Analysis of the Command and Control Segment (CCS) Attitude Estimation Algorithm

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ABSTRACT

This paper categorizes the qualitative behavior of the Command and Control Segment (CCS) differential correction algorithm as applied to attitude estimation using simultaneous spin axis sun angle and Earth cord length measurements. The categories of interest are the domains of convergence, divergence, and their boundaries.

Three series of plots are discussed that show the dependence of the estimation algorithm on the vehicle radius, the sun/Earth angle, and the spacecraft attitude. Common qualitative dynamics to all three series are tabulated and discussed.

Out-of-limits conditions for the estimation algorithm are identified and discussed.

INTRODUCTION

This paper outlines the approach taken to determine the qualitative behavior of the attitude estimation algorithm used by the Command and Control Segment (CCS) system at the Air Force Consolidated Space Test Center (CSTC). For the purposes of this paper, determining the qualitative behavior means defining the regions of convergence and divergence in terms of the Earth, sun, and spacecraft attitude parameters.

This study is an outgrowth of the Information Processing and Analysis System (IPAS) project undertaken by Test Support Complex-1 (TSC-1) at CSTC. The purpose of this project was to evaluate the feasibility of incorporating commercially available hardware and software into an operational mission control center (MCC) design. The result of IPAS was a prototype for a telemetry monitoring system employing a real-time expert system that performs out-of-limits checking and recommends appropriate actions for telemetry anomaly resolution.

This paper supports a follow-on project that determines requirements for an autonomous real-time orbit and attitude estimation expert system prototype that would supplement the telemetry monitoring system. Creating a knowledge base for the estimation expert system requires defining out-of-limits criteria for the estimation algorithms used. This is the motivation for researching the qualitative behavior.

As a first step to understanding the equations that govern the out-of-limits criteria, an attitude estimation package that graphically shows the affects of geometry on attitude estimation was prototyped using Mathematica software on a Sun SPARCstation 1. This analysis tool could also be used for mission planning as well as training.

Work in progress is to mathematically support the visual conclusions drawn from the plots about the relationships between the Earth, sun, and attitude geometry to convergence or divergence of the estimation algorithm. These results will in turn be used as input to the estimation expert system knowledge base.

QUALITATIVE DYNAMICS

The Algorithm. The estimation algorithm chosen to be studied is as follows:

$$\Delta \mathbf{x}_k = [\Sigma_i [\mathbf{P}_i^t [\mathbf{x}_k] \mathbf{P}_i[\mathbf{x}_k]]]^{-1} [\Sigma_i [\mathbf{P}_i^t [\mathbf{x}_k] \Delta \mathbf{y}_i[\mathbf{x}_k]]]$$

with $\Delta\mathbf{x}$ the state vector correction, $\Delta\mathbf{y}$ the residuals vector, and P is the matrix of measurement partials with respect to the state variables. The sum is with respect to the ith time point in the measurement set. For simplicity, this paper considers the case of just one measurement in time consisting of an Earth cord length and a sun angle. For this case, the algorithm reduces to:

$$\Delta x_k = P^{-1}[x_k]\Delta y[x_k]$$

The following standard notation is used:

A is the spacecraft attitude

$x = \{\alpha, \delta\}$ is A in spherical coordinates

$y = \{\Omega, \beta\}$ is the measurement vector

Ω is the earth cord length

η is the nadir angle

β is the sun angle

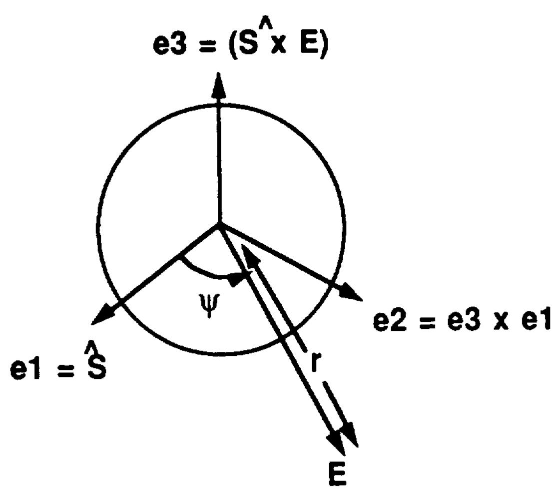
γ is the sensor cant angle

ρ is the apparent earth radius

E is the earth vector

S is the sun vector

To reduce the number of parameters needed to specify the Earth, sun, and attitude geometry, construct the {e1, e2, e3} vehicle centered coordinate system as follows:



In this system, specifying the sun and Earth position requires only two parameters (the sun Earth angle ψ and the vehicle radius r) as compared to five (sun and Earth right ascensions and declinations as well as vehicle radius) in the standard vehicle centered coordinate system.

In the {e1, e2, e3} coordinate system the following relationships hold:

$$\cos[\Omega/2] = \frac{\cos[\rho] - \cos[\eta]\cos[\gamma]}{\sin[\eta]\sin[\gamma]}$$

$$\cos[\eta] = \cos[\alpha-\psi]\cos[\delta]$$

$$\cos[\beta] = \cos[\alpha]\cos[\delta]$$

$$P = \begin{pmatrix} \dfrac{-2(\cos[\rho]\cos[\eta] - \cos[\gamma])}{\sin^3[\eta]\sin[\gamma]\sin[\Omega/2]} & 0 \\ 0 & \dfrac{-1}{\sin[\beta]} \end{pmatrix} \begin{pmatrix} -\sin[\alpha-\psi] & -\cos[\alpha-\psi] \\ -\sin[\alpha] & -\cos[\alpha] \end{pmatrix} \begin{pmatrix} \cos[\delta] & 0 \\ 0 & \sin[\delta] \end{pmatrix}$$

Out-Of-Limits Criteria. The estimation out-of-limits criteria are based on the existence, uniqueness, and convergence behavior of the algorithm. The existence and uniqueness are well defined in terms of the geometry, leaving the convergence behavior for study. The following paragraphs briefly summarize each.

Existence. The algorithm will exist whenever P is invertible. It is easily seen that P is not invertible for the following geometries:

Attitude, sun, and Earth vectors are coplanar. When this happens, δ is zero and the last matrix in the above equation becomes singular.

Attitude has a ± 90 degree declination. For this geometry, the right ascension is undefined and therefor not recoverable. Again, this makes the last matrix singular.

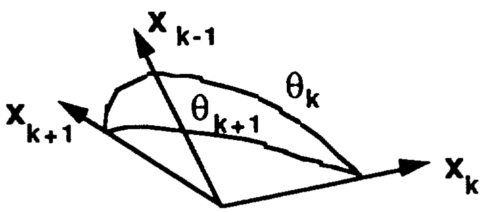
The Sun and Earth are coplanar. This makes the rows of the second matrix dependent and thus singular.

The nadir angle is such that it maximizes the Earth cord length measurement. This causes the first matrix to be singular.

Uniqueness. The algorithm solutions are not unique. Since one Earth cord length corresponds to two possible nadir angles, simultaneous Earth cord length and sun angle measurements define four cones of possible attitude solutions. These cones will intersect in two and possibly four points. Therefor there are two and possibly four choices for the attitude vector that will give zero residuals, resulting in convergence.

Convergence. The estimation algorithm studied here is an iterative algorithm and is therefor susceptible to complicated dynamics. An estimation expert system needs to be able to recognize when divergence will occur due not only to the type of geometry that would make the algorithm not exist, but also due to the choice of algorithm itself. When this occurs, the expert system should know to automatically switch to a better algorithm.

In order to develop the appropriate knowledge base, convergence criteria in terms of the geometry need to be established. For this paper, convergence is defined to be when the arc length of the correction is less than a prescribed tolerance value. Divergence is defined to be when the algorithm doesn't exist or when convergence hasn't occurred within a prescribed number of iterations. The k+1 iteration arc length correction θ is defined as:

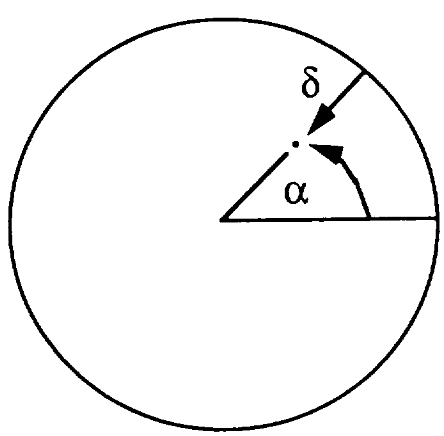


$$\theta_{k+1} = \arccos[\cos[\delta_{k+1}]\cos[\delta_k] + \sin[\delta_{k+1}]\sin[\delta_k]\cos[\alpha_{k+1} - \alpha_k]]$$

The work in progress is to rewrite this expression solely in terms of constants and the k-th itera θ.

The Analysis Tool. In order to gain insight into this rewrite problem, an analysis tool was prototyped using Mathematica software on a Sun SPARCstation 1 that graphically shows the affects of geometry on estimation. User inputs to this tool are the Earth cord length and sun angle measurement, the vehicle and sun position, the Earth horizon sensor cant angle and the choice of positive or negative declination hemisphere.

The output of the tool is a plot showing a colored disk. This disk represents attitude right ascension and declination pairs {α, δ} for the chosen hemisphere as shown below.



Each $\{\alpha, \delta\}$ pair is input to the estimation algorithm and then colored according to what it converged to. The coloring algorithm was Hue[$\alpha c/360$] and Brightness[$1-\delta c/380$], where $\{\alpha c, \delta c\}$ are the converged values. If the algorithm didn't converge, then $\{\alpha c, \delta c\}$ were set to {360, 100}, corresponding to the darkest regions in the plot. With this tool, regions that converge to the same value can be easily spotted and the size of the convergent region can be visually estimated.

Due to memory and time constraints, the plot resolution is limited to two degrees.

Results. In order to understand how each geometric parameter affects estimation, three series of plots were made. Each series tries to hold all parameters fixed except for one. All plots chose the negative declination hemisphere (the brighter colors were chosen since they show more contrast when rendered in black and white). In Figures 1-3, the values for n defined below increase from left to right.

Vehicle radius series. The plots in Figure 1 were made with these inputs:

$\psi = 90$;

r = 8000 + n 500 km: n=0,4;

A = {45, 45};

Sun Earth angle series. The plots in Figure 2 were made with these inputs:

ψ = n 20: n = 4,8;

r = 8000 km;

A = {45, 45};

Attitude series. The plots in Figure 3 were made with these inputs:

$\psi = 90$;

r = 8000 km;

A = {5 + 10 n, -5 n^2 + 40 n + 5}: n = 5,9;

CONCLUSIONS

Plot Features. There appear to be several features common to all the plots.

Figure 1. The vehicle radius series.

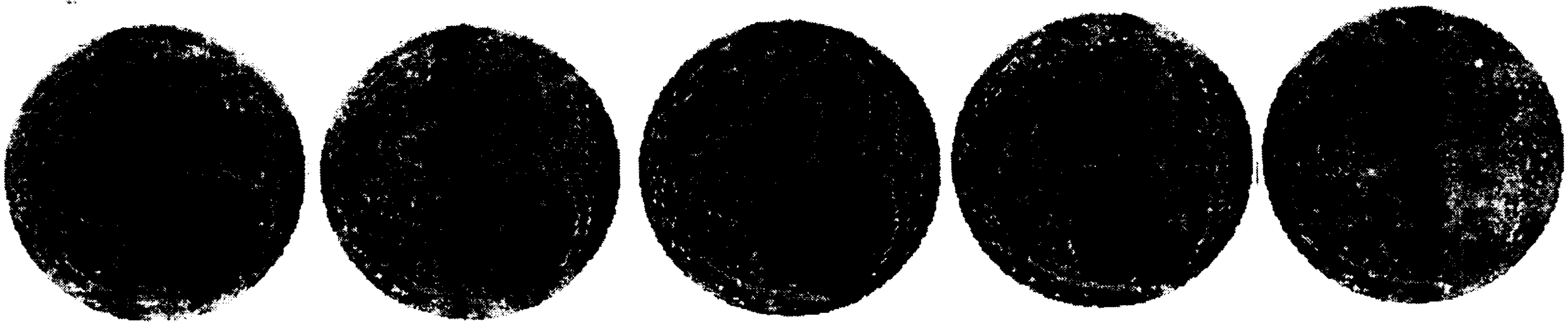


Figure 2. The sun/Earth angle series.

Figure 3. The altitude series.

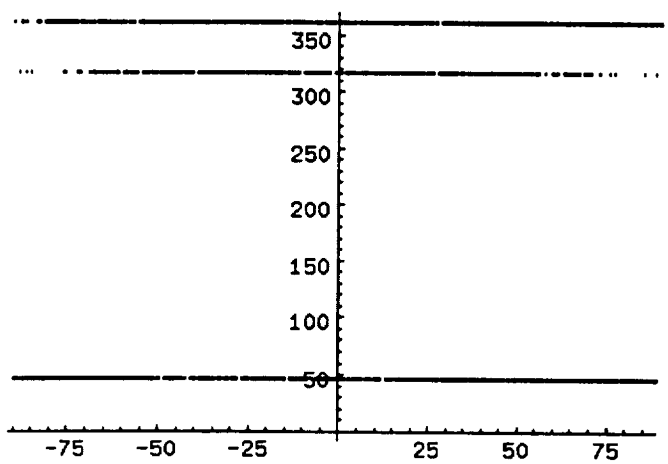
The divergent figure eight. Each plot has a divergent region at the declination poles that is shaped like a figure eight. The size of this region appears to only depend on the vehicle radius and increases as the radius increases. The direction of the long axis lines up with the Earth vector, suggesting that this affect is independent of the sun geometry.

The convergent region. Each possible attitude solution has a convergent region around it. The size and shape of this region appears to be affected by all the geometric parameters, since it varies throughout all the series.

The "image" convergent region. There appears to always be a convergent region 180 degrees away in right ascension from the possible attitude solutions. This region is typically smaller than the region containing the attitude solution.

The Earth perpendicular divergent region. The line perpendicular to the Earth vector has very unstable behavior. Attitudes arbitrarily close to each other converge to different solutions. Figure 4 shows an example of this. Here declination is plotted against the converged right ascension for the input attitudes {0, δ}. The geometry is from n=1 in the vehicle radius series.

Figure 4. Declination vs converged right ascension.



Topology. The dynamics of the topology for each series is difficult to determine from only a two degree resolution. However, there always appears to be alternating layers of convergent and divergent regions, with only the relative size and shapes varying. This suggests that even though the geometry might be in a region far away from the region where the algorithm doesn't exist, the algorithm might still diverge due to the iterative dynamics.

AREAS OF FUTURE RESEARCH

The Analysis Tool. In order to apply this visualization approach to other estimation problems, the analysis tool needs to be extended to estimation problems with more than two variables. For instance, orbit position and velocity estimation would require plotting a six dimensional variable against another six dimensional variable. To get around this plotting difficulty, the dimension of the estimation problem must somehow be reduced. Translating the analysis tool into 'C' code is also planned to make the prototype more operational (ie faster).

More than one measurement. Future work includes extending the results here to the case of more than one measurement in time, or batch estimation. This is the more common case for the algorithm studied here.

Convergence criteria. Work is currently in progress to develop closed form convergence criteria in terms of the geometric parameters. These criteria will then be converted to out-of-limits conditions for the estimation algorithm for input to an estimation expert system knowledge base. Rules will also be developed to determine what algorithm to substitute when out-of-limits conditions occur.

Algorithms. A robust estimation expert system will have several algorithms to choose from, so similar analysis of other estimation algorithms is planned.

SUMMARY

An analysis tool that graphically shows the affects of geometry on attitude estimation has been made. This tool can be used during the readiness phase for a satellite mission to validate or define operational requirements. This tool can also be incorporated into a training program for those needing a high level view of attitude estimation.

Visual inspection of plot output from the analysis tool has lead to an increased understanding of the qualitative behavior of the estimation algorithm. This understanding is currently being quantified mathematically for input to an estimation expert system knowledge base.

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