

# Orbit Determination Support of the Ocean Topography Experiment (TOPEX)/Poseidon Operational Orbit\*

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## ABSTRACT

The Ocean Topography Experiment (TOPEX/Poseidon) mission is designed to determine the topography of the Earth's sea surface over a 3-year period, beginning shortly after launch in July 1992. TOPEX/Poseidon is a joint venture between the United States National Aeronautics and Space Administration (NASA) and the French Centre Nationale d'Etudes Spatiales. The Jet Propulsion Laboratory is NASA's TOPEX/Poseidon project center. The Tracking and Data Relay Satellite System (TDRSS) will nominally be used to support the day-to-day orbit determination aspects of the mission. Due to its extensive experience with TDRSS tracking data, the NASA Goddard Space Flight Center (GSFC) Flight Dynamics Facility (FDF) will receive and process TDRSS observational data.

To fulfill the scientific goals of the mission, it is necessary to achieve and maintain a very precise orbit. The most stringent accuracy requirements are associated with planning and evaluating orbit maneuvers, which will place the spacecraft in its mission orbit and maintain the required groundtrack.

To determine if the FDF can meet the TOPEX/Poseidon maneuver accuracy requirements, covariance analysis was undertaken with the Orbit Determination Error Analysis System (ODEAS). The covariance analysis addressed many aspects of TOPEX/Poseidon orbit determination, including arc length, force models, and other processing options. The most recent analysis has focused on determining the size of the geopotential field necessary to meet the maneuver support requirements. Analysis was undertaken with the full 50x50 Goddard Earth Model (GEM) T3 field as well as smaller representations of this model.

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# 1. INTRODUCTION

## 1.1 Background

Error analysis has been used in a long succession of investigations to evaluate the orbit determination capabilities of the TOPEX/Poseidon mission. Reference 1 gives a mission overview and summarizes the day-to-day operational orbit determination requirements and nominal capabilities. The requirements were provided by the Jet Propulsion Laboratory (JPL), and the capabilities were obtained from previous error analyses presented in References 2 and 3.

The TOPEX mission has been divided into two general phases: the assessment phase, where maneuvers will be used to navigate the spacecraft from the injection orbit to the operational orbit, and the observational phase, where the majority of scientific data will be obtained. Maneuvers will also be required during the observational phase to maintain the stringent groundtrack requirements necessary for the scientific goals.

The TOPEX mission scientific goals require orbit determination accuracies that have spurred the development of new mathematical models for representing the motion of near-Earth satellites. One such improvement is a special 50x50 geopotential field called GEM T3. Approximately 6 months after launch of the satellite, TOPEX tracking data will be added to the observations used to develop GEM T3 to create a gravity field tailored specifically for TOPEX.

## 1.2 Goal of Study

The use of a full 50x50 geopotential field in conjunction with other improved models for representing near-Earth satellite motion is expected to produce orbit ephemerides that will support the scientific goals of the TOPEX mission. However, for operational day-to-day orbit solutions, use of a full 50x50 geopotential field places a significant burden on computer resources. Consequently, the specific goal of this study is to determine if day-to-day operational orbit determination requirements can be achieved with smaller representations of the GEM T3 field.

This investigation uses the Orbit Determination Error Analysis System (ODEAS) to estimate the effect of reducing the size of GEM T3 on day-to-day operational solutions. The most stringent requirements are for support of maneuver evaluations in the observational phase. Consequently, this is the specific area addressed in this study.

## 1.3 Maneuver Support Requirements

The orbit determination requirements specified by JPL for support of maneuver evaluation during the observational phase are given in Table 1. The requirement that is the most difficult to achieve is the 0.2 millimeter/second (mm/sec) change in the alongtrack component of velocity.

**Table 1. TOPEX Orbit Determination Requirements for Evaluation of Changes in Osculating Parameters Due to a Maneuver**

| PARAMETER                                    | MAXIMUM 3- $\sigma$ ERROR |
|--|---------------------------|
| CHANGE IN RADIAL COMPONENT OF VELOCITY       | 10.0 MM/SEC               |
| CHANGE IN CROSSTRACK COMPONENT OF VELOCITY   | 10.0 MM/SEC               |
| CHANGE IN ALONGTRACK COMPONENT OF VELOCITY   | 0.2 MM/SEC                |
| CHANGE IN OSCULATING VALUE OF SEMIMAJOR AXIS | 0.2 M*                    |
| CHANGE IN OSCULATING VALUE OF INCLINATION    | $5.0 \times 10^{-4}$ DEG* |

\*M = METER(S)  
DEG = DEGREE(S)

## 2. ANALYSIS

### 2.1 Geopotential Error Models

This investigation is concerned with the effect of different-size representations of the GEM T3 gravity field on maneuver evaluation capabilities. The GEM T3 field is a full 50x50 set of coefficients developed by S. Klosko (Reference 4).

Covariance analysis estimates the effect of uncertainties in measurement and force model parameters on the solved-for quantities. An error model for the GEM T3 geopotential field has been developed in Reference 5 by creating a 1-sigma standard deviation "clone" geopotential model by a purely mathematical method. The difference between the original GEM T3 and GEM T3<sub>clone</sub> represents a 1-sigma error model for GEM T3. A 3-sigma error model is constructed by simply applying a multiplicative factor of 3 to the 1-sigma error model.

Two additional geopotential models have been generated from the same observations used in the development of the 50x50 GEM T3 model. The additional models solve only for geopotential coefficients up to 20x20 and 30x30. These latter fields are called "folded-over" models. Clone representations for these two additional reduced-size models have not been undertaken because the process requires extensive computer resources on very large systems. Consequently, error models are not currently available for these two folded-over representations, and without error models, these reduced-size fields cannot be used in conjunction with covariance analysis.

Two possibilities exist for developing an error model for the folded-over fields without the use of clone representations. The first is based on analysis presented in Reference 6 and reproduced in Table 2, which summarizes the quality of orbit determination fits to the Starlette, Ajisai, and Lageos satellites with five different geopotential fields based on GEM T3 (the full 50x50 field, folded-over 20x20 and 30x30 fields, and GEM T3 truncated at 20x20 and 30x30). The TOPEX altitude will lie between that of Starlette and Ajisai. Table 2 suggests that the root mean square (RMS) of fit to the observations for a spacecraft between these two altitudes will be best for the full 50x50 field, with a poorer fit for the folded-over fields and the worst fit using the truncated fields.

Error analysis is usually concerned with presenting "worst-case" scenarios. Table 2 suggests that truncated models produce the worst results, so that if error models could be developed for the truncated 20x20 and 30x30 fields, those error models would likely produce error estimates that are larger than those obtained from the folded-over fields. While this procedure may produce excessively pessimistic results, the results would at least indicate a worst-case scenario. Error models for truncated fields *can* be constructed without using clone representations by differencing the original and clone models up to, say, 20x20, and adding to this set of error coefficients 100 percent of the original GEM T3 model from 21x21 up to 50x50.

**Table 2. Fit to Residuals of Different Satellites as a Function of Gravity Field**

| SATELLITE | SATELLITE SEMIMAJOR AXIS (KM*) | RMS OF FIT (CM*)  |                          |                          |                                |                                |
|-----------|--------------------------------|-------------------|--------------------------|--------------------------|--------------------------------|--------------------------------|
|           |                                | FULL 50x50 GEM T3 | FOLDED-OVER 30x30 GEM T3 | FOLDED-OVER 20x20 GEM T3 | FULL GEM T3 TRUNCATED AT 30x30 | FULL GEM T3 TRUNCATED AT 20x20 |
| STARLETTE | 7371                           | 11.4              | 90.9                     | 166.9                    | 141.1                          | 573.3                          |
| AJISAI    | 7820                           | 8.7               | 10.7                     | 22.4                     | 10.3                           | 38.3                           |
| LAGEOS    | 12273                          | 7.5               | 7.5                      | 7.5                      | 7.5                            | 7.5                            |

\*CM = CENTIMETER(S)  
KM = KILOMETER(S)

The second alternative for producing an error model for the folded-over fields without the benefit of clone representations is to assume that the full 50x50 GEM T3 field is absolute truth, and simply difference the 50x50 and 20x20 fields. Two problems arise with this representation. First, the error in GEM T3 itself is ignored. Second, the error model would include 100 percent of the 50x50 coefficients above degree and order 20x20, and yet, to the extent possible, the dynamics of these high-order terms might already be included in the folded-over field. Some of this apparent “excessive error” might be removed when differencing the 50x50 GEM T3 and folded-over 20x20 fields due to inherent correlations, but without evidence to this effect, it appears that the best alternative is to use error models based upon truncated fields and accept a worst-case scenario. Table 3 presents the different geopotential fields and associated error models used in this investigation.

**Table 3. Geopotential Fields and Associated Error Models**

| GEOPOTENTIAL MODEL          | 3- $\sigma$ ERROR MODEL  |
|-----------------------------|--|
| 50 x 50 GEM T3              | 3 x (GEM T3 – GEM T3 <sub>CLONE</sub> )  |
| GEM T3 TRUNCATED AT 30 x 30 | 3 x (GEM T3 – GEM T3 <sub>CLONE</sub> ) UP TO 30 x 30<br>PLUS 100% OF (GEM T3) 31 x 31 UP TO 50 x 50 |
| GEM T3 TRUNCATED AT 20 x 20 | 3 x (GEM T3 – GEM T3 <sub>CLONE</sub> ) UP TO 20 x 20<br>PLUS 100% OF (GEM T3) 21 x 21 UP TO 50 x 50 |

## 2.2 Input Parameters

Epoch conditions for TOPEX and TDRS-East (E) and -West (W) are given in Table 4. Table 5 presents the station locations and Table 6 defines the error sources and associated 3-sigma uncertainties.

Because this investigation is primarily concerned with the effect of geopotential size on maneuver evaluation capabilities, there is no need to propagate errors into the future. Consequently, Table 6 indicates an uncertainty of 2.5 percent for solar flux errors throughout the definitive period, with no errors for prediction periods.

The tracking schedule for determining the orbits of TDRS-E and -W consists of 5 minutes of range and Doppler observations every other hour with a sampling frequency of 60 seconds. The nominal tracking scenario for the observational phase of TOPEX by the two TDRS spacecraft was specified by JPL. It consists of a 7-day arc with 40 minutes per revolution of one-way noncoherent Doppler and a single 20-minute pass per day of two-way coherent range and Doppler. The two-way pass replaces the one-way pass for that particular revolution. For three revolutions before and after a maneuver, the 40-minute one-way Doppler pass is replaced by a 40-minute pass of two-way range and Doppler. TDRS tracking of TOPEX incorporates a 10-second sampling frequency.

## 2.3 Evaluation of Capabilities for Computing the Changes in Osculating Parameters as a Function of Geopotential Field Size

An outline of the procedure used to estimate the error in the change of a parameter due to an instantaneous maneuver is given in Reference 3. In general, the process involves computing and saving the error budget at the time of the maneuver based on the premaneuver solution. A corresponding error budget is obtained at the maneuver time from the postmaneuver solution. If all the error parameters are assumed to be perfectly correlated, the error in the change of a parameter due to an instantaneous maneuver is obtained by differencing the two error budgets, parameter by parameter and component by component. Uncertainties in station position and  $C_D$  can certainly be assumed to be correlated for the premaneuver and postmaneuver solutions.

**Table 4. Epoch Conditions**

| PARAMETER                              | TDRS-E   | TDRS-W   | TOPEX  |
|--|--|--|--|
| EPOCH                                  | 92/06/08 22 <sup>h</sup> 00 <sup>m</sup> 00 <sup>s</sup> | 92/06/08 22 <sup>h</sup> 00 <sup>m</sup> 00 <sup>s</sup> | 92/06/08 22 <sup>h</sup> 00 <sup>m</sup> 00 <sup>s</sup> |
| SEMIMAJOR AXIS (KM)                    | 42168.29724487   | 42163.80284769   | 7706.82281771  |
| ECCENTRICITY                           | 0.00019745860  | 0.00024304387  | 0.0010889678   |
| INCLINATION (DEG)                      | 4.50609744   | 3.72087923   | 66.04679405  |
| ASCENDING NODE (DEG)                   | 70.15012793  | 162.83194621   | 142.72939563   |
| ARGUMENT OF PERIGEE (DEG)              | 337.82089362   | 91.11231707  | 6.09376125   |
| MEAN ANOMALY (DEG)                     | 138.32697568   | 162.69648054   | 358.38472966   |
| E. LONG. (DEG)                         | 318.85566801   | 189.08900083   |  |
| AREA/MASS (M <sup>2</sup> /KG*)        | 0.02   | 0.02   | 0.0064865  |
| C <sub>R</sub>                         | 1.5  | 1.5  | 1.3  |
| C <sub>D</sub>                         | N/A  | N/A  | 2.3  |
| SOLAR FLUX (WATTS/M <sup>2</sup> /HZ*) |  |  | 225.0  |

\*KG = KILOGRAM(S)  
 HZ = HERTZ

**Table 5. Station Locations**

| STATION            | ACRONYM | E. LONGITUDE (DEG, MIN,* SEC) | LATITUDE (DEG, MIN, SEC) | HEIGHT (M) |
|--------------------|---------|-------------------------------|--------------------------|------------|
| WHITE SANDS        | WHSK    | 253 23 29.21                  | 32 30 03.56              | 1430       |
| WHITE SANDS BRTS   | WHSJ    | 253 23 16.92                  | 32 30 22.53              | 1413       |
| ASCENSION BRTS     | ASCJ    | 345 36 33.24                  | -07 55 04.47             | 42         |
| ALICE SPRINGS BRTS | ALSJ    | 133 52 57.36                  | -23 45 31.65             | 547        |

\*MIN = MINUTES

**Table 6. Error Sources and Associated 3-Sigma Uncertainties**

| PARAMETER   | ACRONYM IN LISTING   | 3- $\sigma$ UNCERTAINTY  |  |  |
|---|--|--|--|--|
| GRAVITY FIELD   | GEOERROR   | SEE TABLE 3  |  |  |
| C <sub>D</sub>  | USERDRAG   | 30% IF NOT SOLVED FOR  |  |  |
| SOLAR FLUX  | SOLFLUX  | MEAN SOLAR FLUX = $225 \times 10^{-22}$ WATTS/M <sup>2</sup> /HZ.<br>DAILY ERROR = 2.5%. |  |  |
| C <sub>R</sub><br>TOPEX<br>TDRS-E<br>TDRS-W   | SOLRAD 1<br>SOLRAD 2<br>SOLRAD 3   | 30%  |  |  |
| STATION POSITIONS<br>ASCENSION TRANSPONDER<br>ALICE SPRINGS TRANSPONDER<br>WHITE SANDS TRANSPONDER<br>WHITE SANDS GROUND<br><br>LOCAL X<br>LOCAL Y<br>LOCAL Z | NAME = ACNJ<br>NAME = ALSJ<br>NAME = WHSJ<br>NAME = WHSK<br><br>XLT-NAME<br>YLT-NAME<br>ZLT-NAME |  |  |  |
| TROPOSPHERE   | TRP-NAME   | 45%  |  |  |
| IONOSPHERE<br>FROM STATIONS<br>FROM TDRS-E<br>FROM TDRS-W   | ION-NAME<br>IONSAT 2<br>IONSAT 2   | 100%   |  |  |
| MEASUREMENTS<br><br>BRTS RANGE (M)<br>TDRSS RANGE (M)<br>TDRSS TWO-WAY R/R* (MM/SEC)<br>TDRSS ONE-WAY R/R (MM/SEC)  | MEASBI 3,4,5,6<br>MEASBI 1,2   | NOISE<br>1.5<br>1.5<br>2.82<br>6.29  | WEIGHT $\sigma$<br>$3.0 \times 10^{-4}$<br>90.0<br>100.0<br>6.29 | BIAS<br>7.0<br>7.0<br>0.0<br>SOLVE FOR<br>CLOCK DRIFT<br>AND CLOCK<br>ACCELERATION |

\*RANGE/RATE

Other parameters, such as the uncertainties in the ionospheric and tropospheric refraction, are not necessarily correlated; as a result, errors in the change of the solved-for parameters due to these latter uncertainties are not differenced, but the RSS is computed. The total error in the change of a specific component is obtained by forming the RSS of the individual error sources of the differenced/RSS'ed error budget.

The premaneuver arc was selected to be 7 days, because this will be the typical definitive period for the observational phase. JPL requested deliveries of the changes in parameters at 8 and 24 hours after the maneuver. GSFC personnel indicated that it would take approximately 1 hour to process the data and send the results to JPL. Consequently, postmaneuver data spans of 7 and 23 hours were selected as nominal postmaneuver data arcs. However, the requirements apply only to the 23-hour postmaneuver solution.

Simulations were constructed using the epoch conditions, tracking scenarios, and error models noted in Section 2.2 and Table 3. The maneuver time was selected as exactly 7 days past the epoch time noted in Table 4.

If a maneuver is assumed to be instantaneous, the maneuver will change only the velocity (not the position at this instant of time). It is possible to simulate this scenario by applying the appropriate weight sigmas to the position components of the postmaneuver a priori covariance matrix. This process ensures the same position for the pre- and postmaneuver solutions at the time of the maneuver, but the operational version of ODEAS

does not produce the necessary output to allow the same *error* in the pre- and postmaneuver solution. Consequently, it is not currently possible to properly simulate the process of constrained solutions. The following analysis assumes unconstrained postmaneuver solutions, but the subject of constrained solutions will be addressed in Section 2.4.

Tables 7, 8, and 9 indicate the maneuver evaluation capabilities as a function of postmaneuver data span and gravity model size for semimajor axis, inclination, and crosstrack velocity. The capabilities for evaluating radial and alongtrack velocity components will be discussed later.

Consider first Table 7, which contains the maneuver evaluation capabilities for the semimajor axis. Separate columns are included for the premaneuver and postmaneuver solutions as well as the error in the change of the parameter. The results indicate that for all postmaneuver solutions, the error in the semimajor axis due to the uncertainty in the gravity model is relatively small when compared to the RSS of all other error sources. This is not the case for the premaneuver solutions, where the uncertainties in the gravity field dominate the RSS of all errors. As would be expected, 23-hour postmaneuver data spans produce smaller errors in the change of the semimajor axis than 7-hour data spans. The larger gravity field representations also produce smaller errors in the change of the semimajor axis, but there is relatively little difference between them. The dominant contributor to the error in the change of the semimajor axis is a result of the uncertainty in the geopotential field for the 20x20 gravity model simulations. For the 30x30 and 50x50 gravity fields, the dominant errors are due to the uncertainties in the tropospheric refraction and measurement biases.

**Table 7. TOPEX Observational Phase Maneuver Evaluation Capabilities for the Semimajor Axis With No Constraints on the Postmaneuver Position**

| PREMANEUVER DATA SPAN = 7 DAYS<br>REQUIREMENT ON CHANGE OF SEMIMAJOR AXIS = 0.2 METERS |               |                             |                    |                   |                    |   |
|--|---------------|-----------------------------|--------------------|-------------------|--------------------|---|
| POST-MANEUVER DATA SPAN (HR*)  | GRAVITY MODEL | ERROR IN SEMIMAJOR AXIS (M) |                    |                   |                    | ERROR IN CHANGE OF SEMIMAJOR AXIS DUE TO ALL ERRORS |
|  |               | POSTMANEUVER                |                    | PREMANEUVER       |                    |   |
|  |               | RSS OF ALL ERRORS           | ERROR FROM GRAVITY | RSS OF ALL ERRORS | ERROR FROM GRAVITY |   |
| 23   | 50 × 50       | 0.1969                      | 0.042              | 0.0855            | 0.084              | 0.1968  |
| 7  | 50 × 50       | 0.2716                      | 0.084              | 0.0855            | 0.084              | 0.2592  |
| 23   | 30 × 30       | 0.1961                      | 0.038              | 0.1112            | 0.110              | 0.2055  |
| 7  | 30 × 30       | 0.2722                      | 0.086              | 0.1112            | 0.110              | 0.2603  |
| 23   | 20 × 20       | 0.2087                      | 0.081              | 0.2962            | 0.296              | 0.2885  |
| 7  | 20 × 20       | 0.2723                      | 0.086              | 0.2962            | 0.296              | 0.3332  |

\*HR = HOURS

In summary, no unusual or unexpected results appear in Table 7, and it appears that a 30x30-size gravity field will meet the requirement of 0.2 meters for the 23-hour solutions. However, it must be remembered that errors in the premaneuver and postmaneuver solutions change as a function of time, and the results noted in Table 7 are valid for only a single maneuver epoch. Different maneuver evaluation capabilities may be obtained for different epochs. This concern will be addressed later in Section 2.5. In addition, the geopotential error model used to produce the results in Table 7 represents a truncated geopotential field, whereas operational solutions will probably be based on folded-over fields, which should be superior to the truncated results (see Table 2).

Table 8 presents the corresponding results for the inclination. No unusual results appear, with little or no sensitivity to the gravity model size. The requirement of  $1 \times 10^{-4}$  degrees can be met with a 20x20 geopotential representation. The dominant error source in the change of the inclination is the ionospheric refraction from the ground stations.

**Table 8. TOPEX Observational Phase Maneuver Evaluation Capabilities for the Inclination With No Constraints on the Postmaneuver Position**

| PREMANEUVER DATA SPAN = 7 DAYS<br>REQUIREMENT ON CHANGE IN INCLINATION = $1 \times 10^{-4}$ DEGREES |                |  |                    |                   |                    |  |
|---|----------------|--|--------------------|-------------------|--------------------|--|
| POST-MANEUVER DATA SPAN (HR)  | GRAVITY MODEL  | ERROR IN INCLINATION (DEGREES $\times 10^{-4}$ ) |                    |                   |                    |  |
|   |                | POSTMANEUVER                                     |                    | PREMANEUVER       |                    | ERROR IN CHANGE OF INCLINATION DUE TO ALL ERRORS |
|   |                | RSS OF ALL ERRORS                                | ERROR FROM GRAVITY | RSS OF ALL ERRORS | ERROR FROM GRAVITY |  |
| 23  | 50 $\times$ 50 | 0.205  | -.0064             | 0.272             | .0439              | 0.34   |
| 7   | 50 $\times$ 50 | 0.562  | .0097              | 0.272             | .0439              | 0.63   |
| 23  | 30 $\times$ 30 | 0.202  | -.0031             | 0.271             | .0372              | 0.34   |
| 7   | 30 $\times$ 30 | 0.146  | .0117              | 0.271             | .0372              | 0.63   |
| 23  | 20 $\times$ 20 | 0.211  | -.0602             | 0.275             | -.0587             | 0.34   |
| 7   | 20 $\times$ 20 | 0.560  | .0019              | 0.275             | -.0587             | 0.63   |

Table 9 gives the maneuver evaluation capabilities for the crosstrack component of velocity. Once again, the results are not sensitive to the size of the geopotential, and the requirement of 10 mm/sec can be obtained with a 20x20 gravity field. The dominant error source in the change of the crosstrack component of velocity is again the uncertainty in the ionospheric refraction at the ground stations.

The final set of requirements deals with the errors in the change of the in-plane velocity components. To help explain these results, it is beneficial to first examine the errors in the change of the radial *position* (not the radial *velocity*), which are given in Table 10.

The important feature of this table is that the errors in the change of the radial position are about 15 times larger than those for the semimajor axis noted in Table 7. The semimajor axis reflects the orbital period, while errors in the radial position involve not only errors in the semimajor axis, but also the eccentricity and the eccentric (or true) anomaly. The fact that the radial position error is substantially larger than that of the semimajor axis is due to the errors in these two additional parameters, which produce a tendency to point the velocity vector in the wrong direction. This in turn produces errors in the in-plane velocity components.

Table 10 also indicates that the 20x20 gravity model produces smaller errors than the larger gravity fields, which is opposite to intuition. This feature may be a result of certain correlations in the truncated field, and it might not occur if the maneuver epoch were changed. Since the radial errors are smaller for the 20x20 gravity field, the corresponding in-plane velocity errors are likely to be smaller for the 20x20 geopotential representation, as well.

**Table 9. TOPEX Observational Phase Maneuver Evaluation Capabilities for the Crosstrack Velocity With No Constraints on the Postmaneuver Position**

| PREMANEUVER DATA SPAN = 7 DAYS<br>REQUIREMENT ON CHANGE IN CROSTRACK VELOCITY=10 MM/SEC |                  |                                      |                       |                      |                       |  |
|---|------------------|--------------------------------------|-----------------------|----------------------|-----------------------|--|
| POST-MANEUVER<br>DATA SPAN (HR)   | GRAVITY<br>MODEL | ERROR IN CROSTRACK VELOCITY (MM/SEC) |                       |                      |                       | ERROR IN<br>CHANGE OF<br>CROSTRACK<br>VELOCITY<br>DUE TO ALL<br>ERRORS |
|   |                  | POSTMANEUVER                         |                       | PREMANEUVER          |                       |  |
|   |                  | RSS OF ALL<br>ERRORS                 | ERROR FROM<br>GRAVITY | RSS OF ALL<br>ERRORS | ERROR FROM<br>GRAVITY |  |
| 23  | 50 × 50          | 2.5                                  | 0.0                   | 3.6                  | -0.6                  | 4.3  |
| 7   | 50 × 50          | 7.2                                  | -0.1                  | 3.6                  | -0.6                  | 8.1  |
| 23  | 30 × 30          | 2.5                                  | 0.0                   | 3.5                  | -0.5                  | 4.3  |
| 7   | 30 × 30          | 7.2                                  | -0.1                  | 3.5                  | -0.5                  | 8.1  |
| 23  | 20 × 20          | 2.6                                  | 0.7                   | 3.6                  | 0.7                   | 4.3  |
| 7   | 20 × 20          | 7.2                                  | -0.0                  | 3.6                  | 0.7                   | 8.2  |

**Table 10. TOPEX Observational Phase Maneuver Evaluation Capabilities for the Radial Position With No Constraints on the Postmaneuver Position**

| PREMANEUVER DATA SPAN = 7 DAYS<br>NO REQUIREMENTS ON RADIAL POSITION |                  |                              |                       |                      |                       |   |
|--|------------------|------------------------------|-----------------------|----------------------|-----------------------|---|
| POST-MANEUVER<br>DATA SPAN (HR)                                      | GRAVITY<br>MODEL | ERROR IN RADIAL POSITION (M) |                       |                      |                       | ERROR IN<br>CHANGE OF<br>RADIAL<br>POSITION DUE<br>TO ALL<br>ERRORS |
|  |                  | POSTMANEUVER                 |                       | PREMANEUVER          |                       |   |
|  |                  | RSS OF ALL<br>ERRORS         | ERROR FROM<br>GRAVITY | RSS OF ALL<br>ERRORS | ERROR FROM<br>GRAVITY |   |
| 23   | 50 × 50          | 0.93                         | 0.39                  | 2.92                 | -2.80                 | 3.3   |
| 7  | 50 × 50          | 1.67                         | -0.04                 | 2.92                 | -2.80                 | 3.4   |
| 23   | 30 × 30          | 0.89                         | 0.30                  | 3.01                 | -2.89                 | 3.3   |
| 7  | 30 × 30          | 1.67                         | -0.05                 | 3.01                 | -2.89                 | 3.4   |
| 23   | 20 × 20          | 1.10                         | 0.70                  | 1.17                 | -0.81                 | 1.8   |
| 7  | 20 × 20          | 1.67                         | -0.01                 | 1.17                 | -0.81                 | 2.1   |

Table 11 presents the results for the radial component of velocity. The most notable feature of these results is that they do not meet the requirements. However, as conjectured, superior results are estimated when the gravity model is smaller. The dominant error in the change of the radial component of velocity is due to the uncertainty in the geopotential. Smaller errors in the change of the radical component might be obtained with the use of a shorter premaneuver data span.

**Table 11. TOPEX Observational Phase Maneuver Evaluation Capabilities for the Radial Velocity With No Constraints on the Postmaneuver Position**

| PREMANEUVER DATA SPAN = 7 DAYS<br>REQUIREMENT ON CHANGE IN RADIAL VELOCITY = 10 MM/SEC |               |                                   |                    |                   |                    |  |
|--|---------------|-----------------------------------|--------------------|-------------------|--------------------|--|
| POST-MANEUVER DATA SPAN (HR)   | GRAVITY MODEL | ERROR IN RADIAL VELOCITY (MM/SEC) |                    |                   |                    | ERROR IN CHANGE OF RADIAL VELOCITY DUE TO ALL ERRORS |
|  |               | POSTMANEUVER                      |                    | PREMANEUVER       |                    |  |
|  |               | RSS OF ALL ERRORS                 | ERROR FROM GRAVITY | RSS OF ALL ERRORS | ERROR FROM GRAVITY |  |
| 23   | 50 x 50       | 11.9                              | 1.0                | 11.2              | -10.5              | 17.3   |
| 7  | 50 x 50       | 9.3                               | -0.1               | 11.2              | -10.5              | 14.2   |
| 23   | 30 x 30       | 11.9                              | 1.2                | 11.0              | -10.4              | 17.4   |
| 7  | 30 x 30       | 9.3                               | -0.1               | 11.0              | -10.4              | 14.1   |
| 23   | 20 x 20       | 11.9                              | 0.6                | 7.7               | -6.7               | 14.9   |
| 7  | 20 x 20       | 9.3                               | -0.1               | 7.7               | -6.7               | 11.6   |

The last parameter to be addressed is the alongtrack component of velocity. The requirement for this parameter is, by far, the most stringent (0.2 mm/sec).

JPL requested that the error in the alongtrack component of velocity be estimated with the use of the Vis Viva energy equation. The development of the relationship is straightforward. The energy integral,

$$V^2 = GM * (2/r - 1/a)$$

renders

$$2 * V * (\Delta V) = \frac{\partial V^2}{\partial a} (\Delta a) + \frac{\partial V^2}{\partial r} (\Delta r)$$

Taking the appropriate partial derivatives, this becomes

$$\Delta V = \frac{GM}{2 * V * a^2} (\Delta a) - \frac{GM}{V * r^2} (\Delta r)$$

Substituting typical values for the TOPEX orbit (r = 7698.8 km, a = 7706.8 km, and V = 7.2 km/sec) gives

$$\Delta V = 4.66 \times 10^{-4} * (\Delta a) - 9.341 \times 10^{-4} * (\Delta r) \quad \text{km/sec} \quad (1)$$

The terms ( $\Delta a$ ) and ( $\Delta r$ ) are the errors in the change in a and r respectively. When this equation is used in conjunction with unconstrained postmaneuver solutions, both ( $\Delta a$ ) and ( $\Delta r$ ) must be included in the

computation. If, on the other hand, we could properly simulate a constrained postmaneuver solution where there would be no discontinuity in the position (and consequently, no error in the change of position), then only the  $(\Delta a)$  term would be included in the equation. Table 12 presents the errors in the alongtrack component of velocity. Note that a factor of  $10^6$  has been applied to Equation (1) to convert the units from km/sec to mm/sec.

Table 12 indicates three particular features: first, the requirements are not met; second, there is good agreement between the two methods of computing the error in the change of the alongtrack component of velocity; third, the conjecture that the 20x20 gravity field would produce smaller errors also holds true for the alongtrack component of velocity. The uncertainty in the gravity field is the dominant error source.

**Table 12. TOPEX Observational Phase Maneuver Evaluation Capabilities for the Alongtrack Velocity With No Constraints on the Postmaneuver Position**

| PREMANEUVER DATA SPAN = 7 DAYS<br>REQUIREMENT ON CHANGE IN ALONGTRACK VELOCITY = 0.2 MM/SEC |               |  |   |
|---|---------------|--|---|
| POSTMANEUVER DATA SPAN (HR)   | GRAVITY MODEL | ERROR IN CHANGE OF ALONGTRACK VELOCITY FROM ODEAS (MM/SEC) | ERROR IN CHANGE OF VELOCITY FROM VIS VIVA EQUATION (MM/SEC) |
| 23  | 50 x 50       | 3.1  | 3.0   |
| 7   | 50 x 50       | 3.1  | 3.0   |
| 23  | 30 x 30       | 3.1  | 3.0   |
| 7   | 30 x 30       | 3.2  | 3.1   |
| 23  | 20 x 20       | 1.8  | 1.5   |
| 7   | 20 x 20       | 1.9  | 1.8   |

## 2.4 Constrained TOPEX Maneuver Evaluation Capabilities Implied by Analysis of ERBS Data

The preceding results assume an unconstrained postmaneuver solution, while a constrained postmaneuver position is the proper simulation technique for instantaneous maneuvers. Due to limitations in the output capabilities of the ODEAS program, the proper technique cannot be simulated, but previous analysis presented in Reference 7 indicates what can be expected from constrained solutions.

Reference 7 has used the Goddard Trajectory Determination System (GTDS) in conjunction with actual tracking data of the Earth Radiation Budget Satellite (ERBS) to estimate the accuracy of changes in the velocity components. The technique used was to find an interval of time where a maneuver did *not* occur and to break this tracking interval into premaneuver and postmaneuver solutions. Ideally, there should be no discontinuities in the velocity components for the two solutions at the time chosen for the maneuver. The differences in the velocity components of the pre- and postmaneuver solutions at the maneuver time are a measure of GTDS's ability to resolve the change in the velocity. Reference 7 refers to this as the "Null" maneuver evaluation. GTDS solutions were made that constrained and did not constrain the postmaneuver position. A truncated 30x30 GEM T2 (not T3) gravity field was used in the analysis.

In addition to the GTDS solutions, unconstrained ODEAS simulations were constructed using the same tracking data schedule as incorporated in GTDS. Table 13 indicates the error in the change of the alongtrack velocity. Two important results are apparent. First, constrained GTDS solutions produce errors that are

approximately an order of magnitude smaller than unconstrained solutions. Second, the corresponding ODEAS 3-sigma errors are slightly larger than those obtained with GTDS but follow similar trends, giving a certain measure of credibility to the ODEAS results.

The errors in the changes of the alongtrack component of velocity provided by ODEAS in Table 12 (for TOPEX) and Table 13 (for ERBS) are of the same order of magnitude. This implies that constrained postmaneuver solutions for TOPEX should be of the same order of magnitude as obtained for ERBS, as noted in Table 13, and perhaps even smaller, since the ERBS results were obtained with GEM T2. This in turn suggests the TOPEX requirements for the error in the change of the alongtrack component of velocity should be achievable most of the time with a truncated GEM T3 30x30 geopotential field, with additional improvements for a folded-over representation. Extrapolation to a GEM T3 20x20 geopotential field is difficult due to the unusual results noted in Table 12 for the smaller-size gravity field.

**Table 13. Comparison of GTDS Null Velocity Changes and ODEAS 3-Sigma Error Estimates for Changes in the Alongtrack Velocity for ERBS**

| MANEUVER NUMBER | CHANGES IN ALONGTRACK COMPONENT OF VELOCITY (MM/S) |             |  |
|-----------------|--|-------------|--|
|                 | FROM GTDS  |             | FROM ODEAS WITH NO POST-MANEUVER CONSTRAINTS |
|                 | NOT CONSTRAINED                                    | CONSTRAINED |  |
| 1               | 2.50   | 0.13        | 3.38   |
| 2               | 1.27   | 0.54        | 2.10   |
| 3               | 2.65   | 0.21        | 5.84   |

## 2.5 The Effect of Maneuver Epoch

The preceding results are based on a single maneuver epoch. Reference 3 (Table 2-29) indicates that the error in the change of the semimajor axis is sensitive to time. For three selected epochs, the error in the change of the semimajor axis varied from .12 to .60 meters.

The operational scenario for TOPEX allows for maneuvers to be postponed for one revolution if deemed necessary by the project office. Consequently, error analysis was undertaken for a second epoch, which was chosen as one revolution before the one used in the previous set of results.

Table 14 presents the results for both maneuver epochs. In general, the results indicate relatively small variations in the error of the change of parameters. This is not surprising, given the difference in the epochs of exactly one revolution. Larger variations might be seen if the second maneuver epoch were selected at a different point in the orbit.

## 3. CONCLUSIONS

This study has applied covariance analysis to investigate maneuver evaluation capabilities of the TOPEX satellite in the observational phase as a function of gravity model size. Three representations of the GEM T3 geopotential field have been considered: a full 50x50 model and 30x30 and 20x20 truncated models. Truncated fields were incorporated rather than folded-over representations, because error models for folded-over fields are not available. Orbit solutions using actual tracking data have indicated that folded-over fields should produce results superior to those based on truncated fields.

**Table 14. Maneuver Evaluation Capabilities as a Function of Gravity Model Size and Time of Maneuver Using Unconstrained Postmaneuver Solutions**

| PREMANEUVER DATA SPAN = 7 DAYS<br>POSTMANEUVER DATA SPAN = 23 HOURS |                   |             |            |         |
|---|-------------------|-------------|------------|---------|
| PARAMETER   | GRAVITY MODEL     | REQUIREMENT | CAPABILITY |         |
|   |                   |             | EPOCH 1    | EPOCH 2 |
| SEMIMAJOR AXIS (M)  | FULL 50 × 50      | 0.2         | 0.20       | 0.20    |
|   | TRUNCATED 30 × 30 |             | 0.21       | 0.21    |
|   | TRUNCATED 20 × 20 |             | 0.29       | 0.31    |
| INCLINATION (DEG × 10 <sup>4</sup> )                                | FULL 50 × 50      | 1.0         | 0.34       | 0.35    |
|   | TRUNCATED 30 × 30 |             | 0.34       | 0.35    |
|   | TRUNCATED 20 × 20 |             | 0.34       | 0.35    |
| CROSSTRACK VELOCITY (MM/SEC)  | FULL 50 × 50      | 10.0        | 4.3        | 4.3     |
|   | TRUNCATED 30 × 30 |             | 4.3        | 4.3     |
|   | TRUNCATED 20 × 20 |             | 4.3        | 4.3     |
| RADIAL VELOCITY (MM/SEC)  | FULL 50 × 50      | 10.0        | 17.3       | 15.1    |
|   | TRUNCATED 30 × 30 |             | 17.4       | 15.0    |
|   | TRUNCATED 20 × 20 |             | 14.9       | 12.3    |
| ALONGTRACK VELOCITY (MM/SEC)  | FULL 50 × 50      | 0.2         | 3.0        | 3.2     |
|   | TRUNCATED 30 × 30 |             | 3.0        | 3.2     |
|   | TRUNCATED 20 × 20 |             | 1.5        | 1.9     |

The proper methodology for analyzing instantaneous maneuvers is to incorporate constraints on the postmaneuver position components. The current version of the covariance analysis software cannot properly simulate constrained postmaneuver solutions. The covariance analysis results presented here are therefore limited to unconstrained postmaneuver simulations.

GTDS solutions using tracking data of the ERBS satellite have indicated two important features. First, errors in the change of parameters are substantially smaller for constrained postmaneuver solutions than unconstrained simulations, and, second, covariance analysis corresponding to the unconstrained ERBS solutions gives generally good agreement with the unconstrained GTDS simulations. These two features give credence to the error analysis results and suggest that if the requirements can be met with unconstrained simulations, they should also be met with constrained solutions.

Requirements on errors in the change of parameters have been placed on the semimajor axis, inclination, and three spacecraft-centered components of velocity (radial, crosstrack, and alongtrack). In general, the requirements on the inclination and crosstrack component of velocity can be met with any of the three gravity models using unconstrained postmaneuver solutions. The semimajor axis requirement is slightly exceeded for the 20x20 field, but these results assume the use of an unconstrained postmaneuver solution and a truncated field. The use of constrained postmaneuver solutions and folded-over fields should produce smaller errors.

The in-plane velocity component requirements are not met. The error in the change of the radial component of velocity is exceeded by a factor of approximately 2, while the error in the change of the alongtrack component of velocity is exceeded by a factor of 30. The analysis of tracking data using the GTDS program indicates an order-of-magnitude reduction in the error in the change of the alongtrack component of velocity for constrained solutions compared with unconstrained solutions. If this condition prevails for TOPEX, the requirement for the error in the change of the alongtrack component of velocity should be met or only slightly exceeded.

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