feedback Model (HFM) can be constructed for representing a nonlinear dynamic system. The HFM consists of nonlinear part, which is expressed by a superposition operator $\mathcal{F}$ and contains the nonlinear terms of state variables, followed by a linear system, which is denoted by linear integral operator

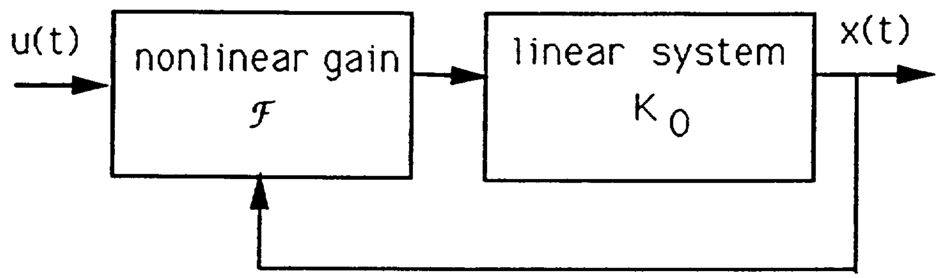


Figure 3.1: The Hammerstein Feedback Model (HFM)

K_0. HFM is a simple block-oriented input-output model for identification of nonlinear structural dynamic systems. It can be illustrated in Fig. 3.1.

CHAPTER IV

IDENTIFICATION OF NONLINEAR STRUCTURAL SINGLE DEGREE OF FREEDOM (SDOF) DYNAMIC SYSTEM BY USING HFM

In chapter 3, the Hammerstein Feedback Model (HFM) of nonlinear dynamic system has been proposed for identification of nonlinear dynamic systems. In this chapter the HFM is used for identification of nonlinear structural single degree of freedom (SDOF) dynamic system. The HFM in discrete time domain of SDOF nonlinear structural dynamic system has been derived. The Singular Value Decomposition Method (SVDM) is used for estimating parameters of HFM of SDOF nonlinear structural dynamic system.

4.1 Hammerstein Feedback Model in Discrete Time Domain of SDOF Nonlinear Dynamic System

For HFM, the response of nonlinear dynamic system is the convolution of a weighting function i.e. Green's function of linear dynamical system and a nonlinear function of the input and the output of nonlinear dynamic system. The Z-transformation of a sequence of function and its properties can be used for deriving the HFM in discrete time domain.

Consider a sequence of function $f(k)$ ($f(k) = 0$ for $k = -1, -2, \cdots$). The Z-transform of $f(k)$ is defined by

$$F(z) = \sum_{k=0}^{\infty} f(k) z^{-k} \tag{4.1.1}$$

and inverse Z-transform is

$$f(k) = Z^{-1}[F(z)] \tag{4.1.2}$$

where z is an arbitrary complex number. The Z-transformation of a sequence of function $f(k)$ has following properties:

Linearty:

If $f(k) = af_1(k) + bf_2(k)$ for $k = 0, 1, 2, \cdots$, where a, b are constants, then

$$F(z) = aF_1(z) + bF_2(z) \tag{4.1.3}$$

for $\mid z \mid > \max(R_1, R_2)$, where R_1, R_2 denote radii of convergence for $F_1(z)$, $F_2(z)$, respectively.

Right-shifting Property:

Consider $f(k)$ ($f(k) = 0$, for $k = 0, -1, -2, \cdots$) and $y(k) = f(k-m)$ for $k = 0, 1, 2, \cdots$. From the definition of Z-transformation, we have

$$\begin{aligned} Y(z) &= \sum_{k=0}^{\infty} f(k-m) z^{-k} \\ &= f(-m) + f(1-m)z^{-1} + \cdots + f(-1)z^{-m+1} \\ &\quad + f(0)z^{-m} + f(1)z^{-m+1} + \cdots \\ &= z^{-m}[f(0) + f(1)z^{-1} + \cdots] \\ &= z^{-m}F(z) \\ &\quad \text{for } \mid z \mid > R \end{aligned} \tag{4.1.4}$$

where R is the radius of convergence for the Z-transformation of $f(k)$. Then the Right-shifting property of Z-transformation can be denoted by the following form.

$$Z[f(k-m)] = z^{-m}F(z) \tag{4.1.5}$$

Convolution-Summation Property:

Consider a convolution $y(k)$ of two sequences $h(k)$ and $u(k)$

$$y(k) = \sum_{i=0}^{\infty} h(i)u(k-i) \tag{4.1.6}$$

The Z-transformation of $y(k)$ is

$$Y(z) = \sum_{k=0}^{\infty}[\sum_{i=0}^{\infty} h(i)u(k-i)]z^{-k} \tag{4.1.7}$$

From the Right-shifting property, we have

$$Z[u(k-i)] = z^{-i}U(z) \tag{4.1.8}$$

Eq.(4.1.7) can be represented as

$$\begin{aligned} Y(z) &= \sum_{i=0}^{\infty} h(i)z^{-i}U(z) \\ &= H(z)U(z) \end{aligned} \tag{4.1.9}$$

If input sequence $u(k)$ is kroneker δ function,

$$u(k) = \delta(k) \tag{4.1.10}$$

the Z-transformation of $u(k)$ is

$$U(z) = 1 \tag{4.1.11}$$

In this case, Eq.(4.1.9) becomes

$$Y(z) = H(z) \tag{4.1.12}$$

Eq.(4.1.12) denotes that response of any linear discrete system for Kronecker δ input is equal to the weighting sequence of the linear dynamic system.

The Hammerstein Feedback Model implys that a nonlinear dynamic system is assumed as a linear system with a nonlinear input, which is a function of the input and the output of nonlinear dynamic system. For a nonlinear dynamic system, if the responses of system are known and steady, and if the parameters of system are time-invariant and linear, the estimating parameters of system reduces to a linear problem.

Let us consider a SDOF linear dynamic system. This SDOF linear dynamic system is represented in a differential equation with order n in general.

$$\begin{aligned}\frac{d^n x(t)}{dt^n} &+ a_1 \frac{d^{n-1} x(t)}{dt^{n-1}} + \cdots + a_n x(t) \\ &= b_1 \frac{d^{n-1} u(t)}{dt^{n-1}} + \cdots + b_n u(t)\end{aligned} \tag{4.1.13}$$

with initial conditions: $x(0)$, $\dot{x}(0)$, $\cdots$, $\frac{d^{n-1}x(0)}{dt^{n-1}}$. If the initial conditions are assumed to be zero,

$$\begin{aligned} x(0) &= 0 \\ \dot{x}(0) &= 0 \\ &\vdots \\ \frac{d^{n-1}}{dt^{n-1}} x(0) &= 0 \end{aligned} \tag{4.1.14}$$

the response of linear dynamic system has the response of following form

$$x(t) = \int_{\Omega} h(t-\tau) u(\tau) d\tau \tag{4.1.15}$$

In discrete time domain, the sequence of response of system is

$$x(k) = \sum_{i=0}^{p} h(k) u(k-i) \tag{4.1.16}$$

The transfer function of linear system in z-domain is

$$\begin{aligned} H(z) &= \frac{X(z)}{U(z)} \\ &= \frac{b_1 z^{n-1} + \ldots + b_n}{z^n + a_1 z^{n-1} + \ldots + a_n} \end{aligned} \tag{4.1.17}$$

The transfer function Eq.(4.1.17) can be rewritten as

$$H(z) = \frac{b_1 z^{-1} + \cdots + b_n z^{-n}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}} \tag{4.1.18}$$

Consider a nonlinear superposition operator $\mathcal{F}$. It is assumed in the form of a polynomial expression for simplicity.

$$\begin{aligned}\mathcal{F} \quad [\quad & x(t), \dot{x}(t), u(t), t] = \gamma_1 u(t) + \gamma_2 x^2(t) + \cdots \\ + \quad & \gamma_p x^p(t) + \mu_2 \dot{x}^2(t) + \cdots + \mu_q \dot{x}^q(t) \end{aligned} \tag{4.1.19}$$

Since the function $f(x, \dot{x}, u, t)$, which is respect with the nonlinear superposition operator, is a polynomial expression, it satisfies the Caratheodory condition. The z-transform of Eq.(4.1.19) is

$$\begin{aligned} Z(y) \quad = \quad & \gamma_1 Z(u) + \gamma_2 Z(x^2) + \cdots + \gamma_p Z(x^p) \\ + \quad & \mu_2 Z(\dot{x}^2) + \cdots + \mu_q Z(\dot{x}^q) \end{aligned} \tag{4.1.20}$$

According to Eq.(4.1.9), we can construct a Hammerstein Feedback Model in dicrete time domain, which is

$$\begin{aligned} Z(x) \quad = \quad & \frac{b_1 z^{-1} + \cdots + b_n z^{-n}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}} [\gamma_1 Z(u) \\ + \quad & \gamma_2 Z(x^2) + \cdots + \gamma_p Z(x^p) \\ + \quad & \mu_2 Z(\dot{x}^2) + \cdots + \mu_q Z(\dot{x}^q)] \end{aligned} \tag{4.1.21}$$

This HFM of SDOF nonlinear dynamic system in Z-domain is illustrated in Fig.4.1. Eq.(4.1.21) can be rewritten as

$$\begin{aligned} Z(x) \quad = \quad & -a_1 Z(x) z^{-1} - \cdots - a_n Z(x) z^{-n} + b_1 \gamma_1 Z(u) z^{-1} \\ + \quad & b_1 \gamma_2 Z(x^2) z^{-1} + \cdots + b_1 \gamma_p Z(x^p) z^{-1} + b_1 \mu_2 Z(\dot{x}^2) z^{-1} \\ + \quad & \cdots + b_1 \mu_q Z(\dot{x}^q) z^{-1} + \cdots + b_n \gamma_1 Z(u) z^{-n} \\ + \quad & \cdots + b_n \mu_q Z(\dot{x}^q) z^{-n} \end{aligned} \tag{4.1.22}$$

By using the Right-shifting Property of Z-transform, Eq.(4.1.22) yields

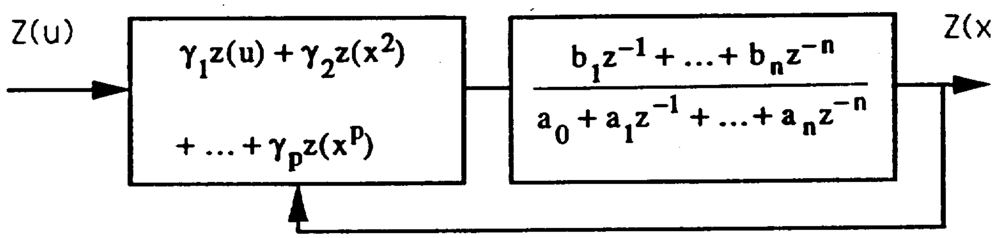


Figure 4.1: The HFM of SDOF nonlinear dynamic system in z-domain.

$$
\begin{aligned}
Z[x(k)] &= -a_1 Z[x(k-1)] - \cdots - a_n Z[x(k-n)] + b_1\gamma_1 Z[u(k-1)] \\
&+ b_1\gamma_2 Z[x^2(k-1)] + \cdots + b_p\gamma_p Z[x^p(k-1)] + b_1\mu_2 Z[\dot{x}^2(k-1)] \\
&+ \cdots + b_1\mu_q Z[\dot{x}^q(k-1)] + \cdots + b_n\gamma_1 Z[u(k-n)] \\
&+ \cdots + b_n\mu_q Z[\dot{x}^q(k-n)] \qquad (4.1.23)
\end{aligned}
$$

Consider the inverse Z-transform, we have

$$
\begin{aligned}
x(k) &= -a_1 x(k-1) - \cdots - a_n x(k-n) \\
&+ b_1\gamma_1 u(k-1) + b_1\gamma_2 x^2(k-1) \\
&+ \cdots + b_1\gamma_p x^p(k-1) + b_1\mu_2 \dot{x}^2(k-1) \\
&+ \cdots + b_n\gamma_1 u(k-n) + b_n\gamma_2 x^2(k-n) \\
&+ \cdots + b_n\gamma_p x^p(k-n) + b_n\mu_2 \dot{x}^2(k-n) \\
&+ \cdots \div b_n\mu_q \dot{x}(k-n) \qquad (4.1.24)
\end{aligned}
$$

Eq.(4.1.24) is the difference form of HFM in discrete time domain of SDOF nonlinear dynamic system. This HFM in discrete time domain, Eq.(4.1.24), is equivalent to following nonlinear differential equation.

$$\begin{aligned} \frac{d^n x(t)}{dt^n} &+ a_1 \frac{d^{n-1} x(t)}{dt^{n-1}} + \cdots + a_n x(t) \\ &= b_1 \frac{d^{n-1} f(t)}{dt^{n-1}} + \cdots + b_n f(t) \end{aligned} \tag{4.1.25}$$

where input $f(t)$ is

$$\begin{aligned} f(t) &= \gamma_1 u(t) + \gamma_2 x^2(t) + \cdots \\ &+ \gamma_p x^p(t) + \mu_2 \dot{x}^2(t) + \cdots + \mu_q \dot{x}^q(t) \end{aligned} \tag{4.1.26}$$

Observably, since the parameters of nonlinear differential equation, Eq.(4.1.25), are indepedent from the initial conditions, the parameters of HFM in dicrete time domain, Eq.(4.1.24), are independent from the initial conditions. This means that we can use the input and output data under any initial conditions to identify the parameters of HFM in discrete time domain of a nonlinear dynamic system.

4.2 The HFM of a Nonlinear Structural SDOF Dynamical System

A nonlinear structural SDOF dynamical system usually is expressed in the form of

$$\ddot{x} + b\dot{x} + cx + f(\dot{x}, x) = u \tag{4.2.27}$$

where f(*) is a nonlinear function of $\dot{x}, x$. It can be also approximately expressed by

$$\begin{aligned} &\ddot{x} + b\dot{x} + cx + \alpha_2 x^2 + \alpha_3 x^3 + \cdots \\ &+\alpha_p x^p + \beta_2 \dot{x}^2 + \cdots + \beta_q \dot{x}^q = u \end{aligned} \tag{4.2.28}$$

In this case, the linear differential equation corresponding to Eq.(4.2.27) or Eq.(4.2.28) is

$$\ddot{x} + b\dot{x} + cx = u \tag{4.2.29}$$

with initial condition: $x(0)$,$\dot{x}(0)$

The transfer function of the linear system in z-domain is

$$H(z) = \frac{X(z)}{U(z)} = \frac{z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{4.2.30}$$

The nonlinear input $f(t)$ in HFM is defined as

$$\begin{aligned} f(t) &= u(t) + \gamma_2 x^2(t) + \cdots + \gamma_p x^p(t) \\ &+ \mu_2 \dot{x}^2 + \cdots + \mu_q \dot{x}^q \end{aligned} \tag{4.2.31}$$

The z-transform of nonlinear input $f(t)$ is

$$\begin{aligned} Z(f) &= Z(u) + \gamma_2 Z(x^2) + \cdots + \gamma_p Z(x^p) \\ &+ \mu_2 Z(\dot{x}^2) + \cdots + \mu_q Z(\dot{x}^q) \end{aligned} \tag{4.2.32}$$

Then we have the HFM of nonlinear structural SDOF dynamic system in z-domain.

$$\begin{aligned} Z(x) &= \frac{z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} [Z(u) \\ &+ \gamma_2 Z(x^2) + \cdots + \gamma_p Z(x^p)] \\ &+ \mu_2 Z(\dot{x}^2) + \cdots + \mu_q Z(\dot{x}^q)] \end{aligned} \tag{4.2.33}$$

Eq.(4.2.33) can be rewritten as

$$\begin{aligned} Z(x) &= -a_1 Z(x) z^{-1} - a_2 Z(x) z^{-2} + Z(u) z^{-2} \\ &+ \gamma_2 Z(x^2) z^{-2} + \cdots + \gamma_p Z(x^p) z^{-2} \\ &+ \mu_2 Z(\dot{x}^2) z^{-2} + \cdots + \mu_q Z(\dot{x}^q) z^{-2} \end{aligned} \tag{4.2.34}$$

By using the Right-shifting Property of Z transform , we have the HFM of nonlinear structural SDOF dynamic system in discrete time domain.

$$\begin{aligned} x(k) &= -a_1 x(k-1) - a_2 x(k-2) + u(k-2) \\ &+ a_3 x^2(k-2) + \cdots + a_{p+1} x^p(k-2) \\ &+ a_{p+2} \dot{x}^2(k-2) + \cdots + a_{p+q+1} \dot{x}^q(k-2) \end{aligned} \tag{4.2.35}$$

where

$$
\begin{aligned}
a_3 &= \gamma_2 \\
a_4 &= \gamma_3 \\
&\vdots \\
a_{p+q+1} &= \mu_q
\end{aligned}
\tag{4.2.36}
$$

The relationship between parameters of nonlinear differential equation and parameters of HFM are

$$
\begin{aligned}
b &= (a_1 + 2)/\Delta t \\
c &= (a_1 + a_2 + 1)/(\Delta t)^2 \\
\alpha_2 &= -a_3/(\Delta t)^2 \\
\alpha_3 &= -a_4/(\Delta t)^2 \\
&\vdots \\
\beta_2 &= -a_{p+2}/(\Delta t)^2 \\
&\vdots \\
\beta_q &= -a_{p+q+1}/(\Delta t)^2
\end{aligned}
\tag{4.2.37}
$$

4.3 Estimation of Parameters of Nonlinear Structural SDOF Dynamic System

When a nonlinear dynamical system is modeled by HFM in discrete time domain, Eq.(4.1.24), and $N+n$ samples of the input and the steady output from $k-n$ to $k+N$ are substituted into Eq.(4.1.24), we have N equations. The set of equations can be represented in matrix as the following form.

$$
X = A\Theta \tag{4.3.38}
$$

where

$$X = \begin{bmatrix} x(k) \\ x(k+1) \\ \vdots \\ x(k+N) \end{bmatrix} \tag{4.3.39}$$

$$A = \left[\begin{matrix} -x(k-1) & \cdots & -x(k-n) & u(k-1) \\ -x(k) & \cdots & -x(k-n+1) & u(k) \\ \vdots & \vdots & \vdots & \vdots \\ -x(k+N-1) & \cdots & -x(k+N-n) & u(k+N-1) \end{matrix}\right.$$
$$\begin{matrix} \cdots & x^2(k-1) & \cdots & x^p(k-n) \\ \cdots & x^2(k) & \cdots & x^p(k-n+1) \\ \vdots & \vdots & \vdots & \vdots \\ \cdots & x^2(k+N-1) & \cdots & x^p(k+N-n) \end{matrix}$$
$$\left.\begin{matrix} \dot{x}^2(k-n) & \cdots & \dot{x}^q(k-n) \\ \dot{x}^2(k-n+1) & \cdots & \dot{x}^q(k-n+1) \\ \vdots & \vdots & \vdots \\ \dot{x}^2(k+N-n) & \cdots & \dot{x}^q(k+N-n) \end{matrix}\right] \tag{4.3.40}$$

$$\Theta = \begin{bmatrix} a_1 \\ \vdots \\ a_n \\ b_1\gamma_1 \\ \vdots \\ b_n\mu^q \end{bmatrix} \tag{4.3.41}$$

Solving the Eq.(4.3.38) we will obtain the parameter vector Θ, which dominate the nonlinear dynamical system.

For identification of nonlinear structural dynamic system, the structural HFM in discrete time domain is considered. When $N+n$ samples of input and output are taken, the Eq.(4.2.35) becomes the following form in matrix.

$$X = A'\Theta' \tag{4.3.42}$$

$$A' = \begin{bmatrix} -x(k-1) & -x(k-2) & u(k-2) & x^2(k-2) & \cdots & x^p(k-2) & \dot{x}^2(k-2) & \cdots & \dot{x}^q(k-2) \\ -x(k) & -x(k-1) & u(k-1) & x^2(k-1) & \cdots & x^p(k-1) & \dot{x}^2(k-1) & \cdots & \dot{x}^q(k-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -x(k+N-1) & -x(k+N-2) & u(k+N-2) & x^2(k+N-2) & \cdots & x^p(k+N-2) & \dot{x}^{(}k+N-2) & \cdots & \dot{x}^q(k+N-2) \end{bmatrix} \tag{4.3.43}$$

$$\Theta' = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{p-1} \\ \vdots \\ a_{p+q+1} \end{bmatrix} \tag{4.3.44}$$

A modeling error noise $e(k)$ usually is considered. This noise is assumed to be white. In this case, Eq.(4.3.38) is

$$X = A\Theta + e \tag{4.3.45}$$

Estimating the parameter vector Θ from Eq.(4.3.45) is a standard least squares problem. The problem is to identify (estimate) the parameter vector Θ which minimizes the $\| A\Theta - X \|$. There are several methods available for estimating parameters from

Eq.(4.3.45). The basic method for estimating Θ is the least squares method. The parameter vector can be estimated by

$$\Theta = (A^T A)^{-1} A^T X \tag{4.3.46}$$

Since matrix A of HFM has elements $x^i(k-j)$ $(i = 1,2,...,p;\ j = 1,2,...,n)$, the determinant of some submatrix of matrix A is the Vandermonde determinant.

$$\begin{aligned} & det \begin{bmatrix} x^2(k-2) & \cdots & x^p(k-2) \\ x^2(k-1) & \cdots & x^p(k-1) \\ \vdots & \vdots & \vdots \\ x^2(k+p-2) & \cdots & x^p(k+p-2) \end{bmatrix} \\ = \ & x^2(k-2) \cdots x^2(k+p-2) \\ & det \begin{bmatrix} 1 & \cdots & x^{p-2}(k-2) \\ 1 & \cdots & x^{p-2}(k-1) \\ \vdots & \vdots & \vdots \\ 1 & \cdots & x^{p-2}(k+p-2) \end{bmatrix} \end{aligned} \tag{4.3.47}$$

The Vandermonde determinant has following value.

$$det \begin{bmatrix} 1 & \cdots & a_1^{n-1} \\ \vdots & \vdots & \vdots \\ 1 & \cdots & a_n^{n-1} \end{bmatrix} = \prod_{1 \le j < i \le n} (a_i - a_j) \tag{4.3.48}$$

if Δt is a very short time period, the difference between $x(k-j)$ and $x(k-j+1)$ is small value, then the value of determinant of maximum square submatrix of matrix A is small. The matrix A becomes an ill-conditioned matrix. Any small round-off errors in the elements of A can cause large changes in $(A^T A)^{-1}$. In this case, Eq.(4.3.46) is not stable for identification of HFM.

4.4 Singular Value Decomposition Method

In this report, the Singular Value Decomposition (SVD) method has been used for estimating the parameters of HFM. For an arbitrary matrix A ($A \in R^{m\times n}$), if there a matrix B which lets AB, BA be a H matrix (H matrix: $m \times n$, $m > n$, and $rank(H) = n$.) and

$$ABA = A \tag{4.4.49}$$

$$BAB = B \tag{4.4.50}$$

the matrix B is defined as a pseudo-inverse matrix A^+ of A .

There is a theorem of singular value decomposition (SVD) for an arbitrary matrix A.

Theorem: if $A \in R^{m\times n}$, there exist orthogonal matrices

$$U = [u_1, \cdots, u_m]$$
$$U \in R^{m\times m} \tag{4.4.51}$$

and

$$V = [v_1, \cdots, v_n]$$
$$V \in R^{n\times n} \tag{4.4.52}$$

such that

$$U^T AV = diag(s_1, \cdots, s_p) \tag{4.4.53}$$

where $s_1 \geq s_2 \geq \cdots \geq s_p \geq 0$. According to this theorem, any matrix A ($m \times n$) with rank r can be decomposed to

$$A = USV^T \tag{4.4.54}$$

where $U(m \times m)$, $V(n \times n)$ are orthogonal matrices. S is a $m \times n$ matrix with all elements of which is equal to zero with the exception of the first r diagonal elements

$$s_1 \geq s_2 \geq \cdots \geq s_r > 0 \tag{4.4.55}$$

We can construct a pseudo-inverse matrix A^+ of A as

$$A^+ = VS^+U^T \tag{4.4.56}$$

where S^+ is a $n \times m$ matrix and has all zero elements except the first r diagonal elements. The nonzero elements of S^+ are $s_1^{-1}, s_2^{-1}, \cdots, s_r^{-1}$. This A^+ satisfies the definition of pseudo-inverse matrix.

A standard least squares problem is finding a vector $\Theta_{ls} \in R^n$ for equation $A\Theta = X$ where $A \in R^{m \times n}$ and $X \in R^m$ and $m > n$, such that

$$min \parallel A\Theta - X \parallel_2 \tag{4.4.57}$$

Here $\parallel A\Theta - X \parallel_2$ is the p-norm ($p = 2$) of vector. It is defined as

$$\parallel A\Theta - X \parallel_2 = [(A\Theta - X)^T(A\Theta - X)]^{\frac{1}{2}} \tag{4.4.58}$$

Denote the minimum sum of squares by ρ_{ls}^2

$$\rho_{ls}^2 = \parallel A\Theta_{ls} - X \parallel_2^2 \tag{4.4.59}$$

When a matrix A ($m \times n$) is decomposed by using the orthogonal matrices U and V, we have

$$\begin{aligned} \parallel A\Theta - X \parallel_2^2 &= \parallel U^T AV(V^T\Theta)_i - U^T X \parallel_2^2 \\ &= \sum_{i=1}^{r}[s_i(V^T\Theta)_i - u_i^T X]^2 \\ &+ \sum_{i=r+1}^{m}(u_i^T X)^2 \end{aligned} \tag{4.4.60}$$

Observably, if Θ has the following form.

$$\Theta_{ls} = \sum_{i=1}^{r}(u_i^T X/s_i)v_i \tag{4.4.61}$$

then the minimum of $\parallel A\Theta - X \parallel_2^2$ is

$$\rho_{ls}^2 = \sum_{i=r+1}^{m}(u_i^T X)^2 \tag{4.4.62}$$

From Eq.(4.4.61) and Eq.(4.4.56), we obtain the solution of least squares problem.

$$\Theta_{ls} = A^{+}X \qquad (4.4.63)$$

where $A^{+} = VS^{+}U^{T}$. Actually, if the rank of matrix A, r is equal to n, then A^{+} can be $(A^{T}A)^{-1}A^{T}$ and the least squares solution Θ_{ls} is

$$\Theta_{ls} = (A^{T}A)^{-1}A^{T}X \qquad (4.4.64)$$

In this report, the U, V and singular values s_i of matrix A are calculated by using the program of IMSL.

4.5 Numerical Examples

Several numerical examples have been presented by using HFM for identification of nonlinear structural SDOF dynamic system in this section. The nonlinear structural dynamic system are described by nonlinear differential equation. The response $x(t)$ and velocity $\dot{x}(t)$ are obtained by using Runge-Kutta Method. The estimated parameters of nonlinear terms are compared with the real parameters.

Example 1:

The simplest case has been considered in this example. A nonlinear dynamic system is assumed to have the following differential equation.

$$2.56x + 0.32\dot{x} + x + 0.05x^{3} = 2.5cost \qquad (4.5.65)$$

It is from Mook's paper. In Mook's paper, [16] a linear model is assumed as

$$2.56x + 0.32\dot{x} + x = 2.5cost \qquad (4.5.66)$$

The real nonlinear dynamic system can be represented as

$$2.56x + 0.32\dot{x} + x = 2.5cost + d(t) \qquad (4.5.67)$$

where $d(t)$ is model error. The model error $d(t)$ between nonlinear dynamic system and assumed linear model is estimated by using Two Point Boundary Value Problem (TPBVP) method. Then the model error is assumed to consist of two nonlinear terms.

$$d(t) = \alpha x^2(t) + \beta x^3(t) \tag{4.5.68}$$

The parameters α and β are estimated by the least squares method from the model error $d(k)$ in discrete time domain.

Since this is a nonlinear dynamic system with one order, the linear dynamic system considered for constructing HFM has following transfer function in z-domain.

$$H(z) = \frac{z^{-1}}{1 + a_1 z^{-1}} \tag{4.5.69}$$

The nonlinear input of HFM is assumed as

$$f(t) = 2.5cos(t) + \gamma_2 x^2(t) + \gamma_3 x^3(t) \tag{4.5.70}$$

Then we can assume a HFM for the nonlinear dynamic system.

$$x(k) = -a_1 x(k-1) + a_2 x^2(k-1) + a_3 x^3(k-1) \tag{4.5.71}$$

The response of system is obtained by Runge-Kutta method from Eq.(4.5.65). The input and response are considered as data for identification of this nonlinear dynamic system by using HFM.

Sampling period Δt is assumed to be 0.01. There are 628 samples taken in a period. 901 samples of input and output of system are used for forming the equation of least squares problem. The parameters a_1, a_2, a_3 are estimated by using SVD method. The nonlinear estimated parameters α, β are obtained from equations:

$$\alpha = a_2/(\Delta t)^2$$
$$\beta = a_3/(\Delta t)^2 \tag{4.5.72}$$

They are compared with mook's results in table 4.1.

Real P.	Est. P. by Mook	Est. P. by HFM
0	0.00001	0.00002
0.05	0.0492(error:1.6%)	0.04995(error :0.09%)

Table 4.1: The estimated nonlinear parameters

Example 2:

A nonlinear SDOF structural dynamic system described by well-known Doffing's equation is considered for this example. The Duffing's equation has the following form.

$$\ddot{x} + 0.225x + 0.0025x^3 = 0.02cos0.5t \tag{4.5.73}$$

The initial conditions are assumed as

$$\begin{aligned} x(0) &= 4.0 \\ \dot{x}(0) &= 0 \end{aligned} \tag{4.5.74}$$

The response $x(t)$ of Eq.(4.5.73) is calculated by using the Runge-Kutta method and illustrated in Fig. 4.2.

The linear dynamic system considered fo HFM has the transfer function in z-domain.

$$H(z) = \frac{z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{4.5.75}$$

The nonlinear input of HFM is assumed as

$$f(t) = 0.02cos0.5(t) + \gamma_2 x^2(t) - \gamma_3 x^3(t) \tag{4.5.76}$$

The HFM of this system is assumed as

$$\begin{aligned} x(k) &= -a_1 x(k-1) - a_2 x(k-2) + a_3 x^2(k-2) \\ &+ a_4 x^3(k-2) + 0.02 \cos 0.5(k-2) \end{aligned} \tag{4.5.77}$$

902 samples of input and output of system are used for parameter estimation. The sampling time period Δt is assumed as $\Delta t = 0.05$. The parameters of HFM are

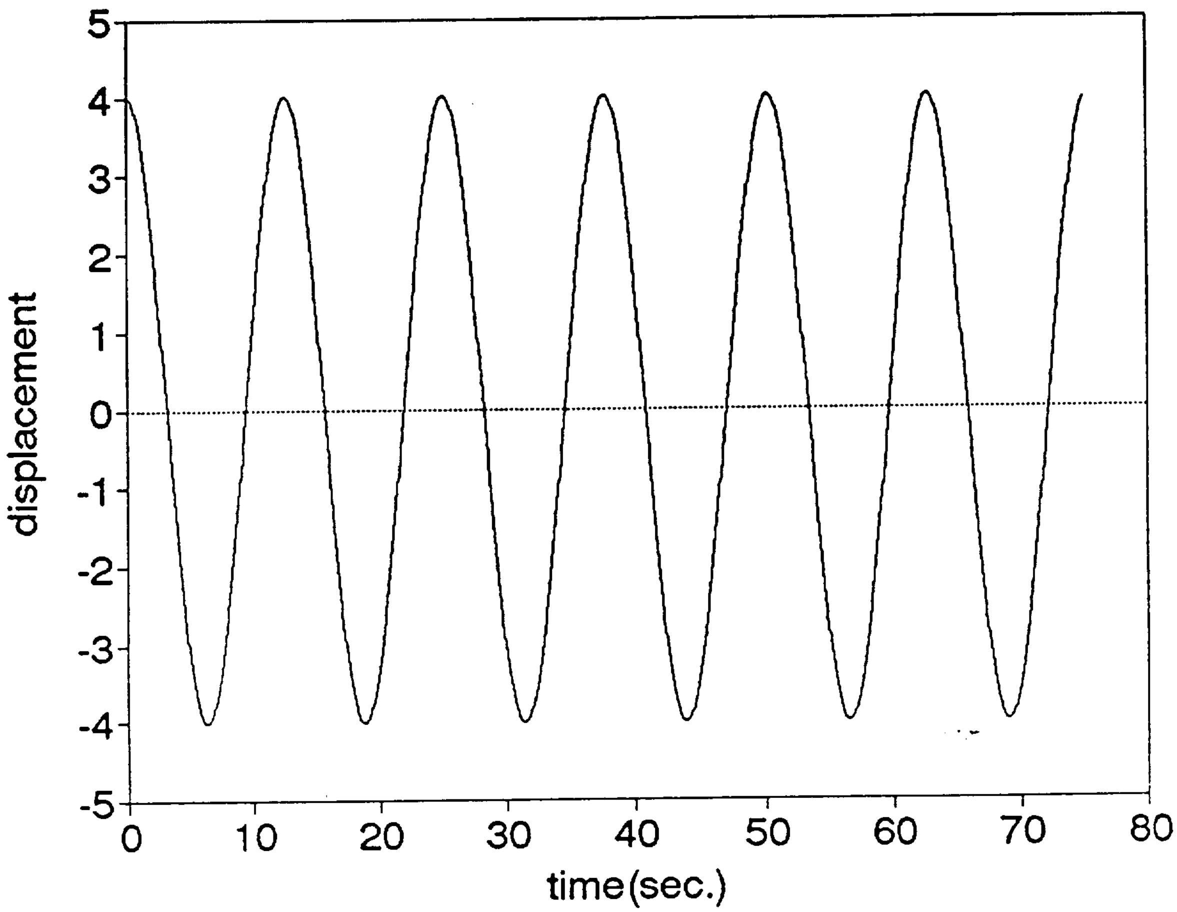


Figure 4.2: The response of a SDOF nonlinear dynamic system

Exact P.	Estimated P.	Error
0.0	0.000012	
0.0025	0.0025030	0.12%

Table 4.2: The estimated nonlinear parameters

estimated by using SVD method.

$$\begin{aligned} a_1 &= -1.999943 \\ a_2 &= 0.999995 \\ a_3 &= 0.320225 \times 10^{-6} \\ a_4 &= -0.625760 \times 10^{-5} \end{aligned} \quad (4.5.78)$$

The parameters of nonlinear terms of nonlinear differential equation are obtained by using Eq.(4.2.37) and shown in table 4.2.

Example 3: The third example is the Duffing's equation, which has linear damping term. This example can be expressed as

$$\ddot{x} + 0.01\dot{x} + 0.225x + 0.0025x^3 = 0.02cos0.5t \quad (4.5.79)$$

The initial conditions are

$$\begin{aligned} x(0) &= 4.0 \\ \dot{x}(0) &= 0 \end{aligned} \quad (4.5.80)$$

The response of this system is illustrated in Fig. 4.3.

The HFM is the same with in Example 2. That is

$$\begin{aligned} X(k) &= -a_1x(k-1) - a_2x(k-2) + a_3x^2(k-2) \\ &+ a_4x^3(k-2) + 0.02cos0.5(k-2) \end{aligned} \quad (4.5.81)$$

In the HFM of this example, a quadric nonlinear term and cubic nonlinear terms are assumed.

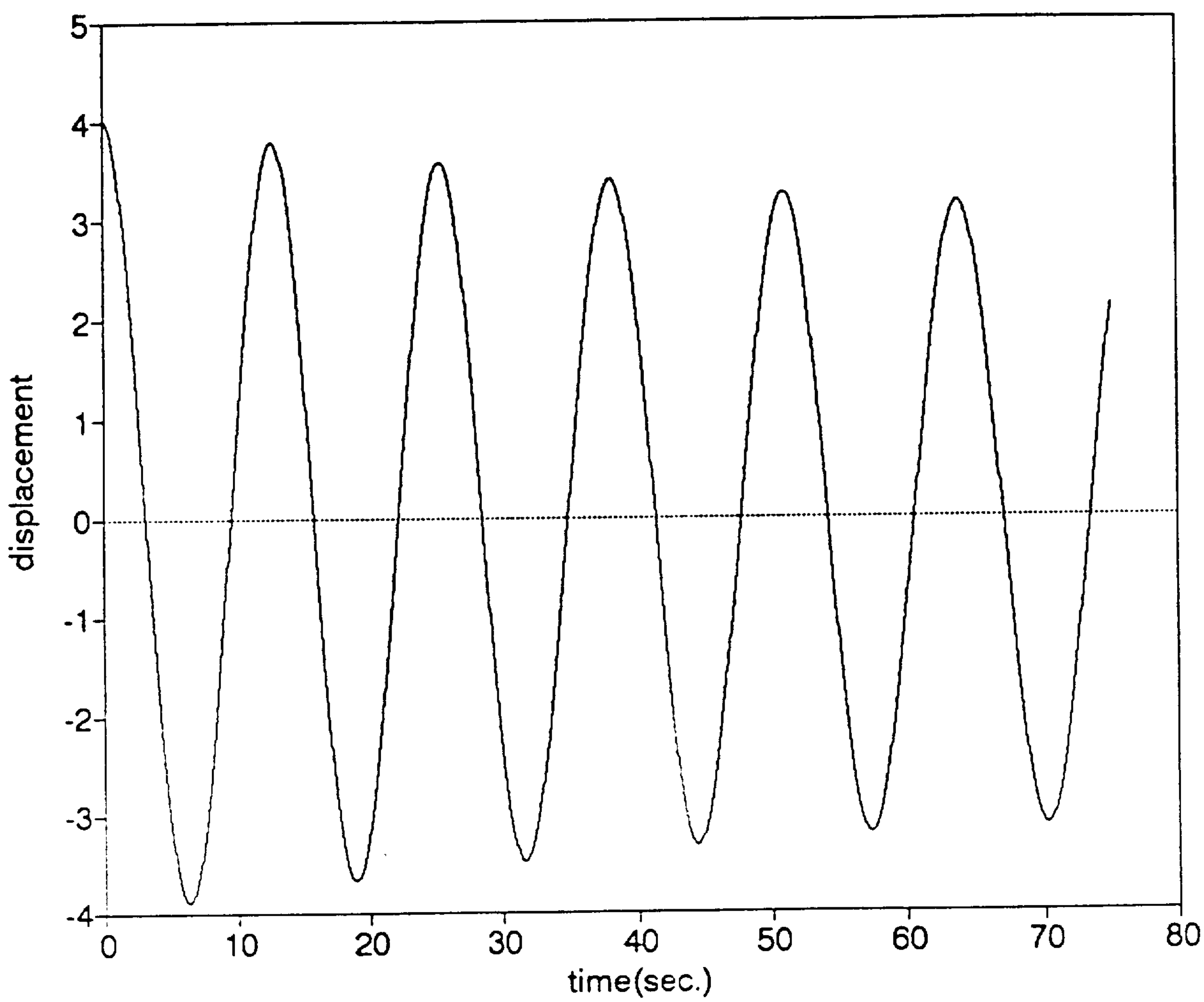


Figure 4.3: The response of SDOF nonlinear structural dynamic system with linear damping

Exact P.	Estimated P.	Error
0.0	0.0000064	
0.0025	0.00256	2.4%

Table 4.3: The estimated nonlinear parameters

Sampling time period $\Delta t = 0.05$ is considered and 755 samples have been used. Estimated sample range approximately is three period of the input and the output. The results obtained are

$$\begin{aligned} a_1 &= -1.99890 \\ a_2 &= 0.999460 \\ a_3 &= 0.162931 \times 10^{-7} \\ a_4 &= -0.6407975 \times 10^{-5} \end{aligned} \qquad (4.5.82)$$

The estimated parameters of nonlinear terms of Duffing's equation, Eq.(4.5.79), are listed in table 4.3. The results show that there is not the quadric nonlinearity of displacement $x(t)$.

Example 4:

Coulomb damping is the friction force between two contact surface. There is a friction drag force.

$$c \mid \dot{x} \mid \dot{x} \qquad (4.5.83)$$

where c is a positive constant. A nonlinear SDOF structural dynamic system with Coulomb damping is considered. This nonlinear dynamic system is denoted as following differential equation.

$$\ddot{x} + 0.225x + 0.1 \mid \dot{x} \mid \dot{x} = 0.02 \cos 0.5t \qquad (4.5.84)$$

The initial conditions are

$$x(0) = 4.0$$
$$\dot{x}(0) = 0 \tag{4.5.85}$$

The responses $x(t)$, $\dot{x}(t)$ of Eq.(4.5.84) are calculated by using Runge-Kutta method and illustrated in Fig.4.4. Sampling time period Δt is assumed to be 0.05 and 755 samples of input and output are considered as data for identification.

The HFM of this system is assumed as

$$x(k) = -a_1 x(k-1) - a_2 x(k-2) + a_3 \mid \dot{x}(k-2) \mid \dot{x}(k-2) \tag{4.5.86}$$

Then a least squares problem is formed for estimating the parameters a_1, a_2, a_3. The damping constant c of Eq.(4.5.84) can be obtained by

$$c = -\frac{a_3}{\Delta t^2} \tag{4.5.87}$$

The parameters, a_1, a_2, a_3 of HFM are estimated by SVD method They are

$$\begin{aligned} a_1 &= -1.99937 \\ a_2 &= 0.999926 \\ a_3 &= -0.00024675 \end{aligned} \tag{4.5.88}$$

The Coulomb damping constant c estimated is 0.0987004. The error is 1.29%.

Example 5:

Usually a linear SDOF structural dynamic system

$$m\ddot{x} + c\dot{x} + kx = u \tag{4.5.89}$$

can be modeled by a discrete difference model for identification of system. (Fig.4.5) The difference model is

$$x(k) = -a_1 x(k-1) - a_2 x(k-2) + u(k) \tag{4.5.90}$$

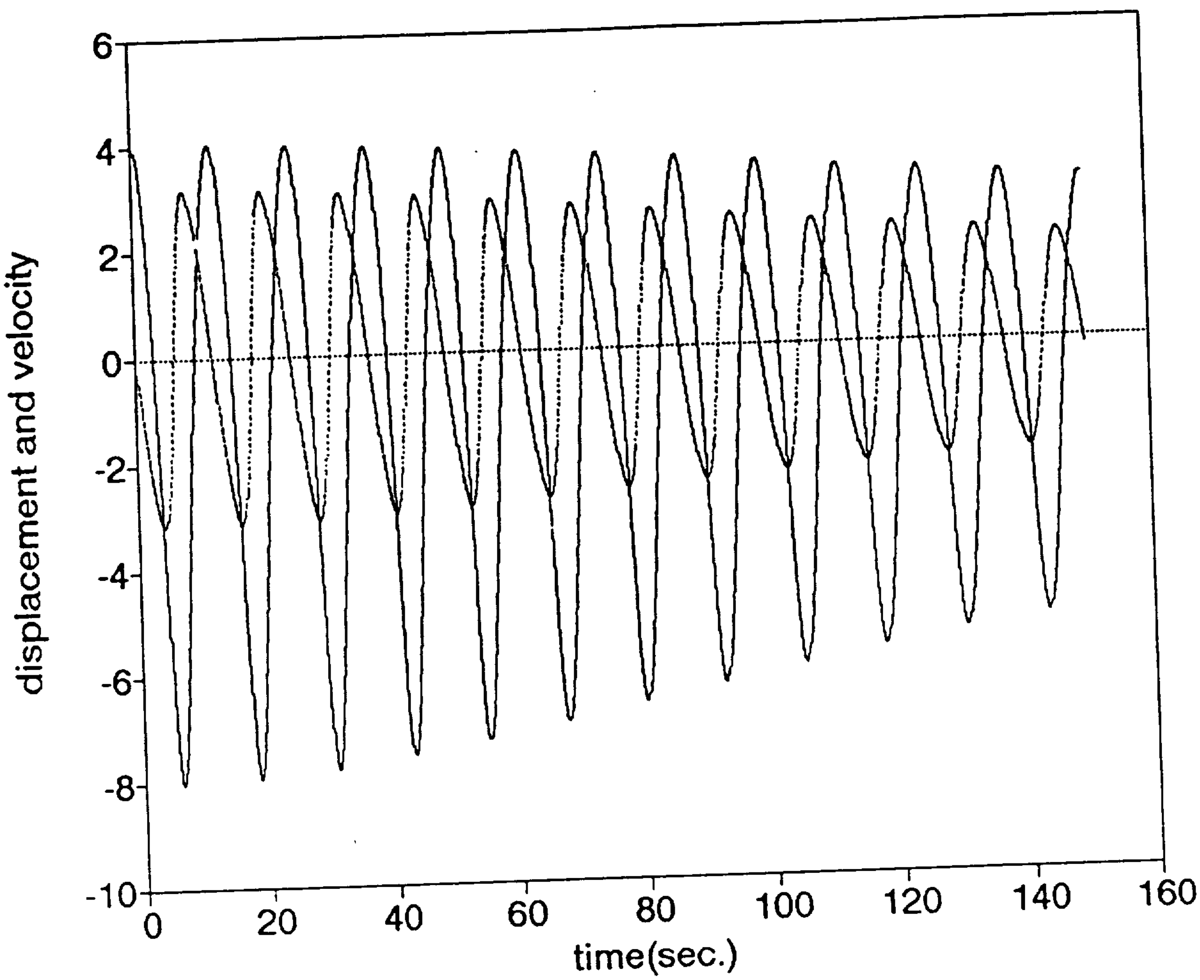


Figure 4.4: The response $x(t)$ and $\dot{x}(t)$ of SDOF nonlinear dynamic system with Coulomb damping.

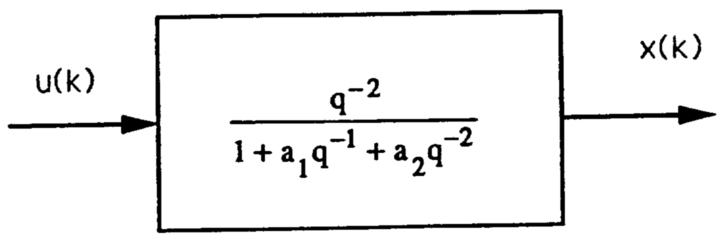


Figure 4.5: The linear dynamic system

where a_1 and a_2 are unknown parameters, which dominate the linear dynamic system. The Model, Eq.(4.5.90) , is an input-output approach of linear dynamic system. It is not possible to obtain the parameters m, k, c in Eq.(4.5.89) from the parameters a_1, a_2.

In engineering, the m, k are easy to be obtained from the real structure. The damping constant c is necessary to estimate. In this case, the Hammerstein Feedback Model can be used for identification of damping constant c. The linear dynamic system is assumed as a feedback linear dynamic system. It is illustrated in Fig.4.6. The parameters a_1 and a_2 are calculated from m. k. The HFM of the system is to be

$$x(k) + a_1 x(k-1) + a_2 x(k-2) = a_3 \dot{x}(k-2) + u(k-2) \tag{4.5.91}$$

Consider a linear dynamic system

$$\ddot{x} + 0.225x + 0.01\dot{x} = 0.02 \cos 0.5t \tag{4.5.92}$$

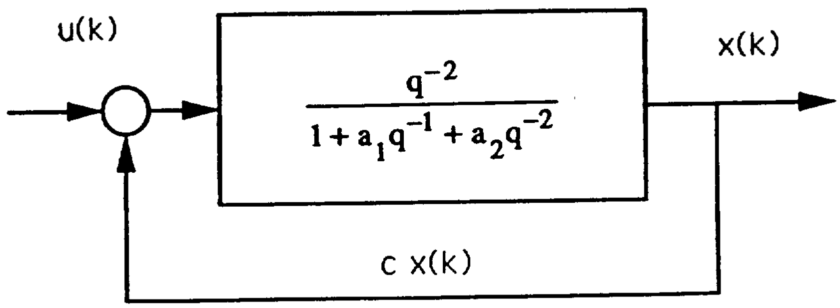


Figure 4.6: The linear model with damping feedback

with initial conditions:

$$x(0) = 1.0$$
$$\dot{x}(0) = 0 \qquad (4.5.93)$$

The responses $x(t)$ and $\dot{x}(t)$ are illustrated in Fig.4.7. The sampling time period $\Delta t = 0.05$ and 755 samples of input and output are taken for identification. The estimated damping constant c is 0.00999692. The error is 0.03 %.

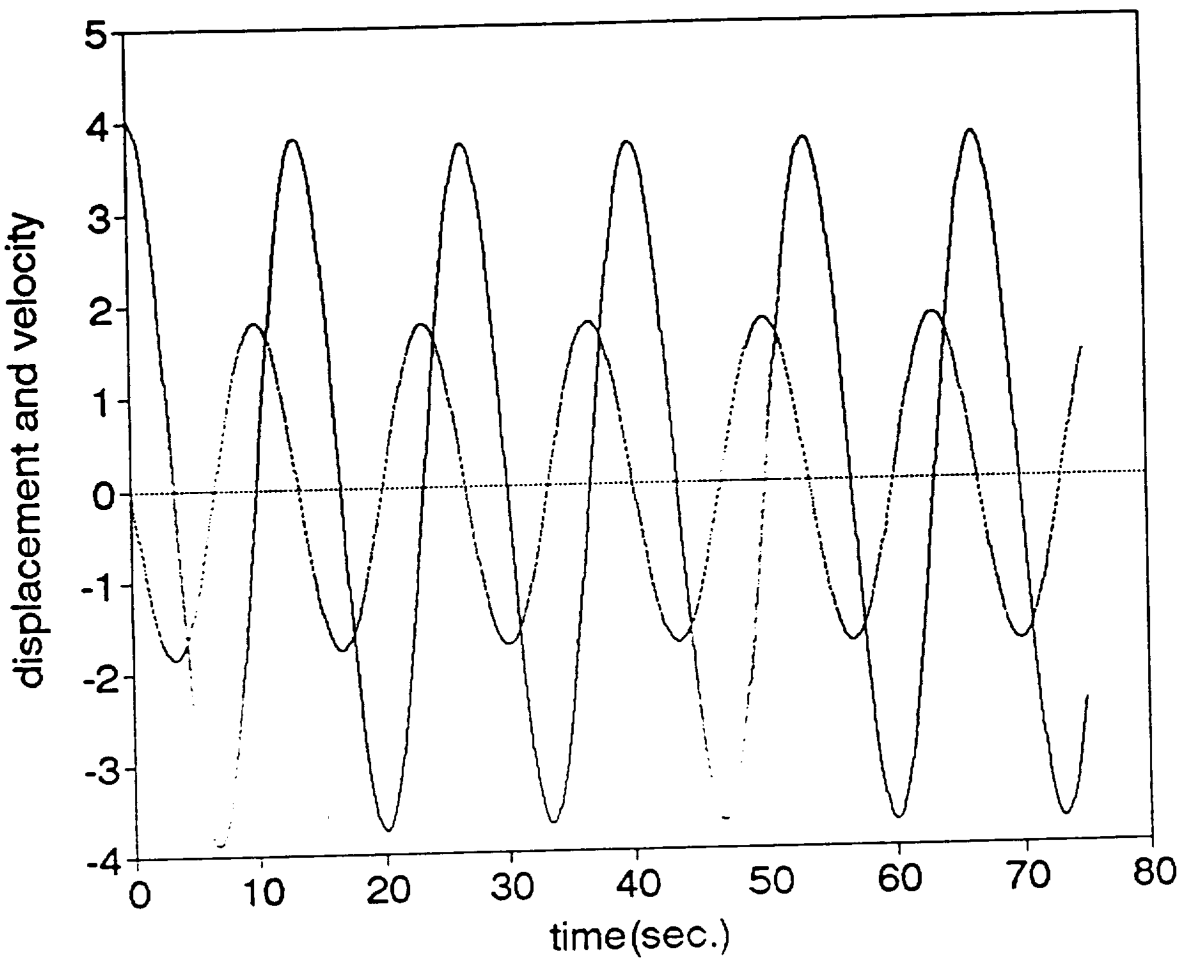


Figure 4.7: The responses of linear dynamic system

CHAPTER V

IDENTIFICATION OF NONLINEAR MDOF DYNAMICAL SYSTEM BY USING HFM

A large and complex structural system is usually approximated by a multiple degrees of freedom (MDOF) dynamical system by using method, like the finite element method. The identification and modeling of nonlinear MDOF dynamical systems by the use of input and output data is then a very important problem in practical structural dynamical system. Masri, Millar, Sand, and Caughey [42] [43] presented a self-starting multistage time-domain procedure for the identification of nonlinear MDOF dynamical systems in free oscillations or subjected to an arbitrary direct force excitations and nonuniform support motions. Yasuda, Kawamura and Watanabe [22] presented a technique in frequency domain for identification of nonlinear MDOF dynamical system. This technique is as follows. The periodic steady state responses data are measured from a MDOF nonlinear dynamical system subjected to a periodic force excitation. The nonlinear terms are expressed in terms of polynomials with unknown coefficients. The parameters are determined by expressing the quantities in a Fourier series and by applying the principle of harmonic balance. Yun and Shnozuka [21] used nonlinear Kalman filtering algorithms for identification of MDOF nonlinear structural dynamical system.

In practical engineering, many systems have multiple input variables and multiple output variables. Such a system can be said to be a multiple input and multiple output (MIMO) dynamical system. In this case, identification of MIMO dynamical system yields the learning problem of mapping between the multiple dimensional input space and multiple dimensional output space. (Fig. 5.1)

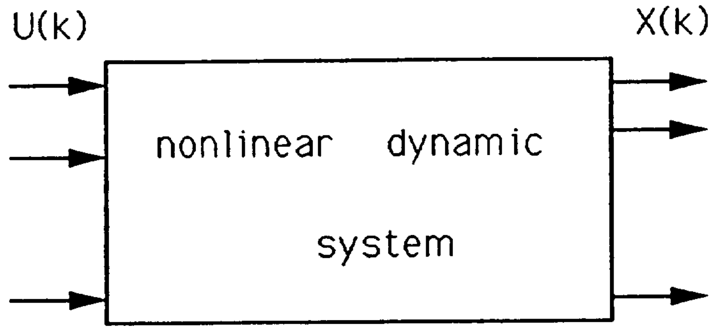


Figure 5.1: MIMO dynamical system

For identification of nonlinear dynamical system, the Hammerstein Feedback Model can be easily extended from SDOF case and SISO case to the MIMO case and MDOF case.

5.1 The Hammerstein Feedback Model of Nonlinear MDOF Dynamic System

The HFM of a nonlinear SDOF dynamical system in discrete time domain is given by Eq.(4.1.24) as follow

$$\begin{aligned} x(k) &= -a_1 x(k-1) - \cdots - a_n x(k-n) \\ &+ b_1 \gamma_1 u(k-1) + b_1 \gamma_2 x^2(k-1) \\ &+ \cdots + b_1 \gamma_p x^p(k-1) + \cdots \end{aligned}$$

$$
\begin{aligned}
&+ \quad b_n\gamma_1 u(k-n) + \cdots + b_n\gamma_p x^p(k-n) \\
&+ \quad b_1\mu_2\dot{x}^2(k-1) + \cdots + b_n\mu_q\dot{x}^q(k-n)
\end{aligned} \tag{5.1.1}
$$

If the input and output in Eq.(5.1.1) are now defined as input vector $U(k)$,

$$
U(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_m(k) \end{bmatrix} \tag{5.1.2}
$$

output vector $X(k)$ and output velocity vector $\dot{X}(k)$

$$
X(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_r(k) \end{bmatrix} \tag{5.1.3}
$$

$$
\dot{X}(k) = \begin{bmatrix} \dot{x}_1(k) \\ \dot{x}_2(k) \\ \vdots \\ \dot{x}_r(k) \end{bmatrix} \tag{5.1.4}
$$

The HFM of nonlinear MDOF dynamical system can now be written in matrix form as:

$$
\begin{aligned}
X(k) &= -\sum_{j=1}^{n} A^j X(k-j) + \sum_{j=1}^{n} B_1^j U(k-j) \\
&+ \sum_{j=1}^{n} B_2^j X^2(k-j) + \sum_{j=1}^{n} B_3^j X^3(k-j) \\
&+ \cdots + \sum_{j=1}^{n} B_p^j X^p(k-j) \\
&+ \sum_{j=1}^{n} \Gamma_2^j \dot{X}^2(k-j) + \cdots \\
&+ \sum_{j=1}^{n} \Gamma_q^j \dot{X}^q(k-j)
\end{aligned} \tag{5.1.5}
$$

where A^j is a $r \times r$ parameter matrix. B_1^j is a $r \times m$ parameter matrix. B_2^j, B_3^j, $\cdots$, B_p^j, Γ_2^j, Γ_q^j are $r \times r$ diagonal parameter matrices ($j = 1 \cdots n$), and

$$X^2(k-j) = \begin{bmatrix} \sum_{i_1=1}^{r} \sum_{i_2=1}^{r} \gamma_{i_1 i_2}^{1} x_{i_1}(k-j) x_{i_2}(k-j) \\ \vdots \\ \sum_{i_1=1}^{r} \sum_{i_2=1}^{r} \gamma_{i_1 i_2}^{r} x_{i_1}(k-j) x_{i_2}(k-j) \end{bmatrix}$$

$$X^3(k-j) = \begin{bmatrix} \sum_{i_1=1}^{r} \sum_{i_2=1}^{r} \sum_{i_3=1}^{r} \gamma_{i_1 i_2 i_3}^{1} x_{i_1}(k-j) x_{i_2}(k-j) x_{i_3}(k-j) \\ \vdots \\ \sum_{i_1=1}^{r} \sum_{i_2=1}^{r} \sum_{i_3=1}^{r} \gamma_{i_1 i_2 i_3}^{r} x_{i_1}(k-j) x_{i_2}(k-j) x_{i_3}(k-j) \end{bmatrix}$$

$$\vdots$$

$$X^p(k-j) = \begin{bmatrix} \sum_{i_1=1}^{r} \cdots \sum_{i_p=1}^{r} \gamma_{i_1 \cdots i_p}^{1} x_{i_1}(k-j) \cdots x_{i_p}(k-j) \\ \vdots \\ \sum_{i_1=1}^{r} \cdots \sum_{i_p=1}^{r} \gamma_{i_1 \cdots i_p}^{r} x_{i_1}(k-j) \cdots x_{i_p}(k-j) \end{bmatrix}$$

$$\dot{X}^2(k-j) = \begin{bmatrix} \sum_{i_1=1}^{r} \sum_{i_2=1}^{r} \mu_{i_1 i_2}^{1} \dot{x}_{i_1}(k-j) \dot{x}_{i_2}(k-j) \\ \vdots \\ \sum_{i_1=1}^{r} \sum_{i_2}^{r} \mu_{i_1 i_2}^{r} \dot{x}_{i_1}(k-j) \dot{x}_{i_2}(k-j) \end{bmatrix}$$

$$\vdots$$

$$\dot{X}^q(k-j) = \begin{bmatrix} \sum_{i_1=1}^{r} \cdots \sum_{i_q}^{r} \mu_{i_1 \cdots i_q}^{1} \dot{x}_{i_1}(k-j) \cdots \dot{x}_{i_q}(k-j) \\ \vdots \\ \sum_{i_1=1}^{r} \cdots \sum_{i_q}^{r} \mu_{i_1 \cdots i_q}^{r} \dot{x}_{i_1}(k-j) \cdots \dot{x}_{i_q}(k-j) \end{bmatrix} \tag{5.1.6}$$

where $j = 1 \cdots n$.

For nonlinear MDOF structural dynamical system, the order n is equal to 2. The HFM of nonlinear structural dynamical system in discrete time domain is

$$\begin{aligned} X(k) &= -A^1 X(k-1) - A^2 X(k-2) + B_1^2 U(k-2) \\ &+ B_2^2 X^2(k-2) + B_3^2 X^3(k-2) + \cdots \\ &+ B_p^2 X^p(k-2) + \Gamma_2^2 \dot{X}^2(k-2) + \cdots \\ &+ \Gamma_q^2 \dot{X}^q(k-2) \end{aligned} \tag{5.1.7}$$

To estimate parameters, the HFM in discrete time domain at i^{th} degree of freedom can

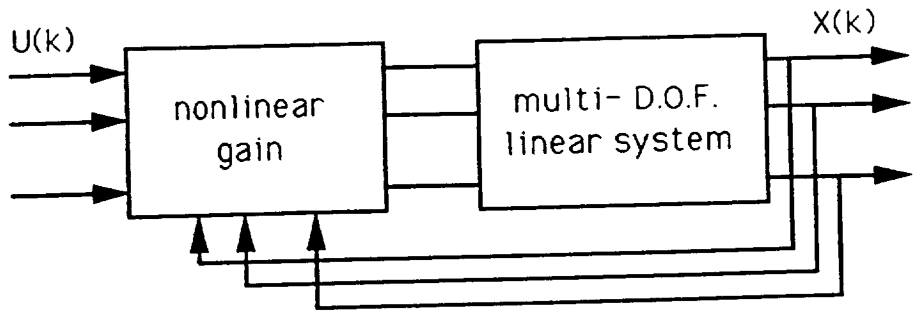


Figure 5.2: The Hammerstein Feedback Model of structural nonlinear MDOF dynamical system

be denoted in following form.

$$\begin{aligned} x_i(k) &= -A_i^1 X(k-1) - A_i^2 X(k-2) + (B_1^2)_i U(k-2) \\ &+ (B_2^2)_{ii} X^2(k-2) + \cdots + (B_p^2)_{ii} X^p(k-2) \\ &+ (\Gamma_2^2)_{ii} \dot{X}^2(k-2) + \cdots + (\Gamma_q^2)_{ii} \dot{X}^q(k-2) \end{aligned} \quad (5.1.8)$$

where A_i^1, A_i^2 and $(B_1^2)_i$ are the i^{th} row of matrices $A^1(r \times r)$, $A^2(r \times r)$ and $B_1^2(r \times m)$ separately. $(B_2^2)_{ii}, \cdots, (B_p^2)_{ii}, (\Gamma_2^2)_{ii}, \cdots, (\Gamma_q^2)_{ii}$ are the i^{th} diagonal elements of matrices $B_2^2, \cdots, B_p^2, \Gamma_2^2, \cdots, \Gamma_q^2$ separately. The HFM of nonlinear MDOF dynamical system in discrete time domain can be illustrated by figure Fig.5.2.

Consider a nonlinear two degree of freedom spring-mass structural dynamical system with cube nonlinear stiffness to show the application of HFM of nonlinear MDOF dynamical system in discrete time domain (Fig.5.3). We have $p = 3$ and $q = 0$ for the

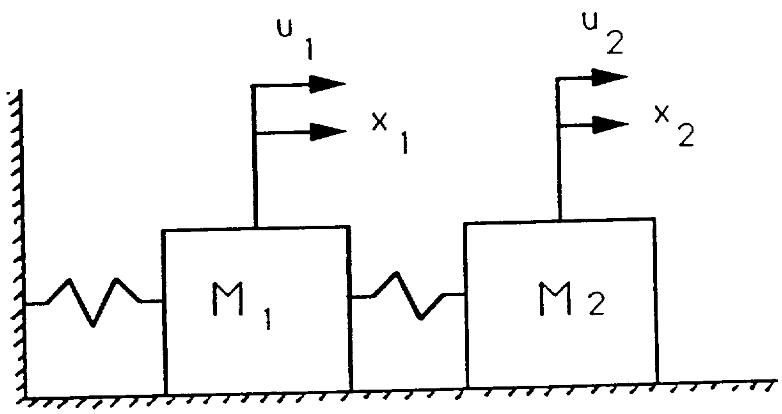


Figure 5.3: A nonlinear spring-mass structural dynamical system

HFM of this system and

$$X(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{5.1.9}$$

$$U(k-2) = \begin{bmatrix} u_1(k-2) \\ u_2(k-2) \end{bmatrix} \tag{5.1.10}$$

$$A^1 = \begin{bmatrix} a^1_{11} & a^1_{12} \\ a^1_{21} & a^1_{22} \end{bmatrix} \tag{5.1.11}$$

$$A^2 = \begin{bmatrix} a^2_{11} & a^2_{12} \\ a^2_{21} & a^2_{22} \end{bmatrix} \tag{5.1.12}$$

$$B^2_1 = \begin{bmatrix} b^{21}_{11} & b^{21}_{12} \\ b^{21}_{21} & b^{21}_{22} \end{bmatrix} \tag{5.1.13}$$

$$B^2_2 = \begin{bmatrix} b^{22}_{11} & 0 \\ 0 & b^{22}_{22} \end{bmatrix} \tag{5.1.14}$$

$$B_3^2 = \begin{bmatrix} b_{11}^{23} & 0 \\ 0 & b_{22}^{23} \end{bmatrix} \quad (5.1.15)$$

$$X^2(k-2) = \begin{bmatrix} x_1^2(k-2) \\ x_2^2(k-2) \end{bmatrix} \quad (5.1.16)$$

where

$$\begin{aligned} x_1^2(k-2) &= \sum_{i_1=1}^{2}\sum_{i_2=1}^{2} \gamma_{i_1 i_2} x_{i_1}^1(k-2) x_{i_2}(k-2) \\ &= \gamma_{11}^1 (x_1(k-2))^2 + (\gamma_{12}^1 + \gamma_{21}^1) x_1(k-2) x_2(k-2) \\ &\quad + \gamma_{22}^1 (x_2(k-2))^2 \end{aligned} \quad (5.1.17)$$

$$\begin{aligned} x_2^2(k-2) &= \sum_{i_1=1}^{2}\sum_{i_2=1}^{2} \gamma_{i_1 i_2}^2 x_{i_1}(k-2) x_{i_2}(k-2) \\ &= \gamma_{11}^2 x_1^2(k-2) + (\gamma_{12}^2 + \gamma_{21}^2) x_1(k-2) x_2(k-2) \\ &\quad + \gamma_{22}^2 (x_2(k-2))^2 \end{aligned} \quad (5.1.18)$$

and

$$X^3(k-2) = \begin{bmatrix} x_1^3(k-2) \\ x_2^3(k-2) \end{bmatrix} \quad (5.1.19)$$

where

$$\begin{aligned} x_1^3(k-2) &= \sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\sum_{i_3=1}^{2} \gamma_{i_1 i_2 i_3}^1 x_{i_1}(k-2) x_{i_2}(k-2) x_{i_3}(k-2) \\ &= \gamma_{111}^1 (x_1(k-2))^3 \\ &\quad + (\gamma_{112}^1 + \gamma_{121}^1 + \gamma_{211}^1)(x_1(k-2))^2 x_2(k-2) \\ &\quad + (\gamma_{122}^1 + \gamma_{212}^1 + \gamma_{221}^1) x_1(k-2)(x_2(k-2))^2 \\ &\quad + \gamma_{222}^1 (x_2(k-2))^3 \end{aligned} \quad (5.1.20)$$

and

$$\begin{aligned} x_2^3(k-2) &= \sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\sum_{i_3=1}^{2} \gamma_{i_1 i_2 i_3}^2 x_{i_1}(k-2) x_{i_2}(k-2) x_{i_3}(k-2) \\ &= \gamma_{111}^2 (x_1(k-2))^3 \end{aligned}$$

$$+(\gamma_{112}^2+\gamma_{121}^2+\gamma_{211}^2)(x_1(k-2))^2 x_2(k-2)$$
$$+(\gamma_{122}^2+\gamma_{212}^2+\gamma_{221}^2)x_1(k-2)(x_2(k-2))^2$$
$$+\gamma_{222}^2(x_2(k-2))^3 \qquad (5.1.21)$$

Substituting $X(k)$, $U(k-2)$, A^1, A^2, B_1^2, B_2^2, B_3^2, $X^2(k-2)$, $X^3(k-2)$ into Eq.(5.1.7), we have the HFM of system in discrete time domain.

$$\begin{aligned} x_1(k) \;=\; & -a_{11}^1 x_1(k-1) - a_{12}^1 x_2(k-1) - a_{11}^2 x_1(k-2) \\ & -a_{12}^2 x_2(k-2) + b_{11}^{21} u_1(k-2) + b_{12}^{21} u_2(k-2) \\ & +b_{11}^{22}\gamma_{11}^1 (x_1(k-2))^2 + b_{11}^{22}(\gamma_{12}^1+\gamma_{21}^1)x_1(k-2)x_2(k-2) \\ & +b_{11}^{22}\gamma_{22}^1 (x_2(k-2))^2 + b_{11}^{23}\gamma_{111}^1 (x_1(k-2))^3 \\ & +b_{11}^{23}(\gamma_{112}^1+\gamma_{121}^1+\gamma_{211}^1)(x_1(k-2))^2 x_2(k-2) \\ & +b_{11}^{23}(\gamma_{122}^1+\gamma_{212}^1+\gamma_{221})^1 x_1(k-2)(x_2(k-2))^2 \\ & +b_{11}^{23}\gamma_{222}^1 (x_2(k-2))^3 \end{aligned} \qquad (5.1.22)$$

$$\begin{aligned} x_2(k) \;=\; & -a_{21}^1 x_1(k-1) - a_{22}^1 x_2(k-1) - a_{21}^2 x_1(k-2) \\ & -a_{22}^2 x_2(k-2) + b_{21}^{21} u_1(k-2) + b_{22}^{21} u_2(k-2) \\ & +b_{22}^{22}\gamma_{11}^2 (x_1(k-2))^2 + b_{22}^{22}(\gamma_{12}^2+\gamma_{21})^2 x_1(k-2)x_2(k-2) \\ & +b_{22}^{22}\gamma_{22} x_2^2(k-2) + b_{22}^{23}\gamma_{111}^2 (x_1(k-2))^2 \\ & +b_{22}^{23}(\gamma_{112}^2+\gamma_{121}^2+\gamma_{211}^2)(x_1(k-2))^2 x_2(k-2) \\ & +b_{22}^{23}(\gamma_{122}^2+\gamma_{212}^2+\gamma_{221}^2)x_1(k-2)(x_2(k-2))^2 \\ & +b_{22}^{23}\gamma_{222}^2 (x_2(k-2))^3 \end{aligned} \qquad (5.1.23)$$

Let

$$\begin{aligned} a_1 &= a_{11}^1 \\ a_2 &= a_{12}^1 \\ &\vdots \\ a_8 &= b_{11}^{22}(\gamma_{12}^1+\gamma_{21}^1) \end{aligned}$$

$$
\begin{aligned}
&\vdots \\
a_{11} &= b^{23}_{11}(\gamma^1_{112} + \gamma^1_{121} + \gamma^1_{211}) \\
a_{12} &= b^{23}_{11}(\gamma^1_{122} + \gamma^1_{212} + \gamma^1_{221}) \\
a_{13} &= b^{23}_{11}\gamma^1_{222} \qquad (5.1.24)
\end{aligned}
$$

and

$$
\begin{aligned}
b_1 &= a^1_{21} \\
b_2 &= a^1_{22} \\
&\vdots \\
b_8 &= b^{22}_{22}(\gamma^2_{12} + \gamma^2_{21}) \\
&\vdots \\
b_{11} &= b^{23}_{22}(\gamma^2_{112} + \gamma^2_{121} + \gamma^2_{211}) \\
b_{12} &= b^{23}_{22}(\gamma^2_{122} + \gamma^2_{212} + \gamma^2_{221}) \\
b_{13} &= b^{23}_{22}\gamma^2_{222} \qquad (5.1.25)
\end{aligned}
$$

Then, the HFM of system can be represented by the following forms.

$$
\begin{aligned}
x_1(k) = {} & -a_1x_1(k-1) - a_2x_2(k-1) - a_3x_1(k-2) \\
& -a_4x_2(k-2) + a_5u_1(k-2) + a_6u_2(k-2) \\
& +a_7(x_1(k-2))^2 + a_8x_1(k-2)x_2(k-2) + a_9(x_2(k-2))^2 \\
& +a_{10}(x_1(k-2))^3 + a_{11}(x_1(k-2))^2x_2(k-2) \\
& +a_{12}x_1(k-2)(x_2(k-2))^2 + a_{13}(x_2(k-2))^3 \qquad (5.1.26)
\end{aligned}
$$

and

$$
\begin{aligned}
x_2(k) = {} & -b_1x_1(k-1) - b_2x_2(k-1) - b_3x_1(k-2) \\
& -b_4x_2(k-2) + b_5u_1(k-2) + b_6u_2(k-2) \\
& +b_7(x_1(k-2))^2 + b_8x_1x_1(k-2)x_2(k-2) + b_9(x_2(k-2))^2 \\
& +b_{10}(x_1(k-2))^3 + b_{11}(x_1(k-2))^2x_2(k-2) \\
& +b_{12}x_1(k-2)(x_2(k-2))^2 + b_{13}(x_2(k-2))^3 \qquad (5.1.27)
\end{aligned}
$$

where parameters $a_1, a_2, \cdots, a_{13}$ and $b_1, b_2, \cdots, b_{13}$ are to be identified.

5.2 Estimation of Parameters of HFM

From Eq.(5.1.8), the HFM of nonlinear MDOF structural dynamical system can be considered to be r submodels at i^{th} degree of freedom ($i = 1, 2, \cdots, r$), in which the $U(k)$ is the input of system, and $X(k)$, $\dot{X}(k)$ are the output of system. If $\gamma^i_{i_1 i_2}$, $\gamma^i_{i_1 i_2 i_3}$, $\cdots$ are equal to zero for $i_1 \neq i_2$, $i_1 \neq i_2 \neq i_3$, $\cdots$, for i^{th} degree, the Eq.(5.1.8) can be represented in following form.

$$
\begin{aligned}
x_i(k) &= -a^i_{11}x_1(k-1) - a^i_{12}x_2(k-1) + \cdots - a^i_{1r}x_r(k-1) \\
&- a^i_{21}x_1(k-2) - \cdots - a^i_{2r}x_r(k-2) \\
&+ a^i_{31}u_1(k-2) + \cdots + a^i_{3m}u_m(k-2) \\
&+ a^i_{41}(x_1(k-2))^2 + \cdots + a^i_{4r}(x_r(k-2))^2 \\
&+ \cdots \\
&+ a^i_{(p+2)1}(x_1(k-2))^p + \cdots + a^i_{(p+2)}(x_r(k-2))^p \\
&+ a^i_{(p+3)1}(\dot{x}_1(k-2))^2 + \cdots + a^i_{(p+3)r}(\dot{x}_r(k-2))^2 \\
&+ \cdots \\
&+ a^i_{(p+q+1)1}(\dot{x}_1(k-2))^q + \cdots + a^i_{(p+q+1)r}(\dot{x}_r(k-2))^q
\end{aligned}
\qquad (5.2.28)
$$

where $i = 1, \cdots, r$, p is the order of nonlinear stiffness and q is the order of nonlinear damping. By taking $N + n$ samples of the input $U(k)$ and the outputs $X(k)$ and $\dot{X}(k)$ and substituting the measured data into Eq.(5.2.28), we obtain a set of linear algebraic equations with unknown variables, which are system parameters. The set of equations can be written as following form in matrix.

$$X^i = A^i\Theta^i \qquad (5.2.29)$$

where $i = 1, \cdots, r$ and

$$X^i = \begin{bmatrix} x_i(k) \\ x_i(k+1) \\ \vdots \\ x_i(k+N) \end{bmatrix} \tag{5.2.30}$$

$$A^i = \begin{bmatrix} -x_1(k-1) & -x_2(k-1) & \cdots & -x_r(k-1) \\ -x_1(k) & -x_2(k) & \cdots & -x_r(k) \\ \vdots & \vdots & \vdots & \vdots \\ -x_1(k+N-1) & -x_2(k+N-1) & \cdots & -x_r(k+N-1) \\ -x_1(k-2) & -x_2(k-2) & \cdots & -x_r(k-2) \\ -x_1(k-1) & -x_2(k-1) & \cdots & -x_r(k-1) \\ \vdots & \vdots & \vdots & \vdots \\ -x_1(k+N-2) & -x_2(k+N-2) & \cdots & -x_r(k+N-2) \\ \vdots \\ u_1(k-2) & u_2(k-2) & \cdots & u_m(k-2) \\ u_1(k-1) & u_2(k-1) & \cdots & u_m(k-1) \\ \vdots & \vdots & \vdots & \vdots \\ u_1(k+N-2) & u_2(k+N-2) & \cdots & u_m(k+N-2) \\ (x_1(k-2))^2 & (x_2(k-2))^2 & \cdots & (x_r(k-2))^2 \\ (x_1(k-1))^2 & (x_2(k-1))^2 & \cdots & (x_r(k-1))^2 \\ \vdots & \vdots & \vdots & \vdots \\ (x_1(k+N-2))^2 & (x_2(k+N-2))^2 & \cdots & (x_r(k+N-2))^2 \\ \vdots \\ (x_1(k-2))^p & (x_2(k-2))^p & \cdots & (x_r(k-2))^p \\ (x_1(k-1))^p & (x_2(k-1))^p & \cdots & (x_r(k-1))^p \\ \vdots & \vdots & \vdots & \vdots \\ (x_1(k+N-2))^p & (x_2(k+N-2))^p & \cdots & (x_r(k+N-2))^p \end{matrix}$$

$$
\begin{array}{llll}
(\dot{x}_1(k-2))^2 & (\dot{x}_2(k-2))^2 & \cdots & (\dot{x}_r(k-2))^2 \\
(\dot{x}_1(k-1))^2 & (\dot{x}_2(k-1))^2 & \cdots & (\dot{x}_r(k-1))^2 \\
\vdots & \vdots & \vdots & \vdots \\
(\dot{x}_1(k+N-2))^2 & (\dot{x}_2(k+N-2))^2 & \cdots & (\dot{x}_r(k+N-2))^2 \\
\vdots & & & \\
(\dot{x}_1(k-2))^q & (\dot{x}_2(k-2))^q & \cdots & (\dot{x}_r(k-2))^q \\
(\dot{x}_1(k-1))^q & (\dot{x}_2(k-1))^q & \cdots & (\dot{x}_r(k-1))^q \\
\vdots & \vdots & \vdots & \vdots \\
(\dot{x}_1(k+N-2))^q & (\dot{x}_2(k+N-2))^q & \cdots & (\dot{x}_r(k+N-2))^q
\end{array}
\Bigg] \quad (5.2.31)
$$

$$
\Theta^i = \begin{bmatrix} a^i_{11} \\ a^i_{12} \\ \vdots \\ a^i_{1r} \\ a^i_{21} \\ \vdots \\ a^i_{2r} \\ a^i_{31} \\ \vdots \\ a^i_{3m} \\ a^i_{41} \\ \vdots \\ a^i_{(p+q+1)r} \end{bmatrix} \quad (5.2.32)
$$

If we have only white noise, the problem of the identification of the parameter vector Θ^i from Eq.(5.2.29) becomes a standard least square problem. Then, the parameter vector Θ^i can be estimated by SVD method.

$$
\Theta^i = (A^i)^+ X^i \quad (5.2.33)
$$

where $(i = 1, 2, \cdots, r)$.

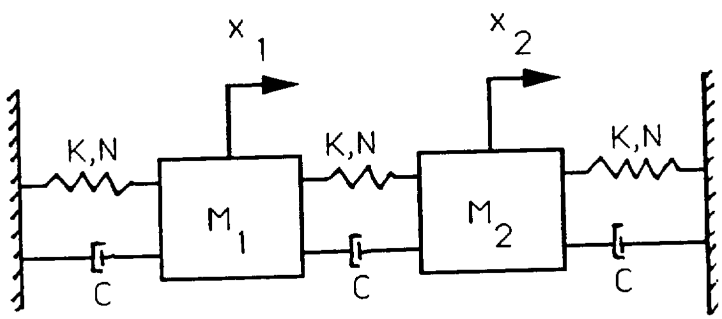


Figure 5.4: A two degree of freedom spring-mass nonlinear dynamical system

5.3 Numerical Examples

A two degree of freedom spring-mass nonlinear dynamical system is considered in Fig.(5.4). Mass and stiffness matrices are [22]

$$[M] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{5.3.34}$$

$$[K] = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \tag{5.3.35}$$

The damping matrix is

$$[C] = \begin{bmatrix} 0.1 & -0.05 \\ -0.05 & 0.1 \end{bmatrix} \tag{5.3.36}$$

and the nonlinear vector {N} and force {F} are

$$\{N\} = \begin{bmatrix} 0.1x_1^3 + 0.1(x_1 - x_2)^3 \\ 0.1x_2^3 + 0.1(x_2 - x_1)^3 \end{bmatrix} \tag{5.3.37}$$

$$[F] = \begin{bmatrix} cos\omega t \\ 0 \end{bmatrix} \tag{5.3.38}$$

The differential equation of motion of the system is

$$[M][\ddot{X}] + [C][\dot{X}] + [K][X] + [N] = [F] \tag{5.3.39}$$

where

$$[\ddot{X}] = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} \tag{5.3.40}$$

$$[\dot{X}] = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \tag{5.3.41}$$

$$[X] = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{5.3.42}$$

Example 1:

In the first example , damping is neglected, Eq. (5.3.39) becomes

$$[M][\ddot{X}] + [K][X] + [N] = [F] \tag{5.3.43}$$

The Eq.(5.3.43) can be rewritten as

$$\begin{aligned} &\ddot{x}_1 + 2x_1 - x_2 + 0.2x_1^3 - 0.3x_1^2x_2 \\ &\qquad +0.3x_1x_2^2 - 0.1x_2^3 = \cos\omega t \end{aligned} \tag{5.3.44}$$

$$\begin{aligned} &\ddot{x}_2 - x_1 + 2x_2 + 0.2x_2^3 - 0.3x_2^2x_1 \\ &\qquad +0.3x_2x_1^2 - 0.1x_1^3 = 0 \end{aligned} \tag{5.3.45}$$

In order to create a set of simulated experimental data, Runge-Kutta method is used to numerically integrate the equations (5.3.44), (5.3.45) and find $x_1(k)$, $x_2(k)$ with initial conditions $x_1(0) = 1$, $\dot{x}_1(0) = 0$, $x_2(0) = 0$, $\dot{x}_2(0) = 0$. Then, we have input data $\cos\omega k$ and the output data $x_1(k)$ (Fig. 5.5), $x_2(k)$ (Fig. 5.6) that can be used for identification of the system.

According to Eq.(5.1.26) and Eq.(5.1.27), the HFM of nonlinear dynamical system Eq.(5.3.43) in discrete time domain can be assumed as following forms.

For x_1:

$$\begin{aligned} x_1(k) &= -a_1x_1(k-1) - a_2x_1(k-2) \\ &+ a_3x_2(k-2) + a_4(x_1(k-2))^3 \\ &+ a_5(x_1(k-2))^2x_2(k-2) \\ &+ a_6x_1(k-2)(x_2(k-2))^2 \\ &+ a_7(x_2(k-2))^3 + \cos\omega(k-2) \end{aligned} \qquad (5.3.46)$$

$$\begin{aligned} x_2(k) &= -b_1x_2(k-1) - b_2x_2(k-2) \\ &+ b_3x_1(k-2) + b_4(x_2(k-2))^3 \\ &+ b_5(x_2(k-2))^2x_1(k-2) \\ &+ b_6x_2(k-2)(x_1(k-2))^2 \\ &+ b_7(x_1(k-2))^3 \end{aligned} \qquad (5.3.47)$$

where $a_1, a_2, \cdots, a_7, b_1, \cdots, b_7$ are unknown parameters. Assume $\omega = 0.5$, sampling time period $\Delta t = 0.05$, and 502 samples of input and output are taken. The results estimated by SVD method have been listed in Table 5.1, and Table 5.2.

Example 2:

The second example is Eq.(5.3.39). A linear damping term is considered. The needed experimented outputs $x_1(k)$ (Fig. 5.7) and $x_2(k)$ (Fig. 5.8) are again simulated by using Runge-Kutta method. The HFM in discrete time domain is

$$x_1(k) = -a_1x_1(k-1) - a_2x_1(k-2)$$

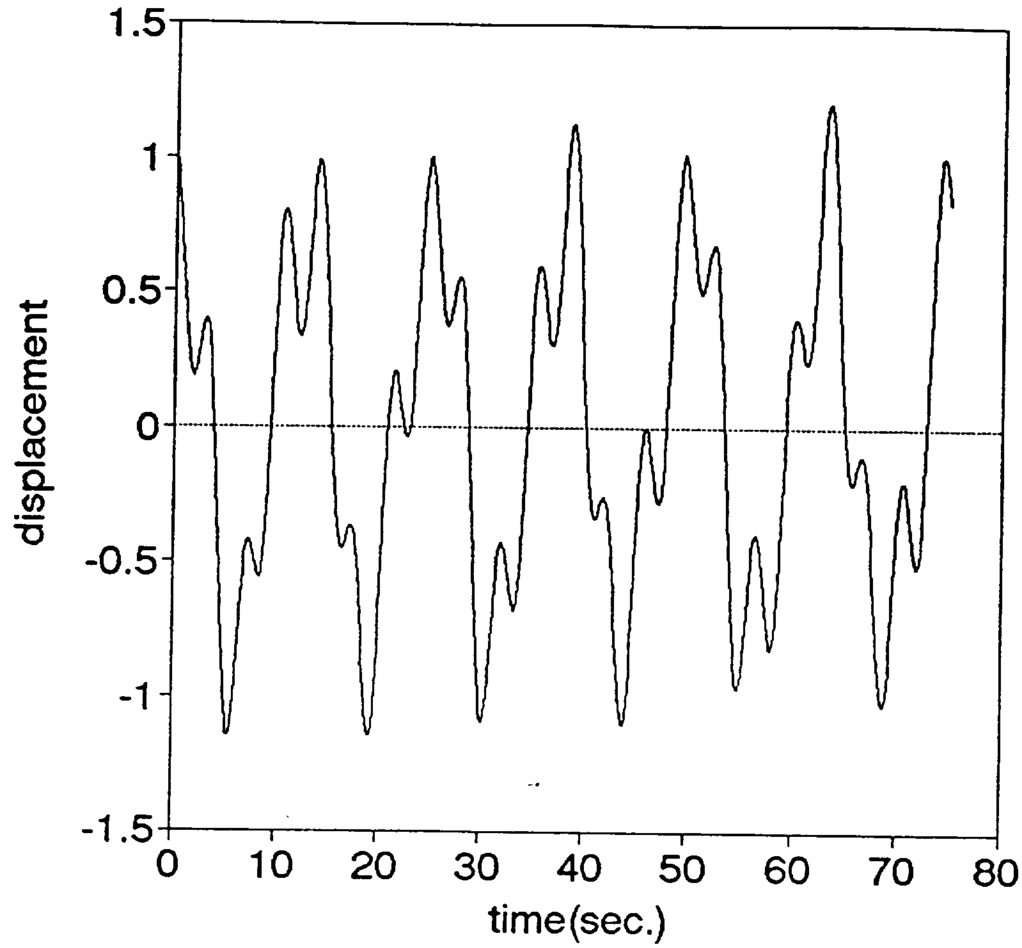


Figure 5.5: The response $x_1(t)$ of two degree of freedom mass-spring nonlinear system without damping

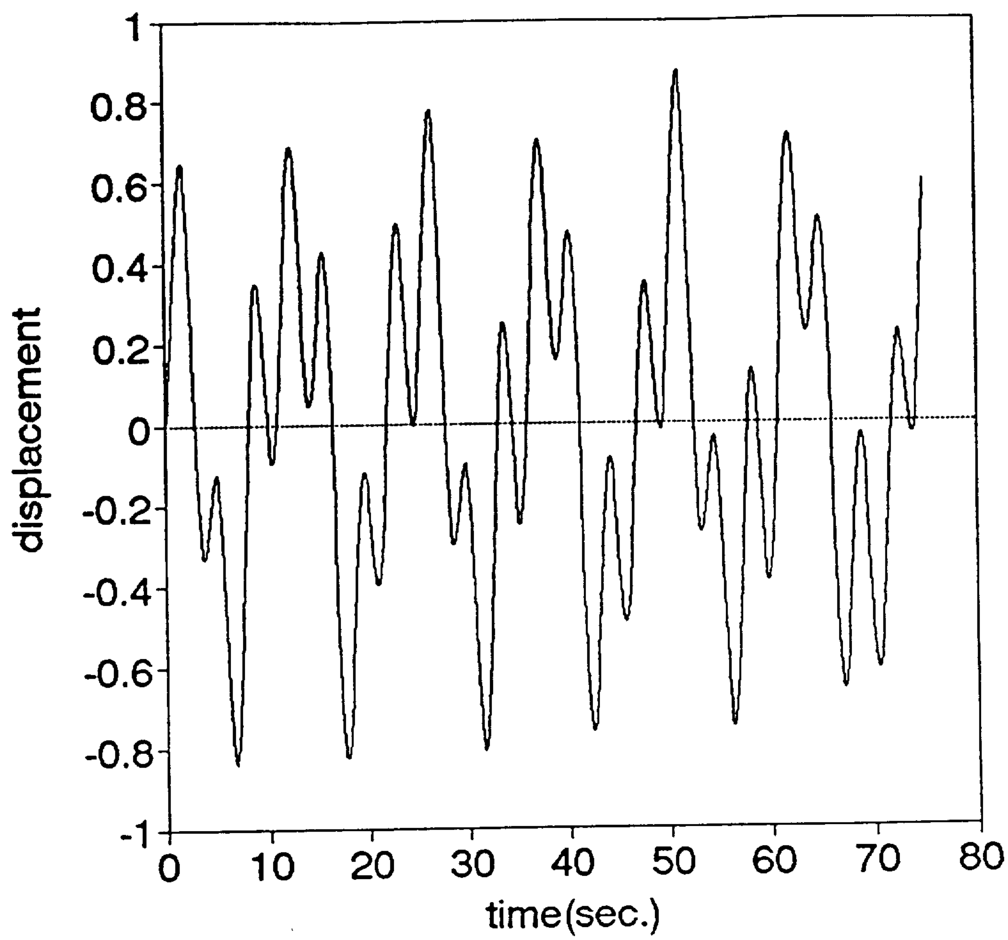


Figure 5.6: The response $x_2(t)$ of two degree of freedom mass-spring nonlinear system without damping.

P	Real P	Est. P	Error
a_1	-1.995	-1.995	0
a_2	1	1	0
a_3	1	0.99864	0.14%
a_4	-0.2	-0.1994	0.06%
a_5	0.3	0.29893	0.35%
a_6	-0.3	-0.29898	0.35%
a_7	0.1	0.0997	0.3%

Table 5.1: Estimated parameters

P	Real P	Est. P	Error
b_1	-1.995	-1.995	0
b_2	1	1	0
b_3	1	0.99952	0.05%
b_4	-0.2	-0.1992	0.07%
b_5	0.3	0.298169	0.6%
b_6	-0.3	-0.298756	0.4%
b_7	0.1	0.099566	0.4%

Table 5.2: Estimated parameters

P	Real P	Est. P	Error
a_1	-1.99	-1.99002	0.005%
a_2	0.995	0.99501	0.0012%
a_3	2	1.98684	0.66%
a_4	-1	-0.99476	0.5%
a_5	-0.2	-0.198628	0.69%
a_6	0.3	0.30208	0.69%
a_7	-0.3	-0.3061	2%
a_8	0.1	-0.94646	5%

Table 5.3: Estimated Parameters

$$\begin{aligned} &+ a_3x_2(k-1) + a_4x_2(k-2) \\ &+ a_5(x_1(k-2))^3 + a_6(x_1(k-1))^2x_2(k-2) \\ &+ a_7x_1(k-2)(x_2(k-2))^2 \\ &+ a_8(x_2(k-2))^3 + \cos\omega(k-2) \end{aligned} \quad (5.3.48)$$

$$\begin{aligned} x_2(k) &= -b_1x_2(k-1) - b_2x_2(k-2) \\ &+ b_3x_1(k-1) + b_4x_1(k-2) \\ &+ b_5(x_2(k-2))^3 + b_6(x_2(k-2))^2x_1(k-2) \\ &+ b_7x_2(k-2)(x_1(k-2))^2 + b_8(x_1(k-2))^3 \end{aligned} \quad (5.3.49)$$

where $a_1, \cdots, a_8, b_1, \cdots, b_8$ are unknown parameters. 402 samples of input and output are taken and $\omega = 0.5$, sampling time period $\Delta t = 0.05$ are assumed. The results estimated by SVD method are shown in Tables 5.3, 5.4.

Example 3:

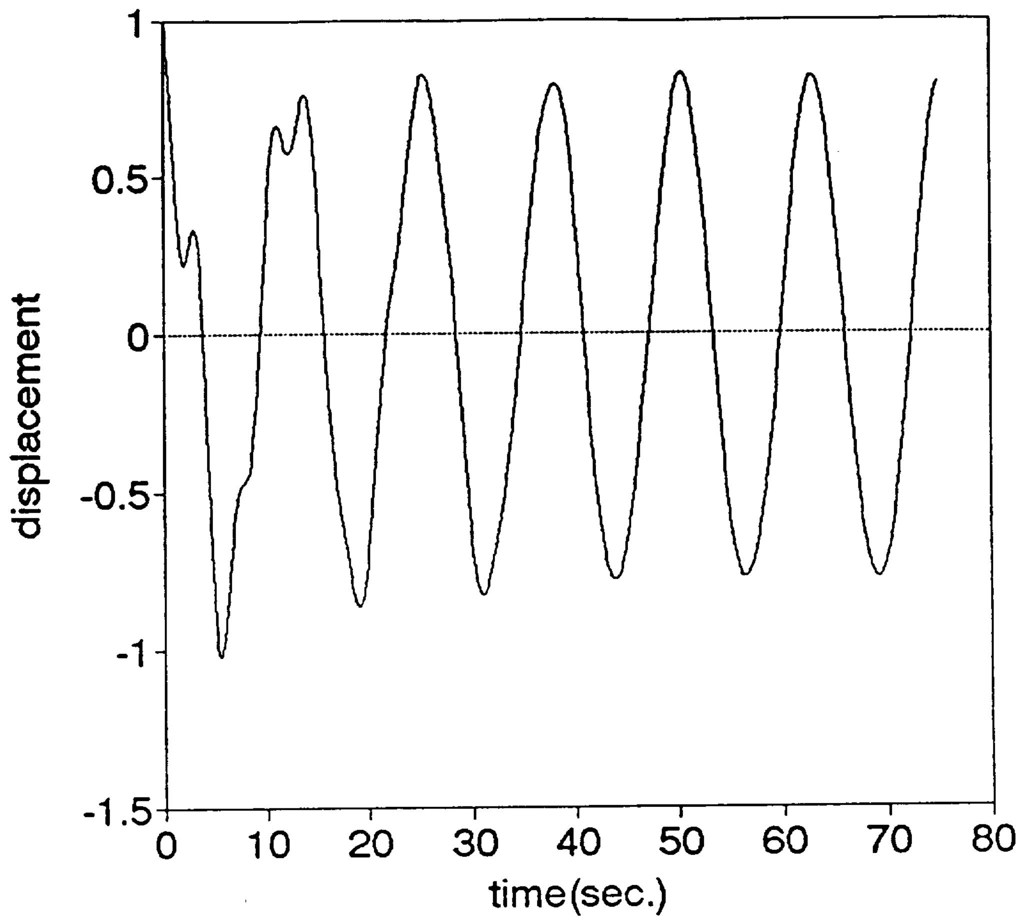


Figure 5.7: The response x_1 of two degree of freedom nonlinear dynamical system with damping

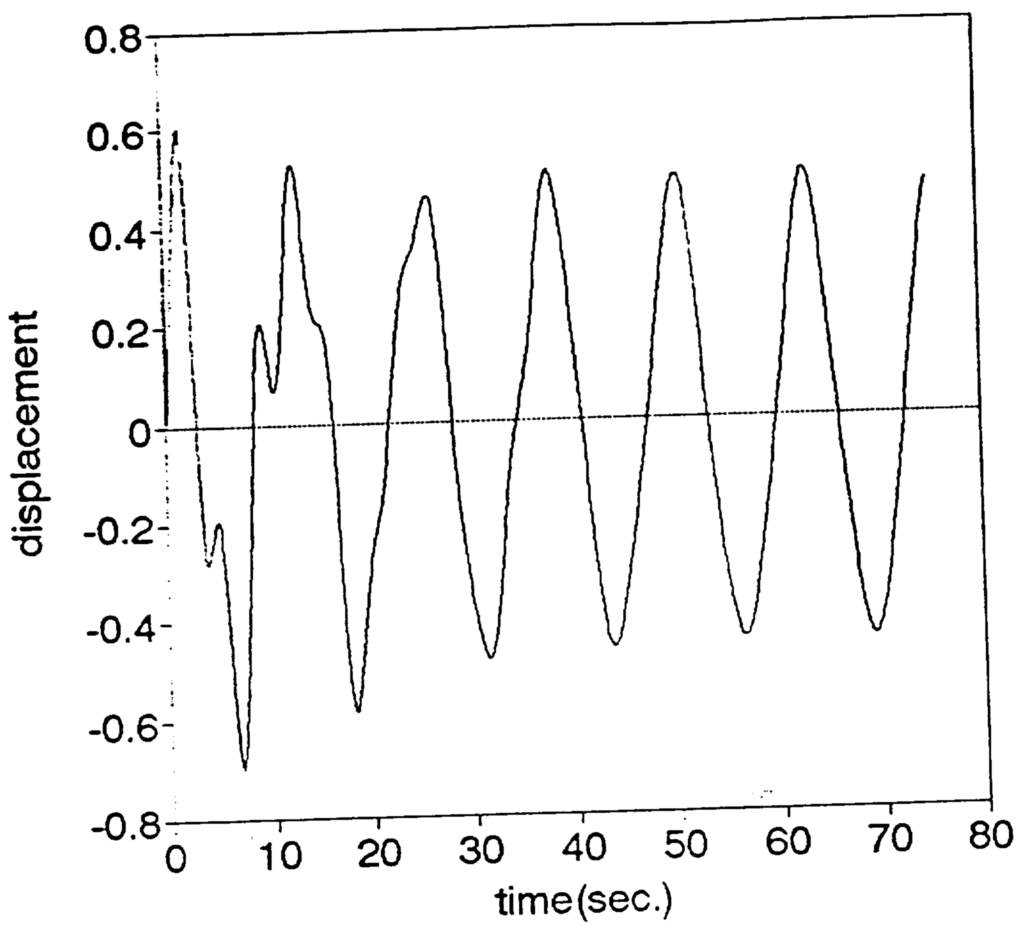


Figure 5.8: The response x_2 of two degree of freedom nonlinear dynamical system with damping

P	Real P	Est. P	Error
b_1	-1.99	-1.99004	0.002%
b_2	0.995	0.995016	0.0016%
b_3	2	1.99138	0.43%
b_4	-1	-0.99566	0.43%
b_5	-0.2	-0.20172	0.8%
b_6	0.3	0.301755	0.6%
b_7	-0.3	-0.30002	0.01%
b_8	0.1	0.09905	0.95%

Table 5.4: Estimated parameters

In this example, a Coulomb damping force is considered. This Coulomb damping force is assumed as a nonlinear term in Eq.(5.3.39).

$$[N] = \begin{bmatrix} 0.2 \mid \dot{x}_1 \mid \dot{x}_1 \\ 0.2 \mid \dot{x}_2 \mid \dot{x}_2 \end{bmatrix} \quad (5.3.50)$$

The equation of motion of this system is

$$[M][\ddot{x}] + [K][x] + [N] = [F] \quad (5.3.51)$$

The initial conditions are:

$$\begin{aligned} x_1(0) &= 1.0 \\ \dot{x}_1(0) &= 0 \\ x_2(0) &= 0 \\ \dot{x}_2(0) &= 0 \end{aligned} \quad (5.3.52)$$

The simulated responses of displacement and velocity are obtained by using Runge-Kutta method from Eq.(5.3.51) and illustrated in Fig. 5.9 and Fig. 5.10. Sampling

P	Real P	Est. P	Error
c_{11}	0.2	0.1988	0.6%
c_{22}	0.2	0.20004	0.002%

Table 5.5: Estimated parameters

time period $\Delta t = 0.05$ and 502 samples of input and output are considered. The estimated results are listed in Table 5.5.

Example 4:

In practical engineering, the real damping usually is different from design damping. Identification of the difference is useful for analysis, design, and control. If the mass matrix $[M]$, stiffness matrix $[K]$, and damping matrix $[C]$ are known, the difference of damping can be estimated by using Hammerstein Feedback Model. The difference of damping is assumed to be $[dC]$. We assume a linear dynamical system as following differential equation.

$$[M][\ddot{x}] + ([C] + [dC])[\dot{x}] + [K][x] = [F] \tag{5.3.53}$$

where difference of damping is assumed as

$$[dC] = \begin{bmatrix} 0.05 & -0.005 \\ -0.005 & 0.05 \end{bmatrix} \tag{5.3.54}$$

The responses of displacement $x_1(t)$, $x_2(t)$ and velocity $\dot{x}_1(t)$, $\dot{x}_2(t)$ are obtained by using Runge-Kutta method from Eq.(5.3.53) and shown in Fig. 5.11 and Fig. 5.12. The HFM of the system is assumed as

$$\begin{aligned} x_1(k) + a_1 x_1(k-1) + a_2 x_1(k-2) + a_3 x_2(k-2) + \cos 0.5(k-2) \\ = a_4 \dot{x}_1(k-2) + a_5 \dot{x}_2(k-2) \end{aligned} \tag{5.3.55}$$

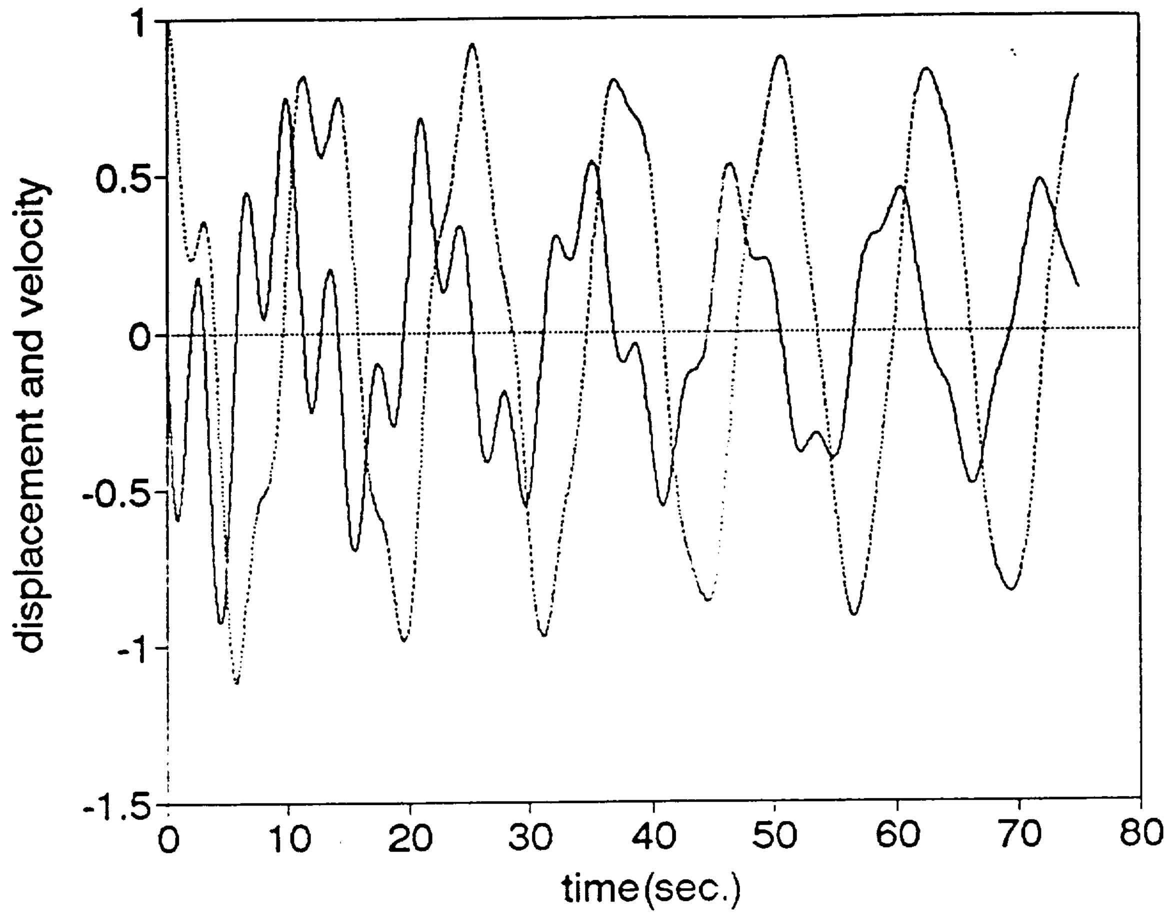


Figure 5.9: The responses x_1 and $\dot{x}_1$ of two degree of freedom nonlinear dynamical system with Coulomb damping

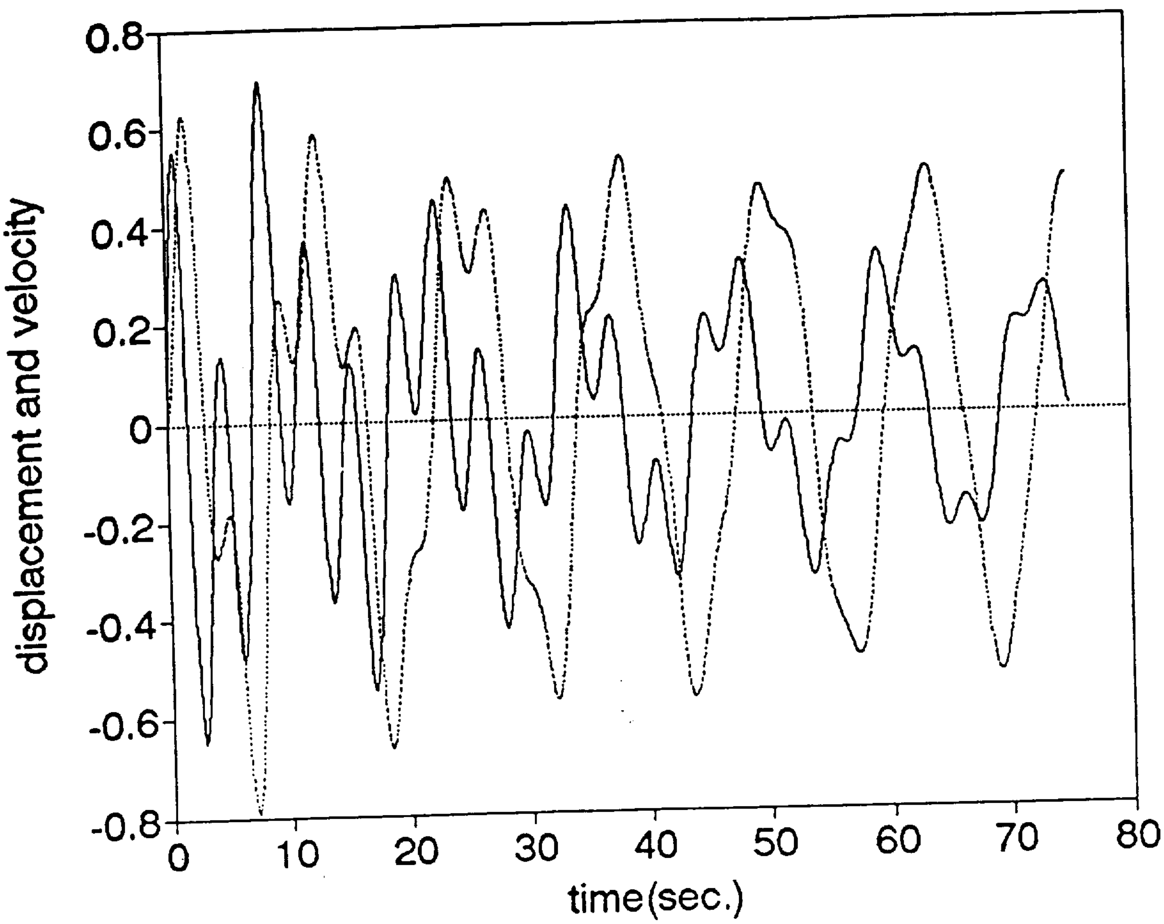


Figure 5.10: The responses x_2 and $\dot{x}_2$ of two degree of freedom nonlinear dynamical system with Coulomb damping

P	Real change	Est. change	Error
dc_{11}	0.05	0.050038	0.008%
dc_{12}	-0.005	-0.0050528	1.1%
dc_{21}	-0.005	-0.00496	0.8%
dc_{22}	0.05	0.04994	0.12%

Table 5.6: Estimated parameters

and

$$\begin{aligned} x_2(k) + b_1 x_2(k-1) + b_2 x_2(k-2) + b_3 x_1(k-1) \\ = b_4 \dot{x}_2(k-2) + b_5 \dot{x}_1(k-2) \end{aligned} \tag{5.3.56}$$

where a_1, a_2, a_3, b_1, b_2, b_3 are calculated from $[M]$, $[K]$ and $[C]$. The $[dC]$ has elements:

$$\begin{aligned} dc_{11} &= \frac{a_4}{(\Delta t)^2} \\ dc_{12} &= \frac{a_5}{(\Delta t)^2} \\ dc_{21} &= \frac{b_5}{(\Delta t)^2} \\ dc_{22} &= \frac{b_4}{(\Delta t)^2} \end{aligned} \tag{5.3.57}$$

$\Delta t = 0.05$ and 502 samples of the input and output are considered, then the estimated parameters are shown in Table 5.6 .

5.4 Sampling time period and estimate range

In this section, the two degree of freedom nonlinear dynamical system with Coulomb damping, numerical example 3, is considered to examine the effect of sampling time period. This example has the equation of motion, Eq.(5.3.51), and initial condition Eq.(5.3.52).

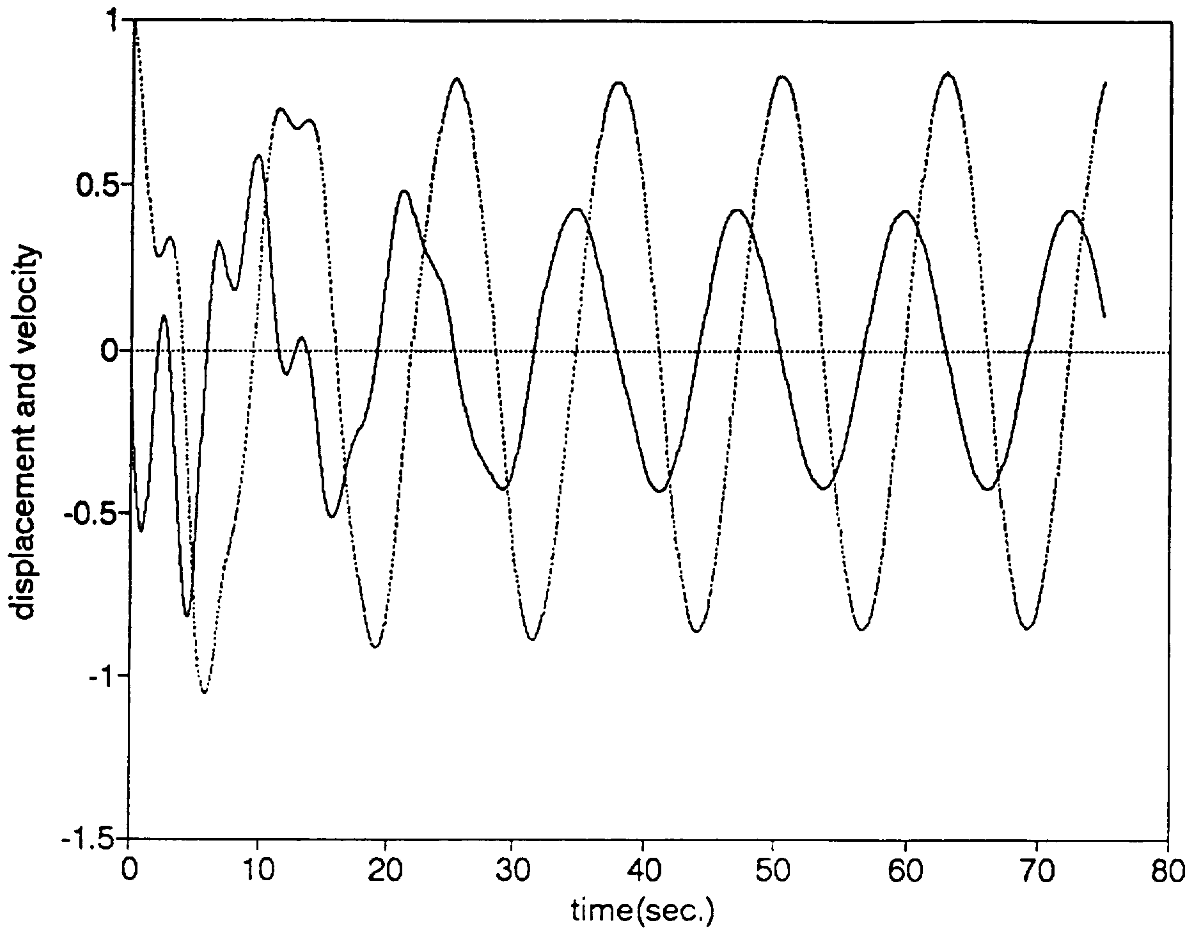


Figure 5.11: The responses x_1, $\dot{x}_1$ of TDOF nonlinear dynamical system

C-2

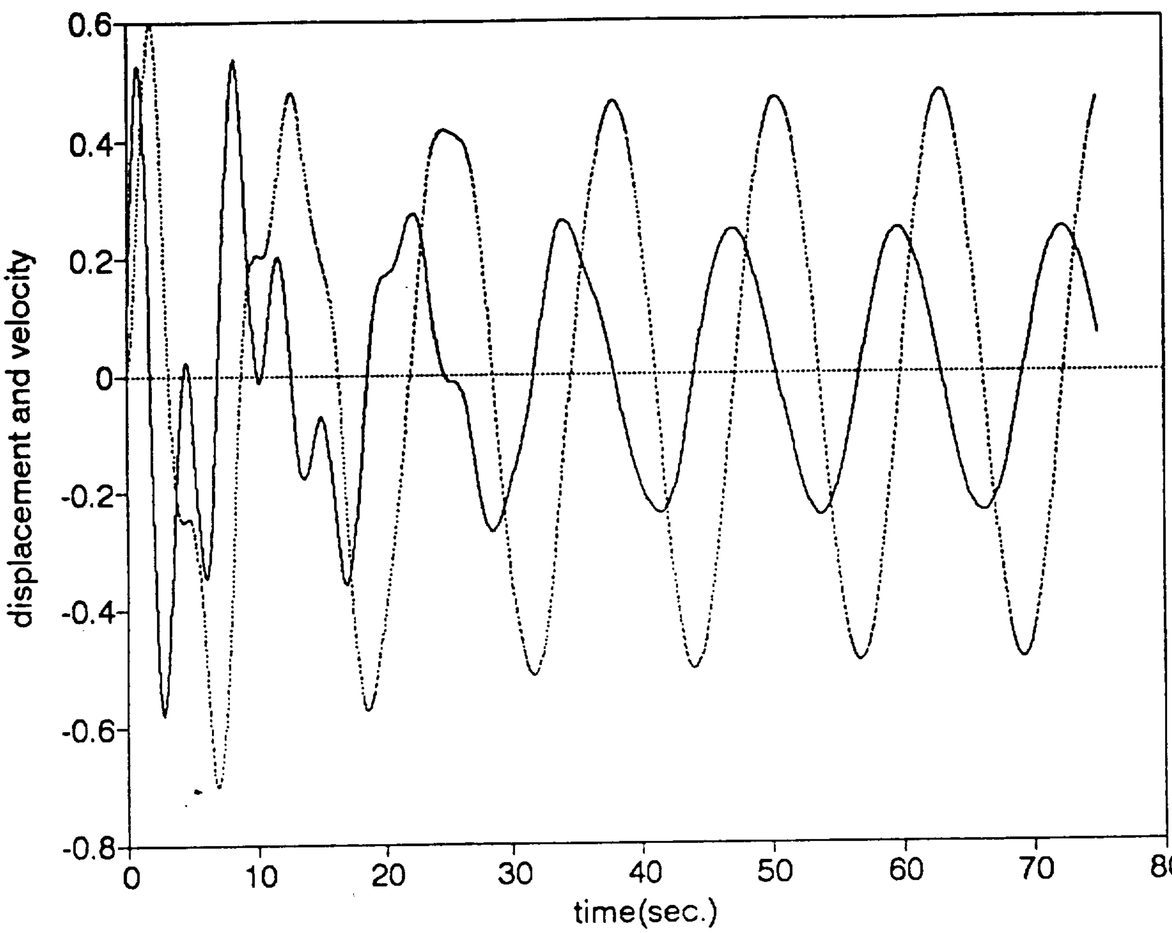


Figure 5.12: The responses x_2. $\dot{x}_2$ of TDOF nonlinear dynamical system

Fig.5.13 and Fig. 5.14 show the estimated Coulomb damping parameters of Eq.(1) and Eq.(2) of Eq.(5.9) vary with sampling numbers in a quarter of period, separately.

Fig.5.15 and Fig.5.16 show the estimated Coulomb damping parameters of Eq.(1) and Eq.(2) of Eq.(5.3.51) vary with sampling numbers in one half of period, separately.

Fig.5.17 and Fig.5.18 show the estimated Coulomb damping parameters of Eq.(1) and Eq.(2) of Eq.(5.3.51) vary with sampling numbers in a period, separately.

The sampling ranges of responses, x_1, $\dot{x}_1$, x_2, and $\dot{x}_2$, of system for estimation are denoted in Fig. 5.19 and Fig.5.20.

The results denote that the nonlinear Coulomb parameters of MDOF nonlinear dynamical system can be estimated even if using few samples in a small estimated range.

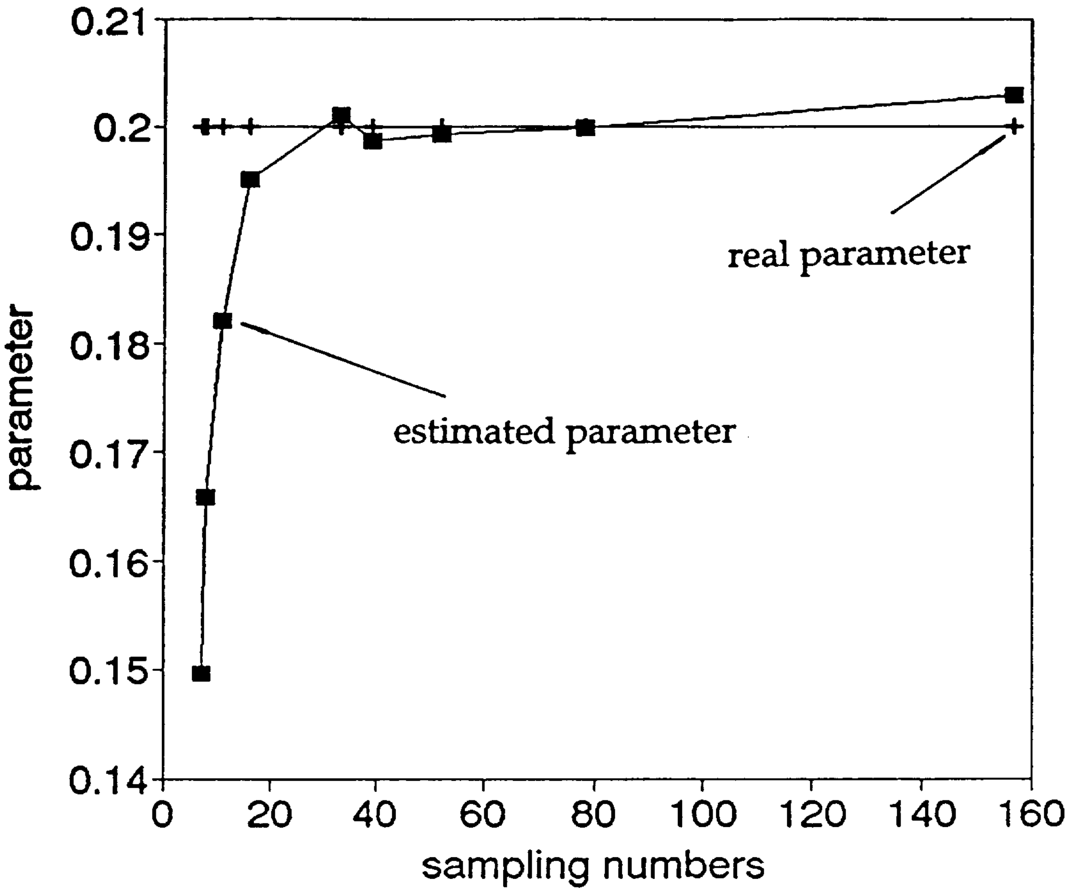


Figure 5.13: The Coulomb damping parameter of Eq.(1) varies with sampling numbers in a quarter of period

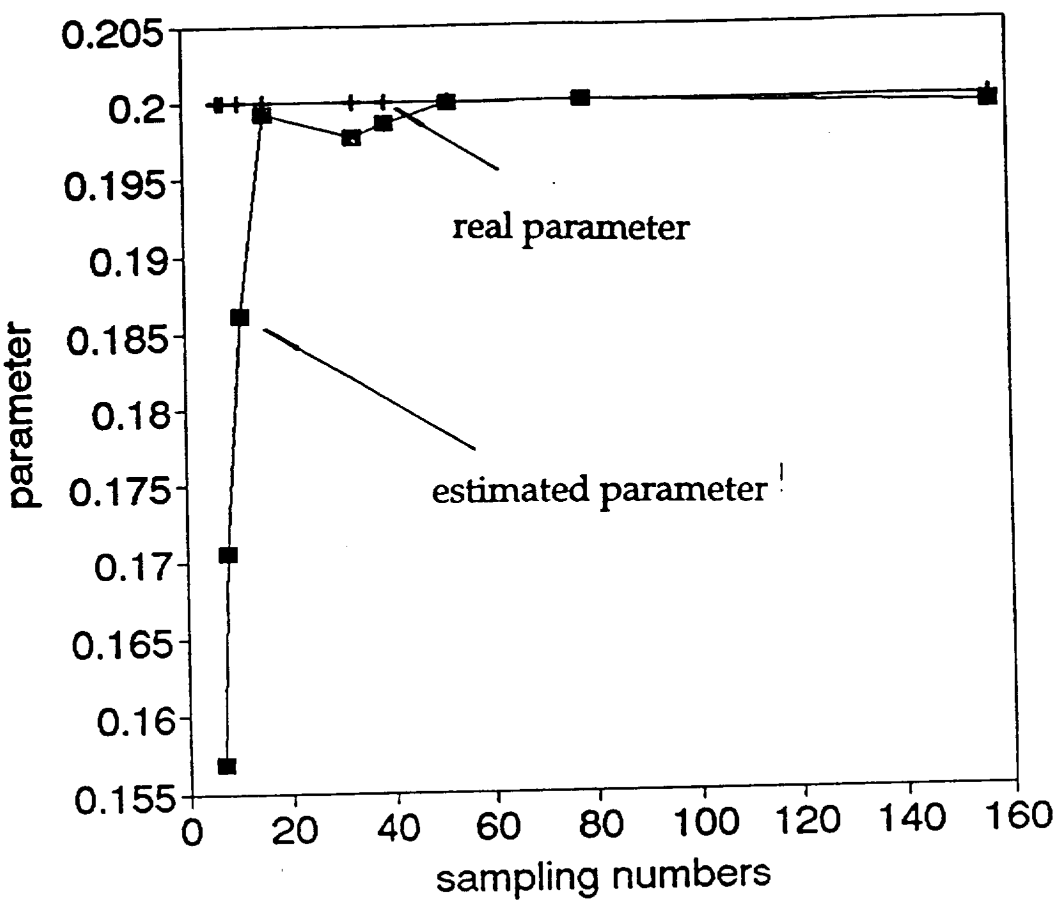


Figure 5.14: The Coulomb damping parameter of Eq.(2) varies with sampling numbers in a quarter of period

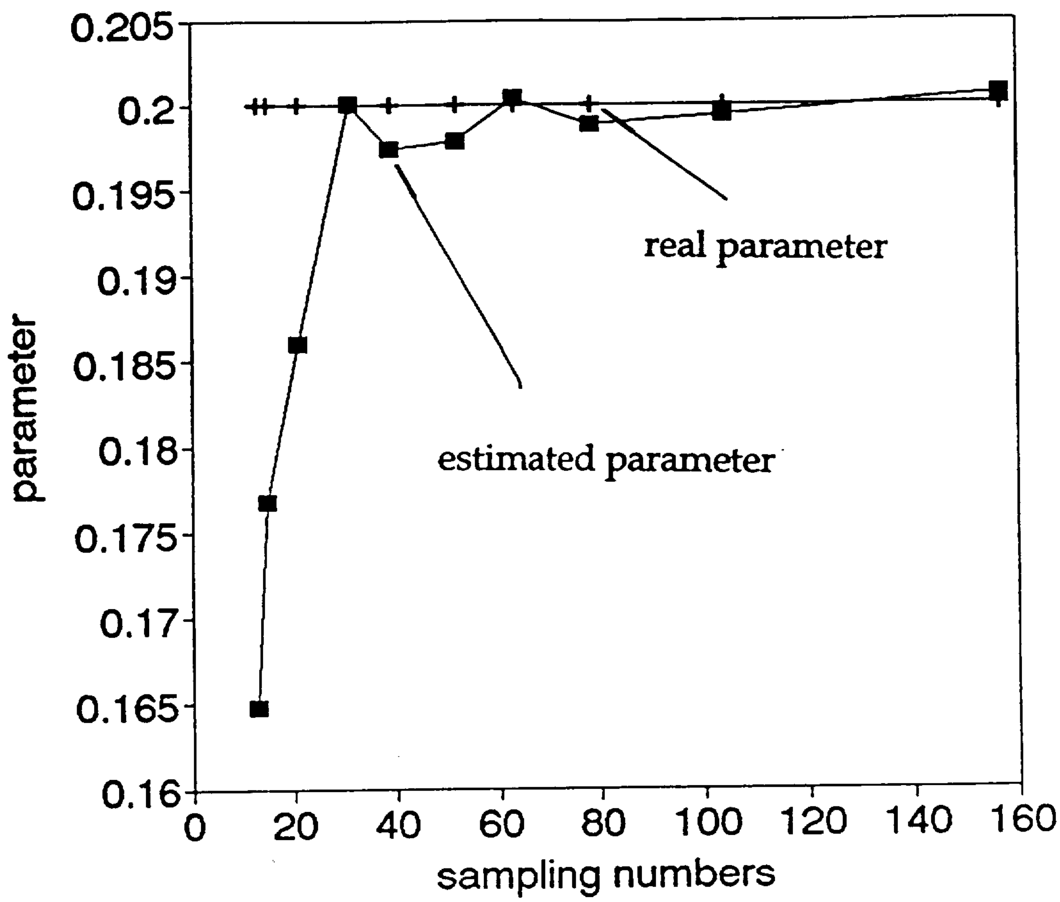


Figure 5.15: The Coulomb damping parameter of Eq.(1) varies with sampling numbers in one half of period

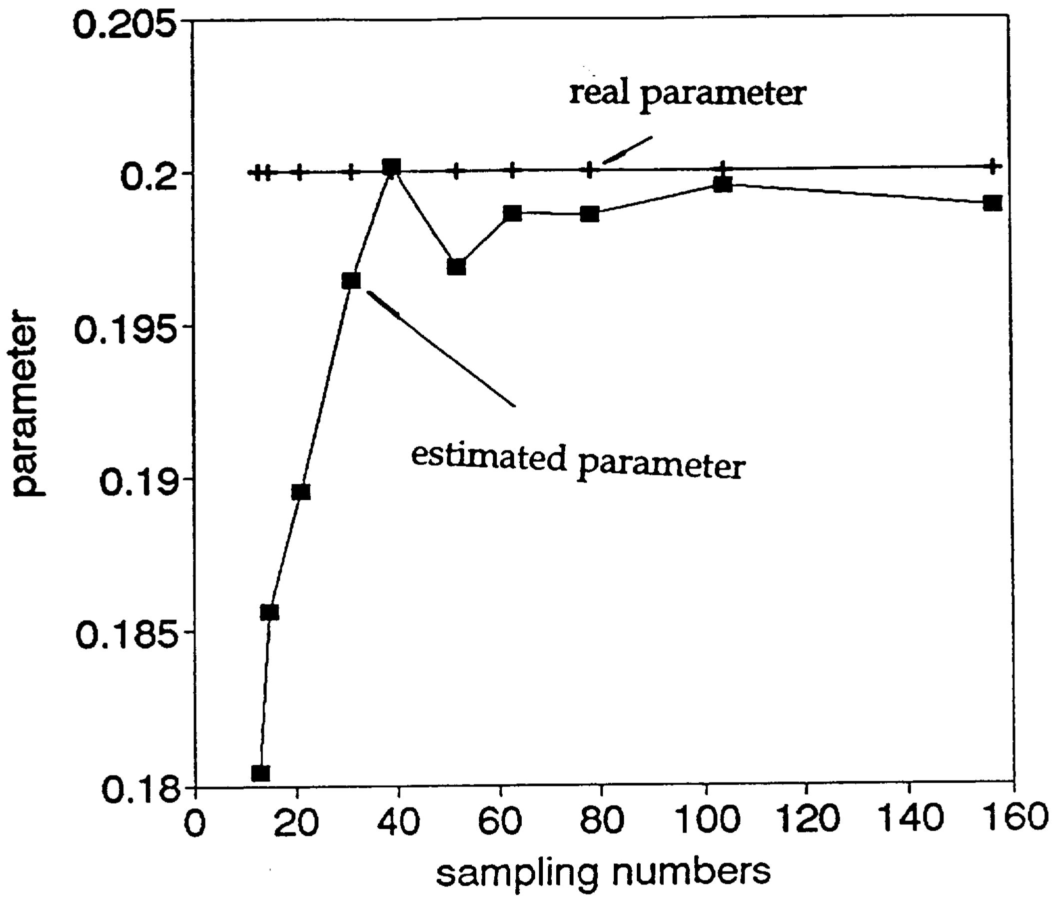


Figure 5.16: The Coulomb damping parameter of Eq.(2) varies with sampling numbers in one half of period

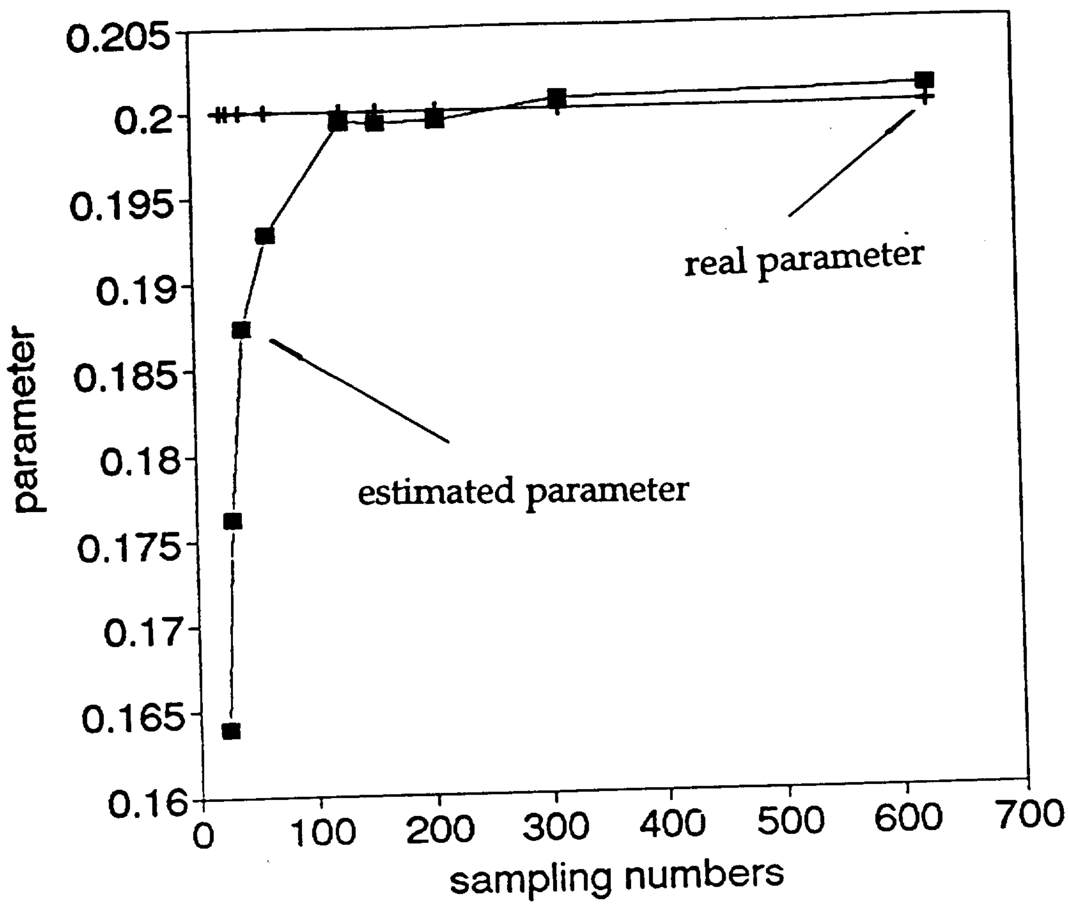


Figure 5.17: The Coulomb damping parameter of Eq.(1) varies with sampling numbers in a period

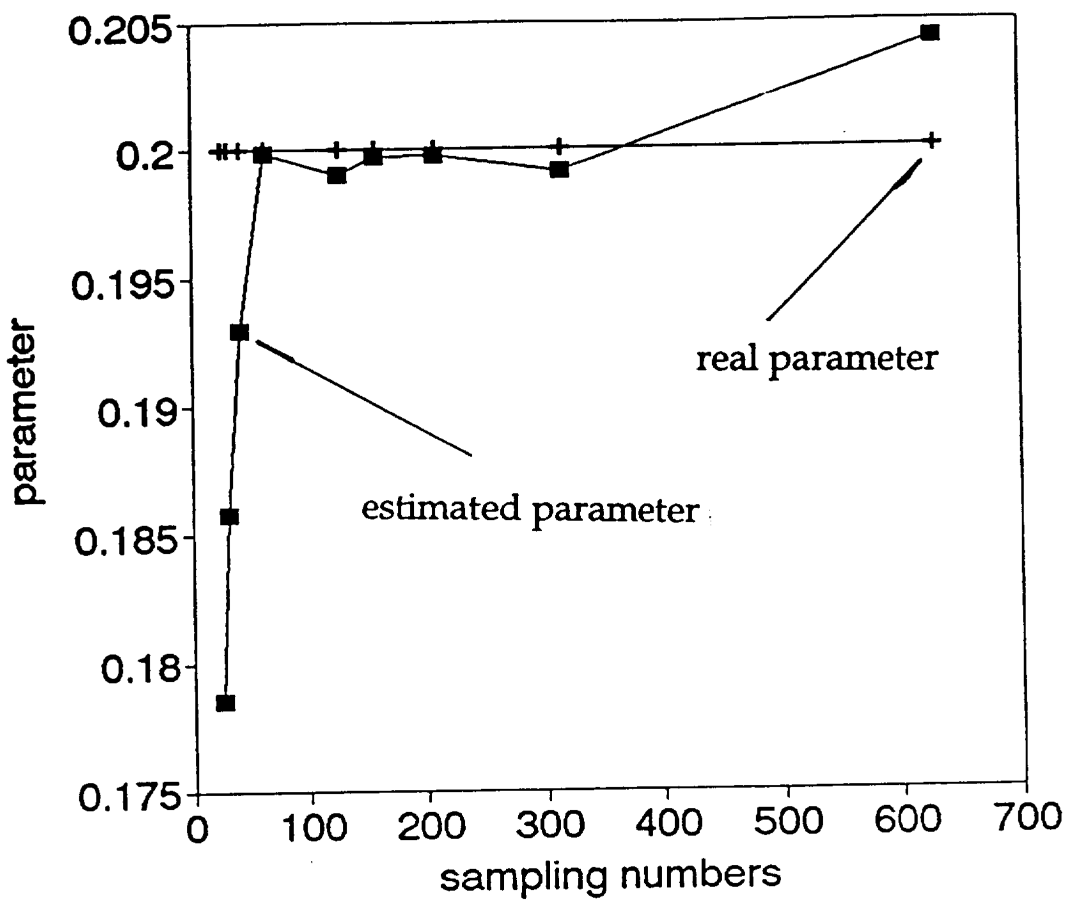


Figure 5.18: The Coulomb damping parameter of Eq.(2) varies with sampling numbers in a period

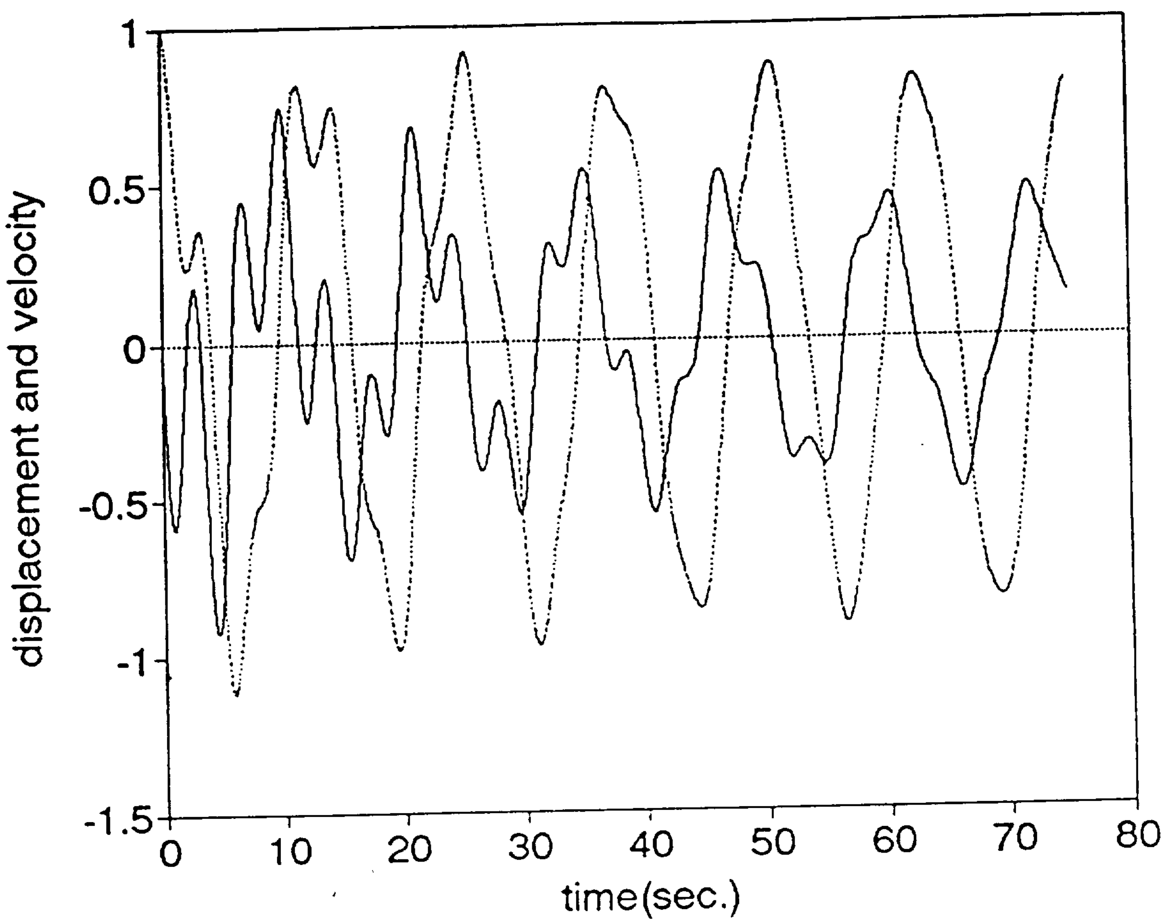


Figure 5.19: The sampling range

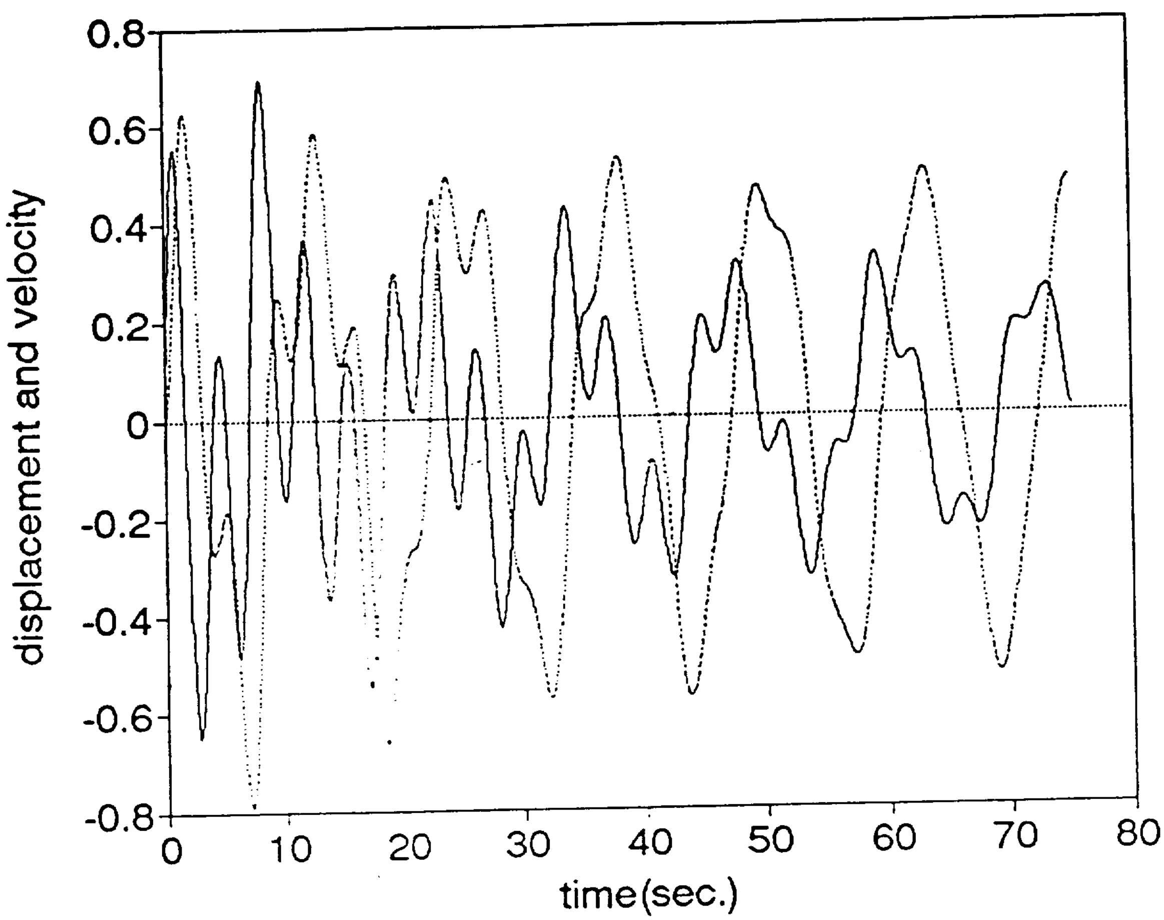


Figure 5.20: The sampling range

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