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System Identification Using Interval Dynamic Models

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ABSTRACT

Motivated by the recent explosive development of results in the area of parametric robust control, this paper presents a new technique to identify a family of uncertain systems. The new technique takes the frequency domain input and output data obtained from experimental test signals and produces an "interval transfer function" that contains the complete frequency domain behavior with respect to the test signals. This interval transfer function is one of the key concepts in the parametric robust control approach and identification with such an interval model allows one to predict the worst case performance and stability margins using recent results on interval systems. The algorithm is illustrated by applying it to an 18 bay Mini-Mast truss structure.

1. INTRODUCTION

It is well known that obtaining an accurate mathematical description of a system is impossible, usually very costly and often increases the complexity of the corresponding control mechanism. Robust control deals with systems described as a family consisting of a nominal model with uncertainty around it. In particular, parametric robust control deals with systems whose parameters of interest vary in known independent intervals. The simplest form of such a system is called an "interval system" wherein the model consists of a family of transfer functions with numerator and denominator coefficients varying in prescribed independent intervals.

A recent trend in the area of system identification is to move toward a new direction in which one tries to model the system uncertainties to fit the available analysis and design tools of robust control. The motivation for this direction of research is clearly stated in [1]. The excellent collection of papers dealing with this important issue extensively is also found in [2] and references therein (also see [3,4,5]). The main emphasis of these results is either to classify the plant uncertainty as a various form of norm bounded uncertainties such as H^∞, ℓ_1, and structured singular values or to find the optimal nominal models to fit their respective robust control design methods.

Commonly, an interval transfer function is interpreted as a family of transfer functions whose coefficients are bounded by some known intervals and centered at the nominal values. In many cases, this type of framework is unnatural and physical parameter perturbations do not correspond to transfer function coefficients. In order to relax this limitation, approaches to deal with linearly or multilinearly correlated perturbations have been proposed. On the other hand, if we observe recent developments in the interval system area we see that the nominal system has very little significance. These results in fact emphasize the boundary properties of the family of systems under consideration. In fact, virtually all the important results we enjoy today in this field are based on the boundary generating extreme points, edges, and segments of the interval system.

Following the motivation described in [1], suppose that the behavior of the plant is described by some known test input and its corresponding measurement output. Due to many reasons such as noise, inaccurate measurements etc., a fixed linear time-

invariant identified model will almost never exactly represent the data obtained from the plant. Our aim in this paper is to obtain a reasonable interval transfer function model around (not necessarily centered in) the nominally identified transfer function so that the entire frequency domain behavior of the physical plant is completely contained in that of the model. We give a systematic and simple algorithm to accomplish this task. We also discuss the issue of design validation of the obtained interval transfer function. Finally we demonstrate this practical technique by applying it to an 18 bay Mini-mast truss which has been extensively used for various large space structure control problems.

2. PROBLEM FORMULATION

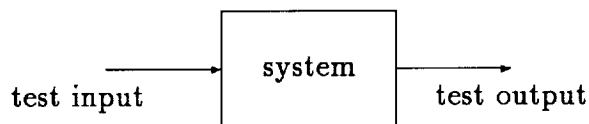Consider the following configuration shown in Figure 1.

Figure 1.

In system identification one applies test inputs and measures the system response in order to identify the parameters of an appropriate proposed mathematical model. It is also common that these test signals are represented in a form of frequency domain data. In system identification literature there are numerous techniques available (see [7] and references therein) to determine a best possible linear time - invariant model that fits this set of frequency domain data. Suppose that the test frequencies are $\omega_1, \omega_2, \cdots \omega_N$ and the complex numbers $u(j\omega_i), y(j\omega_i)$ denote in phasor notation the input-output pair at the frequency ω_i. Let

$$y(j\omega_i) := D(j\omega_i)u(j\omega_i), \quad i = 1, 2, \cdots, N \tag{2.1}$$

denote the test data generated from an identification experiment. Suppose that $G^I(s)$ is the transfer function of a linear time-invariant system which is such that $G^I(j\omega)$ is closest to $D(j\omega)$ in some norm sense. In general it is not possible to find a single rational function $G^I(s)$ for which $G^I(j\omega_i) = D(j\omega_i)$ and the more realistic identification problem is to in fact identify an entire family $\mathbf{G}(s)$ of transfer functions which is capable of "explaining" the data in the sense that for each data point $D(j\omega_i)$ there exists some transfer function $G_i(s) \in \mathbf{G}(s)$ with the property that $G_i(j\omega_i) = D(j\omega_i)$. The family $\mathbf{G}(s)$ can be parametrized in many alternative ways. For instance an unstructured approach to describing $\mathbf{G}(s)$ using a normed algebra is to let each element $G(s)$ of $\mathbf{G}(s)$ be described as $G(s) = G^I(s) + \Delta G(s)$ where the norm $|\Delta G(s)| < \rho$. In such a case the family $\mathbf{G}(s)$ is identified once $G^I(s)$ and ρ are determined. In general the identification algorithm should also be efficient in the sense that the family $\mathbf{G}(s)$ that it produces should be ideally minimal among the set of all such families that

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explain the data. In the unstructured case described above this translates to choosing a small value of ρ.

In this paper our objective is to develop an identification algorithm in a framework where the family of linear time-invariant systems $\mathbf{G}(s)$ is obtained by letting the transfer function coefficients lie in intervals around (not necessarily centered in) those of the nominal $G^I(s)$. Of course the identification requirement is that

$$D(j\omega_i) \in \mathbf{G}(j\omega_i) \qquad \forall \ \omega_i. \tag{2.2}$$

Let

$$G^I(s) := \frac{n_0 + n_1 s + n_2 s^2 + n_3 s^3 + \cdots + n_n s^n}{d_0 + d_1 s + d_2 s^2 + d_3 s^3 + \cdots + d_n s^n}. \tag{2.3}$$

We define

$$G(s) := \frac{\hat{n}_0 + \hat{n}_1 s + \hat{n}_2 s^2 + \hat{n}_3 s^3 + \cdots + \hat{n}_n s^n}{\hat{d}_0 + \hat{d}_1 s + \hat{d}_2 s^2 + \hat{d}_3 s^3 + \cdots + \hat{d}_n s^n} \tag{2.4}$$

and

$$\mathbf{G}(s) := \{ G(s) : \hat{n}_i \in [n_i - \mathbf{w}_{n_i}\epsilon^-_{n_i}, n + \mathbf{w}_{n_i}\epsilon^+_{n_i}], \hat{d}_i \in [d_i - \mathbf{w}_{d_i}\epsilon^-_{d_i}, d + \mathbf{w}_{d_i}\epsilon^+_{d_i}], \qquad \forall \ i \} \tag{2.5}$$

where

$$\begin{aligned} \underline{\mathbf{w}} &:= \begin{bmatrix} \mathbf{w}_{d_0} & \cdots \mathbf{w}_{d_n} & \mathbf{w}_{n_0} & \cdots & \mathbf{w}_{n_n} \end{bmatrix} \\ \underline{\epsilon}^+ &:= \begin{bmatrix} \epsilon^+_{d_0} & \cdots & \epsilon^+_{d_n} & \epsilon^+_{n_0} & \cdots & \epsilon^+_{n_n} \end{bmatrix} \\ \underline{\epsilon}^- &:= \begin{bmatrix} \epsilon^-_{d_0} & \cdots & \epsilon^-_{d_n} & \epsilon^-_{n_0} & \cdots & \epsilon^-_{n_n} \end{bmatrix}. \end{aligned} \tag{2.6}$$

The components of w are to be regarded as weights chosen apriori whereas the ϵs are to be regarded as dilation parameters to be determined by the identification algorithm and the data $D(j\omega_i)$

Remark 1. Note that in the expression in eq. (2.5) we use vectors $\epsilon^\pm_{n_i}$ and $\epsilon^\pm_{d_i}$ instead of a single ϵ. This setting allows n_i and d_i to not necessarily be the center point of the intervals in which $\hat{n}_i$ and $\hat{d}_i$, lie respectively. This flexibility is important to achieve the minimum possible size of the family $\mathbf{G}(s)$.

The requirements on the identified interval model $\mathbf{G}(s)$ become:

1) *Membership Requirement:* $D(j\omega_i) \in \mathbf{G}(j\omega_i) \ \forall \ i$.

2) *Size Requirement:* $\|\underline{\epsilon}^\pm\|$ as small as possible.

3) *Frequency Response Requirement:* the weights $\underline{\mathbf{w}}$ must be chosen so that the frequency response of $\mathbf{G}(j\omega)$ is bounded as tight as possible for every frequency.

It is important to note that both size and frequency response requirements are crucial because smaller intervals do not necessarily map to smaller image sets. The frequency response requirement ensures that the frequency domain image set be as small as possible.

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3. INTERVAL SYSTEM MODELLING

Our objective is to find the set, an interval system $\mathbf{G}(s)$, to satisfy the three requirements given in the previous section. As described above the procedure is divided into two part. First, we identify a linear time - invariant model $G^I(s)$ which represents the test data $D(j\omega)$ as closely as possible. A variety of algorithms are available in the system identification literatures. We use a least squares algorithm which is widely known [7]. Then using this identified model as a nominal model, we create the tightest intervals around each coefficient of the nominal transfer function $G^I(s)$ while satisfying the membership and frequency response requirements.

3.1. Nominal System Identification

In this subsection, Since the purpose of this paper is not to deal with traditional single system identification problem, we briefly describe a standard method to identify a nominal transfer function whose frequency response fits the given test data $D(j\omega_i)$ as closely as possible. The method we use here is widely known as the weighted least square method [7]. The least square approach to curve fitting a transfer function in the frequency domain may be found in [8]. An appropriate order of model may be determined by checking the singular values of the Hankel matrix generated from the impulse response data. Under the assumption that the data is noise free, the number of nonzero singular values determines the order of the system. The details of this approach is found in [9]. After determining the appropriate order of the system, we let the nominal transfer function be

$$G^I(s) := \frac{n(s)}{d(s)} \tag{3.1}$$

The nominal transfer function coefficients must be selected to minimize the following index:

$$\sum_{i=1}^{N} \left\{ W^I(j\omega_i)\{\text{Re}[D(j\omega_i)d(j\omega_i) - n(j\omega_i)]\}^2 + \{\text{Im}[D(j\omega_i)d(j\omega_i) - n(j\omega_i)]\}^2 \right\}. \tag{3.2}$$

This least square problem generates $2N$ linear equation for $2n$ unknown coefficients of the transfer function. The weight W^I may be selected by finding the minimum variance estimator of unknowns. The details of methods for selecting the normalized least square weight $W^I(j\omega)$ and its effect on the identified model are discussed in [7]. The reference [7] also discusses the problem of selecting an appropriate order of the transfer function in detail. In general, the relative error in the valley parts of the frequency response is more significant than the one in the peak parts, it is necessary to assign high weights for the frequency ranges in the valley parts of the frequency response.

Using this idea, we select the normalized least square weight $W^I(j\omega)$ and improve the identified model by comparing the identified model and the test data.

3.2. Weight Selection

As shown in eq. (2.5), the size of the interval of variation for each coefficient of the family $\mathbf{G}(s)$ depends on w and ϵ. In this subsection, we consider the problem of finding an appropriate set of weights $\underline{w}$. The weight selection procedure is extremely important because inappropriate selection of weights may result in an unnecessarily large family. This results in a large image set in the complex plane, even though the intervals themselves may be small.

It is natural to think that a weight represents the average sensitivity of a coefficient of the nominal model with respect to the variation of data points. Thus, we establish the following reasonable algorithm for selecting weights.

Suppose the test data consists of N data points obtained at corresponding frequencies, i.e.,

$$D(j\omega) := \{D(j\omega_i) = \alpha_i + j\beta_i, \quad i = 1, 2, \cdots, N\}. \tag{3.3}$$

Let us define the lth model set as follows:

$$G_l(j\omega) = \begin{cases} D(j\omega_i), & i = l \\ G^I(j\omega_i), & i = 1, 2, \cdots, l-1, l+1, \cdots, N \end{cases} \tag{3.4}$$

In other words, the model $G_l(j\omega)$ is identical to the nominal identified model $G^I(j\omega)$ with the lth data point replaced by the lth component of the test data $D(j\omega)$. Now we construct the lth identified model, which we call $G_l^I(s)$ which is identified from the lth data set $G_l(j\omega)$. Let

$$G_l^I(s) := \frac{n_0^l + n_1^l s + n_2^l s^2 + n_3^l s^3 + \cdots + n_n^l s^n}{d_0^l + d_1^l s + d_2^l s^2 + d_3^l s^3 + \cdots + d_n^l s^n} \tag{3.5}$$

and

$$\mathbf{p} := \begin{bmatrix} n_0 & n_1 & \cdots & n_n & d_0 & d_1 & \cdots & d_n \end{bmatrix}. \tag{3.6}$$

If we assume that $|G_l(j\omega) - G_l^I(j\omega)|$ is small, the sensitivity of the coefficients of the nominal model with respect to variations in the lth data point is described as

$$\frac{\partial \mathbf{p}}{\partial G^I(j\omega_l)} := \begin{bmatrix} |n_0 - n_0^l| & \cdots & |n_n - n_n^l| & |d_0 - d_0^l| & \cdots & |d_n - d_n^l| \end{bmatrix}. \tag{3.7}$$

Collecting the sensitivity of the coefficients of the nominal model with respect to the variation of all the data points, $l = 1, 2, \cdots, N$, we have

$$\frac{\partial \mathbf{p}}{\partial G^I(j\omega)} := \begin{bmatrix} \frac{\partial \mathbf{p}}{\partial G^I(j\omega_1)} \\ \frac{\partial \mathbf{p}}{\partial G^I(j\omega_2)} \\ \vdots \\ \frac{\partial \mathbf{p}}{\partial G^I(\omega_N)} \end{bmatrix}$$

$$= \begin{bmatrix} |n_0 - n_0^1| & \cdots & |n_n - n_n^1| & |d_0 - d_0^1| & \cdots & |d_n - d_n^1| \\ |n_0 - n_0^2| & \cdots & |n_n - n_n^2| & |d_0 - d_0^2| & \cdots & |d_n - d_n^2| \\ \vdots & & \vdots & & \\ |n_0 - n_0^N| & \cdots & |n_n - n_n^N| & |d_0 - d_0^N| & \cdots & |d_n - d_n^N| \end{bmatrix}. \tag{3.8}$$

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The weights are then defined as the average of these for each coefficient:

$$\underline{\mathbf{w}} :=$$

$$\frac{1}{N} \left[\ \sum_{l=1}^{N} |n_0 - n_0^l| \quad \cdots \quad \sum_{l=1}^{N} |n_n - n_n^l| \quad \sum_{l=1}^{N} |d_0 - d_0^l| \quad \cdots \quad \sum_{l=1}^{N} |d_n - d_n^l| \ \right]$$

$$:= \left[\ \mathbf{w}_{n_0} \quad \cdots \quad \mathbf{w}_{n_n} \quad \mathbf{w}_{d_0} \quad \cdots \quad \mathbf{w}_{d_n} \ \right]. \tag{3.9}$$

3.3. Interval System Identification

In this subsection, using the weight that represents sensitivity of the coefficients of the nominal transfer function, we develop an interval model that satisfies three conditions given above. After we determine an appropriate weight vector, we need to find $\underline{\epsilon}^{\pm}$ to satisfy the given requirement. We now first consider the membership requirement. Recall the nominal system given in eq. (2.3) and substitute $s = j\omega$, then we have

$$
\begin{aligned}
G^I(j\omega) &:= \frac{n(j\omega)}{d(j\omega)} \\
&= \frac{n_0 + j\omega n_1 - \omega^2 n_2 - j\omega^3 n_3 + \omega^4 n_4 + j\omega^5 n_5 - \cdots}{d_0 + j\omega d_1 - \omega^2 d_2 - j\omega^3 d_3 + \omega^4 d_4 + j\omega^5 d_5 - \cdots} \\
&= \frac{(n_0 - \omega^2 n_2 + \omega^4 n_4 - \cdots) + j(\omega n_1 - \omega^3 n_3 + \omega^5 n_5 - \cdots)}{(d_0 - \omega^2 d_2 + \omega^4 d_4 - \cdots) + j(\omega d_1 - \omega^3 d_3 + \omega^5 d_5 - \cdots)} \\
&:= \frac{n^{\mathrm{even}}(\omega) + jn^{\mathrm{odd}}(\omega)}{d^{\mathrm{even}}(\omega) + jd^{\mathrm{odd}}(\omega)}.
\end{aligned} \tag{3.10}
$$

Since the nominal model transfer function $G^I(s)$ cannot perfectly represent the data set $D(j\omega)$, we have the following relationships for a particular frequency w_i.

$$
\begin{aligned}
D(j\omega_i) &= \alpha_i + j\beta_i \\
&\approx G^I(j\omega_i) \\
&= \frac{n^{\mathrm{even}}(\omega_i) + jn^{\mathrm{odd}}(\omega_i)}{d^{\mathrm{even}}(\omega_i) + jd^{\mathrm{odd}}(\omega_i)}.
\end{aligned} \tag{3.11}
$$

The difference may be added to the coefficients of the nominal model as follows:

$$
\begin{aligned}
D(j\omega_i) &= \alpha_i + j\beta_i \\
&= \frac{(\hat{n}_0 - \omega_i^2 \hat{n}_2 + \cdots) + j(w_i \hat{n}_1 - \omega_i^3 n_3 + \cdots)}{(\hat{d}_0 - \omega_i^2 \hat{d}_2 + \cdots) + j(w_i \hat{d}_1 - \omega_i^3 d_3 + \cdots)}
\end{aligned} \tag{3.12}
$$

where

$$
\begin{aligned}
\hat{n}_i &:= n_i + \mathbf{w}_{n_i} \epsilon_{n_i} \\
\hat{d}_i &:= d_i + \mathbf{w}_{d_i} \epsilon_{d_i}, \qquad \forall\, i.
\end{aligned} \tag{3.13}
$$

If we rewrite this in terms of a linear matrix equation, we have

$$A(\omega_i, \alpha_i, \beta_i) W \underline{\epsilon}^i = B(\omega_i, \alpha_i, \beta_i) \tag{3.14}$$

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where

$$A(\omega_i, \alpha_i, \beta_i) :=$$

$$\begin{bmatrix} \alpha_i & -\beta_i\omega_i & -\alpha_i\omega_i^2 & \beta_i\omega_i^3 & \cdots & -1 & 0 & \omega_i^2 & 0 & -\omega_i^4 & 0 & \cdots \\ \beta_i & \alpha_i\omega_i & -\beta_i\omega_i^2 & -\alpha_i\omega_i^3 & \cdots & 0 & \omega_i & 0 & -\omega_i^3 & 0 & \omega_i^5 & \cdots \end{bmatrix} \qquad (3.15)$$

$$W := \begin{bmatrix} \mathbf{w}_{d_0} & & & & & \\ & \mathbf{w}_{d_1} & & & & \\ & & \ddots & & & \\ & & & \mathbf{w}_{n_0} & & \\ & & & & \mathbf{w}_{n_1} & \\ & & & & & \ddots \end{bmatrix} \qquad (3.16)$$

$$\underline{\epsilon}^i := \begin{bmatrix} \epsilon_{d_0}^i & \cdots & \epsilon_{d_n}^i & \epsilon_{n_0}^i & \cdots & \epsilon_{n_n}^i \end{bmatrix} \qquad (3.17)$$

and $B(\omega_i, \alpha_i, \beta_i) :=$

$$\begin{bmatrix} -\alpha_i(d_0 - \omega_i^2 d_2 + \omega_i^4 d_4 - \cdots) + \beta_i(\omega_i d_1 - \omega_i^3 d_3 + \omega_i^5 d_5 - \cdots) \\ +(n_0 - \omega_i^2 n_2 + \omega_i^4 n_4 - \cdots) \\ -\beta_i(d_0 - \omega_i^2 d_2 + \omega_i^4 d_4 - \cdots) - \alpha_i(\omega_i d_1 - \omega_i^3 d_3 + \omega_i^5 d_5 - \cdots) \\ +(\omega_i n_1 - \omega_i^3 n_3 + \omega_i^5 n_5 - \cdots) \end{bmatrix} . \qquad (3.18)$$

Here we assume that without loss of generality $A(\omega_i, \alpha_i, \beta_i)$ has full row rank. It is well known in linear algebra literatures [10] that the smallest norm solution $\underline{\epsilon}_i$ can be computed by

$$\underline{\epsilon}_i = A(\cdot)^T [A(\cdot)A(\cdot)^T]^{-1} B(\cdot). \qquad (3.19)$$

After finding $\underline{\epsilon}_i$ for all $i = 1, 2, \cdot, N$, we determine the interval as follows:

$$\epsilon_{n_k}^- := \min_i \{0, \epsilon_{n_k}^i\} \qquad \epsilon_{n_k}^+ := \max_i \{0, \epsilon_{n_k}^i\} \qquad (3.20)$$

$$\epsilon_{d_k}^- := \min_i \{0, \epsilon_{d_k}^i\} \qquad \epsilon_{d_k}^+ := \max_i \{0, \epsilon_{d_k}^i\} \qquad (3.21)$$

for all k. Clearly, the procedure guarantees three requirement given earlier.

4. MODEL VALIDATION

Model validation can be accomplished by comparing the frequency domain characteristics of the obtained interval model and the test data. We use polar plots and Bode plots to accomplish this task. The polar and Bode plots of the test data is assumed to be given. Therefore we here only discuss the problem of obtaining polar and Bode plots of a proposed model interval system. The polar plot of an interval system is obtained by determining the image set of the set of transfer functions at a fixed frequency ω and taking the envelope of such sets obtained by sweeping over ω ranging from $0 \longrightarrow \infty$. This is done via the following theorems. The proofs of these theorems may be found in [11]. Before we stating them some notation is necessary.

Let $\mathbf{G}(s)$ be an interval transfer function family:

$$\mathbf{G}(s) := \left\{ G(s) = \frac{N(s)}{D(s)} \mid N(s) \in \mathcal{N}(s), D(s) \in \mathcal{D}(s) \right\} \qquad (4.1)$$

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where $\mathcal{N}(s)$ and $\mathcal{D}(s)$ are families of interval polynomials. The magnitude and phase of a transfer function $G(s)$ at a frequency ω are defined as $\mu_G(\omega)$ and $\phi_G(\omega)$, respectively. It is natural to define the maximum and minimum values of $\mu_G(\omega)$ and $\phi_G(\omega)$ at each frequency ω.

$$
\begin{aligned}
\underline{\mu}_{\mathbf{G}}(\omega) &:= \inf_{G \in \mathbf{G}} \mu_G(\omega) \\
\bar{\mu}_{\mathbf{G}}(\omega) &:= \sup_{G \in \mathbf{G}} \mu_G(\omega)
\end{aligned}
\tag{4.2}
$$

and

$$
\begin{aligned}
\underline{\phi}_{\mathbf{G}}(\omega) &:= \inf_{G \in \mathbf{G}} \phi_G(\omega) \\
\bar{\phi}_{\mathbf{G}}(\omega) &:= \sup_{G \in \mathbf{G}} \phi_G(\omega).
\end{aligned}
\tag{4.3}
$$

Let

$$
\mathcal{K}_{\mathcal{N}}(s) := \{ K_n^i(s) \mid i = 1, 2, 3, 4 \}.
\tag{4.4}
$$

where $K_{\mathcal{N}}^i(s)$ is the ith Kharitonov polynomial associated with the interval polynomial $\mathcal{N}(s)$ and similarly, we define $\mathcal{K}_{\mathcal{D}}(s)$ to be the set of Kharitonov polynomials associated with $\mathcal{D}(s)$. We now introduce a special set of segments that joint appropriate pairs of Kharitonov polynomials. These segments are known as extremal segments.

$$
\mathcal{S}_{\mathcal{N}}(s) :=
$$

$$
\left\{ \lambda K_n^i(s) + (1 - \lambda) K_n^j(s) \mid \lambda \in [0,1], (i,j) \in \{(1,2),(1,3),(2,4),(3,4)\} \right\}
\tag{4.5}
$$

and $\mathcal{S}_{\mathcal{D}}(s)$ is defined similarly.

Introduce the following transfer function sets:

$$
\mathbf{G_K} := \left\{ \frac{N(s)}{D(s)} \mid N(s) \in \mathcal{K}_{\mathcal{N}}(s), D(s) \in \mathcal{K}_{\mathcal{D}}(s) \right\}
\tag{4.6}
$$

$$
\mathbf{G_E} := \left\{ \frac{N(s)}{D(s)} \mid (N(s), D(s)) \in (\mathcal{N}(s) \times \mathcal{D}(s))_{\mathbf{E}} \right\}
\tag{4.7}
$$

where

$$
(\mathcal{N}(s) \times \mathcal{D}(s))_{\mathbf{E}} := \{ (N(s) \times D(s)) \mid
$$

$$
N(s) \in \mathcal{K}_{\mathcal{N}}(s), D(s) \in \mathcal{S}_{\mathcal{D}}(s) \text{ or } N(s) \in \mathcal{S}_{\mathcal{N}}(s), D(s) \in \mathcal{K}_{\mathcal{D}}(s) \}.
\tag{4.8}
$$

Theorem 1. *[11] For every frequency $\omega > 0$,*

$$
\begin{aligned}
\bar{\mu}_{\mathbf{G}}(\omega) &= \bar{\mu}_{\mathbf{G_E}}(\omega) \\
\underline{\mu}_{\mathbf{G}}(\omega) &= \underline{\mu}_{\mathbf{G_E}}(\omega)
\end{aligned}
$$

Theorem 2. *[11] For every frequency $\omega > 0$,*

$$
\begin{aligned}
\bar{\phi}_{\mathbf{G}}(\omega) &= \bar{\phi}_{\mathbf{G_K}}(\omega) \\
\underline{\phi}_{\mathbf{G}}(\omega) &= \underline{\phi}_{\mathbf{G_K}}(\omega)
\end{aligned}
$$

Theorem 3. *[11] The image set of the interval transfer function* $\mathbf{G}(s)$ *evaluated at* $s = j\omega$ *in the complex plane is bounded by the image of its corresponding extremal subsets:*

$$\partial \mathbf{G}(j\omega) \subset \mathbf{G}_{\mathrm{E}}(j\omega)$$

where $\partial(\cdot)$ *denotes the boundary of a set.*

Once the image set is generated at a fixed value of $\omega = omega_i$ one can verify if the given data point $D(\omega_i)$ is contained in $\mathbf{G}(j\omega_i)$ or not. This verification can be carried out for every value of ω_i to verify whether or not the proposed interval model explains the experimental data.

In the next section, for the sake of illustration we apply these ideas to a large space structure experimental facility developed at NASA's Langley Research Center.

5. APPLICATION TO A MINI - MAST SYSTEM

5.1. Model Description

The Mini-Mast system shown in Figure 2 is a 20.16 meter long deployable truss located in the Structural Dynamics Research Laboratory at NASA Langley Research Center. It is used as a ground test article for the research in the areas of structural analysis, system identification, and control of large space structures. The Mini-Mast was constructed from graphite-epoxy tubes and titanium joints, by using precision fabrication techniques. The 102 measurements shown in Figure 2 were derived using 51 noncontacting displacement sensors distributed from Bay 2 through Bay 18. Three shakers are located circumferentially around the truss at Bay 9 and their locations are selected primarily to excite the low frequency modes below 10 Hz. There are three modes which consist of two bending and one torsion mode in this low frequency range. These three modes are designed to be separated from the other frequency modes. The experimental data used in this example are obtained by using one displacement sensor output at Bay 9 from one input. In this example, we use the experimental data within the 45 radian/sec low frequency range with 180 frequency data points. This low frequency range covers the three low frequency modes described earlier. Figure 3 shows the frequency response test data.

5.2. Interval Model Identification

Using the weighted least square method described earlier, we select $W^I(j\omega)$ shown in Figure 4. The identified model obtained is

$$G^I(s) = \frac{n_0 + n_1 s + n_2 s^2 + n_3 s^3 + n_4 s^4 + n_5 s^5}{d_0 + d_1 s + d_2 s^2 + d_3 s^3 + d_4 s^4 + d_5 s^5 + s^6}$$

where

$$n_0 = -5.78 \times 10^4 \qquad d_0 = 2.96 \times 10^7$$
$$n_1 = 5.88 \times 10^2 \qquad d_1 = 2.15 \times 10^5$$
$$n_2 = -8.74 \times 10^2 \qquad d_2 = 1.10 \times 10^6$$
$$n_3 = 0.073 \qquad d_3 = 2.75 \times 10^3$$
$$n_4 = -0.967 \qquad d_4 = 2.21 \times 10^3$$
$$n_5 = 3.48 \times 10^{-5} \qquad d_5 = 2.58$$

The eigenvalues of the identified model transfer function are as follows:

Model no.	Eigenvalues	Mode
1	$-7.11 \times 10^{-2} \pm j5.356$	first bending mode
2	$-4.22 \times 10^{-1} \pm j26.302$	first torsion mode
3	$-7.99 \times 10^{-1} \pm j38.616$	second bending mode

The magnitude and phase comparison of the test data and the identified model are given in Figures 5 and 6. In Figures 5 and 6, the dashed lines denote the frequency response of $D(j\omega)$ and the solid lines denote the frequency response of $G^I(j\omega)$. The dotted lines in Figure 6 indicates the error in magnitude (i.e., $|D(j\omega) - G^I(j\omega)|$) for illustration.

We now create intervals around this nominal identified model. The weight selection method described in Section 3.2. gives the following weights for each coefficient:

$$\mathrm{w}_{n_0} = 2.7053 \times 10 \qquad \mathrm{w}_{d_0} = 3.9152 \times 10^3$$
$$\mathrm{w}_{n_1} = 1.2041 \qquad \mathrm{w}_{d_1} = 2.2715 \times 10^2$$
$$\mathrm{w}_{n_2} = 2.3214 \times 10^{-1} \qquad \mathrm{w}_{d_2} = 5.8095 \times 10$$
$$\mathrm{w}_{n_3} = 4.0113 \times 10^{-3} \qquad \mathrm{w}_{d_3} = 1.5250$$
$$\mathrm{w}_{n_4} = 2.4768 \times 10^{-4} \qquad \mathrm{w}_{d_4} = 5.9161 \times 10^{-2}$$
$$\mathrm{w}_{n_5} = 2.9620 \times 10^{-6} \qquad \mathrm{w}_{d_5} = 1.2520 \times 10^{-3}$$

This set of weights produced the following interval system:

$$\mathbf{G}(s) := \frac{\hat{n}_0 + \hat{n}_1 s + \hat{n}_2 s^2 + \hat{n}_3 s^3 + \hat{n}_4 + \hat{n}_5 s^5}{\hat{d}_0 + \hat{d}_1 s + \hat{d}_2 s^2 + \hat{d}_3 s^3 + \hat{d}_4 + \hat{d}_5 s^5 + s^6}$$

where

$$\hat{n}_0 \in [-6.0061, -5.6661] \times 10^4 \qquad \hat{d}_0 \in [2.9361, 2.9795] \times 10^7$$
$$\hat{n}_1 \in [\ 4.7696,\ 7.8401] \times 10^2 \qquad \hat{d}_1 \in [2.0663, 2.2335] \times 10^5$$
$$\hat{n}_2 \in [-8.9589, -8.6096] \times 10^2 \qquad \hat{d}_2 \in [1.0923, 1.0986] \times 10^6$$
$$\hat{n}_3 \in [-4.9791,\ 5.6904] \times 10^{-1} \qquad \hat{d}_3 \in [2.6720, 2.8406] \times 10^3$$
$$\hat{n}_4 \in [-9.7491, -9.3729] \times 10^{-1} \qquad \hat{d}_4 \in [2.2090, 2.2186] \times 10^3$$
$$\hat{n}_5 \in [-0.8570,\ 1.1648] \times 10^{-4} \qquad \hat{d}_5 \in [2.5255, 2.6190] \times 10^0$$

5.3. Model Validation

The following figures show that the interval model obtained here is a valid interval model for the given test data set. First, Figure 3 shows the polar plot of the test data for every measured frequency. Each mode of the polar plot has been separated in

Figures 7,8, and 9 for illustration. These figures show that every data point of the test data is bounded by the image set generated by the interval model at the corresponding frequency. Figure 10 was drawn for the entire frequency range. Similarly, Figures 11 and 12 show the magnitude and phase plots of the test data and the interval model. Clearly, both magnitude and phase plots of the test data are contained in the tightly bounded tubes representing the boundary of the frequency responses of the interval system.

6. CONCLUDING REMARKS

A new algorithm to construct an interval transfer function from available frequency domain data of the plant is presented. This model captures the complete frequency domain characteristics that a single identified model cannot capture. This interval model can be directly useful to analyse the robustness properties of any proposed controller using the well developed theory of parametric robust control. Such an interval model can predict the worst case stability margins associated with the controller. This serves as a lower bound on the worst case stability margin of the actual (rather than the model) system.

REFERENCES

[1] R. L. Kosut, G. C. Goodwin, and M. P. Polis, "Introduction: special issue on system identification for control design," *IEEE Transactions on Automatic Control*, vol. 37, p. 899, July 1992.

[2] "Special issue on system identification for robust control design," *IEEE Transactions on Automatic Control*, vol. 37, No. 7, July 1992.

[3] R. G. Hakvoort, "Worst-case system identification in ℓ_1: error bounds, optimal models and model reduction," in *Proceedings of the IEEE Conference on Decision and Control*, (Tucson, AZ), pp. 499 – 504, December 1992.

[4] R. Ravi, J. Chen, and C. N. Nett, "Computing nominal models for uncertain plants," in *Proceedings of the IEEE Conference on Decision and Control*, (Tucson, AZ), pp. 517 – 518, December 1992.

[5] C. M. Belcastro and B. C. Chang, "On parametric uncertainty modeling for real parameter variations," in *Proceedings of the IEEE Conference on Decision and Control*, (Tucson, AZ), pp. 674 – 579, December 1992.

[6] V. L. Kharitonov, "Asymptotic stability of an equilibrium position of a family of systems of linear differential equations," *Differential Uravnen*, vol. 14, pp. 2086 – 2088, 1978.

[7] D. S. Bayard, F. Y. Hadaegh, Y. Yam, R. E. Scheid, E. Mettler, and M. H. Milman, "Automated on-orbit frequency domain identification for large space structures," *Automatica*, vol. 27, pp. 931 – 946, November 1991.

[8] J. L. Adcock, "Curve fitter for pole - zero analysis," *Hewlett - Packard Journal,* Janurary 1987.

[9] J. -N. Juang and R. S. Pappa, "An eigensystem realization algorithm for modal parameter identification and model reduction," *AIAA Journal of Guidance, Control, and Dynamics,* vol. 8, pp. 620 – 627, Sep. - Oct. 1985.

[10] I. Gohberg, P. Lancaster, and L. Rodman, *Invariant Subspaces of Matrices with Applications.* New York: Wiley Interscience, 1986.

[11] L. H. Keel and S. P. Bhattacharyya, "Frequency domain design of interval controllers," in *Control of Uncertain Dynamic Systems,* (S. P. Bhattacharyya and L. H. Keel, eds.), Littleton, MA: CRC Press, September 1991.
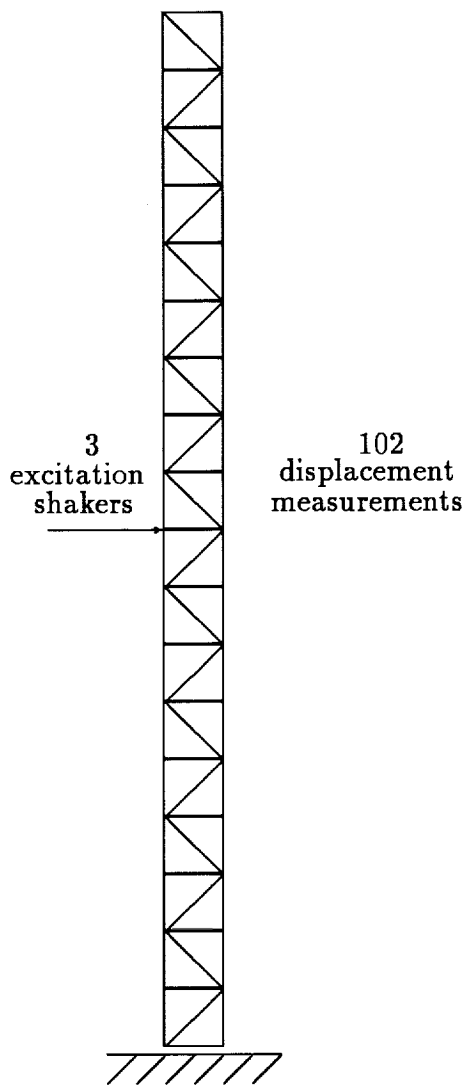
Figure 2. Mini-Mast Structure

Figure 3. Polar Plot of experimental dat

Figure 4. Weighting of least squares

Figure 5. Magnitudes of identified model, experimental data and model error

Figure 6. Phase of identified model and experimental data

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Figure 7. Image set of first mode

Figure 8. Image set of second mode

Figure 9. Image set of third mode

Figure 10. Image set of the system

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Figure 11. Magnitude plots of interval model and experimental data

Figure 12. Phase plots of interval model and experimental data