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Estimating Turbine Limit Load

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ESTIMATING TURBINE LIMIT LOAD

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SUMMARY

This report presents and demonstrates a method that can be used to estimate turbine limit-load pressure ratio from turbine map information. It is a meanline analysis based on assumptions regarding the onset of choke. The required map information includes choke flow at all speeds as well as pressure ratio and efficiency at the onset of choke for design speed. A basic assumption of the analysis is that the turbine initially chokes in the last rotor. Key assumptions required for estimating limit load are the values of exit axial Mach number and exit swirl angle when the throat of the last-stage rotor initially chokes. One- and two-stage turbines were analyzed.

The estimation procedure of this report provides limit-load pressure ratios very close (within 5 percent) to those from a more rigorous flow analysis when correct values for the key assumptions are used. If the key assumptions regarding exit axial Mach number and swirl angle are not known, the limit-load pressure ratio can vary about ±25 percent from an average value over the normal design range. If the basic assumption of initial choke in the last rotor is not correct, the limit-load pressure ratio will be underestimated to a degree that depends on the particular turbine design. However, for airbreathing engine conceptual studies where turbine designs have yet to be established, this procedure provides a consistent and reasonable estimation of the turbine limit-load point.

INTRODUCTION

With given inlet conditions there is a maximum amount of power that can be produced by a turbine and an associated maximum pressure ratio that can be sustained across the turbine. This condition is referred to as turbine limiting load or limit load. Limit load occurs after the last-stage rotor has choked at its throat, and further expansion causes the axial Mach number across its exit to become unity. Beyond this point, increases in turbine pressure ratio have no further effect on the flow within the turbine blading. The limit-load values of power and pressure ratio vary with rotor speed.

The locus of turbine limit-load points is an important constraint for propulsion cycle off-design analyses. Operation of the turbine beyond limit-load pressure ratio does not yield a valid cycle match point. Unfortunately, turbine maps used for cycle analyses often do not identify the limit-load points. A turbine off-design flow analysis, such as is provided by the code of reference 1, does compute limit-load conditions, but the turbine flowpath and blading geometries must be known. For conceptual studies the turbines have yet to be designed, and the maps used are often either scaled experimental maps (i.e., from other turbines) that may not extend to the limit load or parametric maps, such as those from the code of reference 2, that do not identify the limit-load condition.

This report presents a method that can be used to estimate limit-load pressure ratio from turbine map information. It is a meanline analysis based on several assumptions regarding the onset of choke. The analysis methodology is presented. A comparison is made between limit-load pressure ratios

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estimated herein and those obtained from an off-design flow analysis code. Finally, the sensitivities of the computed limit-load pressure ratios to changes in the key assumptions are shown.

SYMBOLS

A annulus area, ft^2

f fraction of design speed

g gravitational constant, 32.17 (lbm)(ft)/(lbf)(sec^2)

M Mach number

P total pressure, lb/ft^2

R gas constant, (ft)(lbf)/(lbm)(°R)

r turbine pressure ratio, inlet to exit

T total temperature, °R

t static temperature, °R

w mass flow rate, lb/sec

α absolute flow angle, deg

β relative flow angle, deg

γ specific heat ratio

η efficiency

Subscripts:

an annulus

c choke

d design speed

ex exit

in inlet

r relative

u blade speed

x axial

θ tangential

METHOD OF ANALYSIS

A typical turbine rotor blade row and an exit velocity diagram with associated symbols are shown in figure 1. The velocity vector angles are positive in the direction of blade speed and negative when opposite to blade speed. Limit load occurs in the following manner: (1) There is a value of turbine pressure ratio r_c at which the last-stage rotor throat initially chokes and the flow Mach number is unity at both the rotor throat and the rotor exit; (2) as pressure ratio increases, the exit Mach number becomes supersonic by virtue of Prandtl-Meyer turning around the rotor trailing edge; (3) the limit-load point is reached when the axial Mach number at the last-rotor exit becomes unity.

This analysis requires as input the choke flow rate w_c at all speeds as well as pressure ratio $r_{c,d}$ and efficiency $\eta_{c,d}$ at the onset of last-stage rotor choke at design speed. Design speed refers to the turbine aerodynamic design point and not the cycle design point, which may or may not be the same. A key assumption of the analysis is that the turbine initially chokes in the last rotor and therefore the required choke information can be read directly from the available turbine map. A typical turbine map with the required input information indicated is shown in figure 2. Because maps are normally presented in terms of corrected flow and speed, standard conditions can be assumed for turbine inlet total temperature T_{in} and pressure P_{in}.

The first step of the analysis is to determine the exit velocity diagram when the last-stage rotor initially chokes and, from that, the exit annulus area. Using the input choke information and assumed choke values for exit axial Mach number $M_{x,c}$ and exit absolute flow (i.e., swirl) angle α_c, the exit flow conditions are

$$P_{ex,c} = \frac{P_{in}}{r_c} \tag{1}$$

$$\frac{\Delta T_c}{T_{in}} = \eta_c\left[1 - r_c^{(1-\gamma)/\gamma}\right] \tag{2}$$

$$T_{ex,c} = T_{in}\left(1 - \frac{\Delta T_c}{T_{in}}\right) \tag{3}$$

$$M_c = \frac{M_{x,c}}{\cos \alpha_c} \tag{4}$$

$$\frac{T_{ex,c}}{t_{ex,c}} = 1 + \frac{\gamma - 1}{2} M_c^2 \tag{5}$$

$$M_{r,c} = 1 \tag{6}$$

$$\cos \beta_c = \frac{M_{x,c}}{M_{r,c}} = M_{x,c} \tag{7}$$

and the exit blade speed is

$$M_{u,d} = M_c \sin \alpha_c - M_{r,c} \sin \beta_c \tag{8}$$

The exit annulus area is

$$A_{an} = \frac{w_c \sqrt{RT_{ex,c}}}{\sqrt{\gamma g}\, P_{ex,c} M_c \cos \alpha_c} \left(\frac{T_{ex,c}}{t_{ex,c}} \right)^{(\gamma+1)/2(\gamma-1)} \tag{9}$$

The limit-load pressure ratio is determined by the following iterative procedure: A value for turbine pressure ratio r greater than the choke pressure ratio r_c is selected. A value for exit relative Mach number M_r greater than 1 is then assumed, and the following calculation sequence (eqs. (10) to (21)) is executed. The value for M_r is incrementally changed until the values of cos α computed from equations (20) and (21) agree. If the value of exit axial Mach number M_x, as computed from equation (14), is not equal to 1, the value of r is incremented and this procedure is repeated until M_x is equal to unity. The calculation sequence is as follows:

$$P_{ex} = \frac{P_{in}}{r} \tag{10}$$

$$\frac{\Delta T}{T_{in}} = \eta \left[1 - r^{(1-\gamma)/\gamma} \right] \tag{11}$$

$$T_{ex} = T_{in} \left(1 - \frac{\Delta T}{T_{in}} \right) \tag{12}$$

The value of efficiency η at pressure ratio r can be read from the turbine map or assumed to be equal to the choke value η_c if the map does not extend to pressure ratio r.

Continuity assuming isentropic flow between the rotor throat (where $M_r = 1$) and the rotor exit locations shown in figure 1 yields the exit relative flow angle β, from which the relative velocity components are computed.

$$\cos \beta = \cos \beta_c \left[\left(\frac{\gamma + 1}{2} \right)^{(\gamma+1)/2(\gamma-1)} M_r \left(1 + \frac{\gamma - 1}{2} M_r^2 \right)^{-(\gamma+1)/2(\gamma-1)} \right]^{-1} \tag{13}$$

$$M_x = M_r \cos \beta \tag{14}$$

$$M_{r,\theta} = M_r \sin \beta \tag{15}$$

Exit static temperature, blade speed, and absolute Mach number are calculated by iterating the following four equations:

$$M_u = M_{u,d} \sqrt{\frac{t_{ex,c}}{t_{ex}}\, f} \tag{16}$$

$$M_\theta = M_{r,\theta} + M_u \tag{17}$$

$$M = \left(M_x^2 + M_\theta^2\right)^{1/2} \tag{18}$$

$$t_{ex} = T_{ex}\left(1 + \frac{\gamma - 1}{2}M^2\right)^{-1} \tag{19}$$

and

$$\cos\,\alpha = \frac{M_x}{M} \tag{20}$$

Choke flow rate w_c at speed fraction f is obtained from the turbine map, and rotor exit continuity yields

$$\cos\,\alpha = \frac{w_c\sqrt{RT_{ex}}}{\sqrt{\gamma g}\;P_{ex}MA_{an}}\left(\frac{T_{ex}}{t_{ex}}\right)^{(\gamma+1)/2(\gamma-1)} \tag{21}$$

As mentioned previously the limit-load pressure ratio is reached when the converged solution yields a value of unity for the exit axial Mach number M_x from equation (14).

RESULTS OF ANALYSIS

This section presents a comparison of results from this analysis with those from a turbine off-design flow analysis. Also presented are the sensitivities of these results to the analysis assumptions.

Comparison With Off-Design Flow Analysis

Limit-load pressure ratios calculated by this analysis for one- and two-stage turbines are compared with values determined by the off-design flow analysis of reference 1. The off-design analysis requires knowledge of turbine flowpath and blading geometries, which are presented in references 3 and 4 for the one- and two-stage turbines, respectively. The comparison of limit-load pressure ratios is shown in table I along with the choke pressure ratios and assumed values of exit axial Mach number $M_{x,c}$ and exit swirl angle α_c used for the analysis of this report. The values assumed for $M_{x,c}$ and α_c were those obtained from the off-design analysis.

The comparison of limit-load pressure ratios in table I is presented for each of the turbines at three speeds: 40, 70, and 100 percent of design. The limit-load pressure ratios from the estimation procedure of this report are all less than but quite close to those from the more rigorous off-design flow analysis. For the one-stage turbine the values compare to within 5 percent; for the two-stage turbine the values compare to within 2 percent.

Sensitivity to Assumptions

The effects of exit axial Mach number $M_{x,c}$ and exit swirl angle α_c on limit-load pressure ratio for the one- and two-stage turbines of table I are presented as the solid curves in figures 3(a) and (b), respectively. Dotted lines show the choke pressure ratios for the turbines. The ranges of values for axial Mach number and swirl angle were selected to cover normal design practice. Also shown in figure 3 are dashed curves representing stage work factor at choke onset and stator/rotor blading angles for a typical 50-percent-reaction turbine stage velocity diagram. The cross-hatched areas between the work-factor curves cover most normally encountered designs.

As seen from figure 3, the limit-load pressure ratio will vary over the normal design space even with a fixed choke pressure ratio and fixed type of velocity diagram. For the one-stage turbine with a choke pressure ratio of 2.09, the limit-load pressure ratio varies from 2.2 to 3.5; for the two-stage turbine with a choke pressure ratio of 3.65, the limit-load pressure ratio varies from 3.9 to 6.2. These represent about a ±25-percent variation around the average of these ranges of limit-load pressure ratio for both turbines.

Although many turbines will choke initially at the last-rotor exit, a turbine can choke in any blade row. As pressure ratio is increased, the last rotor will eventually choke. However, the choking pressure ratio read from the turbine map will be the value for initial choke. When initial choke does not occur in the last rotor, the limit-load pressure ratio will be underestimated because the choking pressure ratio used in the estimation procedure will be too low. The degree of inaccuracy depends on how close the last-rotor choke pressure ratio is to the initial choke pressure ratio, and this, in turn, depends on the particular turbine design. Therefore a turbine design is required if the limit-load pressure is to be known with any certainty.

SUMMARY OF RESULTS

This report presents and demonstrates a method that can be used to estimate turbine limit-load pressure ratio from turbine map information. It is a meanline analysis based on assumptions regarding the onset of choke. The required map information includes choke flow at all speeds as well as pressure ratio and efficiency at the onset of choke for design speed. A basic assumption of the analysis is that the turbine initially chokes in the last rotor. Key assumptions required for the estimation of limit load are the exit axial Mach number $M_{x,c}$ and the exit swirl angle α_c at the onset of last-rotor choke. One- and two-stage turbines were analyzed. This analysis gave the following results:

1. When correct values are used for the key assumptions, the estimation procedure of this report provides limit-load pressure ratios very close to those from a more rigorous flow analysis requiring turbine flowpath and blading geometry. For the one- and two-stage turbines analyzed the estimated limit-load pressure ratios over a range of speeds from 40 to 100 percent of design were within 5 percent of the values from the off-design flow analysis.

2. If the key assumptions regarding exit axial Mach number and swirl angle are not known, the limit-load pressure ratio will vary over the normal range of designs even with a fixed choke pressure ratio and a fixed type of stage velocity diagram. For the one- and two-stage turbines analyzed, the limit-load pressure ratios varied about ±25 percent from an average value.

3. If the basic assumption of initial choke in the last rotor is not correct, the limit-load pressure ratio will be underestimated. The degree of inaccuracy depends on the relationship of last-rotor choke to initial choke, which depends on the particular turbine design.

Turbine maps are being used for airbreathing engine conceptual studies although turbine designs have yet to be established. When these maps are used along with average values for the key assumptions (0.4 to 0.5 for exit axial Mach number $M_{x,c}$ and $-20°$ to $-30°$ for exit swirl angle α_c), this procedure provides a consistent and reasonable estimation of the turbine limit-load point.

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TABLE I.—COMPARISON OF RESULTS WITH OFF-DESIGN ANALYSIS

Parameter	One stage		Two stage	
Design pressure ratio	1.75		3.15	
Choke pressure ratio	2.09		3.65	
Exit axial Mach number, $M_{x,c}$	0.508		0.635	
Exit swirl angle, α_c, deg	−31.4		−21.6	
Speed, percent of design	This analysis	Off-design analysis	This analysis	Off-design analysis
	Limit-load pressure ratio			
100	2.45	2.55	4.01	4.08
70	2.07	2.19	3.52	3.56
40	1.73	1.82	2.97	3.01

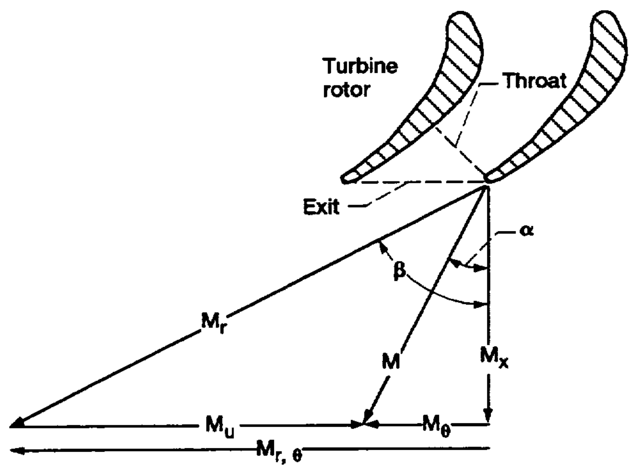

Figure 1.—Turbine rotor blade row and exit velocity diagram.

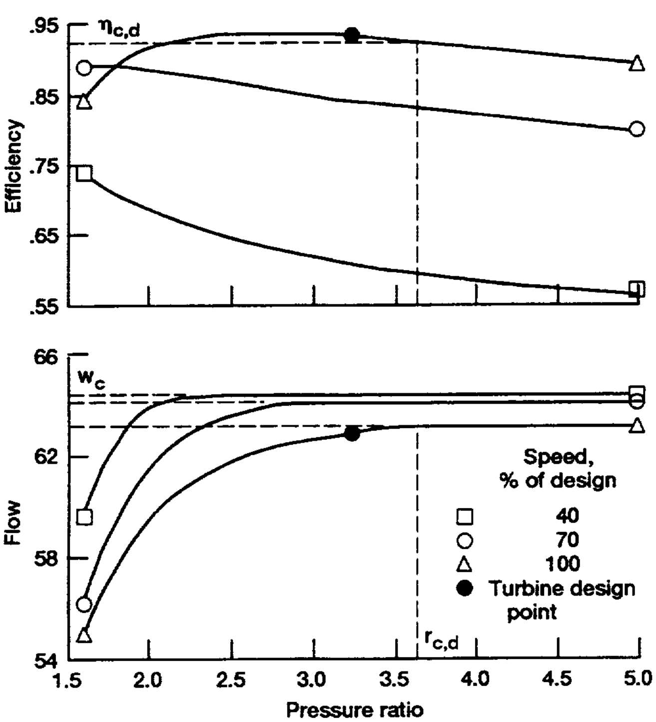

Figure 2.—Turbine map showing limit-load input variables.

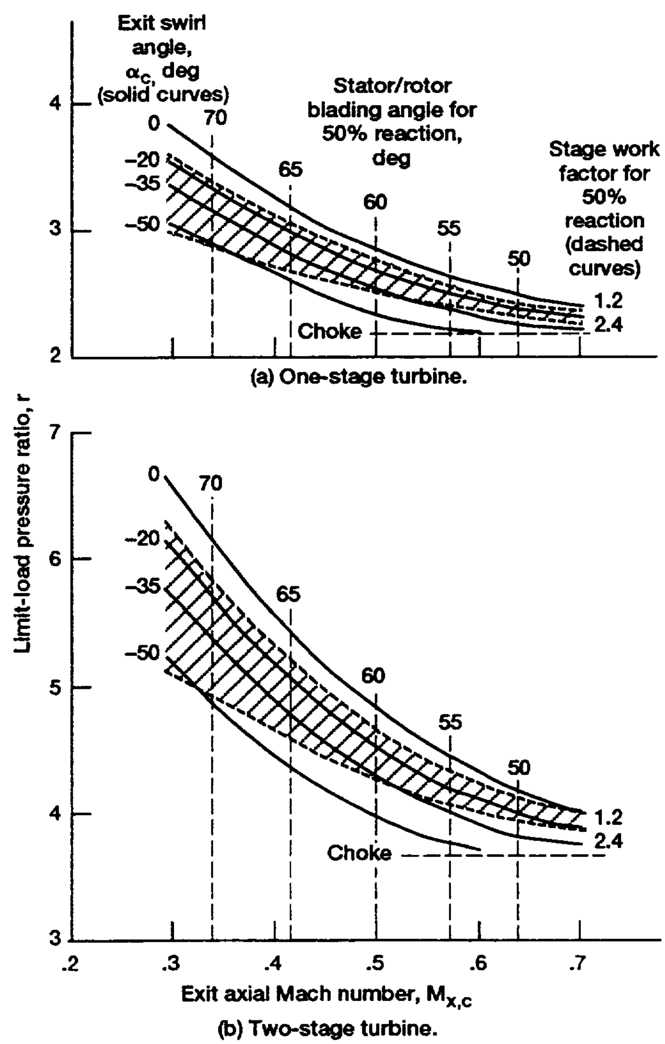

Figure 3.—Effects of exit axial Mach number and exit swirl angle on limit-load pressure ratio (100 % speed).

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A method for estimating turbine limit-load pressure ratio from turbine map information is presented and demonstrated. It is based on a meanline analysis at the last-rotor exit. The required map information includes choke flow rate at all speeds as well as pressure ratio and efficiency at the onset of choke at design speed. One- and two-stage turbines are analyzed to compare the results with those from a more rigorous off-design flow analysis and to show the sensitivities of the computed limit-load pressure ratios to changes in the key assumptions.

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