An Approach For Finding Long Period Elliptical Orbits For Precursor SEI Missions

N93-24720

154746

p. 14

by

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ABSTRACT

Precursors for Solar System Exploration Initiative (SEI) missions may require long period elliptical orbits about a planet. These orbits will typically have periods on the order of tens to hundreds of days. Some potential uses for these orbits may include the following: studying the effects of galactic cosmic radiation, parking orbits for engineering and operational test of systems, and ferrying orbits between libration points and low altitude orbits.

This report presents an approach that can be used to find these orbits. The approach consists of three major steps. First it uses a restricted three-body targeting algorithm to determine the initial conditions which satisfy certain desired final conditions in a system of two massive primaries. Then the initial conditions are transformed to an inertial coordinate system for use by a special perturbation method. Finally, using the special perturbation method, other perturbations (e.g., sun third body and solar radiation pressure) can be easily incorporated to determine their effects on the nominal trajectory.

An algorithm potentially suitable for on-board guidance will also be discussed. This algorithm uses an analytic method relying on Chebyshev polynomials to compute the desired position and velocity of the satellite as a function of time. Together with navigation updates, this algorithm can be implemented to predict the size and timing for ΔV corrections.

1.0 Introduction

During the summer of 1991 the authors were approached (by NASA-JSC) to assist in a trajectory design problem for the "Life Sat" mission. The objective of Life Sat is to determine the biological impact of deep space radiation on the cells of living animals. Data gathered from this mission will be used to estimate the effects of deep space radiation on human beings. Such effects must be well understood prior to sending humans on the necessarily long transfer trajectories to explore Mars.

A major problem in the experiment is that the data can be corrupted by another type of radiation, found in the Van Allen radiation belt region, relatively near the Earth. The trajectory should therefore be designed such that the spacecraft is near the Earth for relatively small amounts of time compared to the time spent in deep space. The ideal trajectory design requirements that would maximize scientific return are:

1.The spacecraft must remain outside the Van Allen region for 60 days.
2.The spacecraft should enter the Van Allen region only twice - once for departure and once for the return
3.The cost of the mission (i.e. ΔV) must be minimized.

This report describes an approach for finding a deep space geocentric orbit which will satisfy the above stated requirements. A key element to this approach is the use of the Double Lunar Swing-by technique first proposed by Farquhar and Dunham (1981). Using this technique the gravitational force of the moon is a significant perturbation to the solution. However, it was found that the moon is of some benefit to mission performance since it can be used to increase the energy of the outbound leg while decreasing the energy on the inbound leg.

A second goal is to present a guidance algorithm, possibly suitable for on-board computations, which keeps the vehicle on the prescribed trajectory even in the presence of other perturbations (e.g. solar third body effects). This algorithm uses a Chebyshev polynomial approach to analytically estimate the desired state as a function of time. This state is then compared to the navigation state and ΔV corrections are applied to maintain the desired trajectory.

2.0 Restricted Three Body Analysis

This section is provided in two parts. First, a description of a restricted three body targeting algorithm which solves the problem of: Given two position vectors and the flight time between these positions find the initial velocity. This is a two point boundary value problem which in the two-body theory is called Lambert's problem. However, since the strong perturbation of the moon must be accounted for, we started with the equations of motion in the restricted three body problem and then solved for the trajectory between the two specified position vectors. The solution is more difficult than in the two-body case since numerical integration is required. The technique for finding a solution is well known (D'Amario and Edelbaum, 1973; Bond and Fraietta, 1991) and will be used in this report. The second part describes how the restricted three body targeting algorithm is used to determine a double lunar swing-by solution suitable for the Life Sat mission.

2.1 Targeting Algorithm

The differential equations of the restricted three body theory are given in a coordinate system whose origin is at the center of mass of the primaries, m_1 and m_2, and is rotating with the line (i.e., the x-axis) connecting the primaries. The z-axis is normal to the plane of motion of m_1 and m_2, and the y-axis lies in the plane of motion. The x-axis rotates about the z-axis at a angular speed which is the mean motion of the primaries. This system appears in Figure 1.

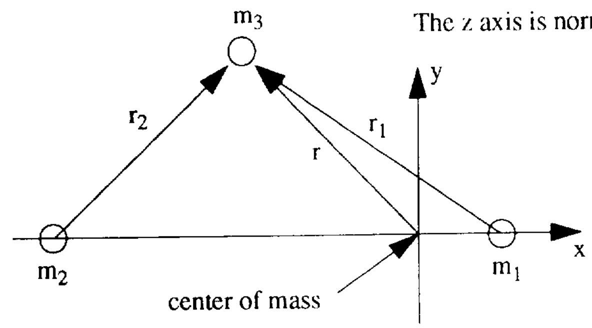

Figure 1 - Rotating (x,y) system

An inertial (X, Y, Z) system remains fixed with respect to the rotating system and is depicted in Figure 2.

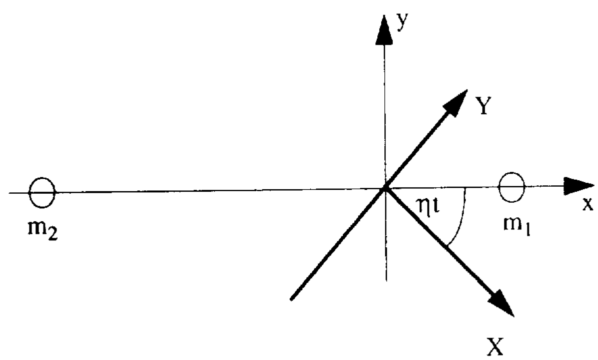

Figure 2 - Rotating (x,y) system in relation to an inertial (X, Y) system

The nonlinear differential equations describing the motion of m_3 (assumed to be massless) in the restricted three body system are given by (Szebehely, 1967)

$$\ddot{x} - 2\dot{y} = \Omega_x = \frac{\partial\Omega}{\partial x}$$

$$\ddot{y} + 2\dot{x} = \Omega_y = \frac{\partial\Omega}{\partial y}$$

$$\ddot{z} = \Omega_z = \frac{\partial\Omega}{\partial z}$$

Where the force function $\Omega(x, y, z)$ is

$$\Omega = \frac{1}{2}(1-\mu)\mu + \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$

and

$$\mu = \frac{m_2}{m_1 + m_2}$$

$$r_1^2 = (x-\mu)^2 + y^2 + z^2$$

$$r_2^2 = (x+1-\mu)^2 + y^2 + z^2$$

An approximate targeting solution, specified by the initial conditions $(\underline{r}_o, \underline{v}_o)$ at t_0, is used as a first guess for solving the restricted three body system of differential equations. The initial velocity is then corrected according to the equation

$$\underline{v}_o^p = \underline{v}_o + \phi_{12}^{-1}(t_f, t_o)\,[\underline{r}_f - \underline{r}(t_f, t_o, \underline{v}_o)]$$

where $\underline{r}_f$, the final or target position is specified. The solution is then recomputed with $\underline{v}_o^p$ instead of $\underline{v}_o$. The matrix ϕ_{12} is a sub-matrix of the transition matrix

$$\Phi(t_o, t_f) = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

which is associated with the differential equations of motion of the restricted three body problem. The matrix ϕ_{12} is found via numerical integration of the differential equations

$$\frac{d\phi_{12}}{dt} = \phi_{22}$$

$$\frac{d\phi_{22}}{dt} = M\phi_{12} + 2J\phi_{22}$$

where M is the matrix of second partials

$$M = \begin{bmatrix} \Omega_{xx} & \Omega_{xy} & \Omega_{xz} \\ \Omega_{xy} & \Omega_{yy} & \Omega_{yz} \\ \Omega_{xz} & \Omega_{yz} & \Omega_{zz} \end{bmatrix}$$

and J is

$$J = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The initial conditions for the transition sub matrices are

$$\phi_{12}(t_o, t_o) = 0 \quad \text{(null matrix)}$$

$$\phi_{22}(t_o, t_o) = I \quad \text{(identity matrix)}$$

This procedure continues iteratively until the computed final position vector $(\underline{r}(t_f, t_o, \underline{v}_o))$ becomes arbitrarily close to the specified final position vector $(\underline{r}_f)$, that is

$$|\underline{r}_f - \underline{r}(t_f, t_o, \underline{v}_o)| \to 0$$

2.2 The Double Lunar Swing-by

A typical trajectory for a double lunar swing-by requires the vehicle to fly by the moon's eastern limb on the outbound leg. The lunar encounter changes the velocity of the vehicle such that a second lunar encounter is achieved after a specified time interval. The second lunar fly by on the inbound leg, occurs on the western limb which acts to decrease the vehicle velocity prior to encountering the earth.

In their paper Farquhar and Dunham (1981) used a closest approach to the moon of approximately 16,000 kilometers. Adopting this value the restricted three body targeting algorithm is employed to find a solution targeting from the east limb of the moon to the symmetrical location on the west limb given a 60 day flight time. The vehicle motion is restricted to the Earth-Moon plane. After convergence the state required at the east limb of the moon that would attain the target conditions on the west limb 60 days later is known.

The next step is to determine the initial conditions required to depart a 400 kilometer altitude circular orbit at the earth (orbit lies in earth-moon plane) such that the state vector at the moon would exactly match the solution found above for the east limb. It is desirable that no additional ΔV corrections be necessary beyond that required for the Trans-Lunar Injection burn. This is essentially a patched solution in the restricted three body system. Again, the restricted three body targeting algorithm was used to determine the solution. However, this problem requires iteration to obtain the solution using two parameters namely, the longitude of the departure orbit and the time of flight to the patch point.

As a result of the symmetry found in the restricted three body system it was not necessary to patch the inbound trajectory from the moon to the earth since it is the mirror of the earth to moon trajectory. The complete double lunar swing-by trajectory in the rotating system is displayed in Figure 3.

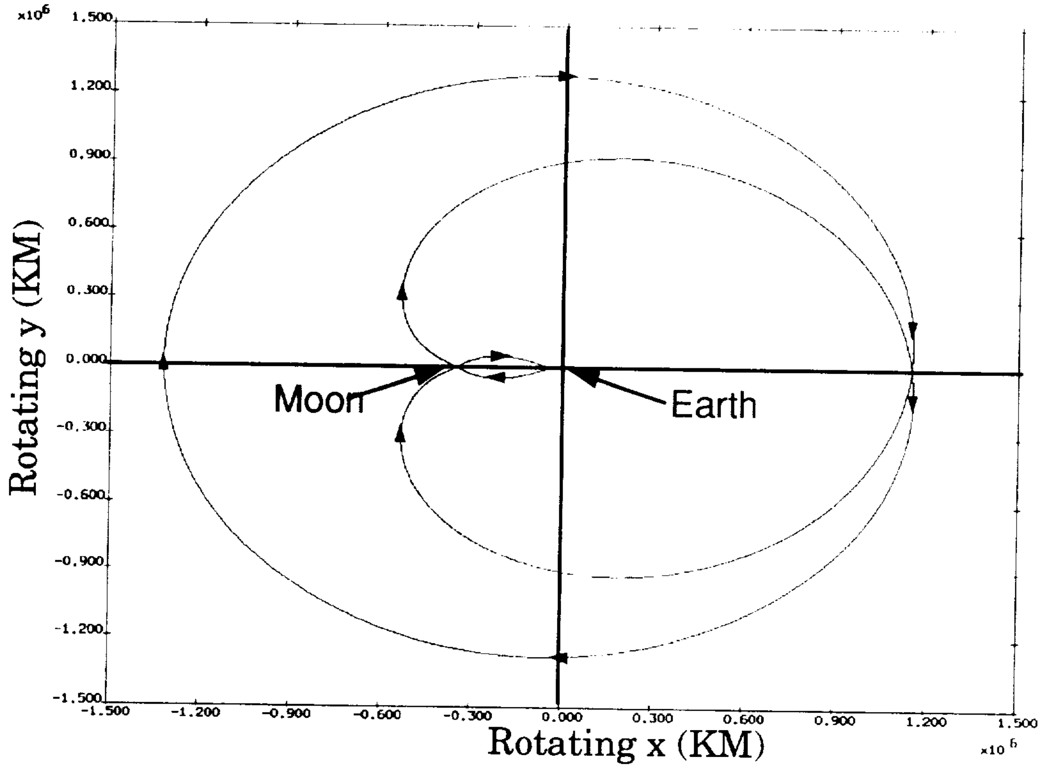

Figure 3 - Double Lunar Swing-by Trajectory

The Earth departure conditions specified by the targeting algorithm are depicted in Figure 4. As shown by the figure the Trans-Lunar Injection (TLI) burn required a ΔV of 3.119 kilometers per second at a longitude of 100.11 degrees with a transfer time of about 2.66 days to the patch point. Using these initial conditions, with no other additional ΔV, the vehicle will arrive at the moon with the required position and velocity for the 60 day moon to moon transfer. It should be noted that the TLI ΔV is very near the Hohmann (minimum energy) value.

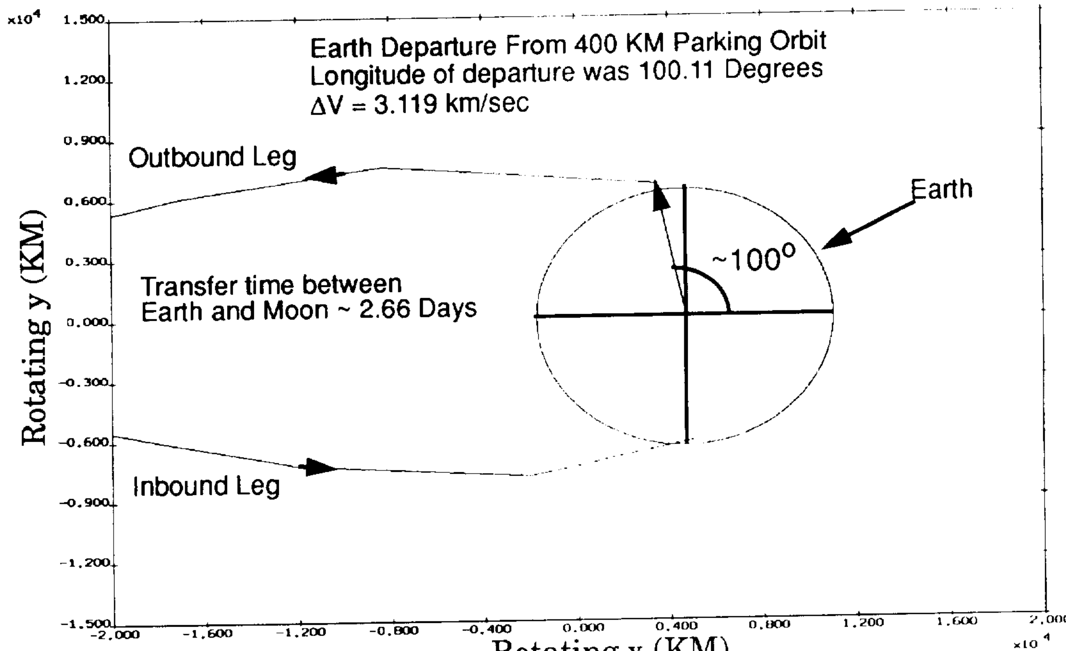

Figure 4 - Earth Departure and Arrival

The vehicle trajectory in the vicinity of the moon is displayed in Figure 5. As shown both lunar encounters have a closest approach to the moon at a relatively safe distance of 16,000 kilometers. The lunar encounter during both flyby's assists the vehicle performance. On the outbound leg the vehicle experiences a net gain in velocity, provided by the lunar gravity, which propels it onto a very large elliptical orbit. On the inbound leg the vehicle experiences a net loss in velocity which is desirable prior to earth encounter.

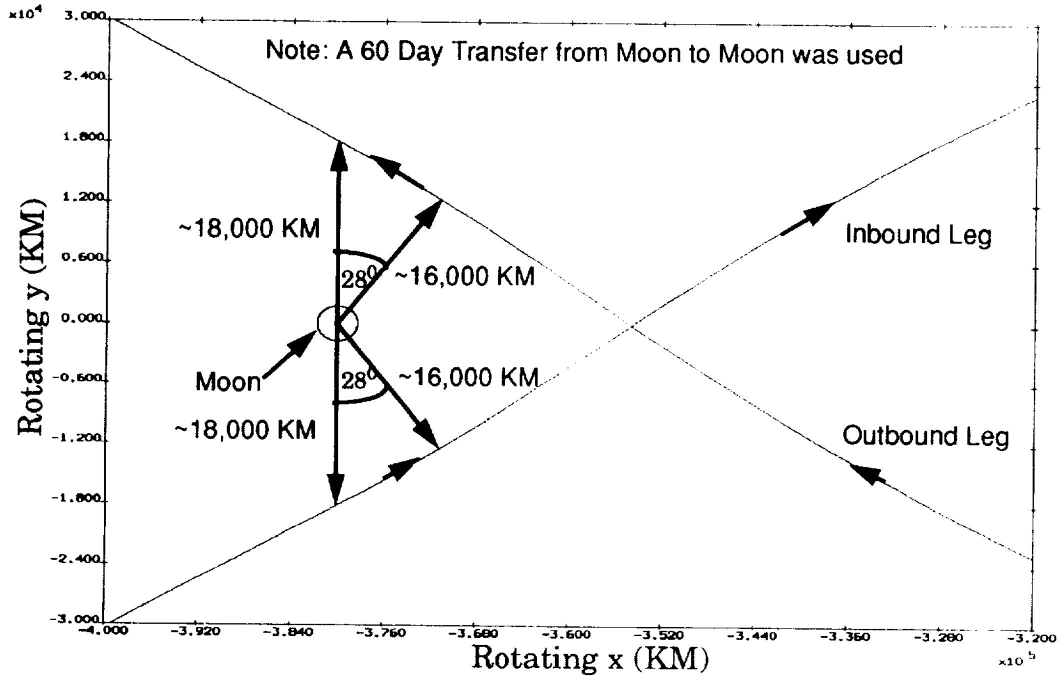

Figure 5 - Outbound/Inbound Lunar Fly by

Preliminary analysis has shown that relatively small amounts of ΔV (about 5 meters per second), applied during the lunar encounter on the inbound leg, would suffice for re-targeting for a specified entry interface (i.e. altitude and longitude). Entry velocities would be similar to those encountered during the Apollo missions (i.e. ~36,000 fps).

3.0 Perturbed Two-Body Analysis

From this point on the problem will be considered as a perturbed two-body problem. Solutions will be found by the special perturbation program, known as BG14, described in Bond and Fraietta (1991). There are several reasons for this change in point of view. For example, navigation, guidance and communication studies are more amenable to standard inertial coordinate systems. Also, even though the most significant perturbation, the moon, is included in the restricted three-body analysis, other significant perturbations such as the solar gravitational perturbations, high order gravitational fields of the Earth and moon, solar radiation pressure are not.

3.1 Transformation to Inertial Coordinates

Once the solution in the restricted three body system is determined the next step is to compute the initial state vector in the inertial coordinate system suitable for use in perturbed two-body analysis. The transformation from the rotating system to the inertial system is a two step process: (1) translate the position vector from the center of mass to the center of the earth and (2) rotate the state vector into the Earth-Moon plane for a particular date. This translation and rotation is given by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = M \begin{bmatrix} x - \mu \\ y \\ z \end{bmatrix}$$

The matrix M provides the rotation into the Earth-Moon plane. For our analysis we used the J2000 inertial system.

3.2 Lunar Perturbation Only

Prior to investigating the effects of other perturbations, the method was first verified by duplicating the results found in the restricted three body system in the perturbed two-body system. To this end a circular lunar orbit, consistent with the computed initial conditions, was implemented in BG14 as a perturbation. The initial conditions were then integrated for the desired flight time (about 65.32 days) in the presence of the lunar third body perturbation. The trajectory, as viewed in the X-Y inertial plane, is displayed in Figure 6.

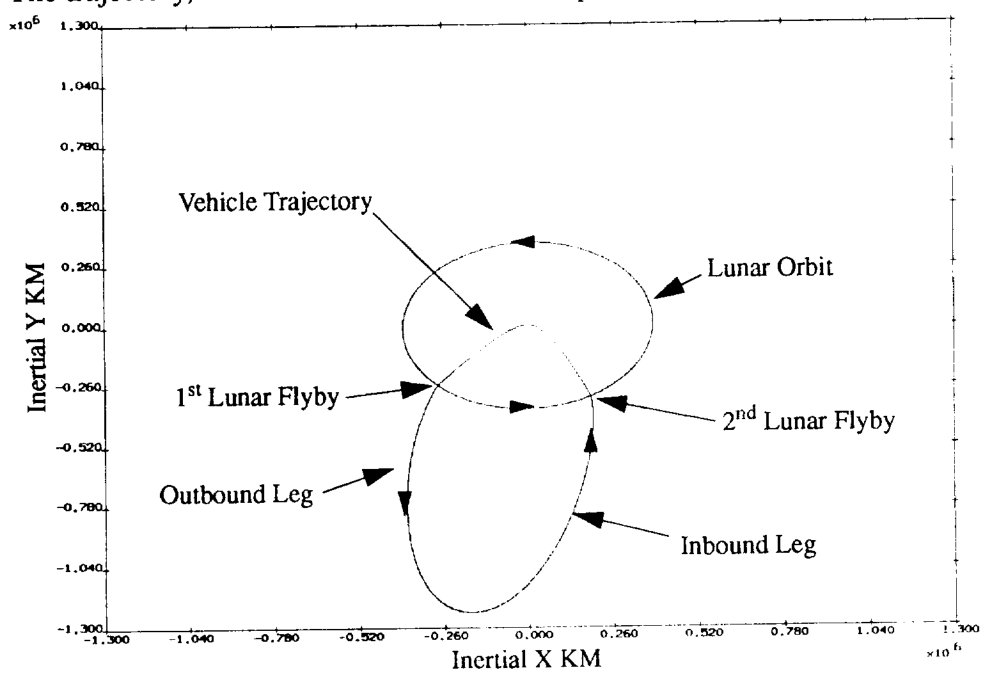

Figure 6 - Double Lunar Swing-by In Inertial Coordinates

As shown by the figure the trajectory experiences a significant bending on both the outbound and inbound legs as a result of the lunar swing-by.

3.3 Lunar and Solar Perturbations

Analysis performed during the preparation of this report has shown that the effects of the solar third body perturbations can not be ignored (especially for elliptical orbits with large semi-major axes.) It will therefore be necessary for the vehicle to periodically apply ΔV corrections to maintain the nominal trajectory. In an effort towards solving this problem a simple guidance control law using Chebyshev polynomials has been developed. This control law is then applied to the problem described in Section 3.2 with the addition of the third body perturbation due to the sun.

3.3.1 Chebyshev Guidance Algorithm

A guidance algorithm using Chebyshev polynomials, which can be expressed as

$$T_{n+1}(x) = 2xT_{n-1}(x) - T_{n-1}(x) \qquad n \geq 1$$

with starting values

$$T_0(x) = 1$$
$$T_1(x) = x$$

has been developed. Using the recursive nature of Chebyshev polynomials, this algorithm analytically provides the required state vector in the restricted three body coordinate system as a function of time using coefficients generated for a particular trajectory. Since for the restricted three body system the motion was restricted to the Earth-Moon plane, only the x-y components of the state vector are required. The restricted three body state vector is then transformation to the J2000 inertial system (in an identical manner to that described in Section 2.3). Once in the inertial system the actual state of the vehicle, as provided by navigation, can be compared to the desired state as provided by the Chebyshev polynomial solution. ΔV corrections are then applied at appropriate intervals to maintain the vehicle on the nominal path.

Using the entire earth to earth trajectory to compute the coefficients for the Chebyshev polynomials, it was found that the accuracy of the approximate state compared to the numerically computed state is a strong function of the number of Chebyshev coefficients used in the approximation. Figure 7 shows the maximum RSS position error between the

Chebyshev approximation and the numerically integrated state as a function of the number of Chebyshev coefficients.

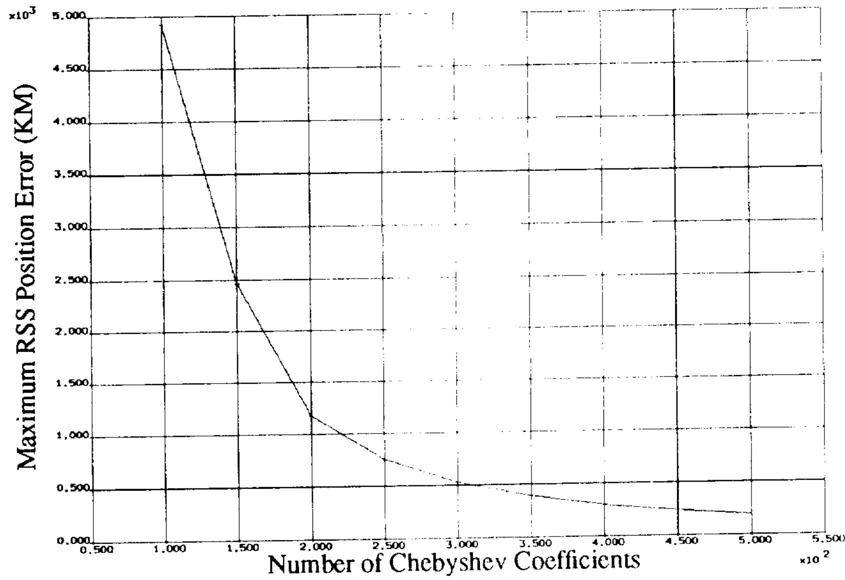

Figure 7 - Maximum RSS Position Error as a function of the Number of Chebyshev Coefficients Used

Note that the RSS position errors are dramatically reduced as the number of coefficients used increases. For example, as shown by the figure, the RSS position errors are reduced by about a factor of 5 by doubling the number of coefficients. Although not shown the velocity error behaves in a similar fashion.

3.3.2 Guidance Algorithm Results with Solar Perturbations

The Chebyshev guidance algorithm presented in Section 3.3.1 was implemented in BG14 along with a function to compute the solar third body perturbation. BG14 was then executed (with the same initial conditions and flight times as described in section 3.2) using both lunar and solar third body perturbations with periodic ΔV corrections being applied by the guidance algorithm. The required ΔV is displayed in Figure 8.

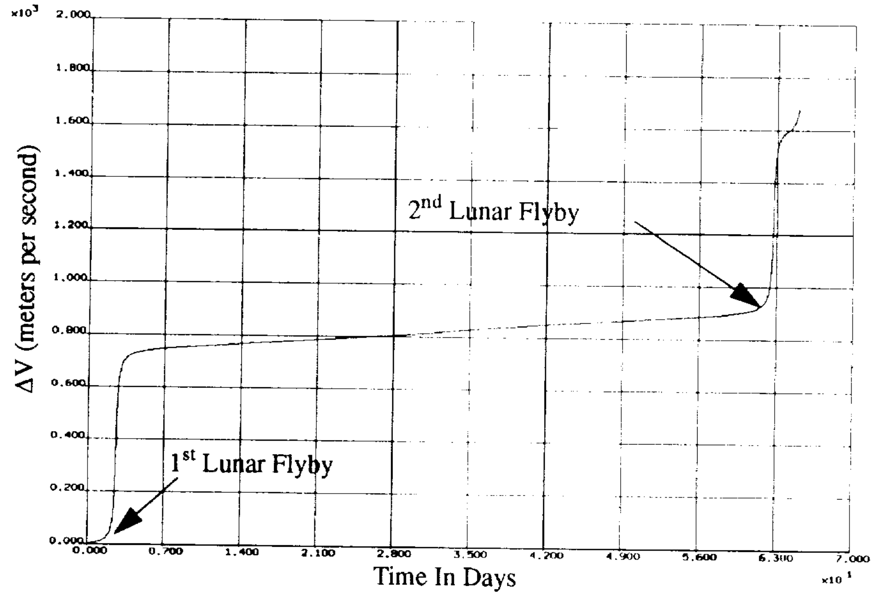

Figure 8 - ΔV vs Time With Sun Perturbations

As shown by the figure the ΔV required to maintain the nominal trajectory during the lunar flyby's approaches several hundred meters per second. These large values do not seem reasonable and are attributed to the fact that the Chebyshev approximation is not doing an adequate job in this region. However, once past the lunar encounter the total ΔV cost (a linear function in time) is only about 182 meters per second during the 60 day moon to moon transfer. It should be noted that it is on this part of the trajectory where the solar perturbation effects are largest.

Although time did not permit it for this study, it is felt that instead of fitting the entire earth to earth transfer with one Chebyshev fit it might be better to break up the trajectory into three different legs (i.e. earth to moon, moon to moon and moon to earth). A Chebyshev fit could then be provided for each leg. It is hoped that this technique would provide the accuracy required for the lunar encounter, thereby reducing the excessively large ΔV's shown in Figure 8.

4.0 Summary

An approach for finding long period elliptical orbits has been presented. The approach uses a targeting algorithm to solve the two point boundary value problem in the restricted three body system. The restricted three body solution found by the targeting algorithm was then transformed to the J2000 inertial system for use in a special perturbation method. This method allows modelling of other perturbations (due to for example the solar third body and solar radiation pressure) which are not easily modelled in the restricted three body system.

The approach was then used, in conjunction with the Double Lunar Swing-by technique, to obtain a candidate trajectory for the Life Sat mission. The candidate trajectory satisfies ideal trajectory design requirements which would maximize scientific return.

Finally, an estimate of the ΔV required to keep a vehicle on the desired trajectory in the presence of the solar third body perturbation was provided using a Chebyshev guidance algorithm. The algorithm was found to work well on the long elliptical trajectory once past lunar encounter. A suggestion for improving the performance of the guidance algorithm during lunar encounter was offered.

5.0 References

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