MORE RAIN COMPENSATION RESULTS

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1. INTRODUCTION

To reduce the impact of rain-induced attenuation in the 20/30 GHz band, the attenuation at a specified signal frequency must be estimated and extrapolated forward in time on the basis of a noisy beacon measurement. Several studies have used model-based procedures for solving this problem in statistical inference. Perhaps the most widely used model-based paradigm leads to the Kalman filter and its lineal variants. In this formulation, the dynamic features of the attenuation are represented by a state process $\{x_t\}$. The observation process $\{y_t\}$ is derived from beacon measurements.

Linear differential (or difference) equations with additive random forcing terms are used in most analytical studies to delineate attenuation variability:

$$dx_t = Ax_t dt + dw_t \tag{1.1}$$

with the observation given at discrete times by a linear function of the state.

$$y_t = Dx_t + n_t \text{ at observation times} \tag{1.2}$$
$$0 \text{ otherwise}$$

In this model, $\{w_t\}$ is a vector Brownian motion process with intensity W ($dw dw' = W dt$), and $\{n_t\}$ is a Gaussian "white noise" sequence with covariance $R_x > 0$, independent of $\{w_t\}$ and the initial condition on (1.1). Equation (1.1) is written in terms of differentials; stochastic and deterministic. In many cases, this level of abstraction is unnecessary; the equation can be formally divided by dt and the result expressed as an ordinary differential equation with a stochastic (white noise) excitation. This more traditional formalism gives considerable insight into the issues of estimation, and leads directly to the Kalman filter. However, when it is necessary to study systems which contain essential nonlinearities, or which are subject to sudden and unpredictable changes, it is expedient to retain the flexibility resident in (1.1).

If the initial conditions are suitably selected, Equations (1.1) and (1.2) delineate the classical linear Gauss-Markov (LGM) model. In many applications, the relations between the
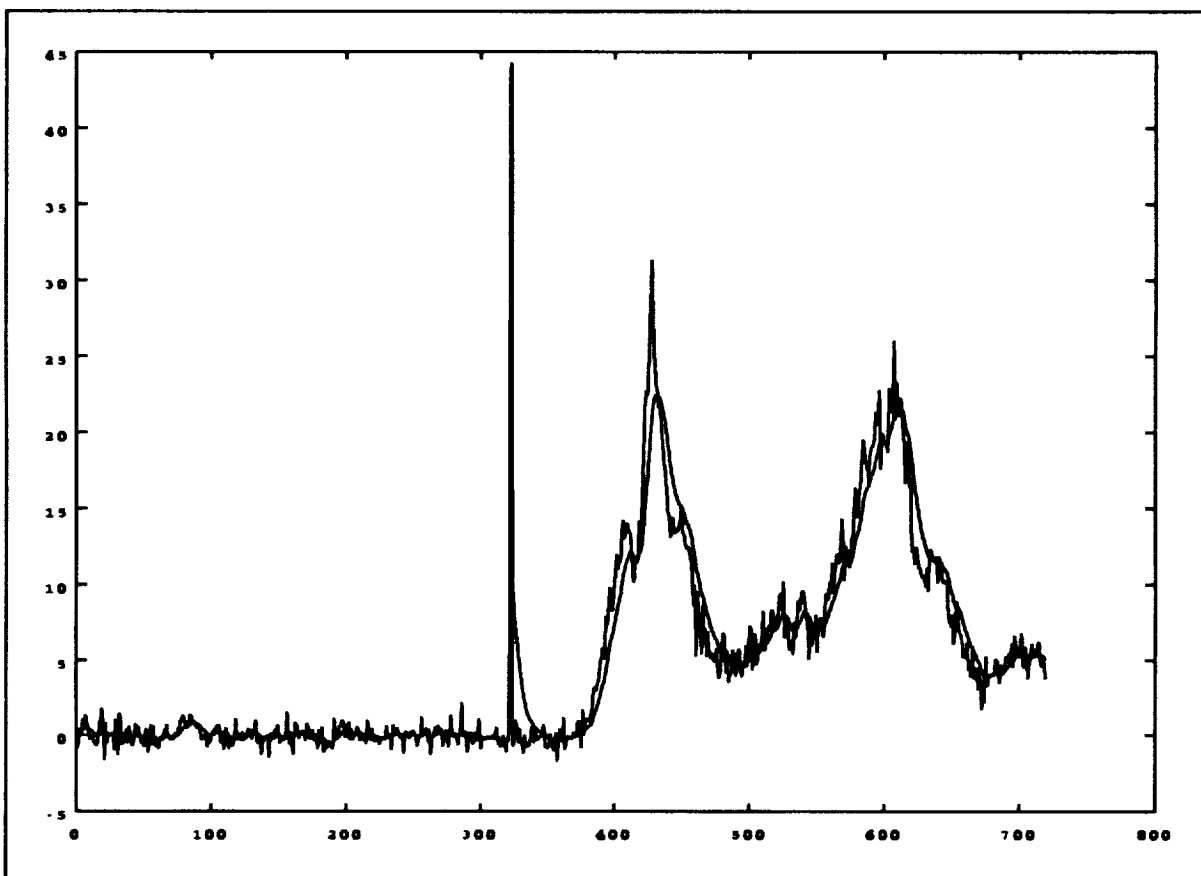
Figure 1: The May 12, 1992 rain event along with the first order algorithm, KF_F.

indicated variables are nonlinear. If the nonlinearity is smooth, it is possible to linearize it about the estimated state, and a quasi-LGM model results. If the linear (or linearized) equations provide an adequate description of the signal and the observation and their interconnection, there is a well known solution to the mean-square inference problem; the (extended) Kalman filter (EKF). Denote the information pattern (filtration) generated by the sensor measurements by $\{Y_t\}$. The best mean-square estimate of the state is given by the Y_t-conditional mean of x_t ($\hat{x}_t = E\{x_t \mid Y_t\}$) where:[1]

A. Between observations: $(d/dt)\hat{x}_t = A\hat{x}_t$ (1.3A)

B. At an observation time: $\Delta\hat{x}_t = P_{xx}D'(DP_{xx}D'+R_x)^{-1}\Delta v_x$ (1.3B)

with (the increment of the innovations process) $\Delta v_t = y_t - D\hat{x}_t$ at the observation times and zero elsewhere, and P_{xx} the error covariance matrix. The appearance of (1.3) is common in applications. It has a suggestive form which transcends the fact that it was derived under the

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[1] For any piecewise continuous process, let $\Delta z_t = z_{t+} - z_{t-}$. Then Δz_t is zero where $\{z_t\}$ is continuous, and gives the jumps in $\{z_t\}$ at points of discontinuity.
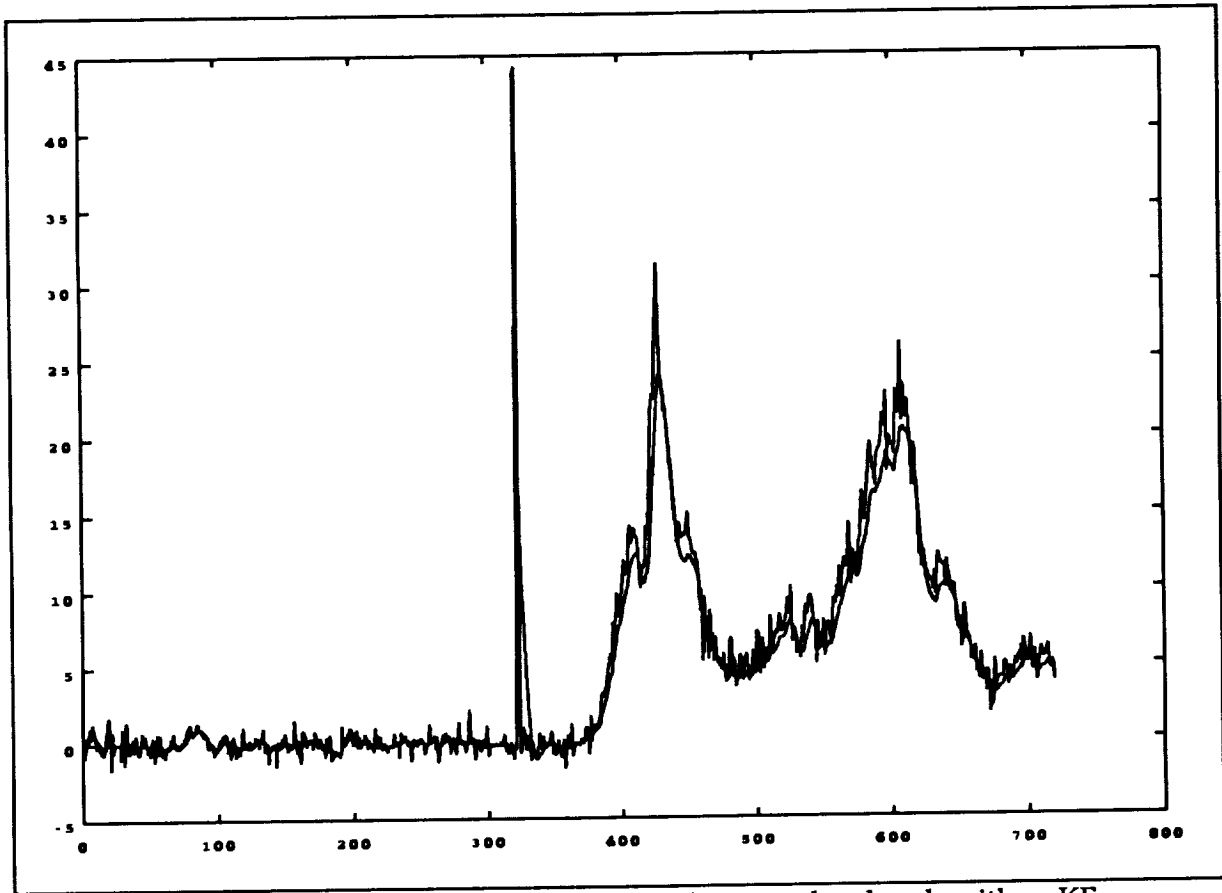
Figure 2: The May 12, 1992 rain event along with the second order algorithm, KFs.

LGM hypothesis. The increment in $\{\hat{x}_t\}$ is expressed as a sum of an extrapolation (1.3A) and a correction (1.3B). The former is in the direction of the mean state increment, and the latter is a multiple the increment in the innovations process. The correction has a gain factor related to the residual uncertainty in the estimate (P_{xx}). It is only this factor that is not given explicitly in the model of the observation link, and indeed P_{xx} is determined jointly by the target state dynamics and observation fidelity.

The error covariance acts to adapt the weight accorded to new information to fit the current circumstances. When P_{xx} is small—little estimation uncertainty—the innovations process is of little note, and the estimate propagates forward along the field of the unexcited system. As the uncertainty in the state estimate increases, new information is accorded increasing value; i.e., as the estimator becomes less sure of the true state, it is more willing to modify its prior estimate in response to new data. It is well known that $\{P_{xx}\}$ is given by the solution to a matrix ordinary differential equation between observations, with jumps at the observation times:

A. Between observations: $(d/dt)P_{xx} = AP_{xx}+P_{xx}A'+W$ (1.4A)

B. At an observation time: $\Delta P_{xx} = - P_{xx}D'(DP_{xx}D'+R_x)^{-1}DP_{xx}$ (1.4B)
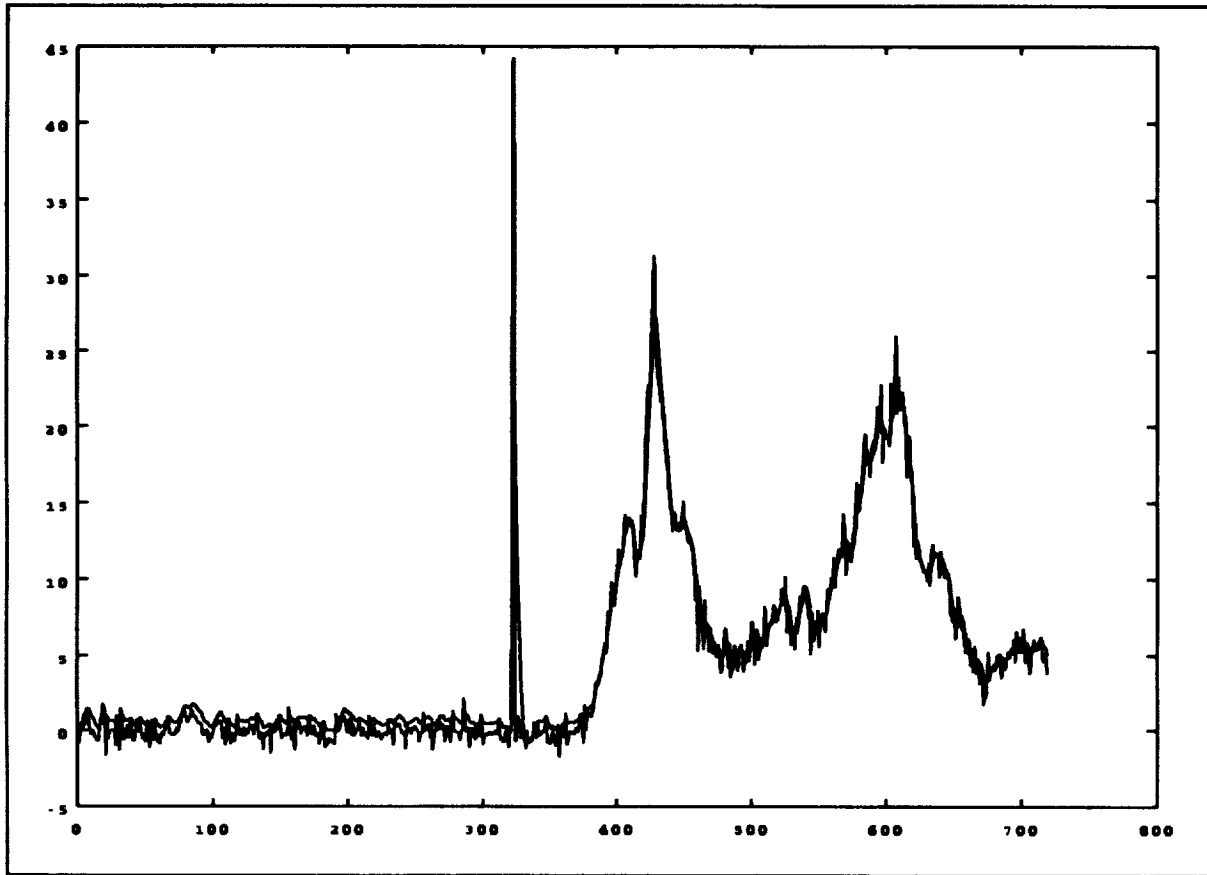
Figure 3: The May 12, 1992 rain event along with the second order algorithm, KF_M.

subject to appropriate initial conditions. The error covariance is contingent upon the intensity of the exogenous processes in both state and observation; e.g., as W increases, the increment in $\{P_{xx}\}$ increases proportionately. This has an intuitive justification. As the state process becomes more volatile, P_{xx} increases, and through this intermediary, the EKF becomes "faster" and more
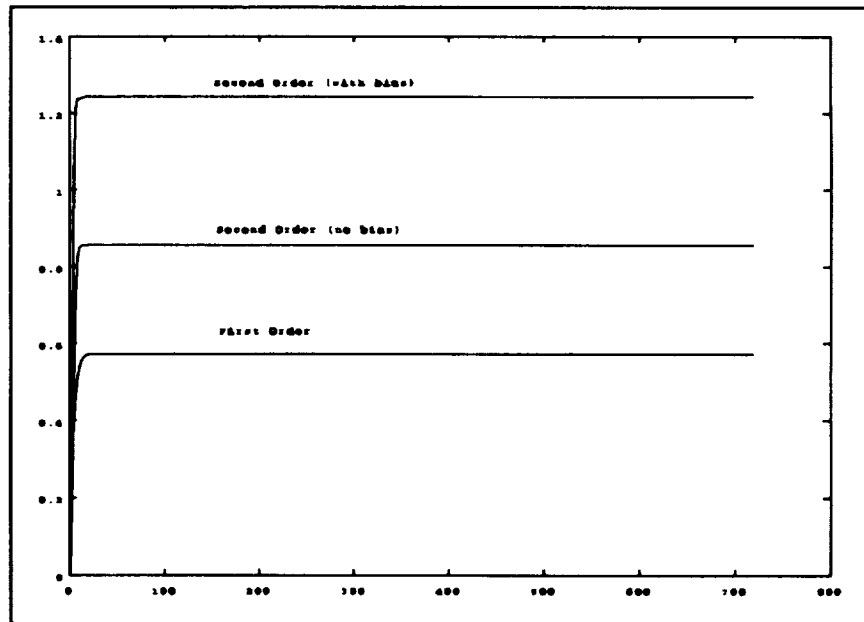

Figure 4: The error covariance for each of the three estimation algorithms.

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responsive to state changes. Of course, as a corollary to this, the same filter will amplify the measurement noise {n_t}. The EKF achieves syncretism with a precomputable gain, and the estimator is linear (if the possible localization of the model is neglected), an advantageous feature in many applications.

There are, however, important situations in which the basic EKF algorithm must be modified in a more fundamental manner. As the name implies, the primitive exogenous processes, and the subordinate state and measurement processes in the LGM model are Gaussian, and a Gaussian distribution has a very thin tail. Sometimes, the statistics of the measurement noise are conspicuously different from those of the normative distribution, and contain numerous outliers. The Kalman filter uses a linear weighting on the increments of the innovation process, and this has the effect of magnifying the outliers; a single anomalous observation may overwhelm the effect of several more typical measurements. Although an isolated occurrence can be accommodated in (1.3), if the filter time constants are long and the occurrences frequent, the estimate generated by the EKF will have significant error. Nonconforming situations arise in the construction of the state space model for rain attenuation. This is discussed in more detail in the next section.

2. MODEL BASED METHODS

To use recursive estimation procedures it is essential that the analytical description used in the model adequately reflect the peculiarities of the signal. It is the purpose of this paper to review some of previous rain fade modeling efforts, and to suggest ways in which they might be generalized. Using some recent samples of rain attenuation gathered by scientists at Virginia Polytechnic Institute and State University (VPI), a comparison can be made between actual rain events and sample functions generated from the proposed models using computer simulation. It is shown that a simply parameterized analytical model provides a natural description of a variety of rain events. There have been a number of investigations of analytical models of rain induced attenuation. Attenuation is intrinsically sign definite—as measured from a quiescent level. As such, it does not fit well within the most common modeling paradigms. In [1], [2], and [3] a novel approach to this problem was proposed. In keeping with the conventional modeling paradigm, consider a stochastic model of the attenuation process. Let {x_t} be the attenuation "state," with $A_t = Hx_t$ the actual attenuation process, and $H=(1,0,...0)$. Because of the event driven nature of attenuation, a generalization of the LGM framework must be used. Let the form of the model be given by:

$$dx_t = A_i x_t dt + B_i \Delta u_t + dw_t \text{ if } \phi_t = e_i \qquad (2.1)$$

where $A_t = (x_t)_1$, $\{u_t\}$ is a Poisson process with rate ρ, and $\{\phi_t\}$ is a Markov Process with generator Q. This model is clearly nonGaussian. The $\{\phi_t\}$ dependance gives the rain structure as different intervals of rain (and clear conditions) occur in succession. The $\{u_t\}$ is selected to model the internal structure of a specific event.
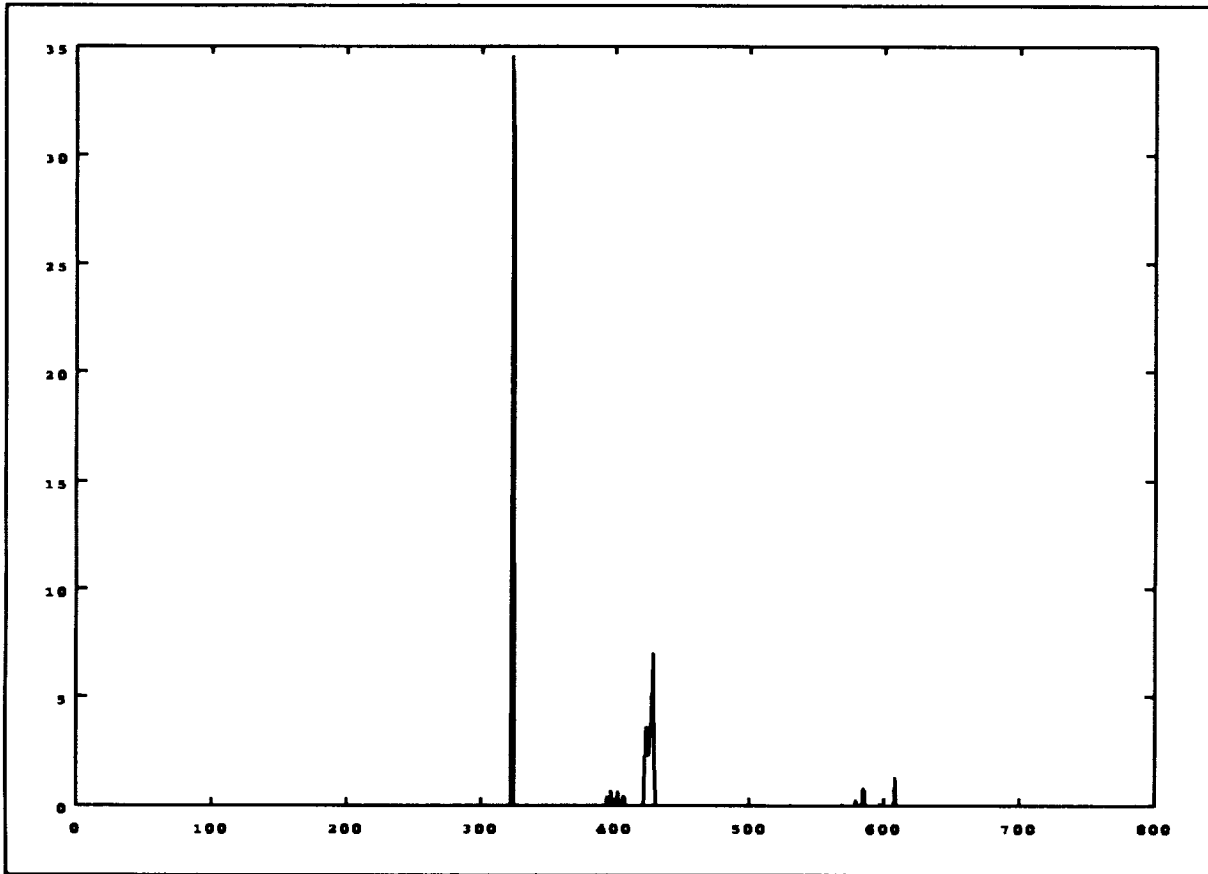

Figure 5: C_t for the May 12, 1992 rain event when using the first order algorithm, KF_F.

Equation (2.1) can be written more concisely as

$$dx_t = \Sigma_i \phi_i (A_i x_t dt + B_i \Delta u_t) + dw_t \tag{2.2}$$

or since a Poisson process admits the decomposition $u_t = \rho t + m_u$ where $\{m_u\}$ is a purely discontinuous martingale.

$$dx_t = \Sigma_i \phi_i (A_i x_t + B_i \rho) dt + \Sigma_i \phi_i B_i \Delta m_u + dw_t \tag{2.3}$$

In this note, only the case in which a 20 GHz beacon is used to estimate a 20 GHz signal will be studied. In this case the model in (1.2) can be used with D=H.

The estimation problem is nonGaussian, but it can be shown that the proper analogue to (1.3)-(1.4) has a similar form. In the monomorphic case this algorithm can be written as $\hat{A}_t = H\hat{x}_t$ where

$$d\hat{x}_t = (A\hat{x}_t + B\rho)dt + P_{xx}D'R_x^{-1}dv_x \tag{2.4A}$$

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subject to

$$dP_{xx} = (AP_{xx} + P_{xx}A' - P_{xx}D'R_x^{-1}DP_{xx} + W + \rho BB')dt + \Sigma_k \pi_{xx}(x_k)d\vartheta_k \qquad (2.4B)$$

where $\Sigma_k \pi_{xx}(x_k)d\vartheta_k$ is an adaptive term selected to adjust the filter time constants in response to changing rainfall conditions. The conditional variance of $\{A_t\}$ is $(P_{xx})_{11} = P_{AA}$.

Equation (2.4) can be integrated into a compensation algorithm as follows. Note that it is worse to underestimate the attenuation than it is to overestimate it; the former can cause a loss of connectivity, while the latter wastes power and can cause cross link interference. Let C_t be the compensating signal and let C_t be given by

$$C_t = \hat{A}_t + 3(P_{AA})^{0.5}$$

The $\{C_t\}$ process compensates for link attenuation by biasing the estimate of attenuation with the standards deviation of the error. When there is uncertainty, the compensator selects a higher power to enhance the fade margin.

3. EXAMPLES

To see how the filters perform, compare three filters in the 20/20 case on the May 12, 1991 rain event measured by VPI. The three filters are:

1) A conventional Kalman filter for a first order model; KF_F

$$dA_t = dw_t$$

This uses the algorithm given in (1.3)-(1.4) with parameters W=0.03, R_x=2, D=1

2) A Kalman filter for second order model; KF_S

$$A = \begin{bmatrix} 0 & 1 \\ -\alpha^2 & -2\alpha \end{bmatrix}; \quad B = \begin{pmatrix} 0 \\ 0.75 \end{pmatrix}$$

This again uses the algorithm given in (1.3)-(1.4) with parameters W=0.03, R_x=2, D=(1,0), and a shaping value α=0.1.

3) The Kalman filter with jump bias; KF_M

$$A = \begin{bmatrix} 0 & 1 \\ -\alpha^2 & -2\alpha \end{bmatrix}; \quad B = \begin{pmatrix} 0 \\ 0.75 \end{pmatrix}$$

This again uses the algorithm given in (2.4), but without adaptivity. The parameters are: W=0.03, R_x=2, D=(1,0), α=0.1 and a rate value ρ= 0.17.

Figure 1, 2 and 3 show the May 12 rain event along with the estimates. The original data was provided at a 10 Hz rate. The event was under sampled to yield a realization at a 0.1 Hz rate. The abscissa in the figures is sample number. Near sample number 3400 (9 Hrs.), a calibration anomaly was recorder. This is an artifact in the process, and requires no compensation.
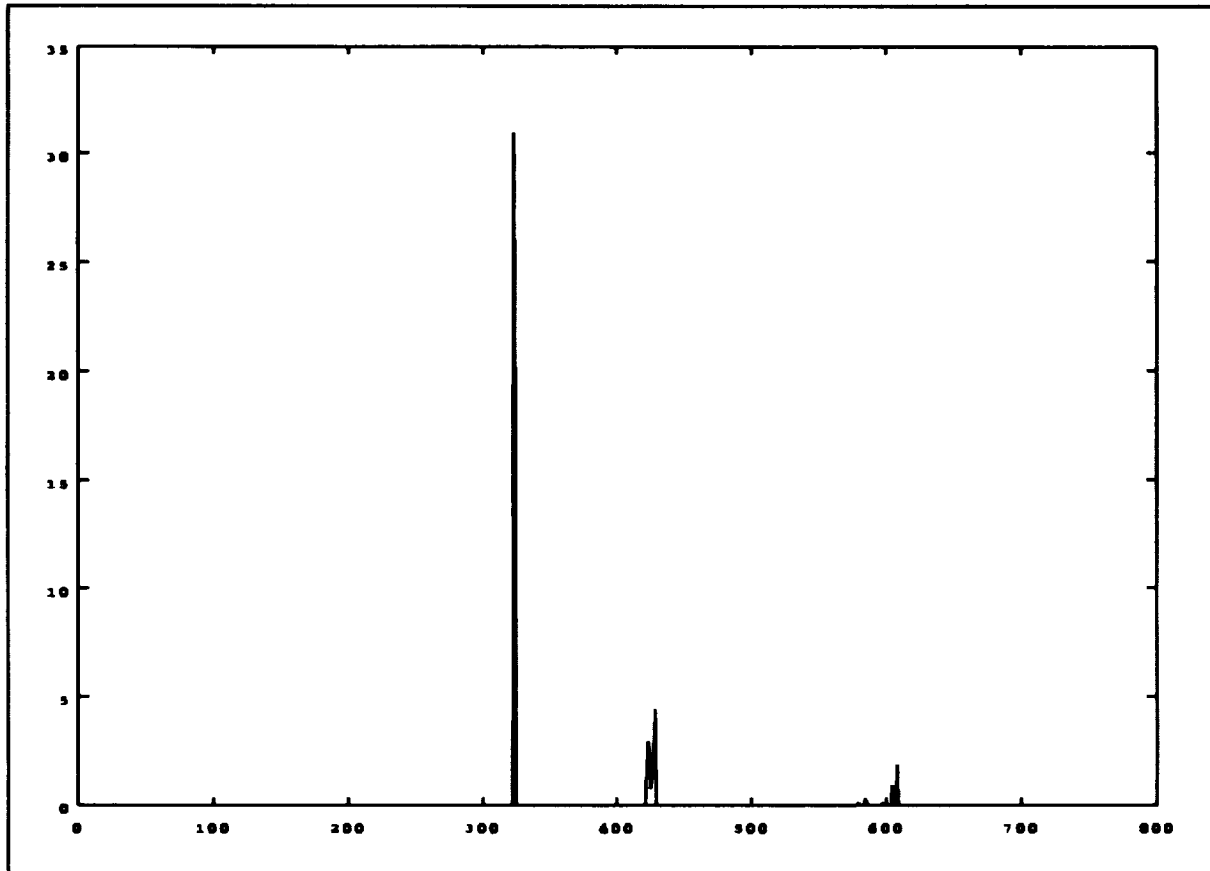
Figure 6: C_t for the May 12, 1992 rain event when using the second order algorithm, KF_S.

Nevertheless, it was retained in the example in order to see how the estimators would handle singular occurrences. The tracking of $\{A_t\}$ is clearly improved as the sophistication of the filters is increased. This is due to two factors. As the model becomes more representative of the attenuation process, the error covariance increases. Figure 4 shows the $\{P_{AA}\}$ process for each filter. Beginning at a null initial condition, it rapidly increases to its steady state value. The error covariance is influenced to a great degree by the intensity of the exogenous influences in the model. The error for KF_M smaller than the others despite the fact that its covariance is larger.

To see more clearly how the different algorithms influence link performance, the fraction of time that the link is unusable is important. As a measure of link connectivity, consider the following criterion:

$$E_t = \max(A_t - C_t - 1, 0)$$

Suppose the uplink power control operated with no delay; a very optimistic assumption. If $\{E_t\} > 0$, link connectivity would be retained if the unperturbed margin were one db. Figures 4, 5 and 6 show $\{E_t\}$ for the three indicated algorithms. The conventional random walk model provides an increase in link availability over that achieved without fade compensation, but has
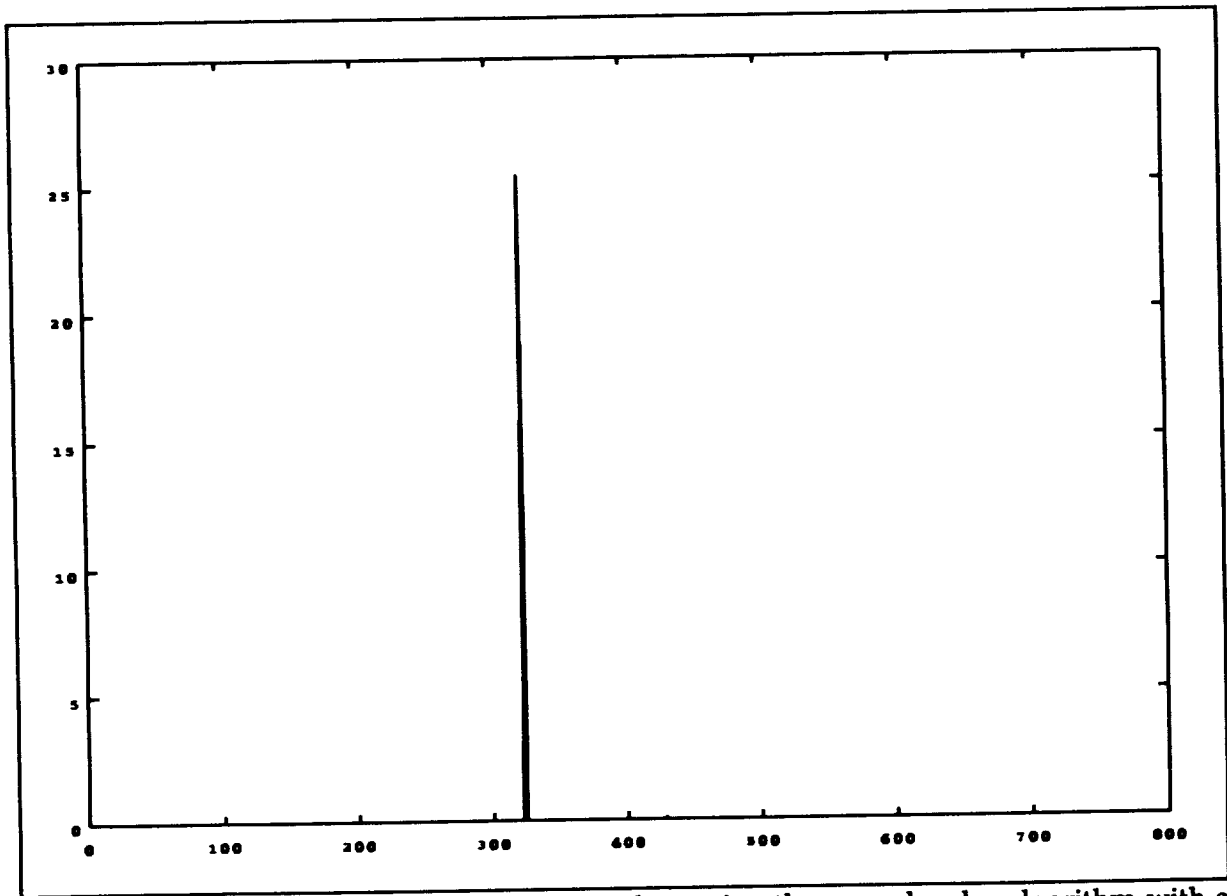
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Figure 7: C_t for the May 12, 1992 rain event when using the second order algorithm with a bias, KF_M.

unsatisfactory periods during the extremes in rain fade. The algorithm with a jump compensation performs the best. Only the pseudotransient brought upon by the recalibration artifact is not eliminated by KF_M.

4. CONCLUSIONS

This paper presents some ideas relating to the signal processing problems related to uplink power control. It is shown that some easily implemented algorithms hold promise for use in estimating rain induced fades. The algorithms have been applied to actual data generated at the VPI test facility. Because only one such event has been studied, it is not clear that the algorithms will have the same effectiveness when a wide range of events are studied. The adaptive rule suggested in (2.4) seems promising, and will be tested on other VPI data. The use of the 20 GHz beacon to predict attenuation in a 30 GHz link is also being explored. These results will be reported in a future report.

REFERENCES

1. R.M. Manning, "A Unified Statistical Rain-Attenuation Model for Communication Link Fade Predictions and Optimal Fade Control Design Using a Location Dependent Rain-Statistics Data Base," *Int. Journal of Satellite Communications*, Vol. 8, 1990, 11-30.

2. R.M. Manning, "A Statistical Rain Attenuation Prediction Model with Application to the Advanced Communication Technology Satellite Project; III-A Stochastic Rain Fade Control Algorithm for Satellite Link Power via Nonlinear Markov Filtering Theory," *NASA Technical Memorandum 100243*, May 1991.

3. D.D. Sworder, and R. Vojak, "Up-Link Power Control," *Proc. of the Third ACTS Propagation Studies Workshop*, (Santa Monica, CA Jan. 1992), Jet Propulsion Laboratory, Pasadena CA, 81-90.

A Review of APSW-III Recommendations and Action Items

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Plans for the ACTS Propagation campaign are drafted and/or revised based on the recommendations made by the participants of the ACTS Propagation Studies Workshops (APSWs). The workshops' two study group chairmen have the responsibility of writing these recommendations and submitting them to the JPL coordinator for inclusion in the workshop proceedings. It should be noted that the recommendations written by the workshop study group chairmen are the only avenue for making (or revising) plans for the ACTS propagation studies. For this reason, these recommendations and their accompanying action items are treated thoroughly and diligently by the JPL coordinator.

The resolution of APSW-III action items is expected to be obtained by the next workshop in December 1992. Therefore, individuals who were assigned action items during APSW-III are expected to prepare a report on their action items and submit the report to study group chairmen. This will allow us to record the resolution of APSW-III action items in the proceedings of APSW-IV.

The working groups joint meeting report contains 14 recommendations [1]. The following presents a brief review of these items.

1. Length of Observation Period

This recommendation addresses the need for extending the data collection period by one or two years. Technical justification of this recommendation will be formally prepared by Robert Crane.

2. Data Sampling Rate

The data sampling rate is 1 Hz. A joint report by Warren Stutzman and Wolf Vogel addresses this issue.

3. Characterization of Polarization Response

No action items were issued on this topic.

4. Observations of Rain Rate

This item makes recommendations regarding rain rate measurements at the data collection sites. The main concerns are the dynamic range of the rain rate measuring device, its performance and its cost . Regarding this item, Julius Goldhirsh has conducted an investigation that was presented by him earlier in this meeting. The final decision will be made by the NASA contractor, Warren Stutzman of VPI, before APSW-IV.

5. Weather Observation Other than Rain Rate

No minimum set of weather observations is recommended. There are no action items.

6. Measurement Values for the Standard Data Files

The working group chairmen have an action item to recommend specific attenuation and rain rate thresholds for which cumulative statistics should be given. A report is due before APSW-IV.

7. Standard Data Formats for ACTS Propagation Terminals

It is recommended that the data formats be supplied as soon as possible to the experimenters selected to receive ACTS propagation terminals, to permit the development of data analysis software required. Warren Stutzman and Wolf Vogel are in charge of this item. It is expected that the data formats will be distributed before APSW-IV.

8. Data Analysis Report Preparation

Data collected by experimenters are NASA property. However, work ethics dictate that the experimenter who has collected the data has the first right to publish them.

9. Data Dissemination

As an action item, the Data Center is to evaluate the best method for long-term storage and dissemination of ACTS propagation data.

10. Beacon Information for Experimenters

The ACTS Project Office will provide relevant information on beacon EIRP variations, satellite orbital elements, and satellite antenna pointing variations that can affect receiver signal levels at the experimenter terminals for the duration of data collection.

11. Lightning Protection for Equipment

VPI will address this issue in their site preparation report.

12. Sparing Philosophy

NASA has not yet decided on a policy regarding spare parts for terminals. It is expected that a decision by NASA will be announced before APSW-IV.

13. UPS Performance

In the event of a power outage, the Uninterruptible Power Supply (UPS) provided with the ACTS propagation terminals provides 40 minutes of coverage. It is recommended that it be the responsibility of individual experimenters to upgrade the coverage period if it is deemed necessary for a given site.

14. Guidelines for Experimenters

It is suggested that the JPL coordinator publish a handbook on good propagation experiments and data handling practices. This handbook will mostly be written by proficient experimenters and edited by the JPL coordinator. VPI will be the main contributor.

Reference

1. F. Davarian, editor, Presentations of the Third ACTS Propagation Studies Workshop (APSW III), JPL D-9443, Feb. 15, 1992, PP 171-175.