# TDRSS-User Orbit Determination Using Batch Least-Squares and Sequential Methods*

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## ABSTRACT

The Goddard Space Flight Center (GSFC) Flight Dynamics Division (FDD) commissioned Applied Technology Associates, Incorporated, to develop the Real-Time Orbit Determination/Enhanced (RTOD/E) system on a Disk Operating System (DOS)-based personal computer (PC) as a prototype system for sequential orbit determination of spacecraft . This paper presents the results of a study to compare the orbit determination accuracy for a Tracking and Data Relay Satellite System (TDRSS) user spacecraft, Landsat-4, obtained using RTOD/E, operating on a PC, with the accuracy of an established batch least-squares system, the Goddard Trajectory Determination System (GTDS), operating on a mainframe computer. The results of Landsat-4 orbit determination will provide useful experience for the Earth Observing System (EOS) series of satellites.

The Landsat-4 ephemerides were estimated for the January 17–23, 1991, timeframe, during which intensive TDRSS tracking data for Landsat-4 were available. Independent assessments were made of the consistencies (overlap comparisons for the batch case and covariances and the first measurement residuals for the sequential case) of solutions produced by the batch and sequential methods.

The forward-filtered RTOD/E orbit solutions were compared with the definitive GTDS orbit solutions for Landsat-4; the solution differences were less than 40 meters after the filter had reached steady state.

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# 1. INTRODUCTION

This paper compares the orbit determination accuracy of a prototype sequential orbit determination system with the accuracy achieved using an established batch least-squares system for a Tracking and Data Relay Satellite (TDRS) System (TDRSS) user spacecraft.

The National Aeronautics and Space Administration (NASA) has completed a transition from tracking and communications support of low Earth-orbiting satellites with a ground-based station network, the Ground Spaceflight Tracking and Data Network (GSTDN), to the geosynchronous relay satellite network, the TDRSS. TDRSS currently consists of three operational geosynchronous spacecraft (TDRS-East, TDRS-West, and TDRS-Spare) and the White Sands Ground Terminal (WSGT) at White Sands, New Mexico. TDRS-East, TDRS-West, and TDRS-Spare are located at 41, 171, and 174 degrees west longitude, respectively. The target TDRSS relay constellation will consist of four operational TDRSs, one each at 174, 171, 62, and 41 degrees west longitude. The ground network can provide only about 15-percent visibility coverage, while TDRSS has the operational capability to provide 85-percent to 100-percent coverage, depending on the spacecraft altitude.

The Bilateration Ranging Transponder System (BRTS) provides range and Doppler measurements for maintaining each TDRS orbit. The ground-based BRTS transponders are tracked as if they were TDRSS user spacecraft. Since the positions of the BRTS transponders are known, their ranging data can be used to precisely determine the trajectory of the TDRSs.

The focus of this paper is an assessment of the relative orbit determination accuracy of the batch least-squares method, used for current operational orbit determination support, with that of a sequential method implemented in a prototype system, used for analysis in the GSFC Flight Dynamics Facility (FDF). The batch weighted least-squares algorithm implemented in the Goddard Trajectory Determination System (GTDS) estimates the sets of orbital elements, force modeling parameters, and measurement-related parameters that minimize the squared difference between observed and calculated values of selected tracking data over a solution arc (Reference 1). GTDS resides and operates on the mainframe computer system at the FDF.

The sequential estimation algorithm implemented in a prototype system, the Real-Time Orbit Determination/Enhanced (RTOD/E), simultaneously estimates the TDRSS user and relay spacecraft orbital elements and other parameters in the force and observation models at each measurement time (Reference 2). RTOD/E performs forward filtering of tracking measurements using the extended Kalman filter with a process noise model to account for serially correlated, geopotential-induced errors, as well as Gauss-Markov processes for drag, solar radiation pressure, and measurement biases. The main features of RTOD/E are summarized in Reference 3.

An orbit determination analysis of Landsat-4 using TDRSS is reported here. Motivation for an orbit determination evaluation of Landsat-4 derives from the fact that the orbital characteristics of Landsat-4 are similar to those of the Earth Observing Satellite (EOS) series of missions, planned for launch starting in 1998. The results of a study for Landsat-4 will provide useful experience and verification of EOS flight dynamics support requirements. Early assessment of conclusions regarding meeting EOS support requirements will provide adequate opportunity to develop comprehensive support scenarios.

The estimated Landsat-4 ephemerides were obtained for the January 17–23, 1991, timeframe. This particular timeframe was chosen because dense TDRSS tracking data for Landsat-4 were available. Independent assessments were made to examine the consistencies (overlap comparisons for the batch case and state error covariances and the first measurement residuals for the sequential case) of results obtained by the batch and sequential methods.

Section 2 of this paper describes the orbit determination and evaluation procedures used in this study, and Section 3 gives the results obtained by the batch least-squares and sequential estimation methods and provides the resulting consistency and cross comparisons. Section 4 presents the conclusions of this study.

## 2. ORBIT DETERMINATION AND EVALUATION PROCEDURE

This section describes the analysis procedures used in this study. The TDRSS and BRTS tracking data characteristics are presented in Section 2.1, and the orbit determination evaluation methodology and options used are described in Section 2.2.

### 2.1 Tracking Measurements

Landsat-4 was deployed by Delta-3920 in July 1982. It has a nearly circular orbit, an altitude of approximately 715 kilometers, an inclination of 98 degrees, and a period of approximately 99 minutes. The time period chosen for this study was from 0 hours Greenwich mean time (GMT) on January 17, 1991, through 10 hours GMT on January 24, 1991.

During this interval, unusually dense TDRSS tracking of the Landsat-4 satellite was made available. The tracking consisted of an average of 15 passes of two-way TDRSS range and Doppler observations each day, each pass ranging from 3 minutes to 45 minutes in duration. The normal TDRSS tracking of Landsat-4 (less dense) typically consists of about six 5-minute passes each day. A timeline plot of the TDRSS tracking data distribution is given in Figure 1.

The typical scenario for BRTS tracking of the TDRSs during the period of study included approximately 4 or 9 minutes of range and two-way Doppler measurements from two ground transponders for each relay every 2 to 3 hours, consisting of an average of 12 BRTS passes per TDRS each day. BRTS stations for TDRS-East are located at White Sands and Ascension Island. BRTS stations for TDRS-West are located at White Sands, American Samoa, and Alice Springs, Australia.

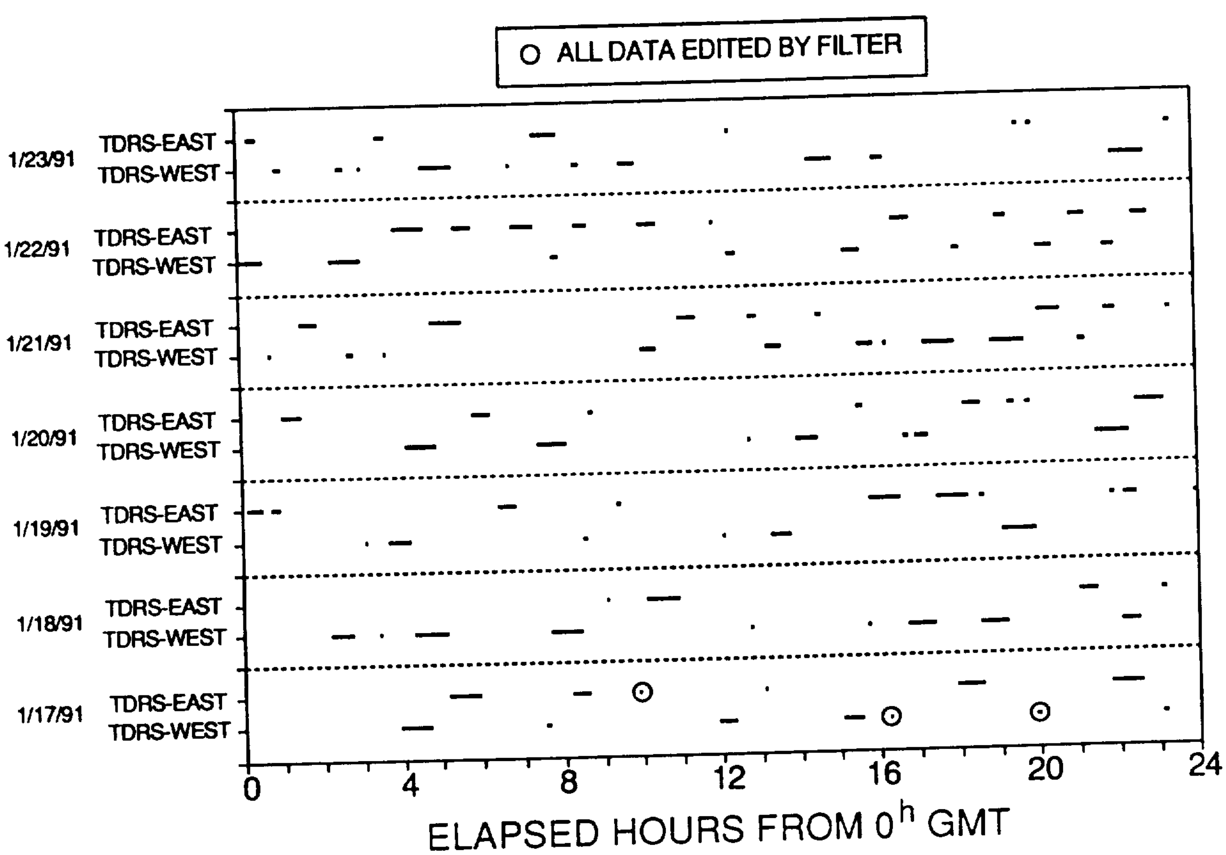


**Figure 1. Tracking Data for Landsat-4**

## 2.2 Evaluation Methodology

The evaluation methodologies for the batch least-squares and sequential estimation methods are described below. Since there are some known differences between the GTDS and RTOD/E force models (geopotential, atmospheric density, solar and planetary ephemerides presentation, solid Earth tides, and process noise modeling), and since the RTOD/E TDRSS and BRTS measurement models were implemented independently from GTDS, the two systems are not expected to provide identical results. Therefore, this study assumes that each system is used in its optimal configuration. Table 1 gives the parameters and options for the simultaneous solutions of the user and relay spacecraft. Table 2 gives the force and measurement model specifications.

### Table 1. Parameters and Options for the Simultaneous Solutions of User and Relay Spacecraft

| ORBIT DETERMINATION PARAMETER OR OPTION | GTDS VALUES | | RTOD/E VALUES | |
|---|---|---|---|---|
| | USER (LANDSAT-4) | RELAY (TDRS-EAST & TDRS-WEST) | USER (LANDSAT-4) | RELAY (TDRS-EAST & TDRS-WEST) |
| ESTIMATED PARAMETERS | STATE, DRAG SCALING PARAMETER ($\rho_1$), RANGE AND DOPPLER MEASUREMENT BIASES FOR TRACKING VIA EACH GROUND STATION | STATE, TRANSPONDER DELAYS FOR EACH BRTS TRANSPONDER | STATE, COEFFICIENT OF DRAG, RANGE AND DOPPLER MEASUREMENT BIASES FOR TRACKING VIA EACH TDRS | STATE, SOLAR REFLECTIVITY COEFFICIENT ($C_R$), RANGE AND DOPPLER MEASUREMENT BIASES FOR TRACKING VIA EACH TRANSPONDER |
| INTEGRATION TYPE | FIXED-STEP COWELL | FIXED-STEP COWELL | VARIATION OF PARAMETERS | VARIATION OF PARAMETERS |
| COORDINATE SYSTEM OF INTEGRATION | MEAN OF 1950.0 | MEAN OF 1950.0 | MEAN OF 1950.0 | MEAN OF 1950.0 |
| INTEGRATION STEP SIZE (SECONDS) | 30.0 | 600.0 | 60.0 | 600.0 |
| TRACKING DATA | TDRSS | BRTS | TDRSS | BRTS |
| DATA RATE | 1 PER 20 SECONDS | 1 PER 10 SECONDS | 1 PER 20 SECONDS | 1 PER 20 SECONDS |
| DC CONVERGENCE PARAMETER | 0.005 | 0.005 | N/A | N/A |
| EDITING CRITERION | $3\sigma$ | $3\sigma$ | $3\sigma$ | $3\sigma$ |
| MEASUREMENT $\sigma$'s: | | | | |
| RANGE | 30.0 METERS | 10.0 METERS | 0.4 METER | 0.4 METER |
| DOPPLER | 0.25 HERTZ | 0.003 HERTZ | 0.004 HERTZ | 0.003 HERTZ |
| GAUSS-MARKOV PARAMETERS: | N/A | N/A | | |
| DRAG HALF-LIFE | | | 1440 MINUTES | N/A |
| DRAG SIGMA | | | 0.207 | N/A |
| $C_R$ HALF-LIFE | | | N/A | 11520 MINUTES |
| $C_R$ SIGMA | | | N/A | 0.2 |
| RANGE BIAS HALF-LIFE | | | 60 MINUTES | 60 MINUTES |
| RANGE BIAS SIGMA | | | 6 METERS | 4.5 METERS |
| DOPPLER BIAS HALF-LIFE | | | 8 MINUTES | 60 MINUTES |
| DOPPLER BIAS SIGMA | | | 0.034 HERTZ | 0.02 HERTZ |
| SATELLITE AREA | 12.26 $METERS^2$ | 40 $METERS^2$ | 12.26 $METERS^2$ | 40.0 $METERS^2$ |
| SATELLITE MASS | 1900 KILOGRAMS | 1991 KILOGRAMS (TDRS-EAST) 1735 KILOGRAMS (TDRS-WEST) | 1900 KILOGRAMS | 1991 KILOGRAMS (TDRS-EAST) 1735 KILOGRAMS (TDRS-WEST) |

N/A = NOT APPLICABLE

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## Table 2. Force and Measurement Model Specifications

| ORBIT DETERMINATION PARAMETER OR OPTION | GTDS VALUES | | RTOD/E VALUES | |
|---|---|---|---|---|
| | USER (LANDSAT-4) | RELAY (TDRS-EAST & TDRS-WEST) | USER (LANDSAT-4) | RELAY (TDRS-EAST & TDRS-WEST) |
| GEOPOTENTIAL MODEL | GEM-T3 (50 x 50) | GEM-T3 (8 x 8) | GEM-10B (30 x 30) | GEM-10B (6 x 6) |
| ATMOSPHERIC DENSITY MODEL | JACCHIA-ROBERTS DAILY SOLAR FLUX VALUES (209, 203, 199, 204, 202, 224, 223) | N/A | CIRA 1972 DAILY SOLAR FLUX VALUES (209, 203, 199, 204, 202, 224, 223) | N/A |
| SOLAR AND LUNAR EPHEMERIDES | JPL DE-118 | JPL DE-118 | ANALYTICAL | ANALYTICAL |
| SOLAR REFLECTIVITY COEFFICIENT ($C_R$) | 1.5 | APPLIED (SEE TEXT) | 1.5 | ESTIMATED |
| COEFFICIENT OF DRAG ($C_D$) | ESTIMATED | N/A | ESTIMATED | N/A |
| IONOSPHERIC REFRACTION CORRECTION | BENT MODEL | BENT MODEL | NO | NO |
| GROUND-TO-SPACECRAFT | N/A | YES | | |
| SPACECRAFT-TO-SPACECRAFT | YES | N/A | | |
| TROPOSPHERIC REFRACTION CORRECTION | YES | YES | YES | YES |
| POLAR MOTION CORRECTION | YES | YES | YES | YES |
| EARTH TIDES | YES | NO | NO | NO |

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GEM = GODDARD EARTH MODEL
JPL = JET PROPULSION LABORATORY
N/A = NOT APPLICABLE

### Batch Least-Squares Method

Except for the variations noted, the computational procedures and mathematical methods used in this study are those used for routine operational orbit determination in the GSFC FDF. The choice to expand the state space of the least-squares solutions to include measurement biases was motivated by the fact that the RTOD/E orbit determination algorithm estimates an equivalent set of bias parameters. The batch weighted least-squares algorithm implemented in GTDS (Reference 1) solves for the set of orbital elements and other parameters that minimizes the squared difference between observed and calculated values of selected tracking data over a solution arc. Parameters solved for, other than the spacecraft state at epoch, include free parameters of the force model and/or the observation model.

A detailed study of the Earth Radiation Budget Satellite (ERBS) with the batch least-squares estimation method was reported in Reference 4, and it was further refined in Reference 5. The models and options found optimal in the previous study of ERBS are used here for Landsat-4. The options used for the study described in this paper are summarized in columns 2 and 3 of Tables 1 and 2.

The solar reflectivity coefficients ($C_R$) for TDRS-East and TDRS-West were not estimated in the simultaneous solutions of Landsat-4, TDRS-East, and TDRS-West but were applied. The values of $C_R$ applied in the present calculations were obtained from a set of separate companion solutions of TDRS-East and TDRS-West using only BRTS tracking data.

To evaluate the orbit determination consistency achievable with a particular choice of options using least-squares estimation, a series of seven 34-hour definitive solutions was performed with 10-hour overlaps between neighboring arcs. The GTDS Ephemeris Comparison Program was used to determine the root-mean-square (RMS) position differences between the definitive ephemerides for neighboring solutions in the 10-hour overlap time period. These "overlap" comparisons measure the adjacent solution consistency, not the absolute accuracy.

### Sequential Estimation Method

RTOD/E was recently developed by Applied Technology Associates, Incorporated (ATA) for the GSFC Flight Dynamics Division (FDD) to respond to the need for a realtime estimation capability, to address future increased TDRSS-navigation accuracy requirements, and to provide automation of some routine orbit determination operations. The goal for future orbit determination accuracy is 10 meters total position error ($1\sigma$) for the user and 25 meters total position error ($1\sigma$) for the TDRSs. RTOD/E provides a proof of concept for the use of sequential estimation techniques for orbit determination with TDRSS tracking data and offers the potential for enhanced accuracy navigation with realtime responsiveness. RTOD/E is a research tool for assessing sequential estimation for FDF navigation applications in realistic operational situations.

RTOD/E uses the extended Kalman filter form for sequential orbit estimation. With the sequential estimation method, each tracking measurement can be processed immediately upon receipt to produce an update of a spacecraft's state vector and auxiliary state parameters. This fact makes it well suited for realtime or near-realtime operation. Sequential estimation is particularly well suited to the development of systems to perform orbit determination autonomously on the spacecraft's onboard computer (Reference 6). Spacecraft orbit determination during and just after a maneuver is a critical support function for which orbit determination is needed in near-realtime. Therefore, sequential estimation is also well suited for such an application. In addition, the forward filter can be augmented with a backward smoothing filter to further improve the overall accuracy, especially during periods without tracking data.

RTOD/E employs a sequential estimation algorithm with a process noise model to stochastically account for gravity model errors (Reference 7). In addition to the spacecraft orbital elements, the filter estimates free parameters of the force model and the measurement model, treating these parameters as random variables whose behavior is governed by a Gauss-Markov stochastic process.

RTOD/E uses a forward-processing extended Kalman filter for sequential orbit estimation. The mathematical algorithms and computational procedures are described in References 2 and 7. The specific options used in RTOD/E for this study are listed in the last two columns of Tables 1 and 2.

A good indicator of the consistency of the sequential estimation results is provided by the state error covariance function generated during the estimation process (Reference 8). In addition, the relationship of the first predicted measurement residual of each tracking pass to the associated predicted residual variance provides an indication of the physical integrity of the state error covariance of the filtered orbits. These parameters were monitored during the sequential estimation process.

## 3. RESULTS AND DISCUSSION

The results of this study for the Landsat-4 and TDRSS relay spacecraft are presented in this section, along with an analysis of the results. Greater emphasis is placed on the Landsat-4 results, since the primary objective is to study TDRSS user orbit determination. The orbit determination results using batch least-squares calculations and sequential estimation are given in Sections 3.1 and 3.2, respectively; the comparisons are presented in Section 3.3.

## 3.1 Batch Least-Squares Results

The RMS values of six Landsat-4 overlap comparisons are summarized in Figure 2. The overlap values vary from about 3 to 5 meters. The mean and sample standard deviation of this distribution, in the form of mean ± standard deviation, is 3.8 ± 1.0 meters. The maximum total position differences over the same distribution vary between 5 and 9 meters, with a mean and standard deviation of 6.1 ± 1.8 meters. The maximum position difference values for Landsat-4 are typically a factor of 1.6 larger than the RMS values.

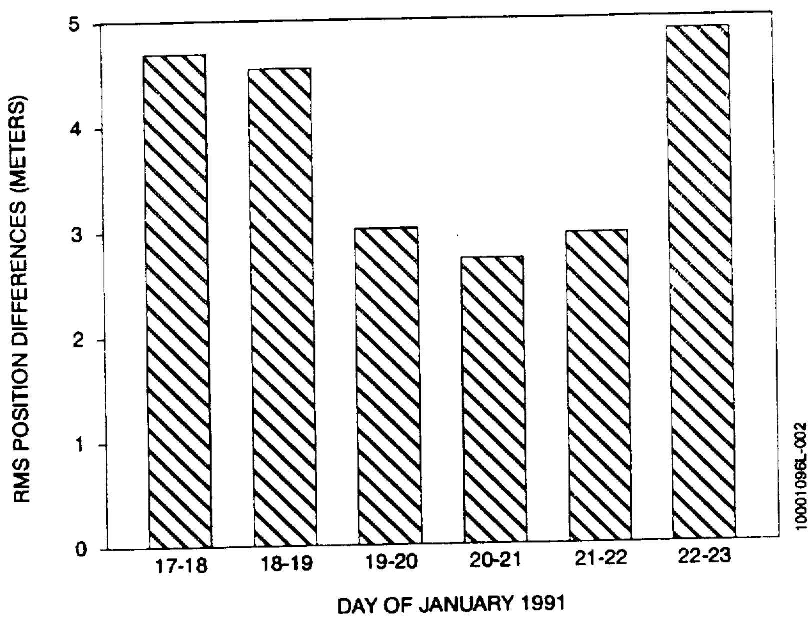


**Figure 2. Landsat Overlap Comparisons**

It should be noted that all data arcs for Landsat-4 solutions consisted of 34 hours, beginning at 0 hours GMT of each day from January 17 to January 23, 1991, with one exception. The exception was made for the arc beginning at 0 hours GMT on January 20, 1991. There is a long data gap of about 5 hours (see Figure 1) at the end of the nominal 34-hour period, resulting in a predicted solution for the last 5 hours instead of a definitive solution. Therefore, for this particular solution, the arc length was extended by 2 hours to 36 hours so that the next tracking pass was included in the solution.

The RMS values of six TDRS-East and TDRS-West overlap comparisons are summarized in Figure 3. The overlap values for TDRS-East vary from about 7 to 30 meters. The mean and sample standard deviation of this distribution is 14.2 ± 7.8 meters. The maximum total position differences over the same distribution vary between 9 and 35 meters, with a mean and standard deviation of 19.1 ± 9.1 meters. The overlap values for TDRS-West vary from about 10 to 55 meters. The mean and the sample standard deviation of this distribution is 21.6 ± 16.9 meters. The maximum total position differences over the same distribution vary between 12 and 74 meters, with a mean and standard deviation of 26.2 ± 23.8 meters. The maximum position difference values for the TDRSs are typically a factor of 1.2 larger than the RMS values.

The possible advantage of estimating a set of bias parameters versus not estimating the set was evaluated. The mean values of the TDRSS range and Doppler measurement residuals (i.e., the observed-minus-computed values for each solution) calculated without estimating biases indicated the existence of a small systematic error. The mean range measurement residuals varied between –0.8 ± 3.0 meters and +1.1 ± 3.5 meters for the seven solution arcs. The mean Doppler measurement residuals varied between –15.8 ± 80.3 millihertz and –3.8 ± 85.3 millihertz. The estimation of a set of bias parameters in the calculations in this study effectively removed the systematic error, thereby significantly reducing the mean range ($0.4 \times 10^{-6}$ to $0.2 \times 10^{-4}$ meters) and mean Doppler measurement ($0.2 \times 10^{-7}$ to $0.3 \times 10^{-3}$ millihertz) residual values, as expected. The standard deviations of the residuals were also somewhat reduced. However, although the removal of a bias may improve accuracy, it was not expected to improve consistency. As a matter of fact, the mean RMS overlap value without estimating for a set of bias parameters was larger for Landsat-4 (4.7 ± 1.1 meters) and for TDRS-East (38.5 ± 13.2 meters) and somewhat smaller for TDRS-West (15.1 ± 10.4 meters).

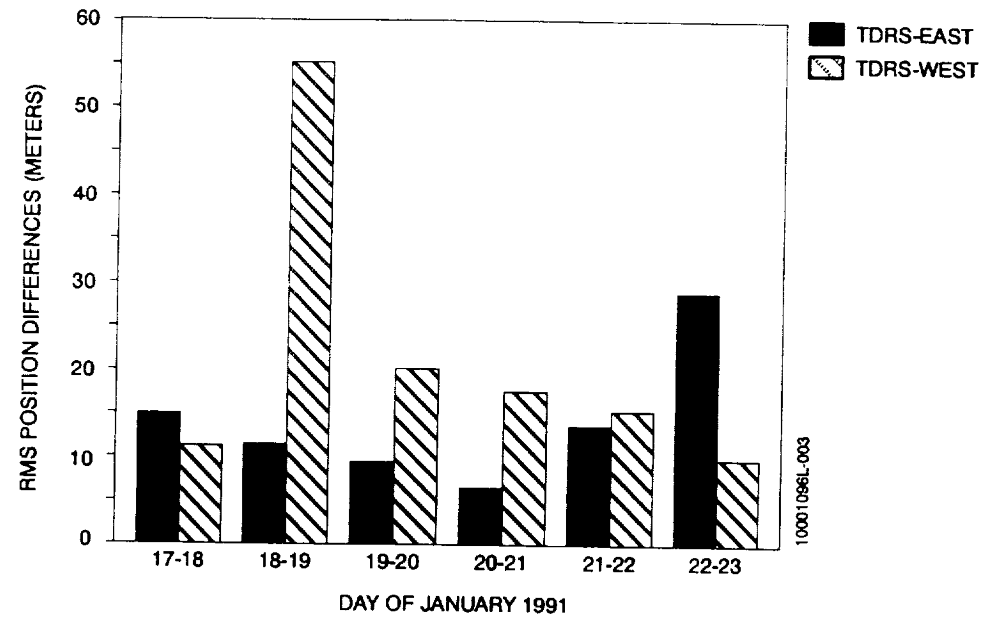


**Figure 3. TDRS-East and TDRS-West Overlap Comparisons**

## 3.2 Sequential Estimation Results

During sequential processing of the TDRSS and BRTS measurements using RTOD/E, the position component standard deviations from the state error covariance function (3σ) were closely monitored. The filter was started with high initial diagonal values in the covariance matrix. In the initial phases of filtering, the 3σ values were as high as 6000 meters for Landsat-4 and were 1200 meters for the TDRSs. This is not unusual before the filter has reached steady-state performance, especially considering that there is no TDRSS data for Landsat-4 in the first 4 hours (see Figure 1). After an initial filter settling period (about 24 hours), the 3σ values varied from about 10 to 40 meters in the RMS position for Landsat-4 and 40 to 60 meters for the TDRSs. The 3σ values for Landsat-4 dropped to their lowest levels during a tracking pass and then gradually rose to the maximum values during the time update phase (propagation phase). (The duration of the time update phases can be seen in Figure 1.) Unlike Landsat-4, the 3σ values for the TDRSs continued to decline gradually for about 4 days. Subsequently, the 3σ values for TDRS-West and TDRS-East remained relatively steady at about 25 meters and 35 meters, respectively.

The first predicted range residuals of Landsat-4 tracking passes after the filter processed the tracking data for 6 days are shown in Figure 4a. The tracking passes via TDRS-East and TDRS-West are plotted separately. The value of the residual varied from nearly −12 meters to about 12 meters. The largest value occurred after about 1 hour of the prediction period following the previous tracking pass. The ratio of the predicted range residual to the predicted residual standard deviation corresponding to Figure 4a is plotted in Figure 4b. The first residual of each pass was within the 3σ bound in the residual space. The postmeasurement-update range residuals were negligibly small, typically of the order of 0.3 meter or less.

The estimated force model parameters varied as a function of time and were updated after each measurement was processed. The time variation of the atmospheric drag coefficient for Landsat-4 is shown in Figure 5. It varied from a low value of 1.6 to a high value of 3.0. The 3σ uncertainty boundary ($C_D$ minus the 3σ uncertainty) in the drag parameter on the lower side is also plotted in NO TAG. The boundary on the upper side ($C_D$ plus the 3σ uncertainty) is not plotted so as not to clutter the figure. The variations in the drag parameter are smaller than the 3σ uncertainty. The 3σ uncertainty converges to an approximate value of 1.2 at

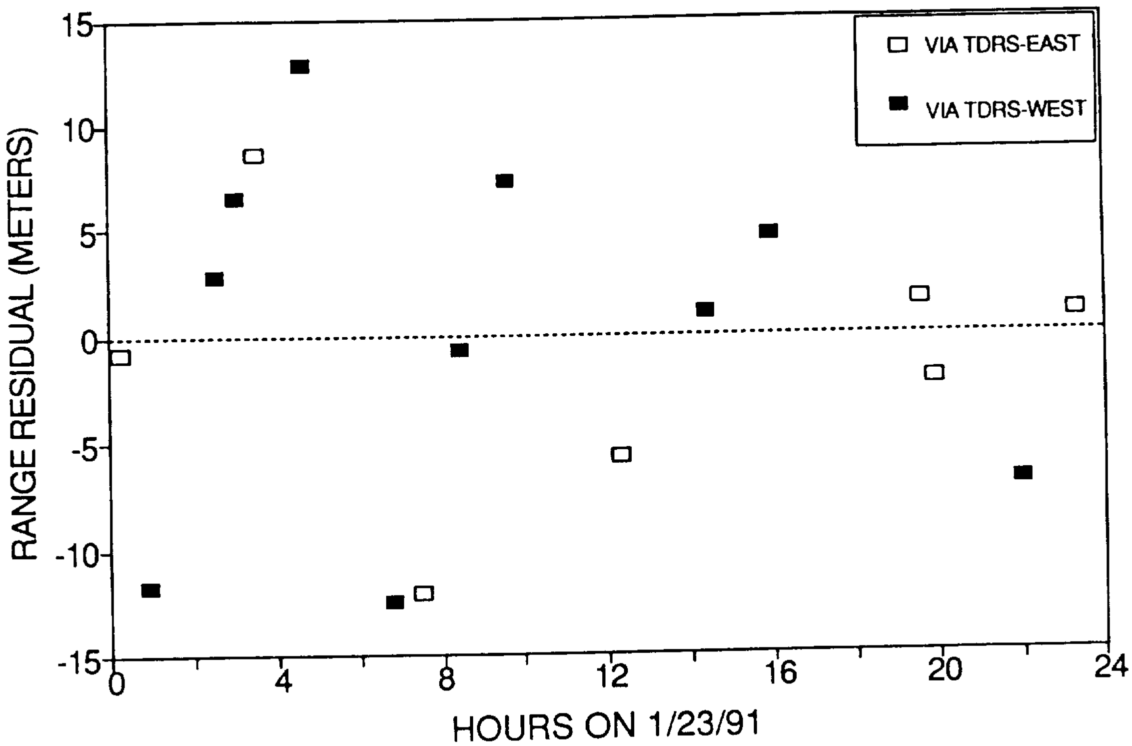


**Figure 4a. First Predicted Range Residual of TDRSS Tracking Passes for Landsat-4**

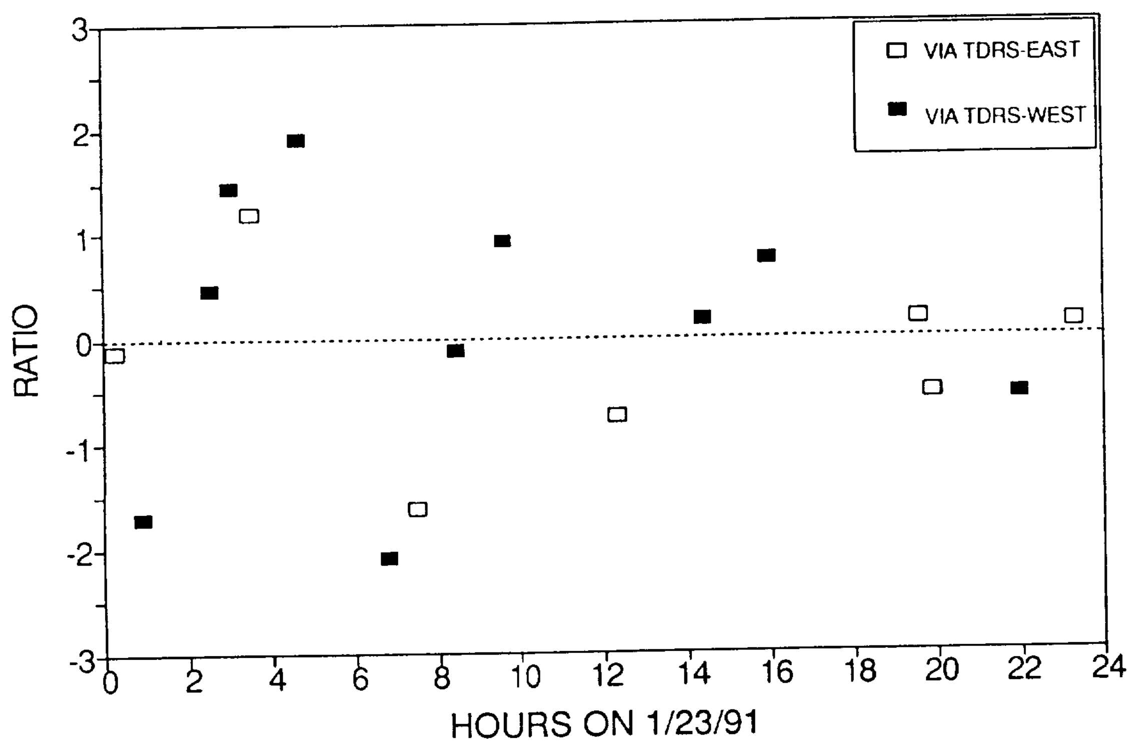


**Figure 4b. Ratio of the Predicted Range Residual to the Predicted Standard Deviation**

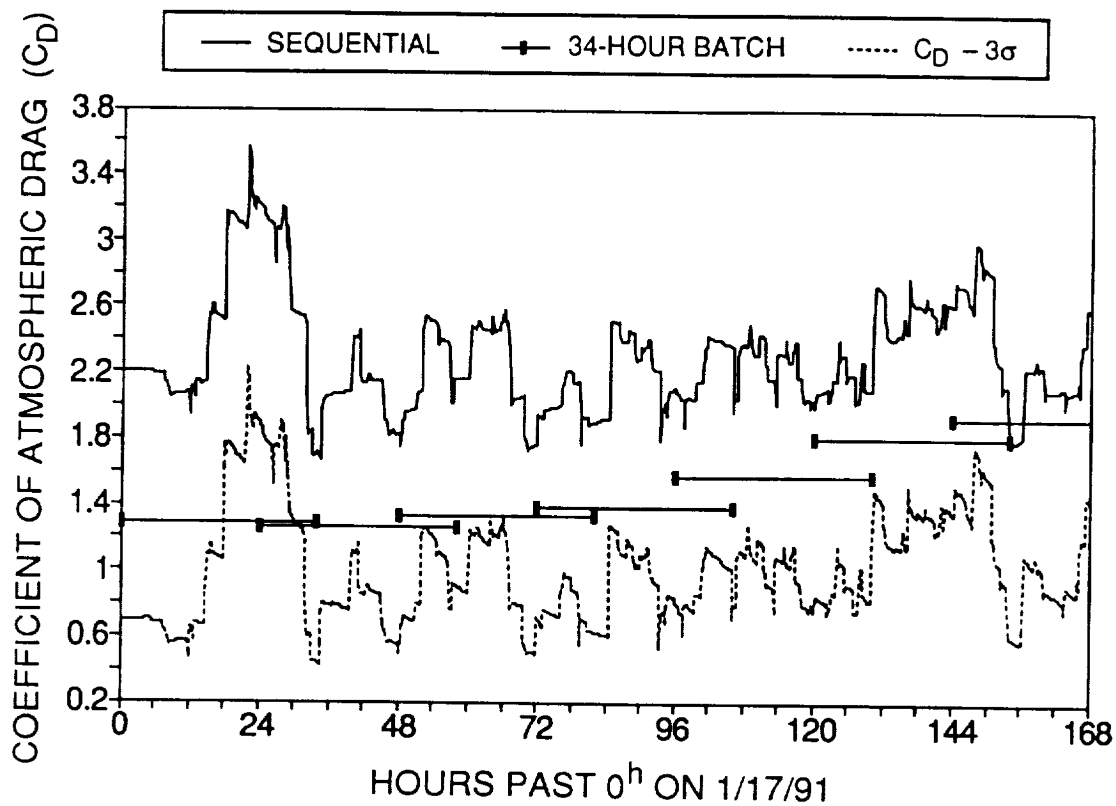


**Figure 5. Coefficient of Atmospheric Drag ($C_D$) for Landsat-4**

steady state. The time variations of the solar radiation pressure coefficient for TDRS-East and TDRS-West are given in Figures 6 and 7, respectively, along with the $3\sigma$ uncertainty boundaries ($C_R \pm 3\sigma$ uncertainty). After the filter reached steady state, the coefficient varied between 1.3 and 1.5. The variations in the estimated solar radiation pressure coefficients are smaller than the $3\sigma$ uncertainty, which varies between 0.15 and 0.2 at steady state. The estimated values obtained from the batch least-squares solutions are also shown in Figures 5 through 7 for comparison.

The solar flux values are input to RTOD/E on a daily basis. The time variation of the flux value over the 24-hour period is not input. Therefore, the atmospheric drag coefficient must be adjusted to compensate for the variation (NO TAG). RTOD/E models the area of the TDRS to be a constant throughout the day, whereas in actuality the TDRS surface area exposed to the solar flux varies with a 24-hour period. The $C_R$ estimated values for TDRS-East, shown in Figure 6, display an approximately repeated variation over 24 hours for the last 5 days during steady-state performance. Such a clear signature of variation is not evident in the $C_R$ values for TDRS-West shown in Figure 7.

The time variation of the estimated range bias values for Landsat-4 via TDRS-East and TDRS-West are shown in Figures 8 and 9, respectively, along with the $3\sigma$ uncertainties. The bias values varied from approximately −15 meters to approximately 10 meters, with an average value of approximately −1 meter. The $3\sigma$ uncertainty is 18 meters during data gaps. During tracking passes, it reduces to about 7 meters; following each tracking pass, it returns to 18 meters, with a half-life of 60 minutes (a priori input; see Table 1). There are some known physical phenomena and considerations that are absorbed in the estimation of the range bias: the time-varying tropospheric refraction delay and ionospheric refraction delay, which are not modeled in the measurement model; static position biases; and TDRS transponder delays.

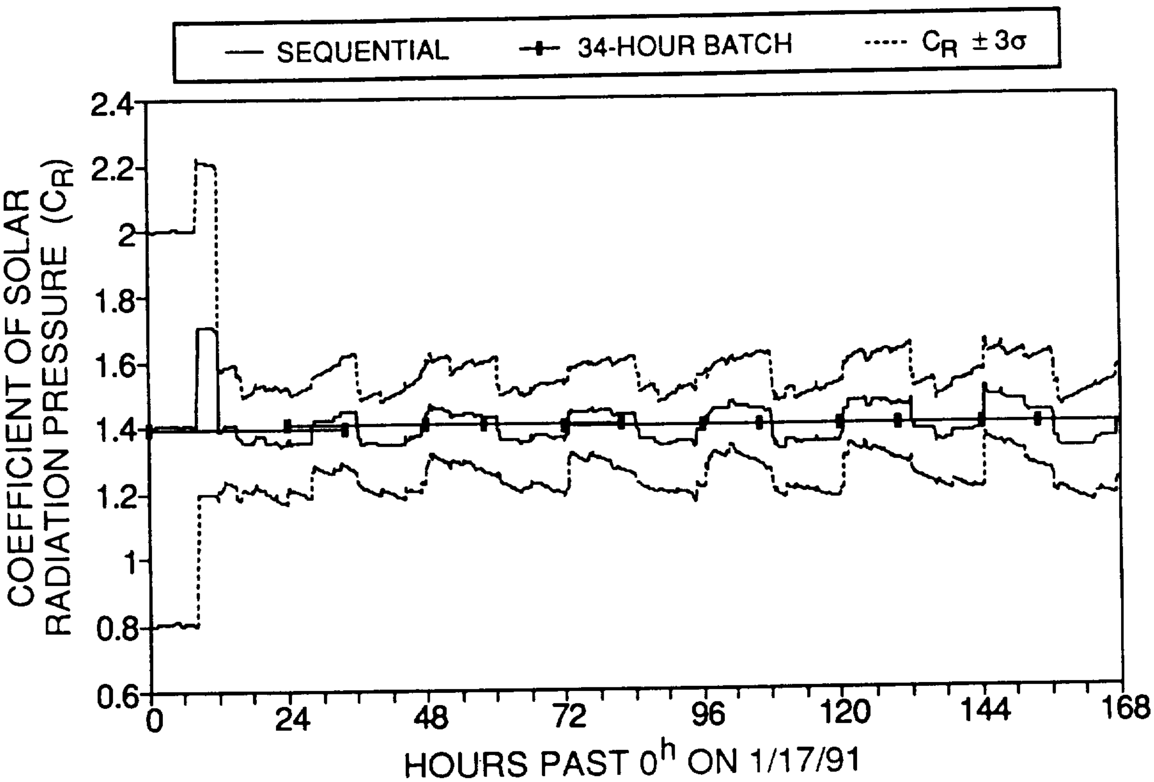


**Figure 6. Coefficient of Solar Radiation Pressure ($C_R$) for TDRS-East**

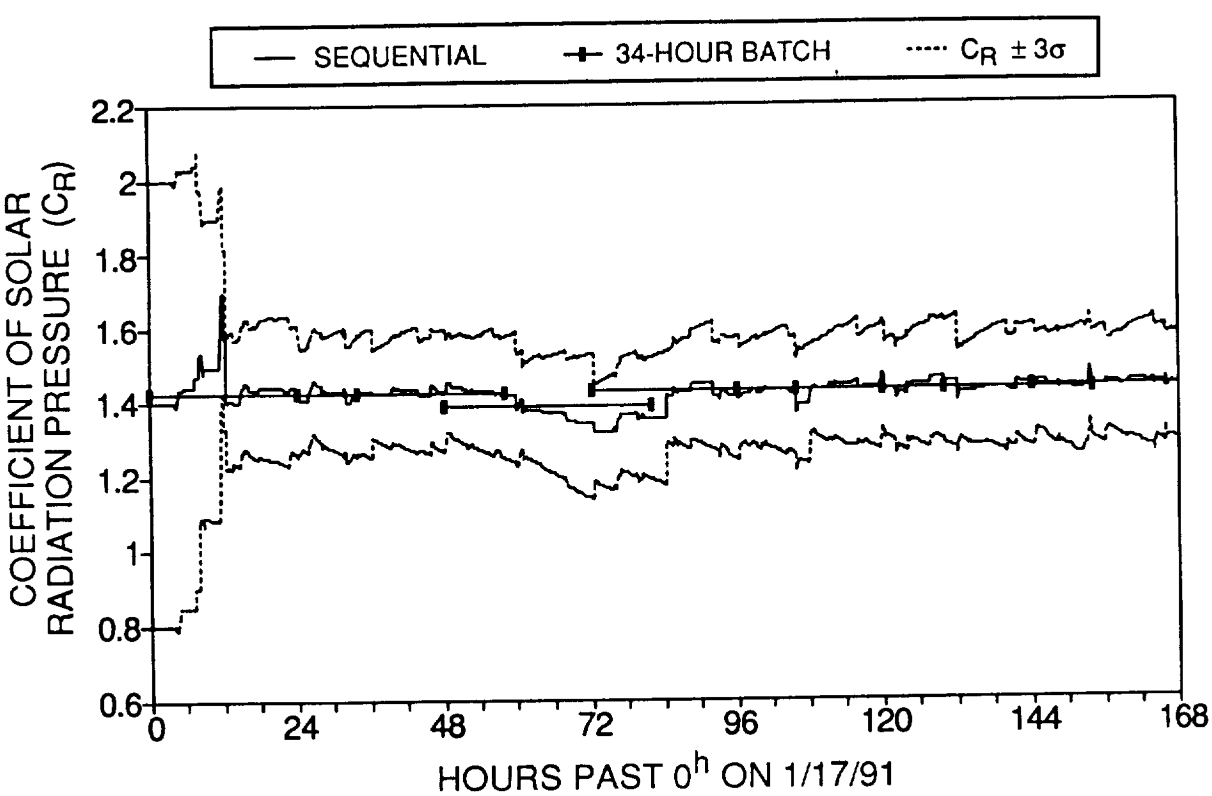


**Figure 7. Coefficient of Solar Radiation Pressure ($C_R$) for TDRS-West**

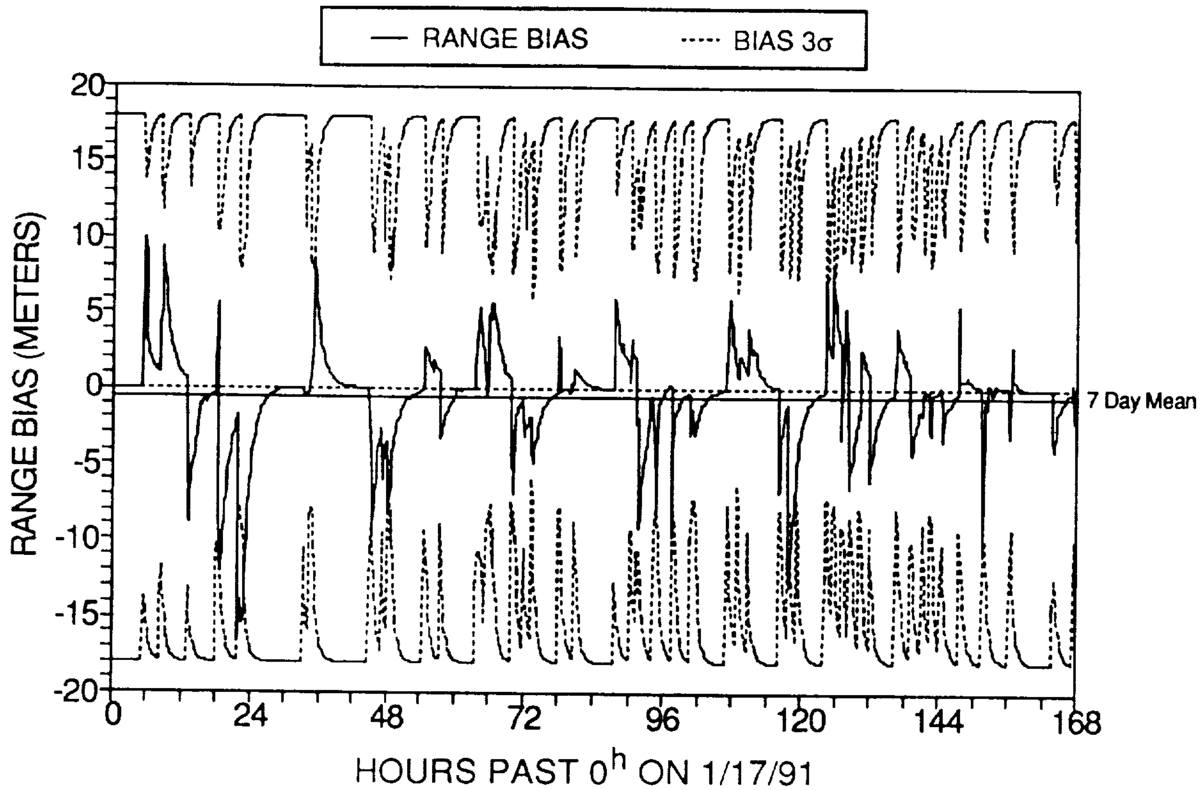


**Figure 8. Range Bias Estimates for Landsat-4 via TDRS-East**

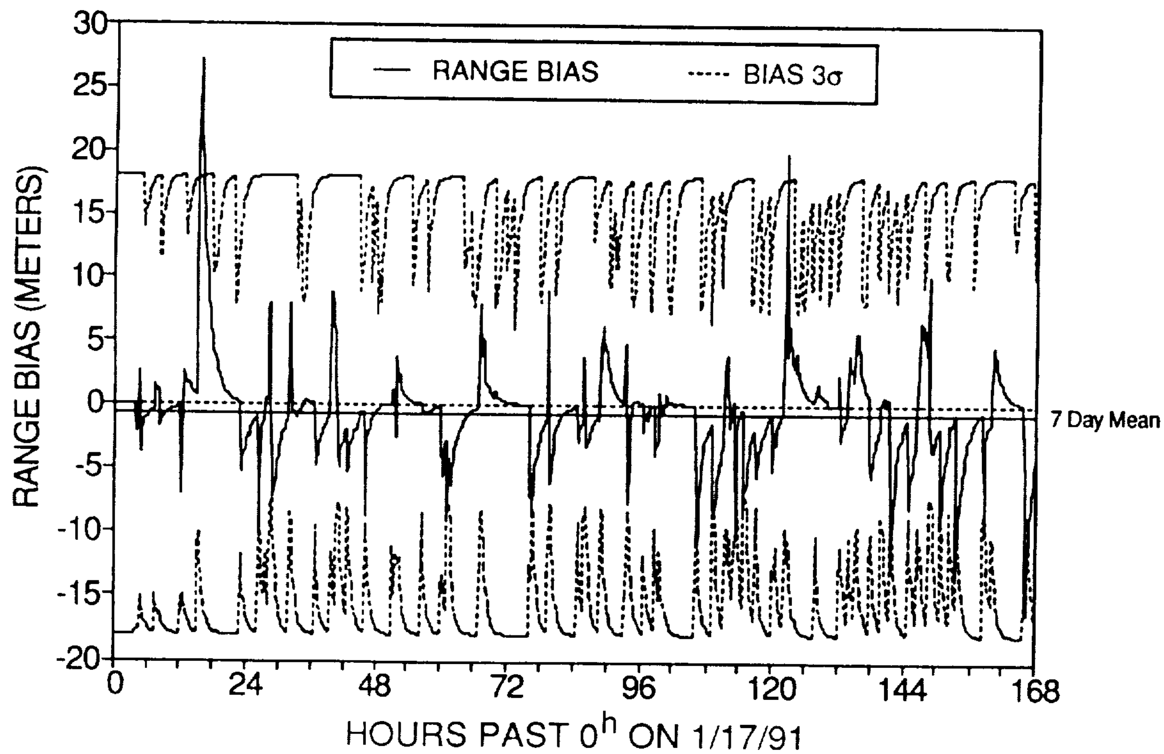


**Figure 9. Range Bias Estimates for Landsat-4 via TDRS-West**

## 3.3 Comparison of Batch and Sequential Estimation Results

Comparisons of the estimated Landsat-4 orbits between GTDS solutions and RTOD/E forward-filtered solutions are presented in Figures 10 and 11. Figure 10 shows the differences during the first day of the filtered solution. Since the filter had not reached steady state during the early phases of this period, the position difference was as large as about 300 meters. However, this difference is not larger than the corresponding state error covariance values of the filter, an indicator of the internal consistency of the filtered solution. After the filter had reached steady state, the differences between the GTDS and RTOD/E solutions were much smaller than on the first day. Therefore, these results are plotted in Figure 11 with a different vertical scale, along with the filter 3σ uncertainty; the position differences (root-sum-square (RSS) of the radial, along-track, and cross-track components) shown in this figure are mostly less than 40 meters. The maximum difference did not increase or decrease toward the end of the 7-day comparison period. Figure 12 shows the position differences on the seventh day, along with the tracking timeline for Landsat-4 and the estimated uncertainty in consistency (3σ covariance function) obtained from RTOD/E.

A few important features shown in Figure 12 are of note. Every time a tracking pass is processed by the sequential filter, the filter's confidence level in the solution increases; conversely, the error covariance function decreases. During the tracking passes, the 3σ position uncertainty estimated by the filter is between 10 and 25 meters. If continuous tracking were available, theoretically it would have been possible to sustain a near-uniform steady-state 3σ uncertainty. Conversely, with a relatively normal gap of about 3 hours in tracking, the 3σ position uncertainty rises to as high as 45 meters. This study was performed during the period of dense Landsat-4 tracking (Figure 1). During normal operation, the tracking is performed with interpass gaps of 4 hours or longer.

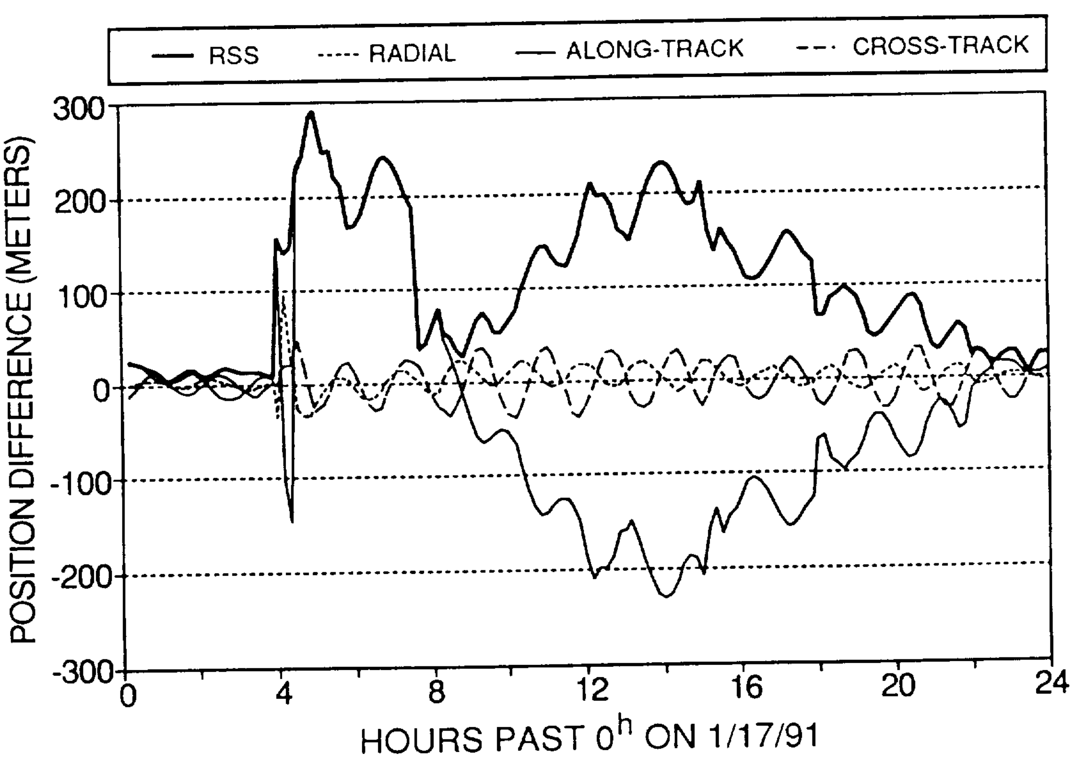


**Figure 10. GTDS and RTOD/E Ephemeris Estimate Differences for Landsat-4 (Day 1)**

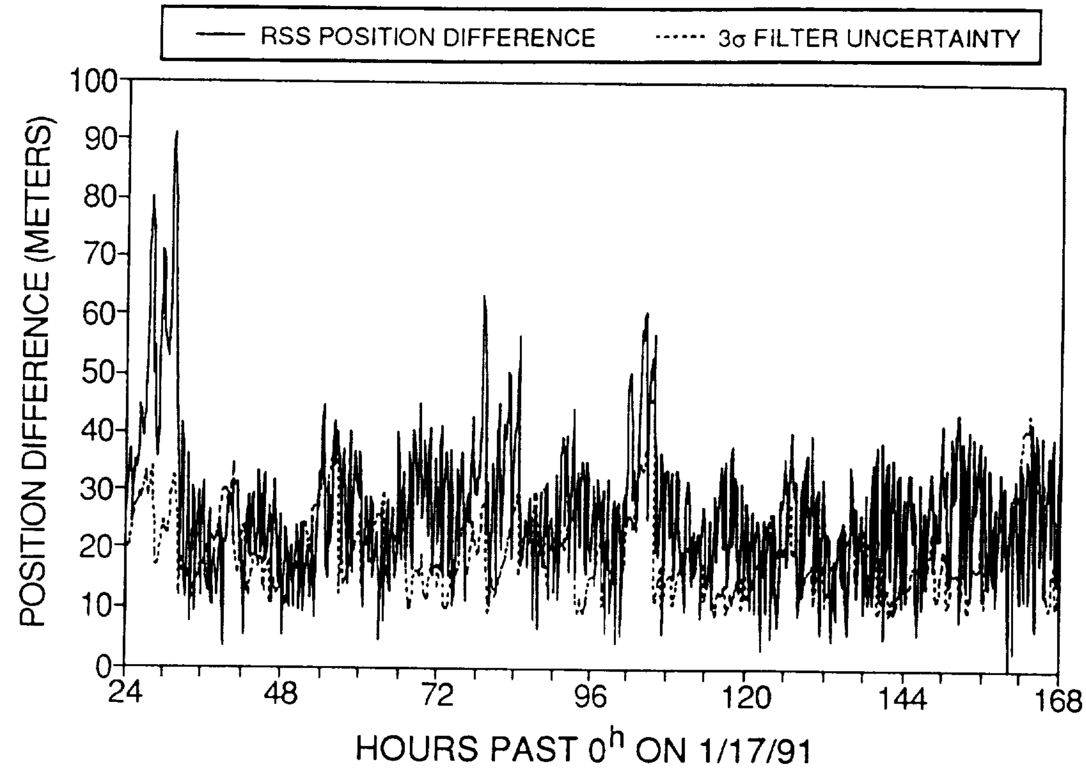


**Figure 11. GTDS and RTOD/E Ephemeris Estimate Differences for Landsat-4 and 3σ Filter Uncertainty (Days 2 Through 7)**

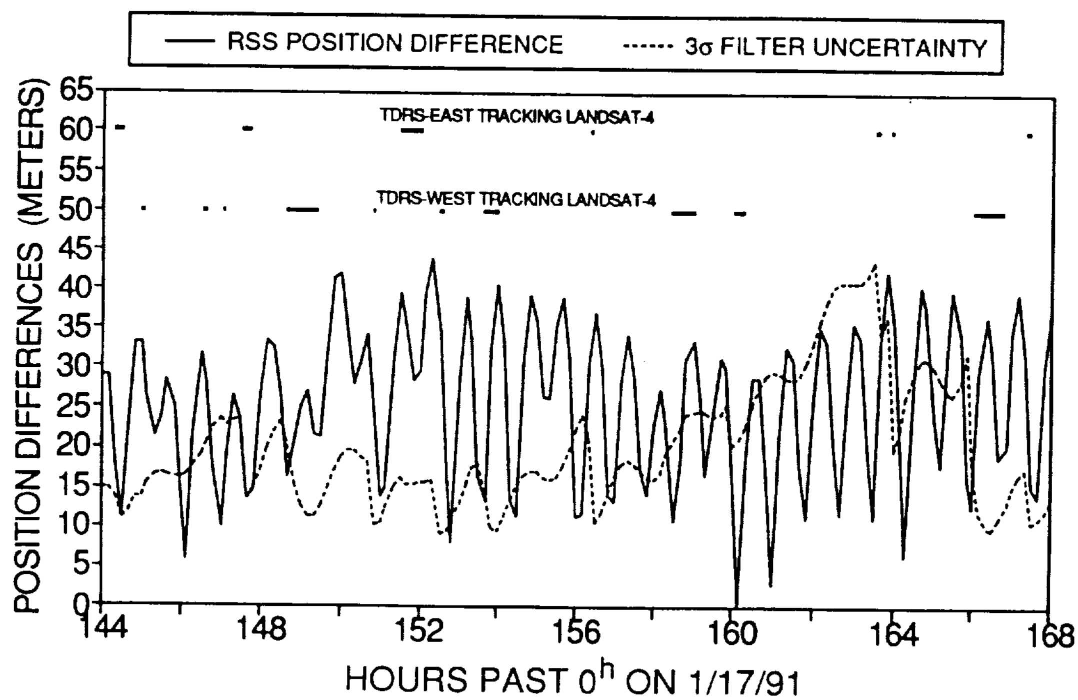


**Figure 12. GTDS and RTOD/E Ephemeris Estimate Differences for Landsat-4 and 3σ Filter Uncertainty and Tracking Schedule (Day 7)**

The position differences between the GTDS and RTOD/E solutions in Figure 12 exceed the estimated uncertainty of the RTOD/E solution more than half the time. The maximum difference of about 40 meters is not consistent within the cumulative consistencies of the batch and sequential solutions. An analysis to identify the source of this discrepancy and resolve it is in progress.

A significant part of the difference between the batch and sequential orbit determination results in Figure 12 can be attributed to the differences in the force and measurement models used for GTDS and RTOD/E. Quantitative estimates for some of these model difference effects are available from previous studies using GTDS. It was reported in Reference 4 that the maximum position difference for 34-hour definitive ERBS solutions using the Goddard Earth Model-T2 (GEM-T2) (50 x 50) and GEM-10B (36 x 36) geopotential models can be as high as 30.1 ± 5.2 meters. RTOD/E uses the GEM-10B geopotential model with order and degree 30. Due to the inclusion of a process noise model for geopotential errors in RTOD/E and its absence in GTDS, the impact of differences in the geopotential models used would be different in the two systems. The maximum position differences observed in the definitive ERBS orbits due to the presence and absence of ionospheric refraction correction in the measurement model for the spacecraft-to-spacecraft leg can be 2.6 ± 0.9 meters (Reference 4). The maximum position difference due to solid Earth tide effects on ERBS were measured at 7.0 ± 3.2 meters. A detailed analysis of the influence of polar motion and solid Earth tides on ERBS orbits is given in Reference 9. ERBS is at an altitude of about 600 kilometers, whereas Landsat-4 is at an altitude of about 715 kilometers. Therefore, all the stated effects above for ERBS should be somewhat diminished in magnitude for Landsat-4. However, Landsat-4 has a polar orbit, which has a significant adverse effect on the tracking geometry.

Another source of the difference between the GTDS and RTOD/E estimated ephemerides is due to the fundamental difference in the way the estimated parameters are obtained in the batch least-squares and sequential estimation techniques. In the batch least-squares method, a single set of parameter values is estimated over an entire arc. In the sequential estimation process, the set of estimated parameter values is updated at each measurement time. The time variations in selected estimated parameters are shown in Figures 5 through 9.

Based on the magnitude of these differences and the differences in the estimation techniques, the maximum position difference of about 40 meters between the GTDS and RTOD/E results is not unusual.

## 3.4 Remarks

The results presented in this paper were obtained using dense-tracking TDRSS measurements for Landsat-4. A previous study of ERBS with single-relay (TDRS-East only) TDRSS tracking has shown that to achieve the highest precision orbit determination using the batch least-square method, the tracking coverage should not fall below 10 minutes every two orbits (Reference 10). The tracking coverage used in the present study, as shown in Figure 1, was well above this criterion. The impact of tracking coverage on accuracy using sequential estimation techniques will be pursued in future studies. In theory, the filter is expected to be more sensitive to large gaps in tracking data than the batch least-squares method; conversely, it would benefit more from more continuous tracking than would the batch least-squares method.

A covariance analysis to further understand the orbit determination results and to identify the major contributing factors to the errors in the estimated orbits is in progress.

# 4. CONCLUSIONS

This study presented an analysis of TDRSS user orbit determination using a batch least-squares method and a sequential estimation method. Independent assessments were performed of the orbit determination consistency within each method, and the estimated orbits obtained by the two methods were also compared. This assessment is applicable to the dense-tracking measurement scenario for tracking Landsat-4.

In the batch least-squares method analysis, the orbit determination consistency for Landsat-4, which was heavily tracked by TDRSS during January 1991, was found to be about 4 meters in the RMS overlap comparisons and about 6 meters in the maximum position differences in overlap comparisons. In the sequential method analysis, the consistency was found to be about 10 to 30 meters in the $3\sigma$ state error covariance function; and, as a measure of consistency, the first residual of each pass was within the $3\sigma$ bound in the residual space.

After the filter had reached steady state, the differences between the definitive batch least-squares ephemerides and the forward-filtered sequentially estimated ephemerides were no larger than 40 meters. Further studies are in progress to investigate the relative qualities of the two methods within this difference.

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