cSc

Configuration Optimization of Space Structures

Carlos Felippa
Luis A. Crivelli
David Vandenbelt

N.B. In my opinion this activity should be funded outside of CSC, as a separate grant from the [illegible] program.

Third Annual Symposium
November 21 & 22, 1991

▷ Objective

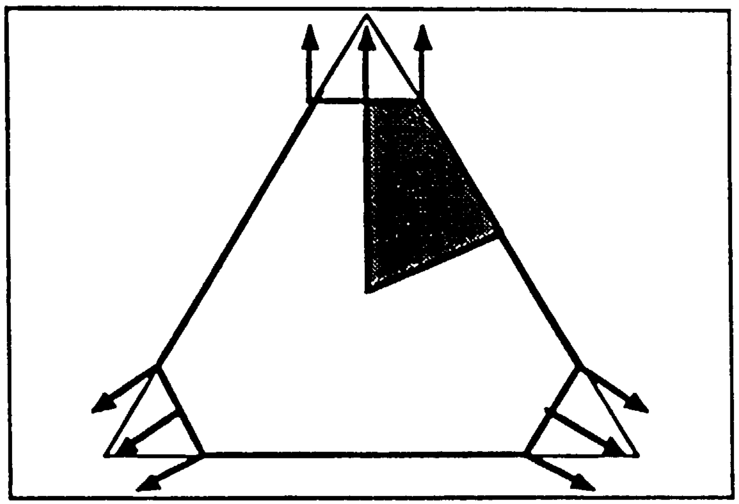
√ Develop a Computer Aid for the Conceptual/Initial Design of Aerospace Structures, Allowing Configuration and Shape to be *a priori* Design Variables.

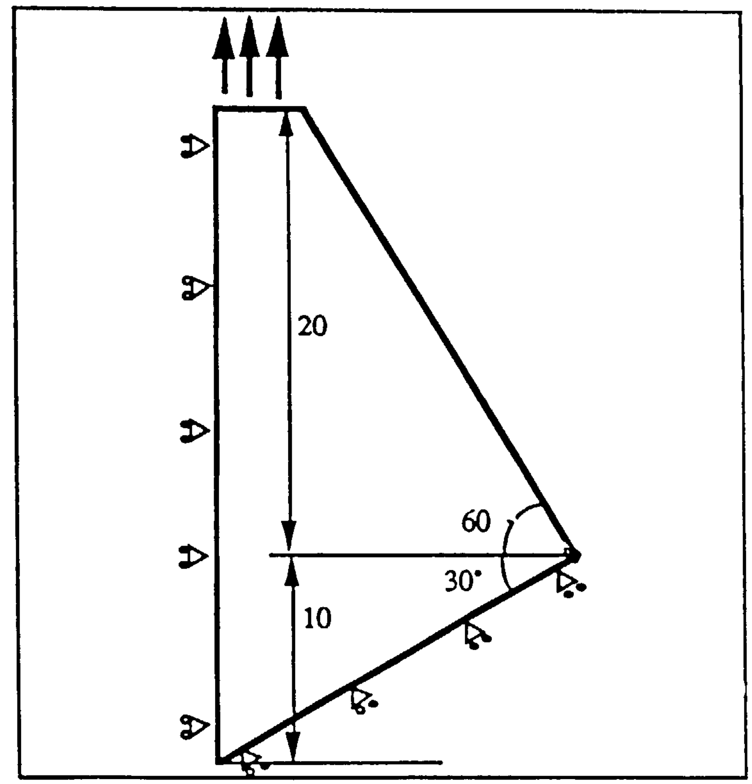
▷ Approach

√ Kikuchi's Homogenization Method:
A "Design Domain Block," Filled Initially with Homogenized Finite Elements, is Gradually "Sculpted" into an Optimal Structure under Control of an Optimization Driver.

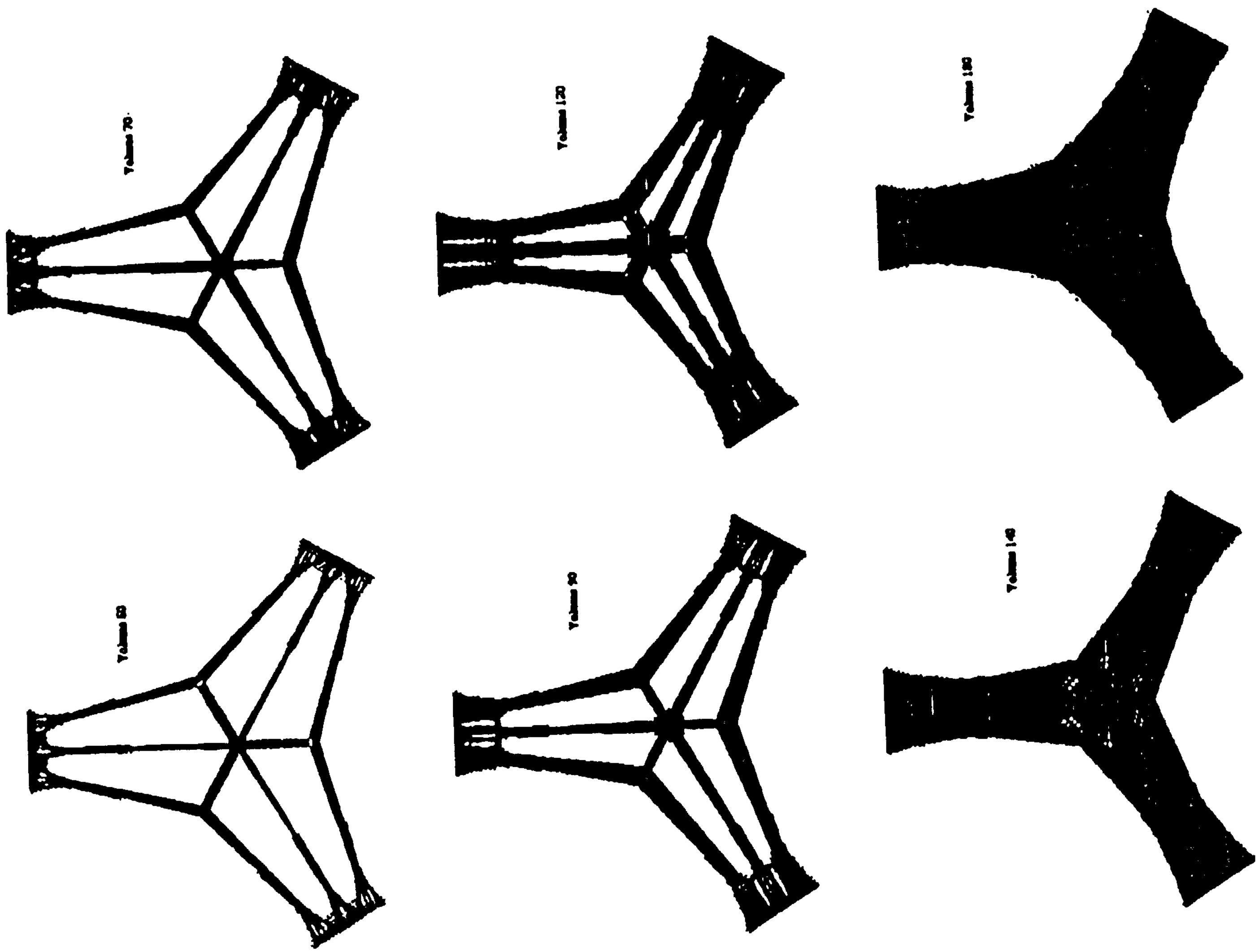
√ A *Sequence* of such Structures may be Obtained.
This can Help the Conceptual Designer.

▷ Example: A Classical Shape Design Problem

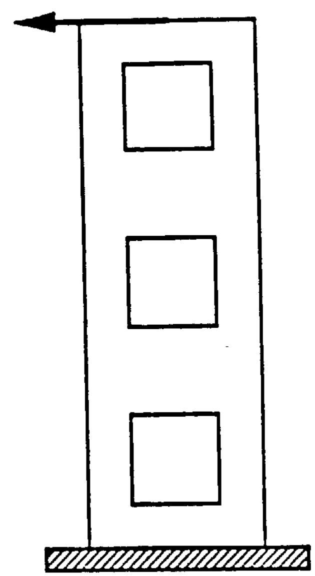


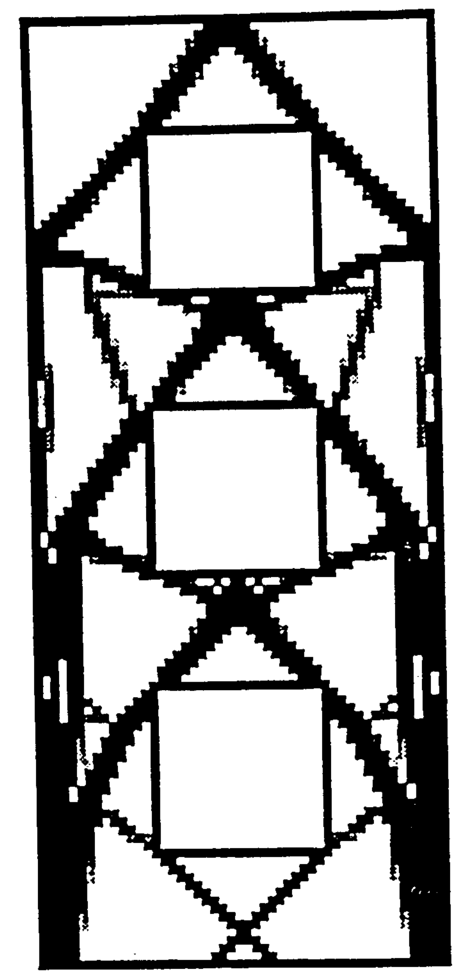


▷ Example (continued)



- Design Domain May Contain Predetermined Holes:

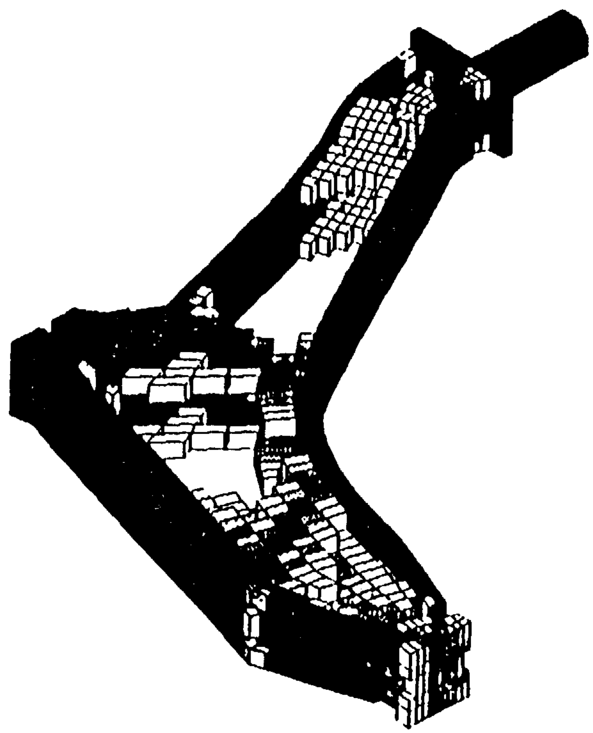


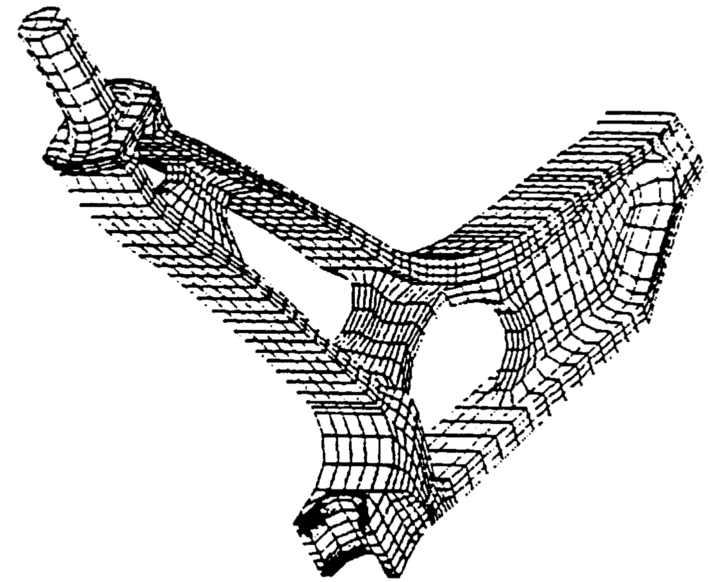


▷ Homogenization Method Steps

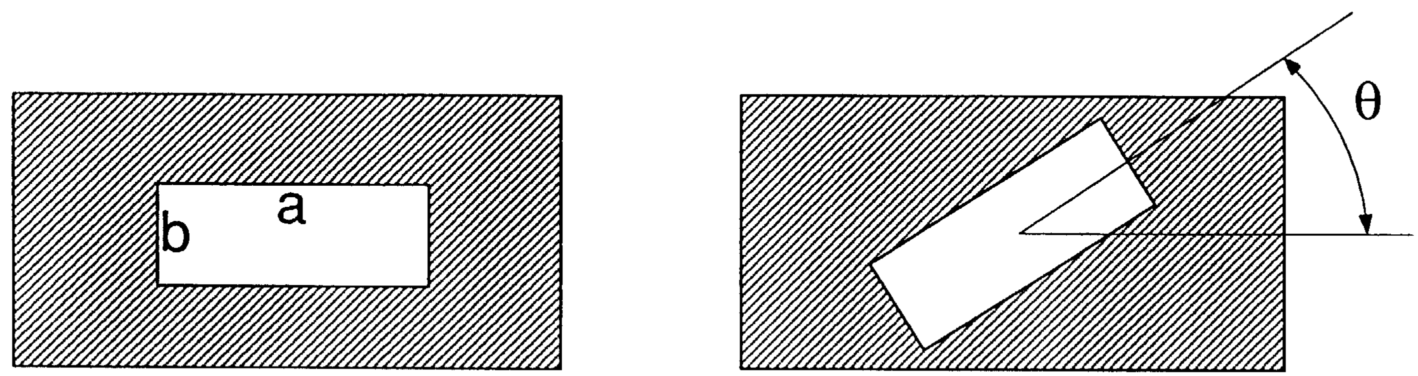
- √ Set up a Design Domain.
- √ Fill it with Homogenized Finite Elements.
- √ Define Loads and Support Conditions.
- √ Minimize an Objective Function (e.g. Compliance) under Maximum-Volume Constraint.
- √ Changing Maximum Volume Yields a Sequence of Designs.
- √ If Satisfied with a Design, Body-Fit-Remesh it, and Proceed with Standard Finite Element Analysis.

▷ Example: 3D Mechanical Component Design





▷ Element-Level Design Variables: MicroHole Dimensions



In two dimensions: a, b, θ in each element (3)

In three dimensions: a, b, c, θ_1, θ_2, θ_3 in each element (6)

100×100 2D mesh: 30,000 Design Variables

$30 \times 30 \times 30$ 3D mesh: 162,000 Design Variables

▷ Taking Advantage of Design-Variable Locality Essential

▷ Forming a Homogenized Finite Element

$$\mathbf{K}^e = \int_A h\mathbf{B}^T\mathbf{C}_H\mathbf{B}\,dA$$

$\mathbf{C}_H = \mathbf{C}_H(a,b,\theta)$ *homogenized* material response matrix

$\mathbf{C} = \mathbf{C}_H(0,0,0)$ full element; no microhole

$\mathbf{C} = \mathbf{C}_H(1,1,\theta) = \mathbf{0}$ void; microhole fills element

▹ 2-D Optimization Problem

- √ OBJECTIVE FUNCTION (COMPLIANCE ≡ INVERSE STIFFNESS)

$$\Pi(\mathbf{a},\mathbf{b},\boldsymbol{\theta}) = \mathbf{p}^T\mathbf{v}$$

- √ STIFFNESS RELATION (DISCRETE FE EQUATION)

$$\mathbf{v}(\mathbf{a},\mathbf{b},\boldsymbol{\theta}) = \mathbf{K}^{-1}(\mathbf{a},\mathbf{b},\boldsymbol{\theta})\,\mathbf{p}, \qquad \mathbf{K} = \sum_e \mathbf{L}^{eT}\mathbf{K}^e(a^e, b^e, \theta^e)\mathbf{L}^e$$

- √ VOLUME INEQUALITY CONSTRAINT

$$V(\mathbf{a},\mathbf{b}) \leq V_T = \kappa V_{domain}, \qquad 0 < \kappa \leq 1$$

- √ MICROHOLE CONSTRAINTS

$$0 \leq a^e \leq 1, \qquad 0 \leq b^e \leq 1, \qquad -45^\circ \leq \theta^e \leq 45^\circ, \qquad e = 1, \ldots N_e$$

▷ Treatment of Volume Inequality Constraint

√ Augmented Lagrangian Formulation

$$L = \Pi - \lambda_V C_- + \sigma_V C_-^2$$

where

$$\lambda_V = \text{Lagrangian multiplier estimate}$$

$$\sigma_V = \text{penalty weight}$$

$$C_- = \begin{cases} V_T - V, & \text{if } V_T < V; \\ 0, & \text{otherwise.} \end{cases}$$

▹ Algorithm for the Volume Inequality Constraint

i) Set $\lambda_V^{(1)} = \lambda_V^0,\ \sigma_V^{(1)} = \sigma_V^0,\ k = 1$

ii) Minimize $\Pi(\mathbf{a}, \mathbf{b}, \boldsymbol{\theta}, \lambda_V^{(k)}, \sigma_V^{(k)})$ keeping λ_V and σ_V fixed, with $(\mathbf{a},\mathbf{b},\boldsymbol{\theta})$ subjected to limit constraints.

iii) Compute $C = C^{(k)} = V_T - V(\mathbf{a}, \mathbf{b}, \boldsymbol{\theta})$.
If $C < 0$ and $|C| > \frac{1}{4}|C^{(k-1)}|$ set $\sigma_V = 10\sigma_V$ and go to ii)

iv) else set

$$k = k + 1$$
$$\lambda_V^{(k)} = \lambda_V^{(k-1)} - \sigma_V C$$

If $C < 0$ go to ii) else done

▷ Object Function Derivatives: Taking Advantage of Design Locality

√ Objective Function Gradients

$$\frac{\partial \mathbf{p}^T \mathbf{v}}{\partial a^e} = -\mathbf{v}^T \frac{\partial \mathbf{K}}{\partial a^e} \mathbf{v}$$

$$\frac{\partial \mathbf{p}^T \mathbf{v}}{\partial b^e} = -\mathbf{v}^T \frac{\partial \mathbf{K}}{\partial b^e} \mathbf{v}$$

√ Stifness (Discrete Equilibrium) Constraints

$$\frac{\partial \mathbf{v}}{\partial a^e} = -\mathbf{K}^{-1} \frac{\partial \mathbf{K}}{\partial a^e} \mathbf{v}$$

$$\frac{\partial \mathbf{v}}{\partial b^e} = -\mathbf{K}^{-1} \frac{\partial \mathbf{K}}{\partial b^e} \mathbf{v}$$

▷ Stiffness Variations

√ For Element Stiffness

$$\mathbf{K}^e = \int_{V^e} \mathbf{B}^T \mathbf{C}(a^e, b^e, \theta^e) \mathbf{B} \, dV^e$$

$$\frac{\partial \mathbf{K}^e}{\partial a^e} = \int_{V^e} \mathbf{B}^T \frac{\partial \mathbf{C}(a^e, b^e, \theta^e)}{\partial a^e} \mathbf{B} \, dV^e$$

$$\frac{\partial \mathbf{K}^e}{\partial b^e} = \int_{V^e} \mathbf{B}^T \frac{\partial \mathbf{C}(a^e, b^e, \theta^e)}{\partial b^e} \mathbf{B} \, dV^e$$

√ For Global Stiffness

$$\frac{\partial \mathbf{K}}{\partial a^e} = \mathbf{L}^{eT} \frac{\partial \mathbf{K}^e}{\partial a^e} \mathbf{L}^e \qquad \frac{\partial \mathbf{K}}{\partial a^e} = \mathbf{L}^{eT} \frac{\partial \mathbf{K}^e}{\partial a^e} \mathbf{L}^e$$

▹ Variations of the Potential

√ Potential

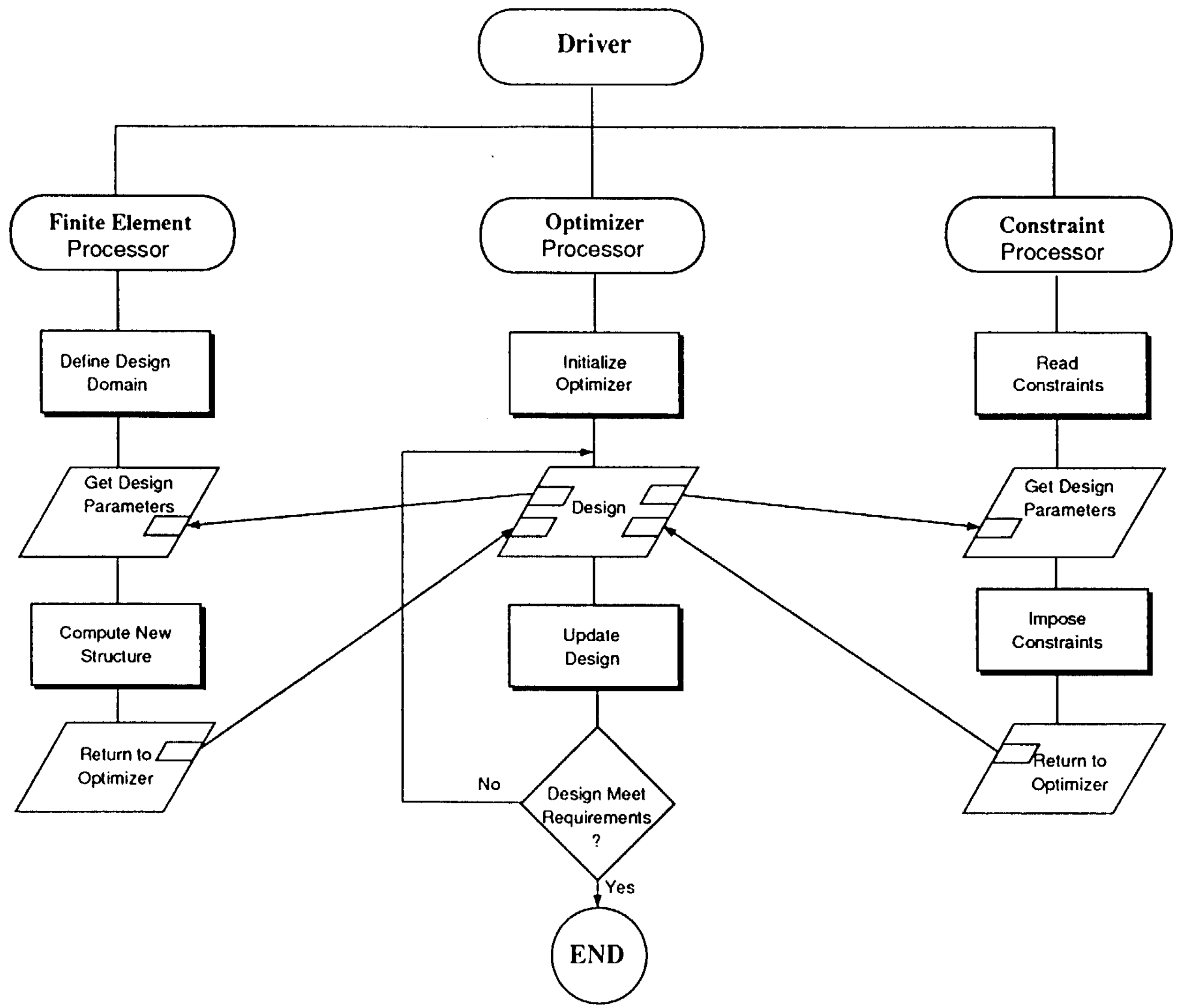
$$\Pi = \mathbf{v}^T \mathbf{K} \mathbf{v}$$

√ First Variation

$$\delta\Pi = -\mathbf{v}^T \delta\mathbf{K}\mathbf{v} \equiv -\int_A \boldsymbol{\epsilon}^T \delta\mathbf{C}\boldsymbol{\epsilon}\, dA = 0$$

√ Second Variation

$$\delta^2\Pi \simeq 2\int_A \boldsymbol{\epsilon}\delta\mathbf{C}\mathbf{C}^{-1}\delta\mathbf{C}\boldsymbol{\epsilon}\, dA - \int_A \boldsymbol{\epsilon}^T \delta^2\mathbf{C}\boldsymbol{\epsilon}\, dA$$



Schematics of the Optimization Program.

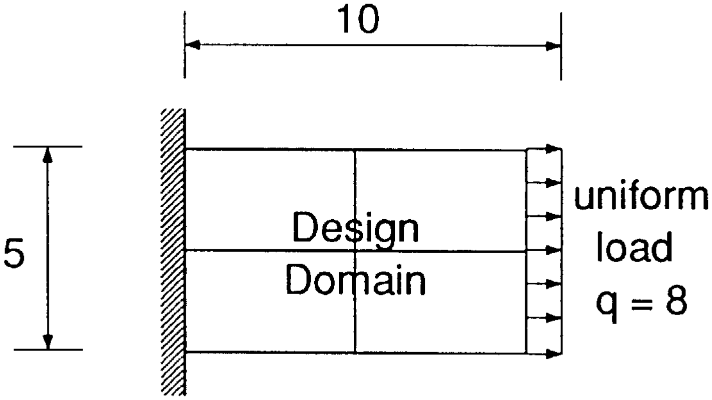
▷ Progress

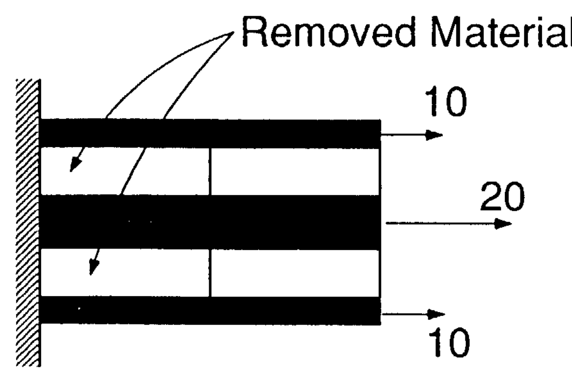
- √ Simple $\mathbf{C}_{H}$ developed and implemented.
- √ Homogenized F.E. Model of Design implemented.
- √ Optimization method:
  - √ Simulating Annealing: did not work.
  - √ Augmented Lagrangian with Conjugate Gradient:

    works for simple problems (next slide)
  - √ Augmented Lagrangian with Newton/Projected Gradient:

    implemented; under testing.

▹ Validation Problem (First Successful Solution)





$V_{ref} = 50$
$E = 10{,}000$
$\nu = 0$
$R = 1$

2x2 mesh over D.D.

Solution for 50% volume reduction
Target volume $V = \frac{1}{2} V_{ref} = 25$
Computed solution agrees with analytical solution from Lagrangian function
Minimization Method: AL + CG + CPT
189 object function evaluations

▷ Computational Issues

- √ Coping with Large Number of Design Variables (10^2–10^6):
  Adaptive Hierarchical Optimization, Domain
  Decomposition, "Hole Dropping"
- √ Handling Design-Following Loads.
- √ Handling Different Materials over Design Domain.
- √ Parallel Computations.

▷ RESEARCH ISSUES

- √ DIFFERENT OPTIMALITY CRITERIA:

  CONCURRENT OBJECT FUNCTIONS OVER DOMAIN

  (E.G. MULTIPLE LOAD CASES)

  DIFFERENT OBJECT FUNCTIONS OVER SUBDOMAINS

  (E.G. MAXIMUM ENERGY ABSORTION ON ONE,

  MINIMUM COMPLIANCE ON ANOTHER)

- √ TENSION/COMPRESSION DESIGN — CABLES, BRITTLE MATERIALS.

- √ ANISOTROPIC DESIGN — COMPOSITES.

- √ VIBRATION/STABILITY CONSTRAINTS.

cSc

Telerobotic Rovers for Extraterrestrial Construction

Why are we doing this under CSC?

Jim Avery

Third Annual Symposium
November 21 & 22, 1991