GENERIC HYPERSONIC VEHICLE PERFORMANCE MODEL

GRANT
IN-05-CR
158848
P-32

Interim Task Report

for

NASA Grant NAG-1-1341

On Research Performed for the

NASA Langley Research Center
Hampton, VA

Principal Investigators

Frank R. Chavez and Dr. David K. Schmidt
Aerospace Research center
College of Engineering and Applied Sciences
Arizona State University
Tempe, AZ 85287-8006

Internal Report No. ARC93-3

April 1993

TABLE OF CONTENTS

SECTION **PAGE**

LIST OF FIGURES

FIGURE **PAGE**

INTRODUCTION

An integrated computational model of a generic hypersonic vehicle has been developed for the purpose of determining the vehicle's performance characteristics, which include the Lift, Drag, Thrust, and Moment acting on the vehicle at specified altitude, flight condition, and vehicular configuration. The material presented herein is based on the prior work reported in [1]. The Lift, Drag, Thrust, and Moment are developed for the body fixed coordinate system. These forces and moments arise from both aerodynamic and propulsive sources. SCRAMjet engine performance characteristics, such as fuel flow rate, can also be determined.

The vehicle is assumed to be a lifting body with a single aerodynamic control surface. The body shape and control surface location are arbitrary and must be defined.

The aerodynamics are calculated using either 2-Dimensional Newtonian or Modified Newtonian theory and approximate high-Mach-number Prandtl-Meyer expansion theory. Skin-friction drag has also been accounted for. The skin-friction drag coefficient is a function of the freestream Mach number. The data for the skin-friction drag coefficient values were taken from NASA Technical Memorandum 102610 [2].

The modeling of the vehicle's SCRAMjet engine is based on quasi 1-Dimensional gasdynamics for the engine diffuser, nozzle, and the combustor with heat addition. The engine has three variable inputs for control. These are the engine inlet diffuser area ratio, the total temperature rise through the combustor due to combustion of the fuel, and the engine internal expansion nozzle area ratio.

The pressure distribution over the vehicle's lower aftbody surface, which acts as an external nozzle, is calculated using a combination of quasi 1-Dimensional gasdynamic theory and Newtonian or Modified Newtonian theory. The exhaust plume shape is determined by matching the pressure inside the plume, calculated from the gasdynamic equations, with the freestream pressure, calculated from Newtonian or Modified Newtonian theory. In this manner, the pressure distribution along the vehicle afterbody expansion surface is then determined.

The following three sections describe the aerodynamic modeling, the engine modeling, and the exhaust plume analysis in more detail. Section four gives a description of the computer code used to perform the above calculations, and an input/output example is then given in section five. The computer code is available on a Macintosh floppy disk.

AERODYNAMIC ANALYSIS

Compression Surface

The non-dimensional pressure coefficient as given by Modified Newtonian theory is given below as [3]

$$Cp_{com}(\gamma, M_\infty, \theta) = \frac{2}{\gamma M_\infty^2}\left\{\left[\frac{(\gamma+1)^2 M_\infty^2}{4\gamma M_\infty^2 - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{2\gamma M_\infty^2 - (\gamma-1)}{\gamma+1}\right] - 1\right\}\sin^2(\theta) \qquad (1)$$

where θ is the local flow deflection angle defined as

$$\theta = \alpha + \tau \qquad (2)$$

The angles α and τ are, respectively, the angle of attack and the local surface slope relative to the body fixed axis as shown in Fig 1. For Newtonian theory, the pressure coefficient value is given by $Cp_{com}(\gamma, M_\infty, \theta) = 2.0 \sin^2(\theta)$. The compression surface is divided into sections or panels each of which has a constant value for τ, say τ_i. The pressure coefficient on the corresponding panel is then given as

$$Cp_i = Cp_{com}(\gamma, M_\infty, \theta_i) \qquad (3)$$

where $\theta_i = \alpha + \tau_i$. The force per unit width acting on the i^{th} panel is given by

$$\vec{F}_i = -P_\infty \left(1 + \frac{1}{2}\gamma M_\infty^2 Cp_i\right) ds_i\, \hat{n}_i \qquad (4)$$

where ds_i is the area per unit width of the i^{th} panel and $\hat{n}_i$ is a unit vector normal to the i^{th} panel pointing outward, also shown in Fig 1. The moment per unit width acting about the vehicle's center of mass is given by

$$\vec{M}_i = \vec{r}_i \times \vec{F}_i \qquad (5)$$

where $\vec{r}_i$ is the vector from the vehicle's mass center to the centroid of the i^{th} panel.

Control Surface

The force per unit width acting on the control surface is given by

$$\vec{F}_{cs} = P_\infty \left[1 + \frac{1}{2}\gamma M_\infty^2 Cp_{com}(\gamma, M_\infty, \theta_{cs})\right] S_{cs}\, \hat{n}_{cs} \qquad (6)$$

where $\theta_{cs} = \alpha + \delta$, and δ is the control surface deflection, S_{cs} is the control surface area per unit width, and $\hat{n}_{cs}$ is a unit vector normal to the control surface as shown in Fig 2. The moment per unit width acting about the vehicle's mass center is given by

$$\vec{M}_{cs} = \vec{r}_{cs} \times \vec{F}_{cs} \qquad (7)$$

where $\vec{r}_{cs}$ is a vector from the vehicle's mass center to the center of the control surface, which is where the aerodynamic center of the control surface is assumed to be located.

Expansion Surface

The pressure coefficient as given by the approximate high Mach number Prandtl-Meyer expansion theory is [4]

$$Cp_{exp}(\gamma, M_\infty, \alpha) = \frac{2}{\gamma M_\infty^2}\left\{\left[\frac{1+\frac{(\gamma-1)}{2}M_\infty^2}{1+\frac{\frac{(\gamma-1)}{2}M_\infty^2}{\left[1-\frac{(\gamma-1)}{2}\alpha M_\infty\right]^2}}\right]^{\frac{\gamma}{\gamma-1}} - 1\right\} \tag{8}$$

where α is the angle of attack. With this, the force per unit width acting on the upper surface is given by

$$\vec{F}_{us} = -P\left[1 + \frac{1}{2}\gamma M_\infty^2\, Cp_{exp}(\gamma, M_\infty, \alpha)\right] L_u\, \hat{n}_u \tag{9}$$

where L_u is the area per unit width of the vehicle's upper surface and $\hat{n}_u$ is a unit vector normal to the upper surface pointing outward as shown in Fig 1. The moment per unit width acting about the vehicle's mass center is given by

$$\vec{M}_{us} = \vec{r}_{us} \times \vec{F}_{us} \tag{10}$$

where $\vec{r}_{us}$ is a vector from the vehicle's mass center to the center of the vehicle's upper surface.

Skin-Friction Drag

The skin-friction drag term is given as

$$D_{sf} = q_\infty\, C_{d_o}(M_\infty)\, S_{ref} \tag{11}$$

where q_∞ is the freestream dynamic pressure. The data used for $C_{d_o}(M_\infty)$ and S_{ref} were obtained from NASA Technical Memorandum 102610 "Hypersonic Vehicle Simulation: Winged Cone Configuration" [2]. This drag term is in the velocity fixed coordinate system. In the body fixed coordinate system, the lift and drag terms are as follows

$$L_o = D_{sf}\sin(\alpha) \tag{12}$$

$$D_o = D_{sf}\cos(\alpha) \tag{13}$$

The Total Aerodynamic Force and Moment

The total aerodynamic force and moment per unit width is determined by adding all the separate contributions from the compression surface panels, the control surface, the upper surface, and the skin-friction drag.

$$\vec{F}_A = \sum_i \vec{F}_i + \vec{F}_{us} + \vec{F}_{cs} - (D_o\,\hat{i} + L_o\,\hat{k}) \tag{14}$$

$$\vec{M}_A = \sum_i \vec{M}_i + \vec{M}_{us} + \vec{M}_{cs} \tag{15}$$

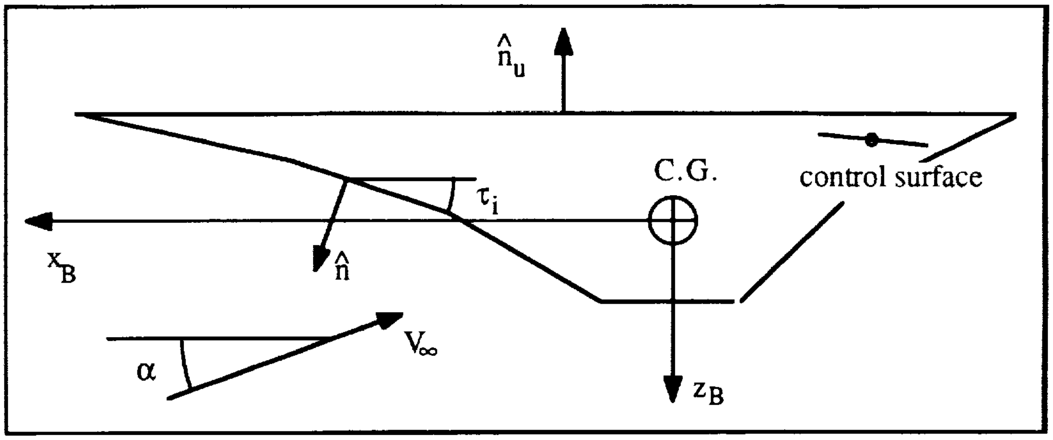


Figure 1. Vehicle Geometry

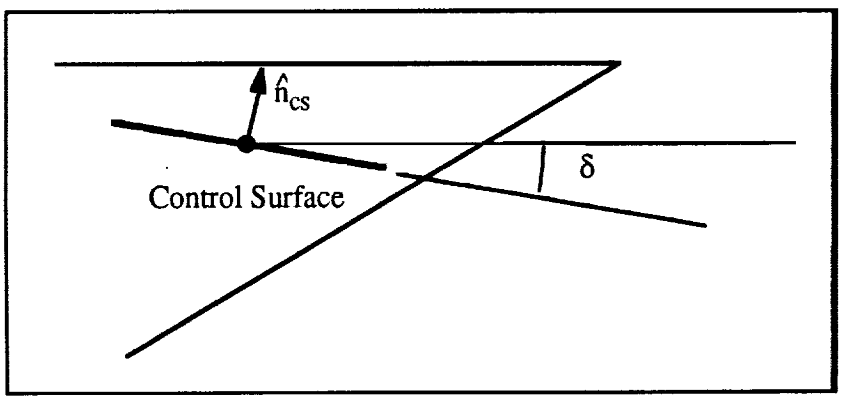


Figure 2. Control Surface Geometry

ENGINE ANALYSIS

The vehicle's SCRAMjet engine consists of three main sections which are the diffuser, the combustor, and the internal expansion nozzle. There are four stations at which the engine gas flow pressure, temperature, and Mach number are calculated, as shown in Fig 3. Station 1 is located at the engine inlet, station 2 is at the diffuser exit/combustor inlet, station 3 is at the combustor exit/nozzle inlet, and station 4 is located at the engine exit. The pressure, temperature, and Mach number at each station are calculated according to the theory described below. The thrust produced by the engine is then calculated according to the following equation [1]

$$\text{Thrust} = \left[\gamma\left(M_4^2\, P_4 - \frac{M_1^2\, P_1}{A_N\, A_D}\right) + \left(P_4 - \frac{P_1}{A_N\, A_D}\right)\right] A_e \qquad (16)$$

where, A_D is the diffuser area ratio, A_N is the internal nozzle area ratio, and A_e is the engine exit area per unit width.

Engine Inlet

At the inlet to the engine, the oncoming freestream flow is turned by the bow shock which, in accordance with Newtonian/Modified Newtonian theory, is parallel to the lower forebody surface at the inlet to the engine. The pressure, temperature, and Mach number behind the shock are then the inlet conditions to the vehicle's sSCRAMjet engine. The inlet pressure, temperature, and Mach number are given in terms of the freestream conditions and the engine inlet slope by the following set of equations which are consistent with Newtonian/Modified Newtonian theory [1]

$$P_1 = P_\infty \left[1 + \frac{1}{2}\gamma M_\infty^2\, Cp_{com}(\gamma, M_\infty, \theta_{inlet})\right] \qquad (17)$$

$$T_1 = T_\infty \left[1 + \frac{1}{2}(\gamma\text{-}1)\, M_\infty^2 \sin^2(\theta_{inlet})\right] \qquad (18)$$

$$M_1 = \frac{M_\infty \cos(\theta_{inlet})}{\sqrt{1 + \frac{1}{2}(\gamma\text{-}1)\, M_\infty^2 \sin^2(\theta_{inlet})}} \qquad (19)$$

where $\theta_{inlet} = \alpha + \tau_{inlet}$ and τ_{inlet} is the inlet slope relative to the vehicle's body fixed axis as shown in Fig 3.

After the flow is turned by the shock, it is assumed to run parallel with the inlet slope angle. This flow is then turned again, by the engine inlet, so that the flow is now parallel to the axis of the engine. This turning as assumed to occur without a change in the mass flow rate momentum; although, there is a significant turning force, which acts at the engine inlet, due to the change in direction of the flow velocity. This turning force per unit width can be calculated as [5]

$$\vec{F}_{inlet} = \gamma M_1^2 P_1 \left[(1-\cos\theta_{inlet})\,\hat{i} + (\sin\theta_{inlet})\,\hat{k}\right] \frac{A_e}{A_D A_N} \tag{20}$$

which can be seen from the vector diagram in Fig 4. Again, it is noted that this turning force is due solely to the change in the flow velocity direction as it turns into the engine. The moment per unit width acting about the vehicle mass center due to this turning force is given by

$$\vec{M}_{inlet} = \vec{r}_{inlet} \times \vec{F}_{inlet} \tag{21}$$

where $\vec{r}_{inlet}$ is the vector from the vehicle mass center to the engine inlet.

Engine Diffuser

The gasdynamic equations which relate the diffuser exit conditions to the diffuser inlet conditions are given by [4]

$$\frac{\left(1+\frac{1}{2}(\gamma-1)\,M_2^2\right)^{\frac{\gamma+1}{\gamma-1}}}{M_2^2} = A_D^2\,\frac{\left(1+\frac{1}{2}(\gamma-1)\,M_1^2\right)^{\frac{\gamma+1}{\gamma-1}}}{M_1^2} \tag{22}$$

$$P_2 = P_1 \left[\frac{1+\frac{1}{2}(\gamma-1)\,M_1^2}{1+\frac{1}{2}(\gamma-1)\,M_2^2}\right]^{\frac{\gamma}{\gamma-1}} \tag{23}$$

$$T_2 = T_1 \left[\frac{1+\frac{1}{2}(\gamma-1)\,M_1^2}{1+\frac{1}{2}(\gamma-1)\,M_2^2}\right] \tag{24}$$

The inlet conditions to the diffuser are simply the inlet conditions to the engine given by Eqns (17), (18), and (19).

Engine Combustor

The gasdynamic equations which relate the combustor exit conditions to the combustor inlet conditions are given by [4]

$$\frac{M_3^2\left[1+\frac{1}{2}(\gamma-1)\,M_3^2\right]}{\left(\gamma M_3^2+1\right)^2} = \frac{M_2^2\left[1+\frac{1}{2}(\gamma-1)\,M_2^2\right]}{\left(\gamma M_2^2+1\right)^2} + \frac{M_2^2}{\left(\gamma M_2^2+1\right)^2}\frac{T_o}{T_2} \tag{25}$$

$$P_3 = P_2 \left[\frac{1+\gamma M_2^2}{1+\gamma M_3^2}\right] \tag{26}$$

$$T_3 = T_2 \left[\frac{\left(1+\gamma M_2^2\right) M_3}{\left(1+\gamma M_3^2\right) M_2}\right]^2 \tag{27}$$

The inlet conditions to the combustor are simply the exit conditions from the diffuser, which are given by Eqns (22), (23), and (24). The quantity T_o, which denotes the total temperature rise across the combustor due to combustion of the fuel, is related to the mass flow rate of fuel by the following equation [6]

$$T_o = \frac{\left(\frac{\dot{m}_{fuel}}{\dot{m}_{air}}\right)}{\left[1+\left(\frac{\dot{m}_{fuel}}{\dot{m}_{air}}\right)\right]} \frac{\eta_c Q}{c_p} \tag{28}$$

where $\dot{m}_{fuel}$ is the mass flow rate of fuel to the engine, $\dot{m}_{air}$ is the mass flow rate of air through the engine, η_c is the combustion efficiency, Q is the heating value of the fuel, and c_p is the specific heat at constant pressure for the fuel/air mixture.

In the above equation for M_3; i.e., Eqn (25), it is important to note that M_3 is essentially a function of diffuser area ratio, A_D, and total temperature rise, T_o. For given values of A_D and T_o, the exit Mach number, M_3, must remain above $M_3 = 1$; i.e., the flow must not be choked.

Engine Internal Expansion Nozzle

The gasdynamic equations which relate the nozzle exit conditions to the nozzle inlet conditions are given by [4]

$$\frac{\left(1+\frac{1}{2}(\gamma-1)\,M_4^2\right)^{\frac{\gamma+1}{\gamma-1}}}{M_4^2} = A_N^2 \frac{\left(1+\frac{1}{2}(\gamma-1)\,M_3^2\right)^{\frac{\gamma+1}{\gamma-1}}}{M_3^2} \tag{29}$$

$$P_4 = P_3 \left[\frac{1+\frac{1}{2}(\gamma-1)\,M_3^2}{1+\frac{1}{2}(\gamma-1)\,M_4^2}\right]^{\frac{\gamma}{\gamma-1}} \tag{30}$$

$$T_4 = T_3 \left[\frac{1+\frac{1}{2}(\gamma-1)\,M_3^2}{1+\frac{1}{2}(\gamma-1)\,M_4^2}\right] \tag{31}$$

The inlet conditions to the nozzle are simply the exit conditions from the combustor which are given by Eqns (25), (26), and (27). These nozzle exit conditions are also the engine exit conditions which are used in Eqn (16), along with the engine inlet conditions given by Eqns (17), (18), and (19), to determine the thrust produced by the engine.

The Total Engine Force and Moment

With the thrust given by Eqn (16), the thrust force per unit width is given as

$$\vec{F}_{Th} = \text{Thrust}\,\hat{i} \tag{32}$$

The moment per unit width acting about the vehicle mass center is given by

$$\vec{M}_{Th} = (z_{inlet}\ \text{Thrust})\,\hat{j} \tag{33}$$

where z_{inlet} is the distance from the mass center to the centerline of the engine, which is assumed to be parallel with the vehicle's body fixed x-axis.

The total engine force and moment per unit width is then given by the sum of the separate contributions as

$$\vec{F}_E = \vec{F}_{Th} + \vec{F}_{inlet} \tag{34}$$

$$\vec{M}_E = \vec{M}_{Th} + \vec{M}_{inlet} \tag{35}$$

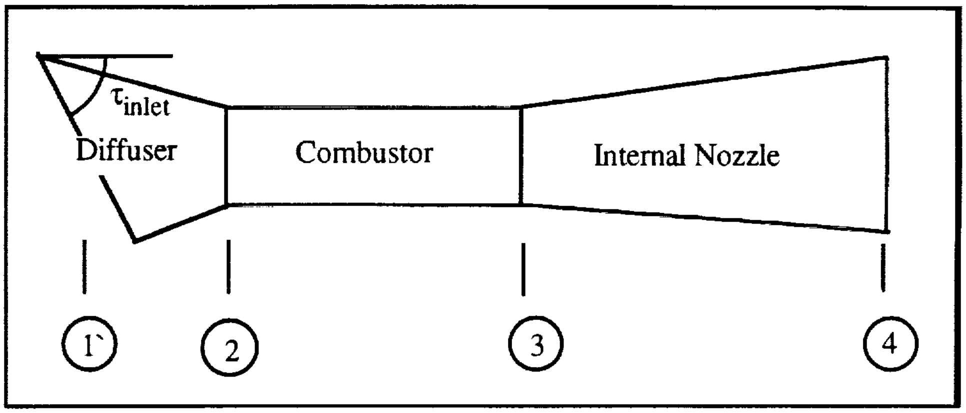


Figure 3. Engine Station Locations

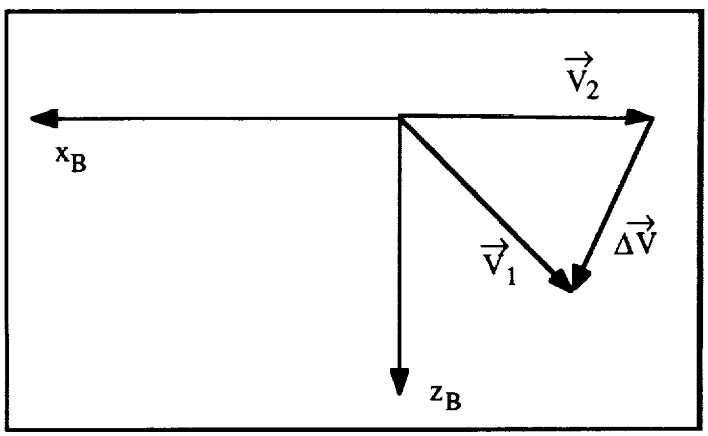


Figure 4. Engine Inlet Velocity Vector Diagram

EXHAUST PLUME ANALYSIS

Plume Position and Pressure Calculation

In Fig 5, the geometry used to perform the plume shape calculation is shown. The region enclosed by the vehicle's lower aftbody expansion surface and the plume surface is discretized into several sections. The plume shape is assumed to have a quadratic form along each of these sections given by

$$y_i = a_i + b_i\,(x - x_{i-1}) + c_i\,(x - x_{i-1})^2 \tag{36}$$

The slope of the plume shape, denoted as β_i, is then simply given by the derivative of Eqn (36)

$$\tan(\beta_i) = b_i + 2\,c_i\,(x - x_{i-1}) \tag{37}$$

The Mach number and pressure at station (i) is given in terms of the Mach number and pressure at station (i-1) through the gasdynamic equations for an expansion [4]

$$\frac{\left(1 + \frac{1}{2}(\gamma-1)\,M_i^2\right)^{\frac{\gamma+1}{\gamma-1}}}{M_i^2} = (A_D)_i^2\,\frac{\left(1 + \frac{1}{2}(\gamma-1)\,M_{i-1}^2\right)^{\frac{\gamma+1}{\gamma-1}}}{M_{i-1}^2} \tag{38}$$

$$(P_i)_{inner} = (P_{i-1})_{inner}\left[\frac{1 + \frac{1}{2}(\gamma-1)\,M_{i-1}^2}{1 + \frac{1}{2}(\gamma-1)\,M_i^2}\right]^{\frac{\gamma}{\gamma-1}} \tag{39}$$

The area ratio for section (i) is given by

$$(A_D)_i = \frac{y_i(x_i) - s_i}{y_i(x_{i-1}) - s_{i-1}} \tag{40}$$

where s_i is the position of the vehicle's lower aftbody surface at station (i). The pressure given by Eqn (39) is the pressure on the inside of the plume at station (i). The pressure on the outside of the plume at station (i) is given by Newtonian/Modified Newtonian theory as

$$(P_i)_{outer} = P_\infty\left[1 + \frac{1}{2}\gamma\,M_\infty^2\,Cp_{com}(\gamma, M_\infty, \theta_i)\right] \tag{41}$$

where $\theta_i = \alpha + \beta_i$.

The plume shape at each section has three coefficients to be determined, a_i, b_i, and c_i. Two of the coefficients, a_i and b_i, can be determined by enforcing that the plume position and slope at each station is continuous; i.e.,

$$y_i(x_{i-1}) = a_i = y_{i-1}(x_{i-1}) \tag{42}$$

$$\frac{dy_i}{dx}(x_{i-1}) = b_i = \frac{dy_{i-1}}{dx}(x_{i-1}) \tag{43}$$

The coefficient c_i is determined by imposing the condition that the inner pressure value determined from Eqn (39) and the outer pressure determined from Eqn (41) are equal. An iterative approach is used to determine the proper value of c_i at each station.

The solution procedure begins at the engine exit, where the plume position and the pressure are known. The initial values for a_1 and b_1 are given by

$$a_1 = \frac{1}{2} A_e \tag{44}$$

$$b_1 = \tan\left[\sin^{-1}\sqrt{\frac{\frac{P_4}{P_\infty} - 1}{\frac{1}{2}\gamma M_\infty^2 Cp_{com}(\gamma, M_\infty, \frac{\pi}{2})}} - \alpha\right] \tag{45}$$

The solution at each station is then determined successively using the solution obtained at the previous station.

The Total Exhaust Gas Force and Moment

With the pressure at each station now known from the above procedure, the force per unit width acting on each panel of the vehicle's lower aftbody expansion surface can be written as

$$\vec{F}_i = -\frac{1}{2}(P_{i-1} + P_i)\, ds_i\, \hat{n}_i \tag{46}$$

where ds_i is the area per unit width of the i^{th} panel and $\hat{n}_i$ is a unit vector normal to the i^{th} panel pointing outward. The moment per unit width acting about the vehicle mass center due to the i^{th} panel is given by

$$\vec{M}_i = \vec{r}_i \times \vec{F}_i \tag{47}$$

where $\vec{r}_i$ is a unit vector from the mass center to the centroid of the i^{th} panel.

The total force and moment per unit width is then just the sum of the separate contributions from each of the panels

$$\vec{F}_{Ex} = \Sigma \vec{F}_i \tag{48}$$

$$\vec{M}_{Ex} = \Sigma \vec{M}_i \tag{49}$$

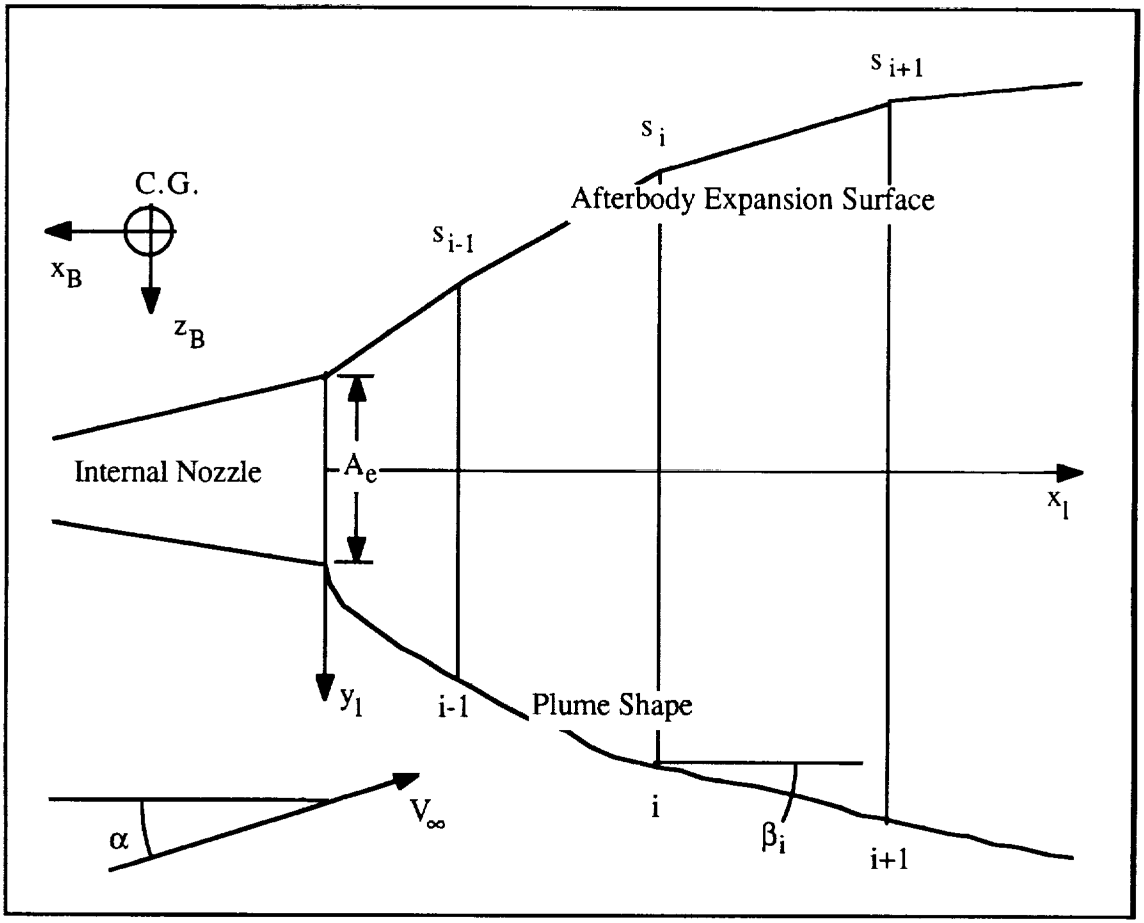
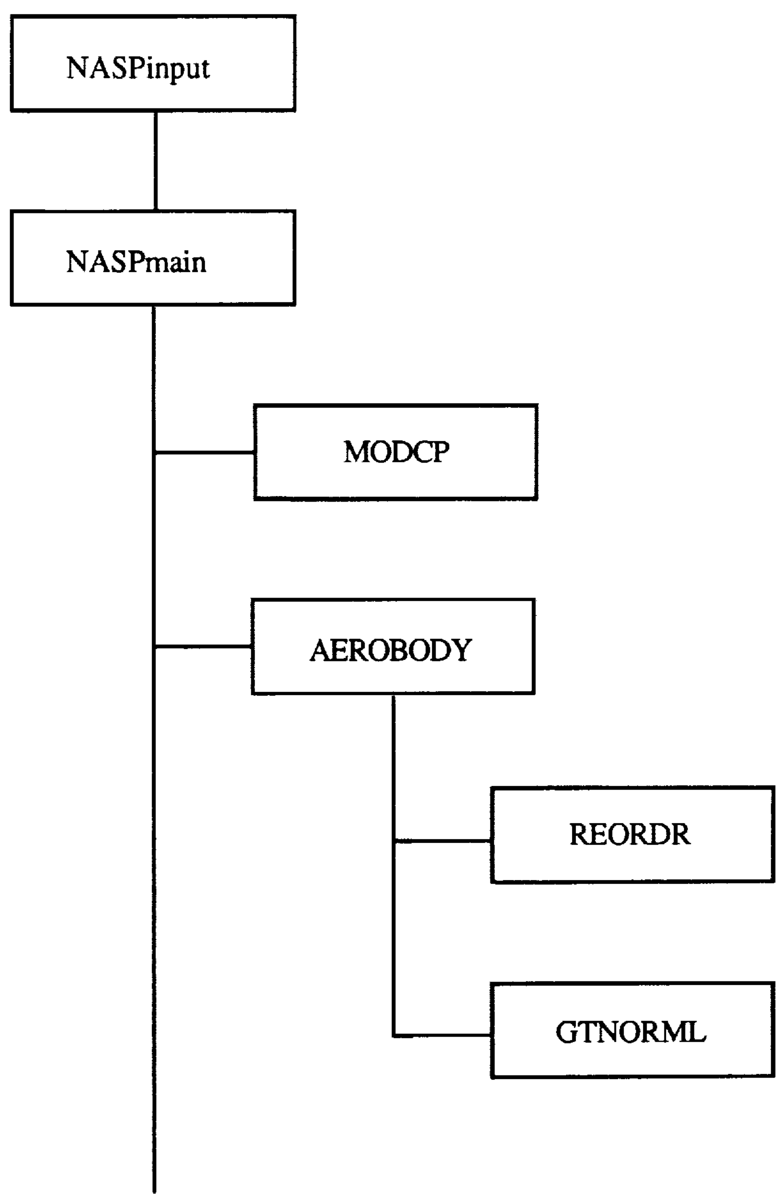


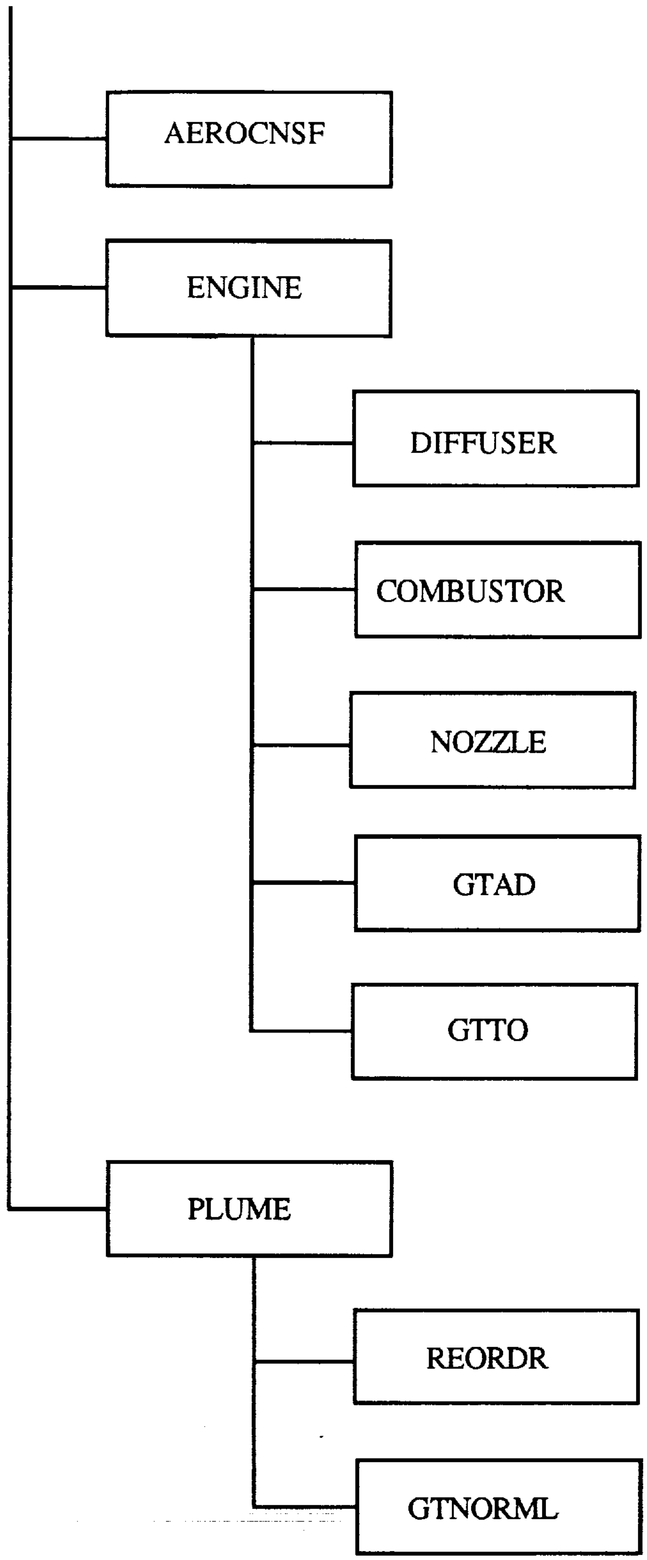
Figure 5. Plume Shape Calculation Coordinate Sysytem

MATLAB® PROGRAM DESCRIPTION

In this section, a program flow chart is given followed by a description of the subroutines called by the main program, NASPmain. The program is written in MATLAB®. The input file is named NASPinput and contains the information as shown in the example case of the next section.

Program Flow Chart




AEROCNSF
ENGINE
DIFFUSER
COMBUSTOR
NOZZLE
GTAD
GTTO
PLUME
REORDR
GTNORML

Program NASPmain

The inputs are:

given in the input file NASPinput

The outputs are:

any of the following variables described below

Variable names description:

Cpmax	- Maximum pressure coefficient value determined from Newtonian or Modified Newtonian Theory
gama	- Ratio of specific heats, set to 1.43
dtr	- Conversion factor for converting from degrees to radians
rtd	- Conversion factor for converting from radians to degrees
R	- Gas constant for air, set to 1686.0 [(lb ft)/(slug ° K)]
To	- Total temperature rise across the combustor [° K]
Ad	- Engine diffuser area ratio
Vi	- Freestream velocity [ft/s]
Machi	- Freestream Mach number
Ti	- Freestream temperature [° K]
dynpres	- Dynamic pressure [lb/ft^2]
Pi	- Freestream pressure [lb/ft^2]
arotyp	- Parameter used to determine type of 'Cpmax' calculation arotyp = 0: Newtonian theory arotyp = 1: Modified Newtonian theory
uspts	- Array of two (x,z) data points defining the upper surface [ft]
lspts	- Array of (x,z) data points defining the lower forebody compression surface [ft]
alpha	- Angle of attack [degrees]
Lb	- Lift per unit width due to upper and lower compression surface[lb/ft]
Db	- Drag per unit width due to upper and lower compression surface [lb/ft]
Mb	- Moment per unit width about the mass center due to upper and lower compression surface [lb]
delta	- Control surface deflection, positive down [degrees]
Scs	- Control surface area per unit width (includes both sides) [ft]

xcs	- x-axis location of the control surface [ft]
zcs	- z-axis location of the control surface [ft]
Lcs	- Lift per unit width due to the control surface [lb/ft]
Dcs	- Drag per unit width due to the control surface [lb/ft]
Mcs	-Moment per unit width about the mass center due to the control surface[lb]
Laero	- Total lift per unit width due to aerodynamics [lb/ft]
Daero	- Total drag per unit width due to aerodynamics [lb/ft]
Maero	- Total moment per unit width about the mass center due to aerodynamics [lb/ft]
inslp	- Engine inlet angle [degrees]
xinlt	- x-axis position of the engine inlet [ft]
zinlt	- z-axis position of the engine inlet [ft]
ctyp	- parameter used in ENGINE to set the type of engine control ctyp = 0: calculate flow quantities based on given Ad and To ctyp = 1: For the given To, Ad is calculated which will give a choked flow condition at the combustor exit ctyp = 2: For the given Ad, To is calculated which will give a choked flow condition at the combustor exit
An	- Engine internal nozzle area ratio
Ae	- Engine exit area per unit width [ft]
Leng	- Lift per unit width due to the engine [lb/ft]
Teng	- Thrust per unit width due to the engine [lb/ft]
Meng	- Moment per unit width about the mass center due to the engine [lb]
Engdat	- Matrix containing the engine flow quantity values

$$\text{Engdat} = \begin{matrix} \text{P1 [lb/ft}^2] & \text{P2 [lb/ft}^2] & \text{P3 [lb/ft}^2] & \text{P4 [lb/ft}^2] \\ \text{T1 [K]} & \text{T2 [K]} & \text{T3 [K]} & \text{T4 [K]} \\ \text{M1} & \text{M2} & \text{M3} & \text{M4} \end{matrix}$$

exspts	- Array of (x,z) data points defining the lower aftbody expansion surface [ft]
Lex	- Lift per unit width due to the exhaust gas [lb/ft]
Tex	- Thrust per unit width due to the exhaust gas [lb/ft]
Mex	- Moment per unit width about the mass center due to the exhaust gas [lb]
plmpts	- Array of (x,z) data points defining the calculated plume position [ft]

Lift	- Total lift per unit width on the vehicle [lb/ft]
Drag	- Total drag per unit width on the vehicle [lb/ft]
Thrust	- Total thrust per unit width on the vehicle [lb/ft]
Momnt	- Total moment per unit width about the mass center on the vehicle [lb]
igraph	- draws the vehicle and plume shape for igraph = 1
scale	- sets the axis scale for the vehicle/plume diagram [ft]

The maximum value for the compression surface pressure coefficient is first determined based on Newtonian or Modified Newtonian theory depending on the value of 'arotyp'. Next, the program calls AEROBODY, which calculates the lift, drag, and moment values due to the vehicle's upper expansion surface, lower compression surface, and the skin-friction drag, and AEROCNSF, which calculates the lift, drag, and moment values due to the control surface, and then adds them to obtain the total aerodynamic contribution to the vehicle's total lift, drag, and moment. Note that the 'uspts' array contains only two data points which are the coordinates of the trailing edge and leading edge of the upper surface, which is assumed to be parallel with the body fixed x-axis. The engine contribution to the total vehicle lift, thrust, and moment are then calculated by calling ENGINE. Finally, PLUME is called which calculates the lift, thrust, and moment due to the exhaust gas flow over the vehicle's lower aftbody expansion surface. The plume shape is also calculated and returned for graphing if desired. The total vehicle lift, drag, thrust, and moment is then calculated by adding the separate contributions from the aerodynamics, engine, and exhaust gas flow.

Subroutine MODCP

The inputs are:

M	- Freestream Mach number

The outputs are:

cpvalue	- The maximum pressure coefficient value calculated from Modified Newtonian theory

This subroutine calculates the maximum pressure coefficient value based on Modified Newtonian theory which is the coefficient on the $\sin^2(\theta)$ term in Eqn (1).

Subroutine AEROBODY

The inputs are:

uspts	- Array of (x,z) data points defining the upper surface [ft]
lspts	- Array of (x,z) data points defining the lower forebody compression surface [ft]
Mi	- Freestream Mach number
Pi	- Freestream pressure [lb/ft^2]
qi	- Freestream dynamic pressure [lb/ft^2]
alpha	- Angle of attack [degrees]

The outputs are:

L	- Lift per unit width due to upper and lower compression surface[lb/ft]
D	- Drag per unit width due to upper and lower compression surface [lb/ft]
M	- Moment per unit width about the mass center due to upper and lower compression surface [lb]

The subroutine first determines if the upper surface is an expansion surface ($\alpha \geq 0$) or a compression surface ($\alpha < 0$) and determines the upper surface pressure accordingly. The two points defining the upper surface are then reordered, through a call to REORDR, so that the GTNORML subroutine returns the correct direction for the upper surface unit normal vector. The upper surface force and moment is then calculated according to Eqns (9) and (10). The same procedure is followed for the lower compression surface panels. For each panel, the unit normal vector is determined from which the panel force and moment are calculated according to Eqns (4) and (5). The separate panel forces and moments are then summed to obtain the total force and moment due the compression surface. Next, the skin-friction drag coefficient is obtained for the particular freestream Mach number input. The skin-friction drag force is then calculated and transformed from the velocity fixed coordinate system to the body fixed coordinate system using Eqns (11), (12), and (13). The total lift and drag is then determined from the components of the total force vector.

Subroutine AEROCNSF

The inputs are:

delta	- Control surface deflection, positive down [degrees]
Scs	- Control surface area per unit width (includes both sides) [ft]

xcs	- x-axis location of the control surface [ft]
zcs	- z-axis location of the control surface [ft]
Mi	- Freestream Mach number
Pi	- Freestream pressure [lb/ft^2]
qi	- Dynamic pressure [lb/ft^2]
alpha	- Angle of attack [degrees]

The outputs are:

L	- Lift per unit width due to the control surface [lb/ft]
D	- Drag per unit width due to the control surface [lb/ft]
M	-Moment per unit width about the mass center due to the control surface[lb]

This subroutine calculates the lift, drag, and moment about the vehicle mass center due to the control surface. The force acting on the control surface is first calculated in the velocity fixed coordinate system. This force is then transformed to the body fixed coordinate system depending on which side (upper or lower) of the control surface is exposed to the oncoming freestream flow. The lift and drag are then determined from the components of the force vector. The moment about the vehicle mass center is calculated according to Eqn (7).

Subroutine ENGINE

The inputs are:

alpha	- Angle of attack [degrees]
inslp	- Engine inlet angle [degrees]
xinlt	- x-axis position of the engine inlet [ft]
zinlt	- z-axis position of the engine inlet [ft]
Pi	- Freestream pressure [lb/ft^2]
Ti	- Freestream temperature [° K]
Mi	- Freestream Mach number
ctyp	- Parameter used in ENGINE to set the type of engine control ctyp = 0: calculate flow quantities based on given Ad and To ctyp = 1: For the given To, Ad is calculated which will give a choked flow condition at the combustor exit ctyp = 2: For the given Ad, To is calculated which will give a choked flow condition at the combustor exit
An	- Engine nozzle area ratio

Ae	- Engine exit area per unit width [ft]

The outputs are:

L	- Lift per unit width due to the engine [lb/ft]
T	- Thrust per unit width due to the engine [lb/ft]
My	- Moment per unit width about the mass center due to the engine [lb]
data	- Matrix containing the engine flow quantity values

$$\text{data} = \begin{matrix} P1\ [lb/ft^2] & P2\ [lb/ft^2] & P3\ [lb/ft^2] & P4\ [lb/ft^2] \\ T1\ [K] & T2\ [K] & T3\ [K] & T4\ [K] \\ M1 & M2 & M3 & M4 \end{matrix}$$

P1	- Pressure at engine/diffuser inlet
T1	- Temperature at engine/diffuser inlet
M1	- Mach no. at engine/diffuser inlet
P2	- Pressure at diffuser exit/combustor inlet
T2	- Temperature at diffuser exit/combustor inlet
M2	- Mach no. at diffuser exit/combustor inlet
P3	- Pressure at combustor exit/nozzle inlet
T3	- Temperature at combustor exit/nozzle inlet
M3	- Mach no. at combustor exit/nozzle inlet
P4	- Pressure at nozzle/engine exit
T4	- Temperature at nozzle/engine exit
M4	- Mach no. at nozzle/engine exit

The inlet conditions to the engine are first calculated using Eqns (17), (18), and (19). Next, depending on the type of engine control; i.e. 'ctyp' value, the flow conditions across the diffuser, combustor, and internal nozzle are calculated through calls to the subroutines DIFFUSER, COMBUSTOR, NOZZLE, GTAD, and GTTO. The subroutines GTAD and GTTO are called to determine the diffuser area ratio, A_D, or total temperature rise required, T_o, to give a choked flow condition; i.e., M3 = 1.1, at the combustor exit. The values are then stored in the matrix 'data'. The engine thrust and turning force are then determined using Eqns (16) and (20) respectivley. The total moment about the vehicle's mass center is then calculated from Eqn (35).

Subroutine PLUME

The inputs are:

exspts	- Array of (x,z) data points defining the lower aftbody expansion surface [ft]
Pi	- Freestream pressure [lb/ft^2]
Mi	- Freestream Mach number
Engdat	- Matrix containing the engine flow quantity values

$$\text{Engdat} = \begin{matrix} P1\ [lb/ft^2] & P2\ [lb/ft^2] & P3\ [lb/ft^2] & P4\ [lb/ft^2] \\ T1\ [K] & T2\ [K] & T3\ [K] & T4\ [K] \\ M1 & M2 & M3 & M4 \end{matrix}$$

Ae	- Engine exit area per unit width [ft]
alpha	- Angle of attack [degrees]

The outputs are:

L	- Lift per unit width due to the exhaust gas [lb/ft]
T	- Thrust per unit width due to the exhaust gas [lb/ft]
My	- Moment per unit width about the mass center due to the exhaust gas [lb]
plmpts	- Array of (x,z) data points defining the calculated plume position [ft]

The lower aftbody surface points are first transformed to the local plume calculation coordinate system as shown in Fig 5 and the initial values for the coefficients 'a' and 'b' are set according to Eqns (44) and (45).

The iterative process for determing the plume position begins by initialy setting the coefficient 'c' to c = 0. The section area ratio is then determined form Eqn (40) and an inner iteration method is used to solve Eqn (38) for the current station Mach number. The inner pressure is then determined from Eqn (39). The slope of the plume is calculated from Eqn (37) and used in Eqn (41) to determine the outer pressure due to aerodynamics. A Newton Method is then used to obtain an updated value for 'c' based on the difference in inner and outer pressure value. The above procedure is repeated until the difference in inner and outer pressure values is within a specified tolerance. The (x,z) data points defining the position of the plume are then transformed back to the body fixed coordinate system and stored in the array 'plmpts'.

The force on each lower surface panel is determined according to Eqn (46) and the total lift and thrust is determined from the total force vector components. The total moment about the vehicle mass center is determined according to Eqns (47) and (49).

Subroutine DIFFUSER

The inputs are:

P1	- Diffuser inlet pressure [lb/ft^2]
T1	- Diffuser inlet temperature [° K]
M1	- Diffuser inlet Mach number

The outputs are:

P2	- Diffuser exit pressure [lb/ft^2]
T2	- Diffuser exit temperature [° K]
M2	- Diffuser exit Mach number

The subroutine simply solves Eqns (22), (23), and (24) for the diffuser exit conditions for a specified diffuser area ratio. An iterative method is used to solve Eqn (22) for 'M2'.

Subroutine COMBUSTOR

The inputs are:

P1	- Combustor inlet pressure [lb/ft^2]
T1	- Combustor inlet temperature [° K]
M1	- Combustor inlet Mach number

The outputs are:

P2	- Combustor exit pressure [lb/ft^2]
T2	- Combustor exit temperature [° K]
M2	- Combustor exit Mach number

The subroutine simply solves Eqns (25), (26), and (27) for the combustor exit conditions for a specified total temperature rise across the combustor. The subroutine checks for choked flow conditions. If the inlet conditions and total temperature rise value combination yields a subsonic exit Mach number, then an error message is displayed. An iterative method is used to solve Eqn (25) for 'M2'.

Subroutine NOZZLE

The inputs are:

P1	- Nozzle inlet pressure [lb/ft^2]
T1	- Nozzle inlet temperature [° K]
M1	- Nozzle inlet Mach number
An	- Engine nozzle area ratio

The outputs are:

P2	- Nozzle exit pressure [lb/ft^2]
T2	- Nozzle exit temperature [° K]
M2	- Nozzle exit Mach number

The subroutine simply solves Eqns (29), (30), and (31) for the internal nozzle exit conditions. An iterative method is used to solve Eqn (29) for 'M2'.

Subroutine GTAD

The inputs are:

P1	- Diffuser inlet pressure [lb/ft^2]
T1	- Diffuser inlet temperature [° K]
M1	- Diffuser inlet Mach number

The outputs are:

P2	- Diffuser exit/Combustor inlet pressure [lb/ft^2]
T2	- Diffuser exit/Combustor inlet temperature [° K]
M2	- Diffuser exit/Combustor inlet Mach number
P3	- Combustor exit pressure [lb/ft^2]
T3	- Combustor exit temperature [° K]
M3	- Combustor exit Mach number

This subroutine determines the diffuser area ratio required for combustor choked flow exit conditions given engine/diffuser inlet conditions and the total temperature rise through the combustor. Eqn (24) is first substituted into Eqn (25) for 'T2', then the resulting equation is solved for 'M2' which gives a choked flow combustor exit Mach number M3 = 1.1. With 'M2' known, Eqn (22) is used to determine 'Ad', the diffuser area ratio. The diffuser exit pressure and temperature are determined from Eqns (23) and (24) and the combustor exit pressure and temperature are determined from Eqns (26) and (27).

Subroutine GTTO

The inputs are:

P1	- Combustor inlet pressure [lb/ft^2]
T1	- Combustor inlet temperature [° K]
M1	- Combustor inlet Mach number

The outputs are:

P2	- Combustor exit pressure [lb/ft^2]
T2	- Combustor exit temperature [° K]
M2	- Combustor exit Mach number

This subroutine determines the total temperature rise through the combustor required for combustor choked flow exit conditions given the combustor inlet conditions. Eqn (25) is solved for 'To', the total temperature rise through the combustor. The choked flow exit Mach number is set at M2 = 1.1. Eqns (26) and (27) are used to determine the exit pressure and temperature.

Subroutine REORDR

The inputs are:

xypts -An array containing (x,z) data points to be reordered

sgn - A parameter specifying the type of reordering to be performed

sgn > 0 points ordered such that the x coordinates are in ascending order

sgn < 0 points ordered such that the x coordinates are in descending order

The outputs are:

xyptso -An array containing the reordered (x,z) data points

This subroutine simple orders the (x,z) data points such that the x coordinate is in ascending or descending order, depending on the sign of 'sgn'.

Subroutine GTNORML

The inputs are:

x2	- x coordinate of data point $(x,z)_2$
z2	- z coordinate of data point $(x,z)_2$

x1	- x coordinate of data point $(x,z)_1$
z1	- z coordinate of data point $(x,z)_1$

The outputs are:

r	- vector from the coordinate system origin to the center of a line connecting points $(x,z)_1$ and $(x,z)_2$
n	- unit vector normal to the line pointing away from the origin.
ds	- length of the line

This subroutine first determines the components of the vector 'r' and the length of the line 'ds', from the input data points. The unit normal is then calculated according to the following set of equations

$$\hat{n} \times \vec{t} = -|\vec{t}|$$

$$\hat{n} \cdot \vec{t} = 0$$

where $\vec{t}$ is the vector from $(x,z)_1$ to $(x,z)_2$.

INPUT/OUTPUT EXAMPLE

In this section an example case is given. The input file, NASPinput, is shown below. The output forces and moments are given on pages 24 and 25.

Sample NASPinput File

```
% Flight Conditions at 100,000 ft altitude
Machi=8;
Pi=44.95;
Ti=396.36;
%
% Vehicle Configuration
alpha=2;
delta=5;
Scs=22.5;
xcs=-35;
zcs=-10;
%
% Engine Configuration
inslp=18.4349;
Ae=5;
xinlt=5;
zinlt=10;
Ad=0.1482;
To=2000.0;
An=6.35;
%
% Newtonian (arotyp=0) or Modified Newtonian (arotyp=1)
arotyp=0;
%
% Type of engine control
% ctyp=0 - straight calculation
% ctyp=1 - determine Ad to choke the flow for given To
% ctyp=2 - determine To to choke the flow for given Ad
ctyp=0;
```

```
%
% Body Geometry
uspts=[-35,  -10;
       65,  -10];
%
lspts=[65.0000, -10.0000;
      59.0000,  -8.0000;
      53.0000,  -6.0000;
      47.0000,  -4.0000;
      41.0000,  -2.0000;
      35.0000,  -0.0000;
      29.0000,   2.0000;
      23.0000,   4.0000;
      17.0000,   6.0000;
      11.0000,   8.0000;
       5.0000,  10.0000;
      -5.0000,  10.0000];
%
exspts=[-5.0000,  10.0000;
       -8.0000,   6.6761;
      -11.0000,   3.6464;
      -14.0000,   0.9109;
      -17.0000,  -1.5304;
      -20.0000,  -3.6775;
      -23.0000,  -5.5304;
      -26.0000,  -7.0891;
      -29.0000,  -8.3536;
      -32.0000,  -9.3239;
      -35.0000, -10.0000];
%
igraph=0;
scale=65;
```

Sample NASP output

Lb = 3.2443e+04
Db = 7.4344e+04
Mb = 1.1016e+06

Lcs = 2.3770e+03
Dcs = 207.9627
Mcs = -8.1116e+04

Laero = 3.4820e+04
Daero = 7.4552e+04
Maero = 1.0205e+06

Leng = -2.7557e+04
Teng = 1.4958e+04
Meng = 1.1791e+04

Lex = 5.9956e+03
Tex = 5.2346e+03
Mex = -6.0108e+04

Lift = 1.3258e+04
Drag = 7.4552e+04
Thrust = 2.0192e+04
Momnt = 9.7221e+05

REFERENCES

1) Chavez, F.R. and Schmidt, D.K., " An Integrated Analytical Aeropropulsive / Aeroelastic Model for the Dynamic Analysis of Hypersonic Vehicles", Final Task Report ARC92-2, June 1992.

2) Shaughnessy, J.D., Pinckney, S.Z., McMinn, J.D., Cruz, C.I., and Kelley, M., "Hypersonic Vehicle Simulation Model: Winged Cone Configuration", NASA Technical Memorandum 102610, November 1990.

3) Lees, L., "Hypersonic Flow", Fifth International Aeronautical Conference, Los Angeles, CA, 1955.

4) Anderson, John D., Modern Compressible Flow with Historic Perspective, McGraw-Hill, 1982.

5) Bilimoria K.D., Schmidt D.K., "An Integrated Development of the Equations of Motion for an Elastic Hypersonic Flight Vehicle", Final Task Report ARC92-3, July 1992.

6) Bossard, J.A., Peck, R.E., and Schmidt D.K., "An Extended Supersonic Combustion Model for the Dynamic Analysis of Hypersonic Vehicles", Interim Task Report ARC93-2, March 1993.