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Dynamic Gas Temperature Measurements Using a Personal Computer for Data Acquisition and Reduction

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Dynamic Gas Temperature Measurements Using a Personal Computer for Data Acquisition and Reduction

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Abstract

This report describes a dynamic gas temperature measurement system. It has frequency response to 1000 Hz, and can be used to measure temperatures in hot, high pressure, high velocity flows. A personal computer is used for collecting and processing data, which results in a much shorter wait for results than previously. The data collection process and the user interface are described in detail. The changes made in transporting the software from a mainframe to a personal computer are described in appendices, as is the overall theory of operation.

Introduction

The dynamic gas temperature measurement system (refs 1-4) consists of a probe with two thermocouples of different diameters (fig. 1) and special software. The signals from the thermocouples are fed through differential amplifiers and into the data collection/digitizing board in a PC. There are 3 signal channels. The first two are AC coupled and are the fluctuating part of the signal from the two thermocouples. The third channel is DC coupled and is the mean temperature signal from the large thermocouple. By comparing the amplitudes of the two signals at low frequencies, the software is able to provide the dynamic gas temperature, frequency compensated to 1 kHz. A brief description is given here; the full details are in appendix A.

The software begins by calculating the frequency response function for each thermocouple using the one dimensional model shown in figure 2. Because the thermocouples are beadless, they can be treated as cylinders in cross-flow, so that the temperature distribution in the wire obeys the one dimensional heat transfer equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + \frac{4h}{\rho c D} (T_s(t) - T) \quad (1)$$

where $T = T(x,t)$ = wire temperature at location x & time t , K
 α = wire thermal diffusivity, m^2/s
 ρ = wire density, kg/m^3
 c = wire specific heat, $J/kg/K$
 D = wire diameter, m
 $T_g(t)$ = instantaneous gas temperature, K
 h = convective heat transfer coefficient, $W/m^2/K$

The first term on the right is required because the low length to diameter ratios of the thermocouples require that heat conduction down the wire be accounted for in calculating the wire temperature.

The frequency response depends on the heat transfer coefficient h as well as on D , the wire diameter. The value of D is known, and based upon user input and the measured mean gas temperature, the program makes an initial estimate of the value of h . Actually, a related parameter, Γ , is used because it is independent of diameter and has the same value for both thermocouples. It then goes on to calculate the ratio of the frequency response functions for the two thermocouples for values of Γ ranging from 0.2 of the estimated value to 1.8 times the estimated value in steps of 0.1. Each of the calculated ratios, together with the corresponding value of Γ is then stored, for later comparison with the measured ratio. Determining measured frequency response ratio is the next step to be performed.

The temperature data consists of the time varying part of the signal from each thermocouple, and the mean value from the large thermocouple. Typically in a run, about 15 seconds of data are digitized and recorded. This time record is then broken into blocks of 1/2 second each, and each of the blocks is Fourier transformed. Next, the auto and cross spectral density functions are calculated from sums of the transformed data blocks, and finally, the in situ frequency response ratio is obtained from the ratio of the cross spectral density to ^{one} of the auto spectral densities. The program then compares this ratio to the set of ratios previously calculated, in order to determine the corresponding value of Γ , interpolating if necessary. The frequency response then calculated using this value of Γ in the mathematical model is taken to be the true frequency response of the thermocouple under the prevailing flow conditions and is used to frequency compensate the thermocouple signals.

As originally implemented, data reduction for dynamic gas temperature measurements required the services of a central computing facility and a mainframe computer. The time interval between the collection of data and the availability of results could be as long as a week.

Due to the availability of more powerful personal computers, and to hardware which permits rapid collection and storage of data on the hard disc, it is now possible to collect and reduce data using a personal computer located at the test facility. The results of a measurement are now available in minutes instead of days, and the process is more user friendly.

In transferring the Dynamic Gas Temperature Measurement System to a personal computer, several changes were made to the software. The first was to replace the numerical technique used to calculate the frequency response function with a closed form solution. This one change resulted in a reduction of execution time from about 7 minutes of CPU time to about 90 seconds. The derivation of the formula is in Appendix B. The second change was to the fast Fourier transform routine, which resulted in a further reduction in execution time to 23 seconds. There are at least 60 transforms to be performed, 30 for each thermocouple, and each transform is 2048 points. By making use of the fact that the data consists of real numbers rather than complex numbers, it was possible to cut in half the execution time for performing a transform. The method works by treating the 2048 real data points as 1024 complex data points, performing the transform, and unscrambling the results. The method is described more fully in Appendix C.

In addition, major changes were made to the way the program accepts user input, and the way data is acquired from the thermocouples. These changes are described in the following section.

Software

The data acquisition specification for the Dynamic Gas Temperature Measurement System requires the acquisition of three channels. These channels are the AC part of the signal from the small diameter thermocouple, and the AC and DC parts of the large diameter thermocouple, called respectively: small thermocouple-AC, large thermocouple-AC, large thermocouple-DC. These three signals must be simultaneously sampled at intervals of approximately 244.1 micro-seconds for a duration of up to 60 seconds.

Performing this task is a Data Translation DT2829 Acquisition Card used in conjunction with a Compaq 486/25 Personal Computer. Some important features of the DT2829 are the eight single-ended analog inputs with simultaneous sample-and-hold capability, the 16-bit A/D converter operating at a throughput rate of 30Khz, the external clock and trigger options and the dual DMA transfer capability which allows continuous acquisition to disk.

The program written to use the Data Translation Acquisition Card is graphically based and menu driven. The user begins the program by typing *Acquire*. Once inside the operating environment, the user selects *OPEN* from the list of options in the main window menu named *FILE* (figure 3). After opening a file for the data, the user can start filling in acquisition parameters such as the clock source, the scan time interval and the channel selections (figure 4).

For each of the eight channels there is a BNC connector icon. There also is a connection point for each of the eight locations in the scan list. The DT2829 works its way down the scan list reading the channels assigned to each scan list location. Any channel can be in any scan list location or in several scan list locations, but the first channel is usually defaulted to the first scan location because the hardware samples-and-holds the current values in the other seven channels upon each acquisition of the first channel. In order to connect a particular channel to a particular scan location, the user simply draws a wire starting from the desired scan location to the desired BNC channel icon. Afterwards, the user is prompted for an

external gain and a descriptive comment for that particular channel. If the user desires to change the gain or the comment fields, the cursor is clicked in either field and a prompt for new information will appear. The scan list must be filled in ascending order to prevent the user from creating voids where a scan location has no associated channel. In order to remove a channel from the list, the user clicks the cursor twice at the associated scan location.

The requested acquisition time duration and requested time interval between acquisitions can be changed by once again clicking the cursor in these fields and responding to the prompts. After entering the requested time duration and time interval, the actual values for these quantities are displayed in an adjacent field. If an illegal value is entered, the user will be notified. The clock mode (internal or external) is selected by the clicking the cursor in the desired field. Selecting the external clock mode will cause the program to prompt the user for the external clock speed. Similar to the clock mode, the trigger mode (internal, external and external scan) is selected by clicking the cursor in the desired field. The user is prompted for period information when the external-scan trigger mode is selected.

Once the data storage file is chosen and all of the parameters are setup, the user can acquire data by selecting *ACQUIRE* from the list of options in the main window menu named *OPTIONS*. After completing the acquisition, the data can be displayed by selecting *DISPLAY* from the same main window menu named *OPTIONS*.

All recorded channels can be viewed individually and scaled along the time axis by using the appropriate display window menu options (figure 5). When satisfied that the data is suitable, the user exits the display window. Afterward, the user can either close the data file and request that it be saved or not saved, quit without saving the data file or quit and save the data file. Any of these choices can be selected from the main window menu named *file*. All saved files will have the form '*Filename.acq*' and can not be re-displayed with the program *Acquire* because the data file will be erased upon re-entry into the program.

The data files created by the program *Acquire* are read by another program called *Convert* which allows the user to reconfigure the data into a text file, an IEEE real file or a data file compatible with the Dynamic Gas Temperature Program. The user simply types *Convert* to enter the program and then selects an acquisition file using *OPEN* from the main window menu named *FILE* (figure 6). Then the conversion type (*ASCII*, *BINARY* or *ANALYZE*) is selected from the main window menu named *OPTIONS*. Individual channels are selected for conversion by clicking the cursor on the desired conversion output field and then clicking the cursor at the desired input channel field (figure 7). Once all of the desired channels have been selected for conversion, select *CONVERT* from the main window menu named *OPTIONS*. The user will be asked if the output data is to be divided by the external gain for an actual reading or kept in its original form. After converting the data, the program is terminated using the *QUIT* option from the main window menu named *OPTIONS*. The resulting output file will be in one of three forms (*Filename.txt*, *Filename.bin* or *Filename.dat*) determined by the conversion type selected by the user. If the data file for the Dynamic Gas Temperature Program was created (*Filename.dat*), the menu program *Thermo* can be used to access its information.

The original program was written in FORTRAN, a language designed with rudimentary Input/Output capabilities. Because of this, the I/O section of the data reduction software is cumbersome to use.

Rather than rewrite the I/O section of the FORTRAN code to allow for the improved I/O capabilities, we decided to develop a program which would become the input interface to the FORTRAN program from the user. This code, written in Borland Turbo Pascal Version 5.5, is called **Thermo**, and is described here.

After acquiring and converting the data, and assigning a data set name, the user calls 'Thermo' to set up the input file for the data reduction software. The first concern of the code is to find and list all the data sets found on the disk. While it is doing this, the program displays the message shown in fig 8. After the files are sorted, the Main Menu is displayed. With the pressing of the 'Enter' key, the Main Menu screen appears as in fig. 9. The Directory is shown in the bottom half of the screen, in alphabetical order. Those files marked with an asterisk are files for which the data reduction software has already been run. After choosing a data file, the user must decide to either run the FORTRAN program (*EXECUTE*); edit the input (*MODIFY*), quit the program (*QUIT*) or ask for help (*HELP*).

If the user requests help in the Main Menu, the screen displays the contents of figure 10. This screen displays the available functions for the program, along with the keystrokes which activate them.

If the user edits the input file, a full-screen data editor is invoked. Pressing 'Help' (F3) from the data editor displays fig. 11. Fig 11 shows the data to be modified, and each entry is individually modifiable. After accepting any changes, the data editor generally takes the user through a series of questions to finish the data entry process. These questions are variable, based on the entries in the first part of the data editor. If the user wants to save the modified input file, the program will require verification of the file name. This is simply a safeguard to ensure that the proper data set was edited. The stored input file is functionally equivalent to the file described in reference 2. This is the file used by the processing software to determine the data manipulation and display required by the user. The seven possible display outputs, corresponding to the 7 example display types in ref. 2 are shown in Appendix D.

References

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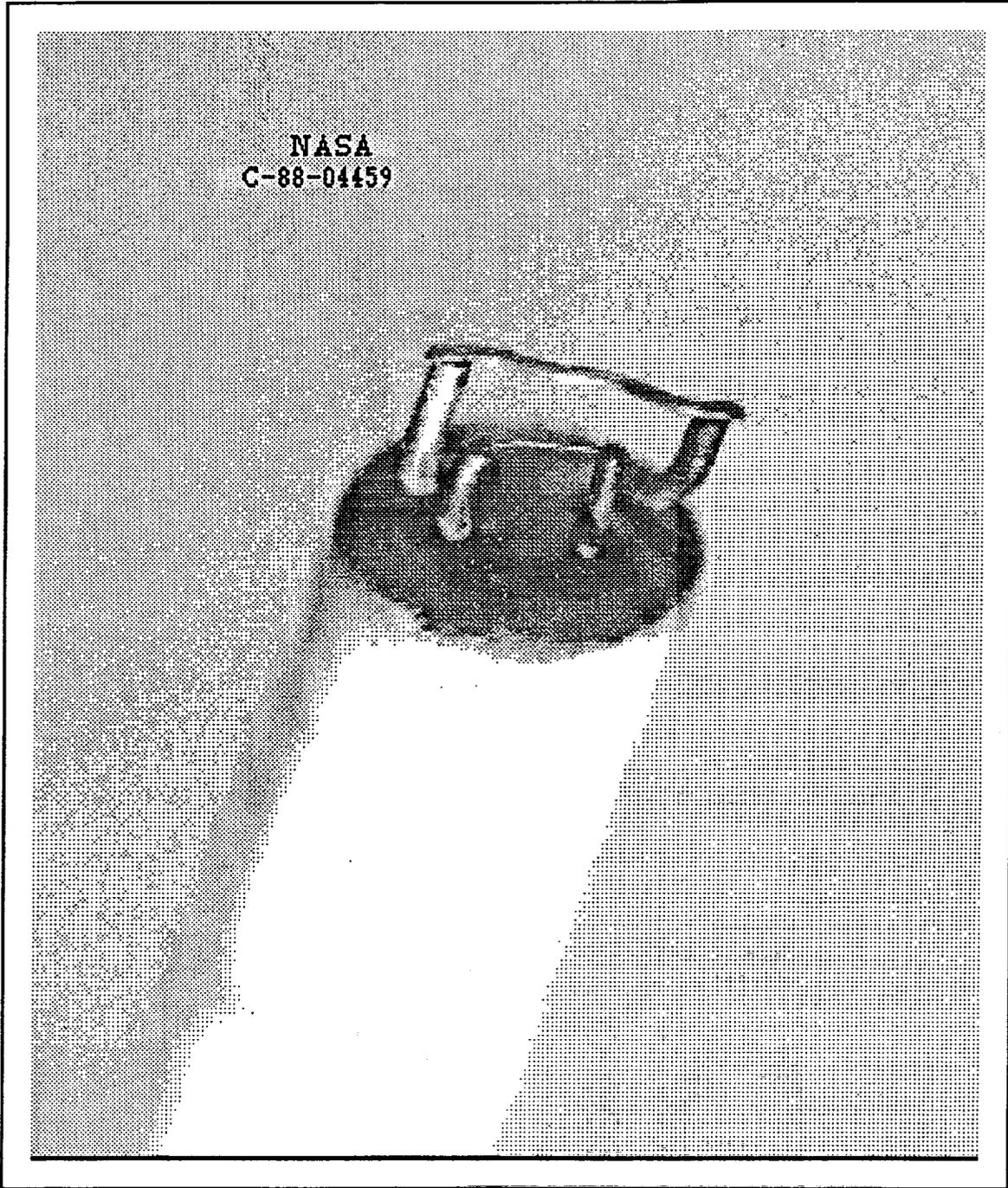


Figure 1: Dual Wire Thermocouple Probe

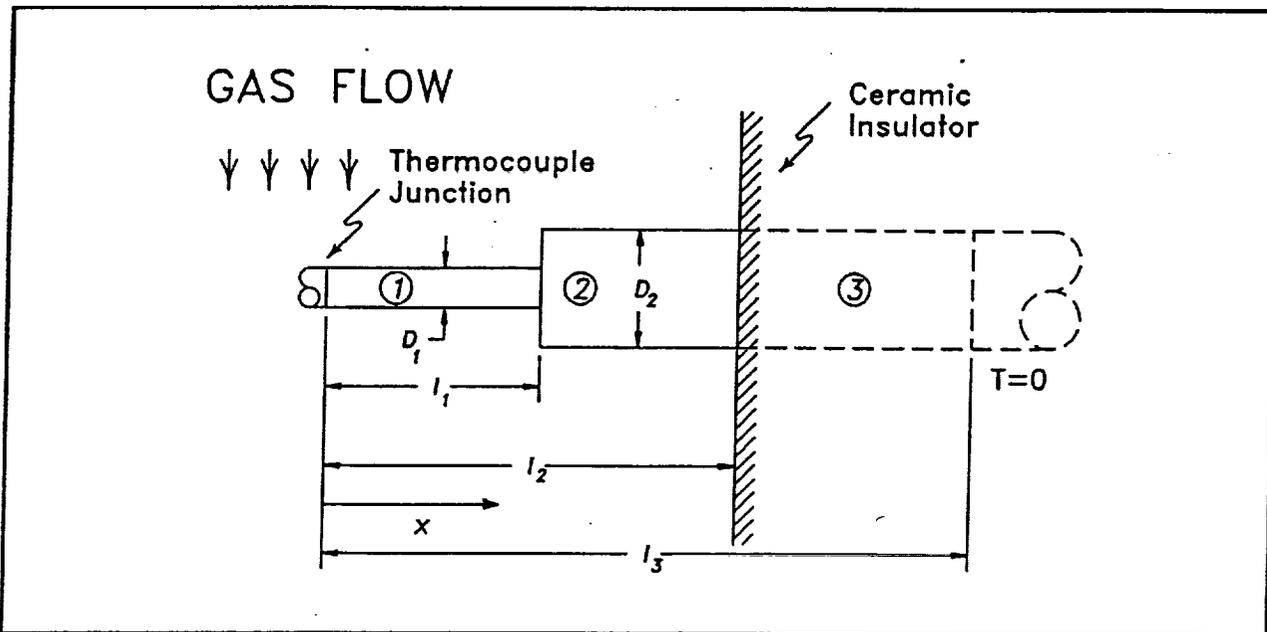


Figure 2: One Dimensional Model of Thermocouple. The model is symmetric about the junction.

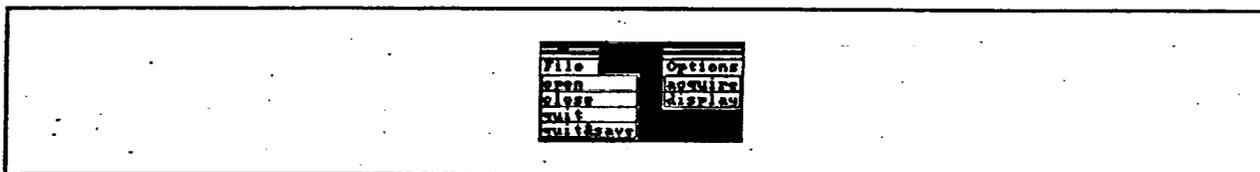


Figure 3: Data Acquisition Window – Menu Choices

Data Acquisition			
File		Options	
File: Data1			
	Channel	Gain	Comment
Clock	0	1	test point 1
internal	1	10	thermocouple A
external	2	-	-
	3	-	-
Trigger	4	-	-
internal	5	-	-
external	6	-	-
ext scan	7	-	-
requested	Time (sec) 99	Interval (u-sec) 998	Clock (kHz) 197
actual	100	1000	200

Figure 4: Data Acquisition Window -- Window Format

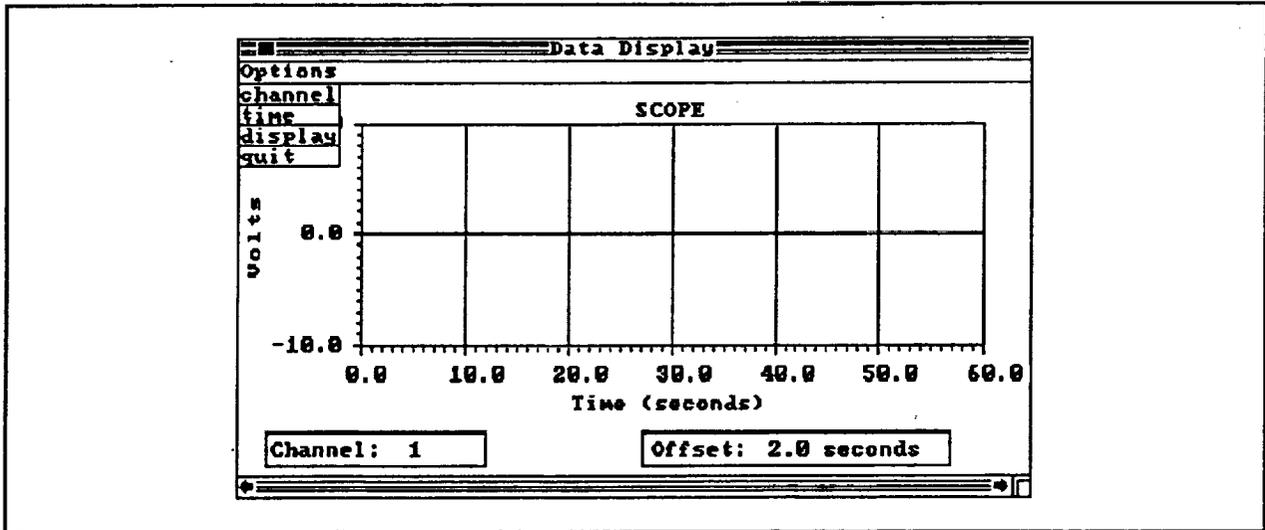


Figure 5: Data Display Window -- Window and Menu Choices

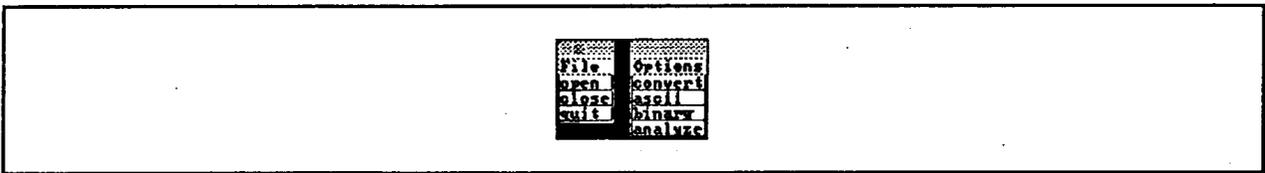


Figure 6: Data Conversion Window -- Menu Choices

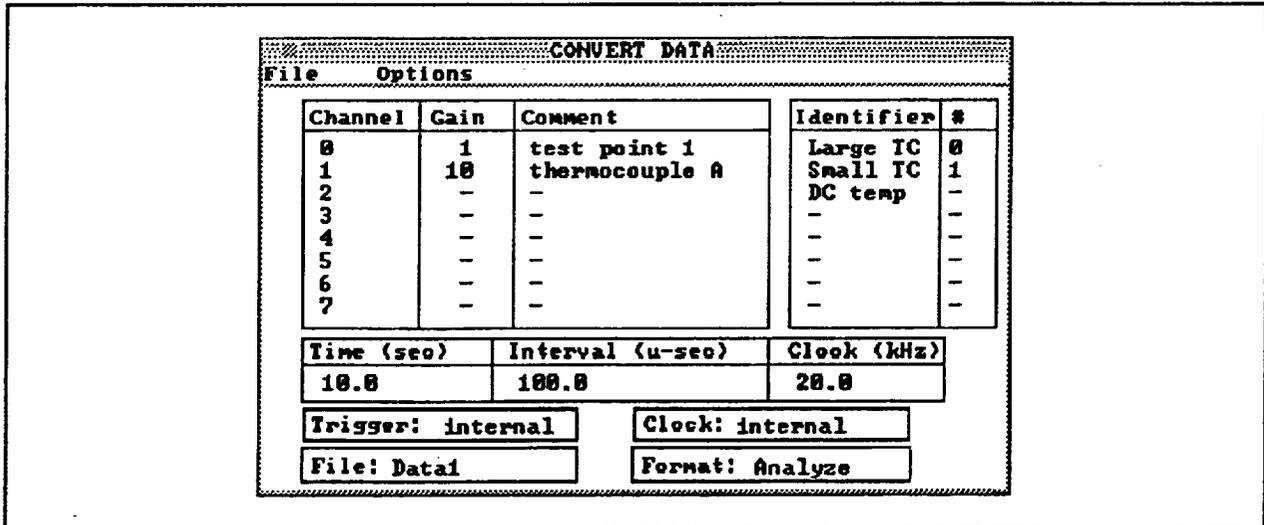


Figure 7: Data Conversion Window -- Window Format

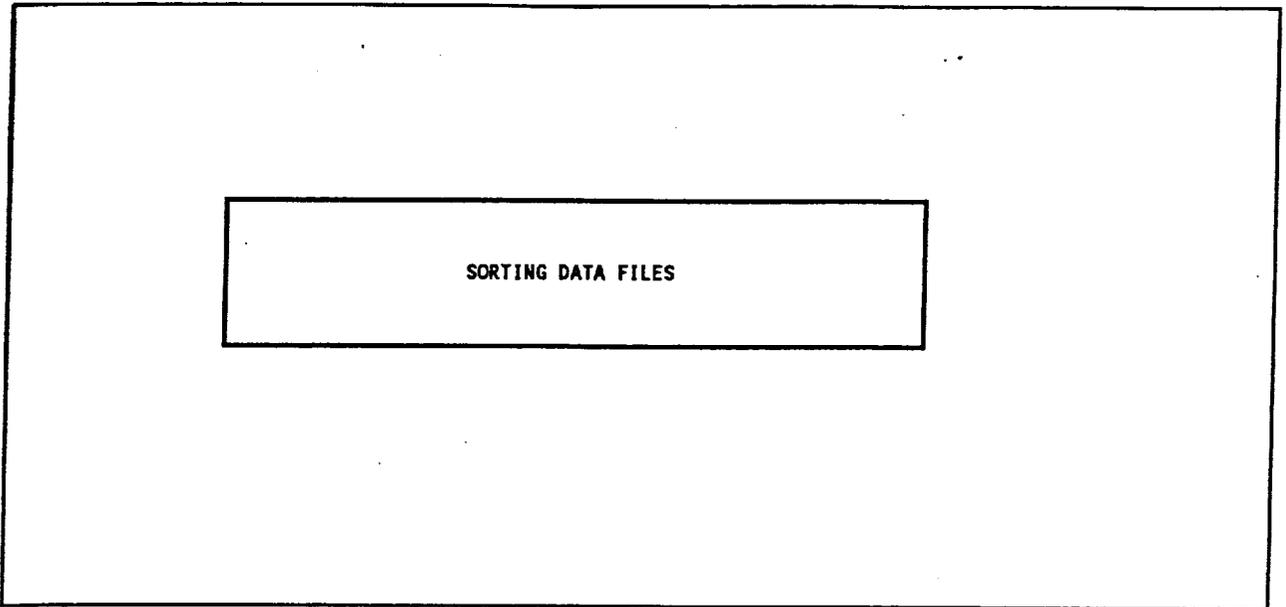


Figure 8: Initial Screen; Waiting for Sort to Finish

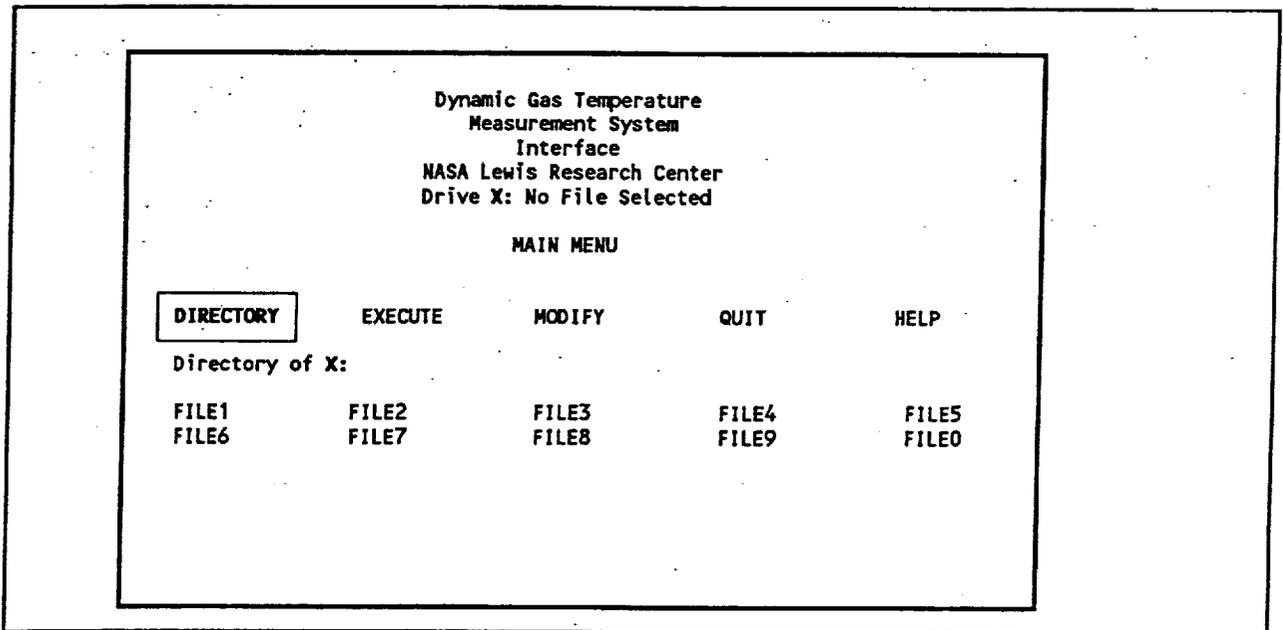


Figure 9: Main Menu with Directory Option Chosen

Dynamic Temperature Measurement System Available Functions:

F1 Quit	F2 Edit	F3 Help	F4	F5 File	F6	F7 Exec	F8	F9	F10 FFT
------------	------------	------------	----	------------	----	------------	----	----	------------

F1- Quit - Exit w/o Save/Execute.	Esc - Escape - Exit w/o Save/Execute.
F2- Edit - Edit a file.	^C - Ctrl C - Exit w/o Save/Execute.
F3- Help - Show this screen.	Hom - Home - beg of entry/line/screen.
F4- -	End - End - end of entry/line/screen.
F5- File - Get Input File.	PgU - PgUp - Toggle betwen num/text.
F6- -	PgD - PgDn - Toggle betwen num/text.
F7- Exec - Exit & Save/Execute.	Ins - Insert - Toggle insert mode
F8- -	Del - Delete - Delete character.
F9- -	BkSp-BackSpce- Delete prev. character.
F10- FFT - Delete FFT files.	

Files	list the data files on C:, and load a saved file.
Execute	runs the analysis software using your selected file
Edit	will allow you to edit a loaded file or create a new one
Quit	terminates the program

Press any key to continue:

Figure 10: Main Menu Help Screen

```

TCD(1,1) TCD(2,1) TCD(3,1) TCD(4,1) TCD(1,2) TCD(2,2) TCD(3,2) TCD(4,2)
GAS(1) GAS(2) GAS(4) GAS(3) BLSZ -(1) -(2)
FREQ(1) FREQ(2) FREQ(3) FREQ(4) PLTFRQ * CHANL(1) CHANL(2)
CHANL(3) CHANL(4) CHANL(5) CHANL(6) CHANL(7) CHANL(8) CHANL(9) #
TEXT DATA GOES HERE
    
```

KEY: - = IAVDAT, * = IFLAGS(2), # = PltChc
 See data editor for more information.

Press any key to continue:

Figure 11: Help Screen Available in the Editor

Appendix A

Theory of Operation of the Dynamic Gas Temperature Measurement System

For purposes of computation each of the thermocouples is represented by the one dimensional model shown in figure 12. The thermocouples are constructed without beads, so can be treated as cylinders in crossflow. The temperature distribution in the thermocouple then obeys the one dimensional heat transfer equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + \frac{4h}{\rho c D} (T_g(t) - T) \quad (\text{A.1})$$

where

- α = thermal diffusivity of the wire, m^2/s
- ρ = wire density, kg/m^3
- c = wire specific heat, $\text{J}/\text{kg}\cdot\text{K}$
- D = wire diameter, m
- $T_g(t)$ = instantaneous gas temperature, K
- h = heat transfer coefficient, $\text{W}/\text{m}^2\cdot\text{K}$

The first four quantities are known, either from handbooks or by direct measurement. The gas temperature, $T_g(t)$, is what we are trying to measure, and is not known. And the value of h , the heat transfer coefficient, since it depends on local flow conditions, is not known either. However, although the exact value of h for the particular operating conditions is not known, it can be estimated based on the mean temperature from the large thermocouple (ref. 5).

Having chosen a starting value for h , the thermocouple is treated as a linear system (ref. 6). From linear system theory, the response to a sinusoidal input $e^{2\pi i f t}$ is $H_\Gamma(f) e^{2\pi i f t}$ where $H_\Gamma(f)$ is the frequency response function. The subscript Γ refers to the parameter

$$\Gamma = h \frac{\sqrt{D}}{\rho c} \quad (\text{A.2})$$

which is used instead of h because it has the same value for both thermocouples.

The next step, is to use the mathematical model of the thermocouple to generate the ratio of frequency response function magnitudes $|H_{\Gamma,10}(f)| / |H_{\Gamma,3}(f)|$ for values of Γ from $0.2\Gamma_e$ to $1.8\Gamma_e$, in steps of $0.1\Gamma_e$. The quantity Γ_e is the value of Γ based on the estimated value of h . The subscripts 3 & 10 refer to the diameters of the thermocouple wires. This set of ratios is then saved, to be compared with the measured ratio obtained from the data.

To measure the value of Γ in the flow, a time history of length t_r seconds is recorded from each thermocouple. It is assumed that flow conditions are steady during this period. The thermocouple data is linearized using a polynomial appropriate to the thermocouple material (ref. 7). The thermocouple can now be regarded as a black box producing an output $y(t)$, which is directly proportional to the input $x(t)$, the gas temperature.

The t_r seconds of data are separated into n_r records, each of length t seconds, so that $n_r t = t_r$. Typically 15 seconds of data is recorded, which is broken up into 30 blocks of $\frac{1}{2}$ second each. Breaking up the total time record into small pieces tends to improve the statistics (see for instance section 8.6 of ref. 6)

The Fourier transform of each record is now computed numerically

$$\begin{aligned} Y_{k,3}(f) &= FFT[y_{k,3}(t)] \\ Y_{k,10}(f) &= FFT[y_{k,10}(t)] \\ k &= 1, 2, \dots, n_r \end{aligned} \quad (A.3)$$

where FFT stands for fast Fourier transform. Note that capital letters are used for frequency domain data.

The auto and cross spectral density functions (ref. 8, sections 3.3.2 and 3.3.4) are then calculated from the Fourier transforms:

$$\begin{aligned} G_{3,3}(f) &= \frac{2}{n_r T} \sum_{k=1}^{n_r} |Y_{k,3}(f)|^2 \\ G_{3,10}(f) &= \frac{2}{n_r T} \sum_{k=1}^{n_r} Y_{k,3}^*(f) Y_{k,10}(f) \end{aligned} \quad (A.4)$$

The reason for calculating these quantities is that they are related to the frequency response functions of the thermocouples via the relations

$$\begin{aligned} G_{3,3}(f) &= |H_3(f)|^2 G_{xx}(f) \\ G_{3,10}(f) &= H_3^*(f) H_{10}(f) G_{xx}(f) \end{aligned} \quad (\text{A.5})$$

The quantity $G_{xx}(f)$ in the above equations is the unknown auto spectral density of the gas temperature fluctuations.

The measured gain ratio of the two thermocouples is now found by merely dividing, i.e.,

$$\frac{G_{3,10}(f)}{G_{3,3}(f)} = \frac{H_3^*(f) H_{10}(f)}{|H_3(f)|^2} = \frac{|H_{10}(f)|}{|H_3(f)|} ((\cos\phi_3 - \phi_{10}) + i\sin(\phi_3 - \phi_{10})) \quad (\text{A.6})$$

and the desired ratio $|H_{10}(f)| / |H_3(f)|$ is just the square root of the sum of the squares of the real and imaginary parts of $G_{3,10}(f)/G_{3,3}(f)$.

The measured value of the gain ratio is then compared with the set of gain ratios calculated earlier. Once a match is found, the in-situ value of Γ has been found. This value of Γ is used with the mathematical model of the thermocouple to generate the frequency response function for the small thermocouple, $H_3(f)$. Then the Fourier transform of the gas temperature fluctuation, $X(f)$, is related to the Fourier transform of the signal from the thermocouple, $Y_3(f)$, by the relation

$$X(f) = \frac{Y_3(f)}{H_3(f)} \quad (\text{A.7})$$

Finally, the unknown gas temperature is just

$$T_g(t) = x(t) = FFT^{-1}[X(f)]$$

Appendix B Frequency Response Function

This appendix contains a derivation of the analytic expression for the frequency response of a supported thermocouple wire in which there is conductive heat transfer down the wire (fig. 12). With reference to the photograph of the probe shown in figure 1, the smaller wire in figure 12 represents the horizontal cross wire, & the larger diameter wire represents the vertical support wire, part of which is exposed to the flow and part of which extends into the ceramic probe housing. The expression derived in this section replaces the finite difference method originally used.

The temperature in the thermocouple wire, $T_w(x,t)$, satisfies the one dimensional heat transfer equation

$$\rho c \frac{\partial T_w}{\partial t} = k \frac{\partial^2 T_w}{\partial x^2} + \frac{4h}{D} [T_g(t) - T_w] \quad (\text{B.1})$$

where

- ρ = wire density, kg/m³
- c = wire specific heat, J/kg-K
- k = thermal conductivity, W/m-K
- h = convective heat transfer coefficient, W/m²-K
- D = wire diameter, m
- T_g = gas temperature, K

In our case, the gas temperature is assumed to be a function of time only, and to be described by a sinusoidal fluctuation superimposed on a mean value,

$$T_g(t) = \bar{T} + T_o e^{i\omega t} \quad (\text{B.2})$$

where

- \bar{T} = mean gas temperature
- T_o = amplitude of sinusoidally varying part

The wire temperature is then also the sum of a time independent and a time dependent part. Let

$$T_w(x,t) = T_{dc}(x) + T(x,t) \quad (\text{B.3})$$

and substitute equations (B.2) and (B.3) into equation (B.1):

$$\frac{\partial T_w}{\partial t} = \frac{\partial T}{\partial t} = \alpha \frac{d^2 T_{dc}}{dx^2} + \alpha \frac{\partial^2 T}{\partial x^2} + \omega_n [T + T_o e^{i\omega t} - T_{dc} - \bar{T}]$$

where α = thermal diffusivity of the wire, m²/s.

This equation can be separated into two, one of which is independent of time and one which is not:

$$\alpha \frac{d^2 T_{dc}}{dx^2} + \omega_n (T - T_{dc}) = 0 \quad (\text{B.4})$$

$$\alpha \frac{\partial^2 T}{\partial x^2} + \omega_n (T_\omega e^{i\omega t} - T) = \frac{\partial T}{\partial t} \quad (\text{B.5})$$

It is equation B.5 which is of interest here. The steady state solution of equation (B.5) is obtained by separating variables. Let

$$T(x,t) = X(x)e^{i\omega t} \quad (\text{B.6})$$

and substitute (B.6) into (B.5). After cancelling the common factor $e^{i\omega t}$, and gathering terms, the result is

$$\left[1 + i \frac{\omega}{\omega_n} \right] X = \frac{\alpha}{\omega_n} \frac{d^2 X}{dx^2} + T_\omega$$

Upon dividing through by T_ω to normalize, the result is

$$\gamma f'' - G(\omega)f = -1 \quad (\text{B.7})$$

where

$$\gamma = \frac{\alpha}{\omega_n}$$

$$f = \frac{X}{T_\omega}$$

$$G = 1 + i \frac{\omega}{\omega_n} \quad (\text{B.8})$$

Equation (B.7) is an ordinary differential equation which has the solution

$$f(x) = A \cosh(qx) + B \sinh(qx) + \frac{1}{G} \quad (\text{B.9})$$

where G is defined in equation (B.8), $q = (G/\gamma)^{1/2}$, and A and B depend on the boundary conditions.

Thus for the three regions shown in figure 12,

$$f_j = A_j \cosh(q_j x) + B_j \sinh(q_j x) + \frac{1}{G_j}, \quad j=1,2,3$$

except that $1/G_3 = 0$. That is because the part of the support wire in region 3 is not exposed to the flow, so $h_3 = 0$. Then $\omega_3 = 4h_3/\rho c D = 0$ and

$$\frac{1}{G_3} = \left[1 + i \frac{\omega}{\omega_3} \right]^{-1} = \frac{\omega_3}{(\omega_3 + i\omega)} = 0$$

However,

$$q_3^2 = \frac{G_3}{\gamma_3} = \left[\frac{\omega_3}{\alpha} \right] \left[1 + i \frac{\omega}{\omega_3} \right] = i \frac{\omega}{\alpha}$$

The unknown constants A_j & B_j are determined by the requirement that

- a) the temperature is continuous everywhere
- b) the heat flux is continuous everywhere
- c) the temperature fluctuations at $x=l_3$ are zero, i.e., $f_3(l_3) = 0$
- d) the material properties are the same on either side of the junction, so the solution is symmetric about $x=0$.

(This latter simplification of uniform material properties is adequate for the type B thermocouples used in high temperature environments, but not for thermocouples such as type K used for lower temperatures. In such cases the solution given here must be extended to take into account the differences in material properties, as has been done in refs 9 & 10).

Condition d) states that $f_1(-x) = f_1(x)$, which in turn implies

$$f'(0) = (q_1 A_1 \sinh(q_1 x) + q_1 B_1 \cosh(q_1 x)) \Big|_{x=0} = q_1 B_1 = 0$$

so that

$$f_1(x) = A_1 \cosh(q_1 x) + \frac{1}{G_1} \quad (\text{B.10})$$

Because of the boundary condition c) on f_3 it is convenient to write f_3 as

$$f_3(x) = A_3 \cosh(q_3(l_3 - x)) + B_3 \sinh(q_3(l_3 - x))$$

Then $f_3(l_3) = A_3 = 0$, and

$$f_3(x) = B_3 \sinh(q_3(l_3 - x)) \quad (\text{B.11})$$

The six unknowns $A_1, B_1, \dots, A_3, B_3$ have been reduced in number to four, which will now be found by applying the requirements a) & b) at $x=l_1$ & at $x=l_2$. Continuity of temperature and heat flux at $x=l_1$,

$$\begin{aligned} f_1(l_1) &= f_2(l_1) \\ D_1^2 f_1'(l_1) &= D_2^2 f_2'(l_1) \end{aligned}$$

and at l_2

$$\begin{aligned} f_2(l_2) &= f_3(l_2) \\ f_2'(l_2) &= f_3'(l_2) \end{aligned}$$

give four equations in four unknowns:

$$A_1 \cosh(q_1 l_1) + \frac{1}{G_1} = A_2 + \frac{1}{G_2} \quad (\text{B.12})$$

$$A_1 q_1 \sinh(q_1 l_1) = R q_2 B_2 \quad (\text{B.13})$$

$$A_2 \cosh(q_2(l_2 - l_1)) + B_2 \sinh(q_2(l_2 - l_1)) = A_3 \sinh(q_3(l_3 - l_2)) \quad (\text{B.14})$$

$$A_2 q_2 \sinh(q_2(l_2 - l_1)) + B_2 q_2 \cosh(q_2(l_2 - l_1)) = -A_3 q_3 \cosh(q_3(l_3 - l_2)) \quad (\text{B.15})$$

where $R = D_2^2/D_1^2$ and $f_2(x)$ has been written in the form

$$f_2(x) = A_2 \cosh(q_2(x - l_1)) + B_2 \sinh(q_2(x - l_1)) + \frac{1}{G_2}$$

The desired quantity is the temperature at the thermocouple junction, which from equation

(B.10) is $f_1(0) = A_1 + 1/G_1$. The quantity G_1 is defined in equation (B.8), and A_1 is extracted from the above equations (B.12) to (B.15). The result is

$$A_1 = Rq_2 \frac{\left[\frac{1}{G_2} - \frac{1}{G_1} \right] q_2 \sinh(q_3(l_3 - l_2)) \sinh(q_2(l_2 - l_1))}{\Delta} \quad (\text{B.16})$$

$$+ Rq_2 \frac{q_3 \cosh(q_3(l_3 - l_2)) \left[\left[\frac{1}{G_2} - \frac{1}{G_1} \right] \cosh(q_2(l_2 - l_1)) - \frac{1}{G_2} \right]}{\Delta}$$

where

$$\Delta = q_2 \sinh(q_3(l_3 - l_2)) \left(Rq_2 \cosh(q_1 l_1) \sinh(q_2(l_2 - l_1)) + q_1 \sinh(q_1 l_1) \cosh(q_2(l_2 - l_1)) \right) \quad (\text{B.17})$$

$$+ q_3 \cosh(q_3(l_3 - l_2)) \left(Rq_2 \cosh(q_1 l_1) \cosh(q_2(l_2 - l_1)) + q_1 \sinh(q_1 l_1) \sinh(q_2(l_2 - l_1)) \right)$$

Equations (B.16) and (B.17) form the basis for the changes made in subroutines **TRFP** and **TRFEM**. They replace the finite difference equations formerly used.

Appendix C Modified FFT Algorithm

Typically, there are 30 blocks of data taken for each run, with 2048 data points per block, and each of these blocks of data is Fourier transformed. Since there are two thermocouples, there are 60 transforms for each data run. Because of the large number of transforms to be calculated, an increase in computational efficiency could significantly reduce execution time.

Let a typical data point in one of these blocks of data be written $x(k)$, where $k=0,1,2,\dots,2N-1$ is a free index denoting location within the data block. The size of the data block is $2N$, which is a power of 2 (ref. 11). In our case $2N = 2048$.

Fast Fourier Transform programs are designed to calculate the transform of an array of complex numbers. Each of the $x(k)$ must be converted to complex form by writing it as a complex number $x_c(k)$, whose imaginary part is zero:

$$x_c(k) = [x(k), 0] = x(k) + i0 \quad (\text{C.1})$$

The transform of the array $x_c(k)$ then consists of the array

$$X(n) = \frac{1}{2N} \sum_{k=0}^{2N-1} x_c(k) e^{\frac{2\pi i kn}{2N}} \quad (\text{C.2})$$

$$n = 0, 1, \dots, 2N-1$$

and $4N$ numbers are calculated, the real and imaginary parts of each of the $X(n)$. Since we started with an array of size $2N$, half the $4N$ numbers must be superfluous. This is, in fact, the case:

First, $X(0)$ is real, which follows directly from the definition (C.2); i.e., with $n=0$, the expression is just a sum of real numbers, which must be real. Further, since the $x_c(k)$ are real, the transformed array satisfies the relation

$$X(N+m) = X^*(N-m), \quad m = 0, 1, \dots, N-1 \quad (\text{C.3})$$

where the $*$ denotes complex conjugation. Setting $m = 0$ in equation (C.3) shows that $X(N)$ is real, and for $m \neq 0$, the second half the array $X(n)$ is just the complex conjugate of the first half. Thus the Fourier transform $X(n)$, $n=0,1, \dots, 2N-1$, of the real array $x(k)$, $k=0,1, \dots, 2N-1$, has only the $2N$ independent quantities $X_r(0), X_i(1), \dots, X_r(N), X_i(1), X_i(2), \dots, X_i(N-1)$, where r & i denote the real and imaginary parts of $X(n)$.

Thus padding each real number with a zero as in equation C.1 is not a very efficient procedure for calculating the discrete Fourier transform. Fortunately there is a better way,

(refs 11 & 12). By assembling the real array into pairs, and treating each pair as a single complex number, one calculates an N point transform instead of a $2N$ point transform, cutting the computing time required roughly in half. The details of how this is done follows.

As above, given the real array $x(k)$ of size $2N$, the discrete Fourier transform is the array

$$X(n) = \frac{1}{2N} \sum_{k=0}^{2N-1} x(k) e^{\frac{2\pi i k n}{2N}}, \quad n = 0, 1, \dots, 2N-1 \quad (\text{C.4})$$

Grouping the sum into the even and odd numbered parts,

$$\begin{aligned} X(n) &= \frac{1}{2N} \left[\left(x(0) + x(2) e^{\frac{2\pi i 2n}{2N}} + \dots + x(2N-2) e^{\frac{2\pi i (2N-2)n}{2N}} \right) \right. \\ &\quad \left. + \left(x(1) e^{\frac{2\pi i n}{2N}} + \dots + x(2N-1) e^{\frac{2\pi i (2N-1)n}{2N}} \right) \right] \\ &= \frac{1}{2N} \left[\sum_{j=0}^{N-1} x(2j) e^{\frac{2\pi i (2j)n}{2N}} + \sum_{j=0}^{N-1} x(2j+1) e^{\frac{2\pi i (2j+1)n}{2N}} \right] \\ &= \frac{1}{2} \left[\frac{1}{N} \sum_{j=0}^{N-1} h(j) e^{\frac{2\pi i j n}{N}} + e^{\frac{\pi i n}{N}} \frac{1}{N} \sum_{j=0}^{N-1} g(j) e^{\frac{2\pi i j n}{N}} \right] \quad (\text{C.5}) \end{aligned}$$

where

$$\begin{aligned} h(j) &= x(2j) \\ g(j) &= x(2j+1) \\ j &= 0, 1, \dots, N-1 \end{aligned} \quad (\text{C.6})$$

The two sums which appear in equation (C.5) are the Fourier transforms of h and g ,

$$\begin{aligned} H(n) &\equiv \frac{1}{N} \sum_{j=0}^{N-1} h(j) e^{\frac{2\pi i j n}{N}} \\ G(n) &\equiv \frac{1}{N} \sum_{j=0}^{N-1} g(j) e^{\frac{2\pi i j n}{N}} \\ n &= 0, 1, \dots, N-1 \end{aligned} \quad (\text{C.7})$$

so

(Note that the range of the free index n is different in equations C.7 and C.8. This doesn't

$$X(n) = \frac{1}{2} \left[H(n) + e^{\frac{\pi i n}{N}} G(n) \right] \quad (C.8)$$

$$n = 0, 1, \dots, 2N-1$$

matter because the functions H and G start to repeat after n exceeds $N-1$).

The functions H and G are not computed separately, since that would offer no savings in computer time. Instead, define the complex array

$$y(j) = h(j) + ig(j) \quad (C.9)$$

and calculate the single N - point transform

$$Y(n) = \frac{1}{N} \sum_{j=0}^{N-1} y(j) e^{\frac{2\pi i j n}{N}}$$

$$= \frac{1}{N} \sum_{j=0}^{N-1} (h(j) + ig(j)) e^{\frac{2\pi i j n}{N}}$$

$$Y(n) = H(n) + iG(n), \quad n = 0, 1, \dots, N-1 \quad (C.10)$$

The trick now is to express $H(n)$ & $G(n)$ in terms of $Y(n)$. First, we need the result that, since $g(j)$ and $h(j)$ are real,

$$G_r(N-n) = G_r(n) \quad (C.11)$$

$$G_i(N-n) = -G_i(n)$$

and similarly for H . This result is the same as $G(N-n) \equiv G^*(n)$, which follows from equation (C.7). Then, since

$$Y(n) = H(n) + iG(n) = [H_r(n) + iH_i(n)] + i[G_r(n) + iG_i(n)]$$

$$Y_r(n) = H_r(n) - G_i(n) \quad (C.12)$$

$$Y_i(n) = H_i(n) + G_r(n) \quad (C.13)$$

Then, from equations (C.12) and (C.11),

$$\begin{aligned} Y_r(n) &= H_r(n) - G_i(n) \\ Y_r(N-n) &= H_r(n) + G_i(n) \end{aligned}$$

$$\begin{aligned} H_r(n) &= \frac{1}{2}(Y_r(n) + Y_r(N-n)) \\ G_i(n) &= -\frac{1}{2}(Y_r(n) - Y_r(N-n)) \end{aligned} \quad (\text{C.14})$$

Similarly, equation (C.13) gives

$$\begin{aligned} G_r(n) &= \frac{1}{2}(Y_i(n) + Y_i(N-n)) \\ H_i(n) &= \frac{1}{2}(Y_i(n) - Y_i(N-n)) \end{aligned} \quad (\text{C.15})$$

so that

$$\begin{aligned} H(n) &= H_r(n) + iH_i(n) \\ &= \frac{1}{2}(Y_r(n) + Y_r(N-n)) + \frac{i}{2}(Y_i(n) - Y_i(N-n)) \end{aligned} \quad (\text{C.16})$$

and

$$\begin{aligned} G(n) &= G_r(n) + iG_i(n) \\ &= \frac{1}{2}(Y_i(n) + Y_i(N-n)) - \frac{i}{2}(Y_r(n) - Y_r(N-n)) \end{aligned} \quad (\text{C.17})$$

Equations (C.16) and (C.17) can now be substituted into equation (C.8) to give the elements of the desired transform $X(n)$ in terms of the downsized transform $Y(n)$.

The process can also be reversed, so that the original array $x(k)$ can be recovered from the array $X(n)$ via an efficient inverse transform. After some rearranging, $H(n)$ and $G(n)$ can be written

$$H(n) = \frac{1}{2}(Y(n) + Y^*(N-n)) \quad (\text{C.18})$$

$$G(n) = -\frac{1}{2}(Y(n) - Y^*(N-n)) \quad (\text{C.19})$$

and equation (C.8) becomes

$$X(n) = \frac{1}{4}(Y(n)+Y^*(N-n)) - \frac{i}{4}e^{-\frac{i\pi n}{N}}(Y(n)-Y^*(N-n)) \quad (\text{C.20})$$

Then

$$X(N-n) = \frac{1}{4}(Y(N-n)+Y^*(n)) + \frac{i}{4}e^{-\frac{i\pi n}{N}}(Y(N-n)-Y^*(n))$$

and

$$X^*(N-n) = \frac{1}{4}(Y(n)+Y^*(N-n)) + \frac{i}{4}e^{\frac{i\pi n}{N}}(Y(n)-Y^*(N-n)) \quad (\text{C.21})$$

Adding and subtracting (C.20) and (C.21),

$$\begin{aligned} X(n)+X^*(N-n) &= \frac{1}{2}(Y(n)+Y^*(N-n)) \\ X(n)-X^*(N-n) &= \frac{i}{2}e^{\frac{i\pi n}{N}}(Y(n)-Y^*(N-n)) \end{aligned}$$

from which

$$Y(n) = (X(n)+X^*(N-n)) + ie^{-\frac{i\pi n}{N}}(X(n)-X^*(N-n)) \quad (\text{C.22})$$

The inverse N point transform is then performed on $Y(n)$ to give the array $y(j)$, from which the array $x(j)$ is available via equations (C.9) and (C.6). Equations (C.20) and (C.22) are the basis for the subroutine **REALFT**, which performs all of the Fourier transforms in the dynamic gas temperature measurement software.

Appendix D

Plotting Choices available from User Interface

These plotting choices correspond to the example plotting outputs found in ref 2. They are shown as figures D.1 through D.7.

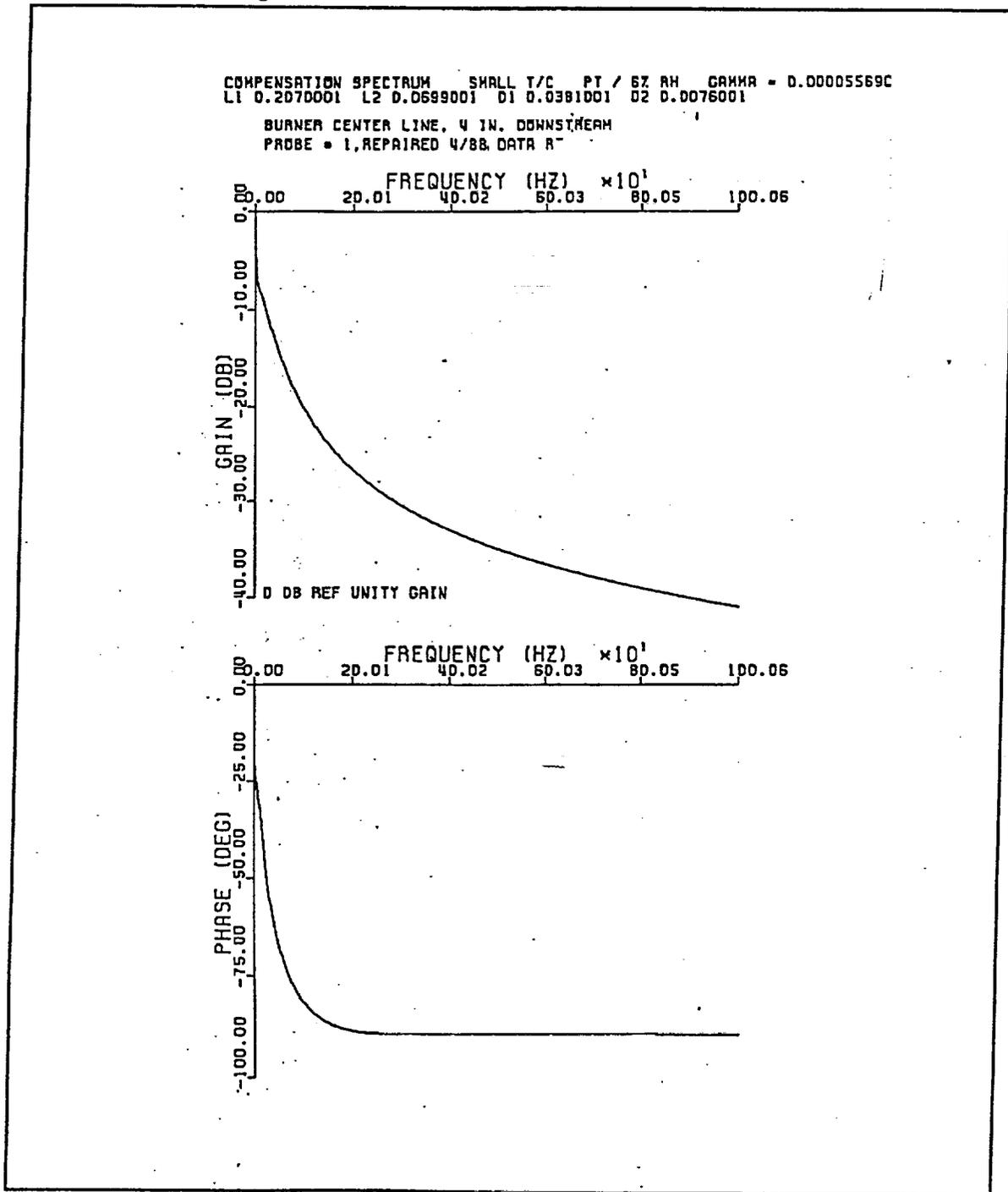


Figure D.1: Compensation Spectrum, Gain and Phase

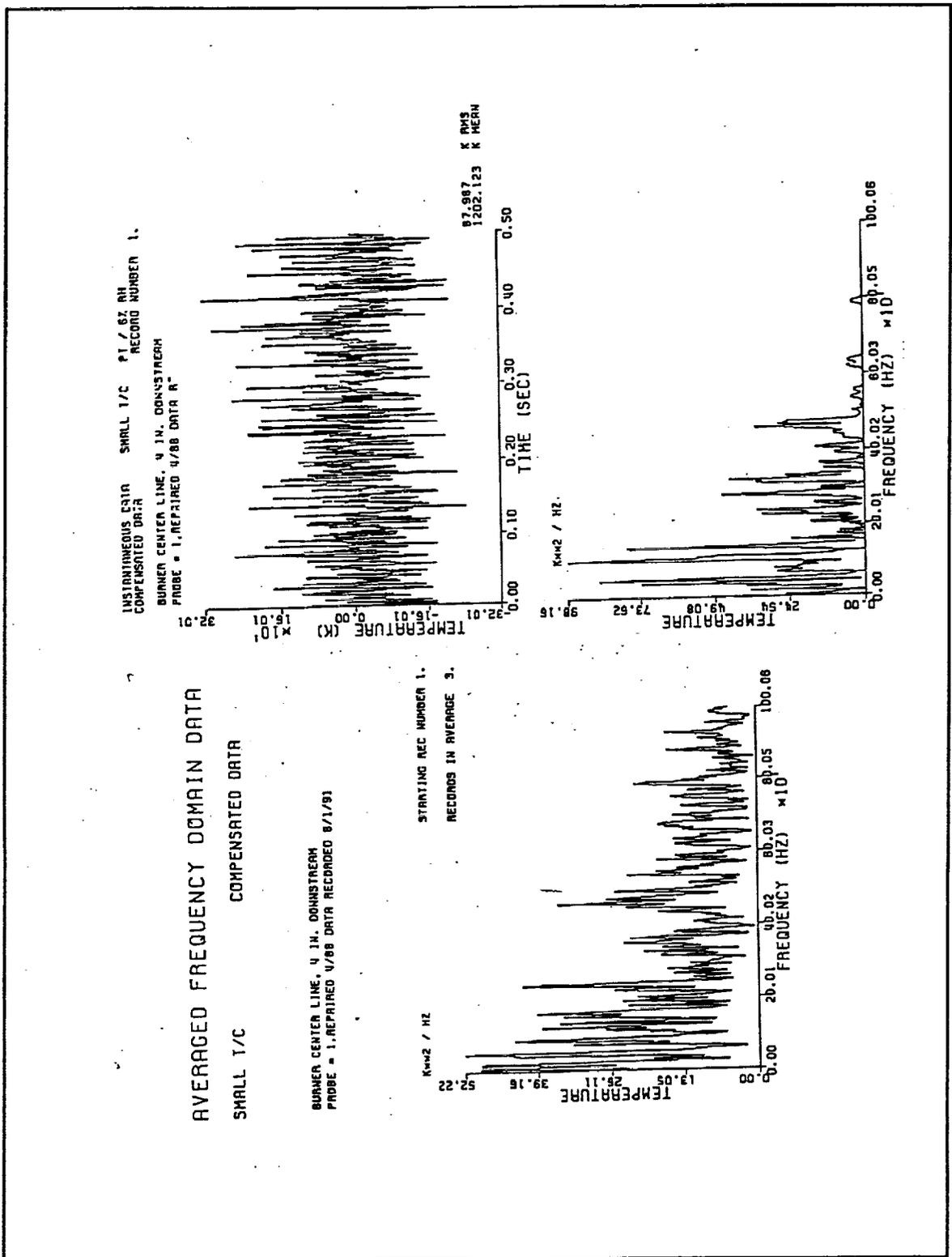


Figure D.2: Compensated Data: Average Power Spectral Density (K^2/Hz), Instantaneous Temperature (K), and Instantaneous Power Spectral Density (K^2/Hz)

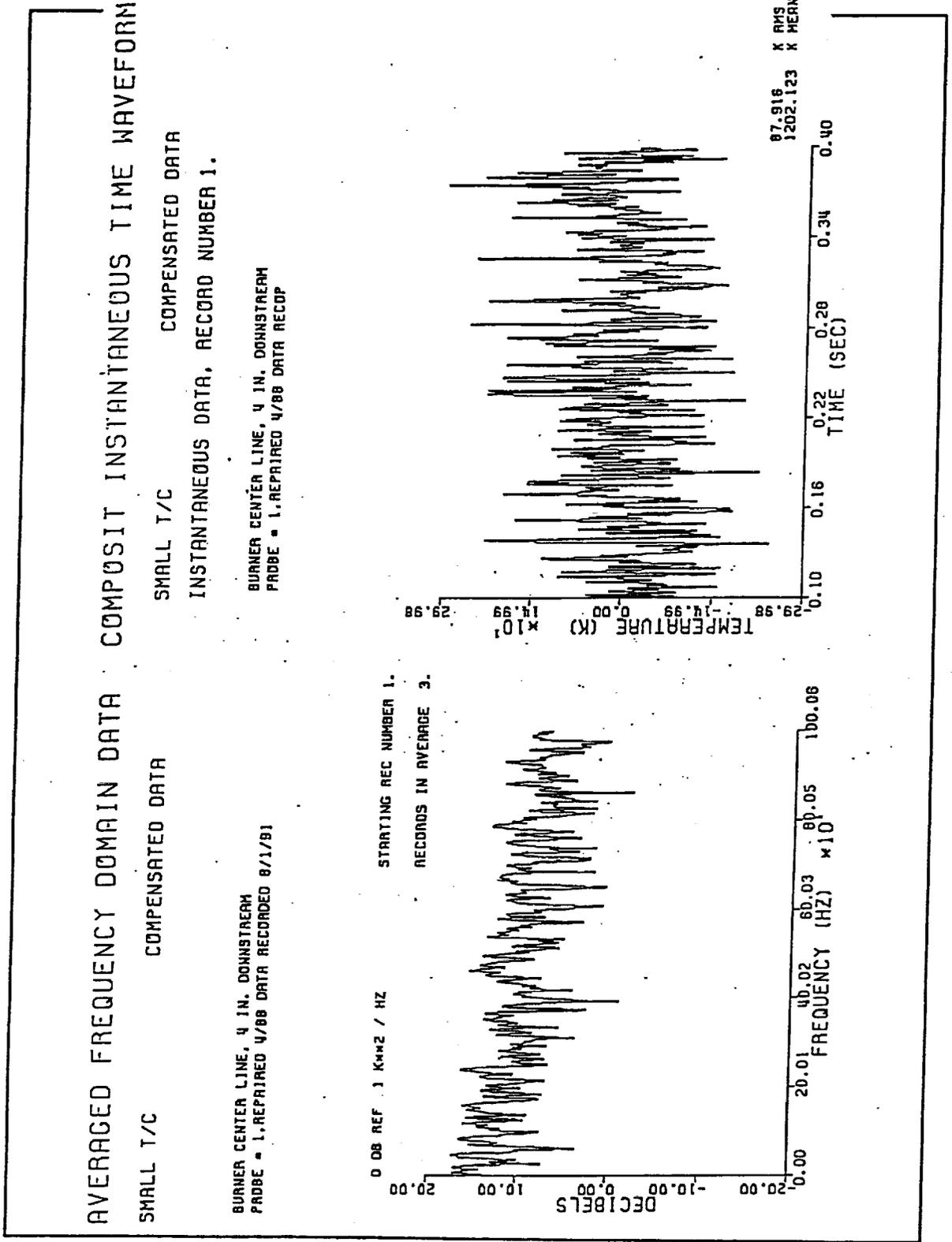
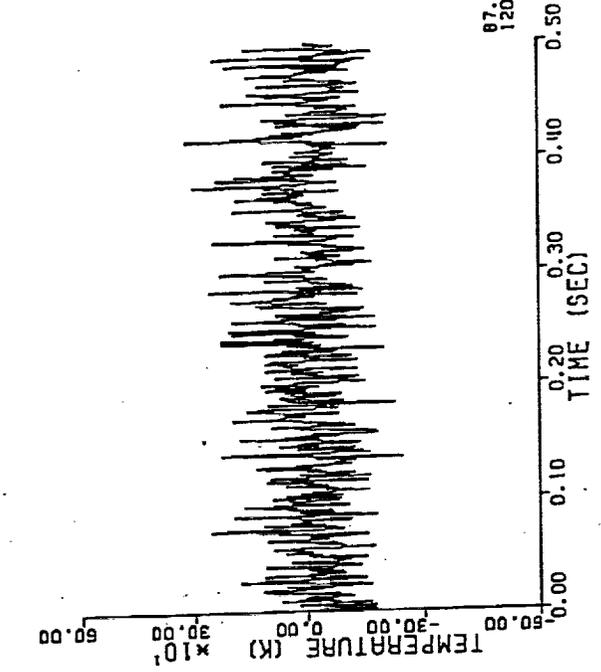
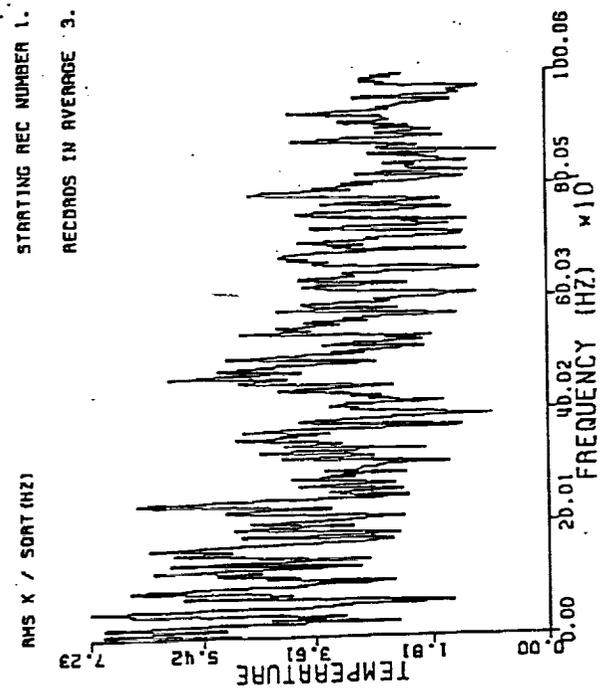


Figure D.3: Compensated Data: Average Log Power Spectral Density (dB), Instantaneous Temperature (K)

AVERAGED FREQUENCY DOMAIN DATA COMPOSIT INSTANTANEOUS TIME WAVEFORM
 SMALL T/C COMPENSATED DATA SMALL T/C COMPENSATED DATA
 INSTANTANEOUS DATA, RECORD NUMBER 1.

BURNER CENTER LINE, 4 IN. DOWNSTREAM
 PROBE # 1. REPAIRED 4/88 DATA RECORDED 8/1/91

BURNER CENTER LINE, 4 IN. DOWNSTREAM
 PROBE # 1. REPAIRED 4/88 DATA RECORDED 9



07.987 K RMS
 1202.123 X MEAN

Figure D.4: Compensated Data: Averaged Linear Power Spectral Density (K/\sqrt{Hz}), Instantaneous Temperature (K)

AVERAGED FREQUENCY DOMAIN DATA

SMALL T/C

COMPENSATED DATA

BURNER CENTER LINE, 4 IN. DOWNSTREAM
PROBE # 1. REPAIRED 4/88 DATA RECORDED

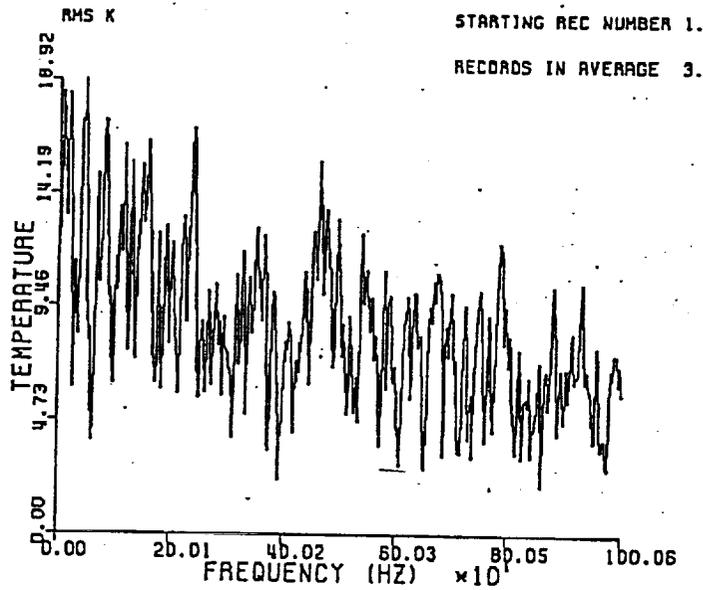


Figure D.5: Compensated Data: Averaged Narrowband Power Spectral Density (K rms)

November 17, 1992

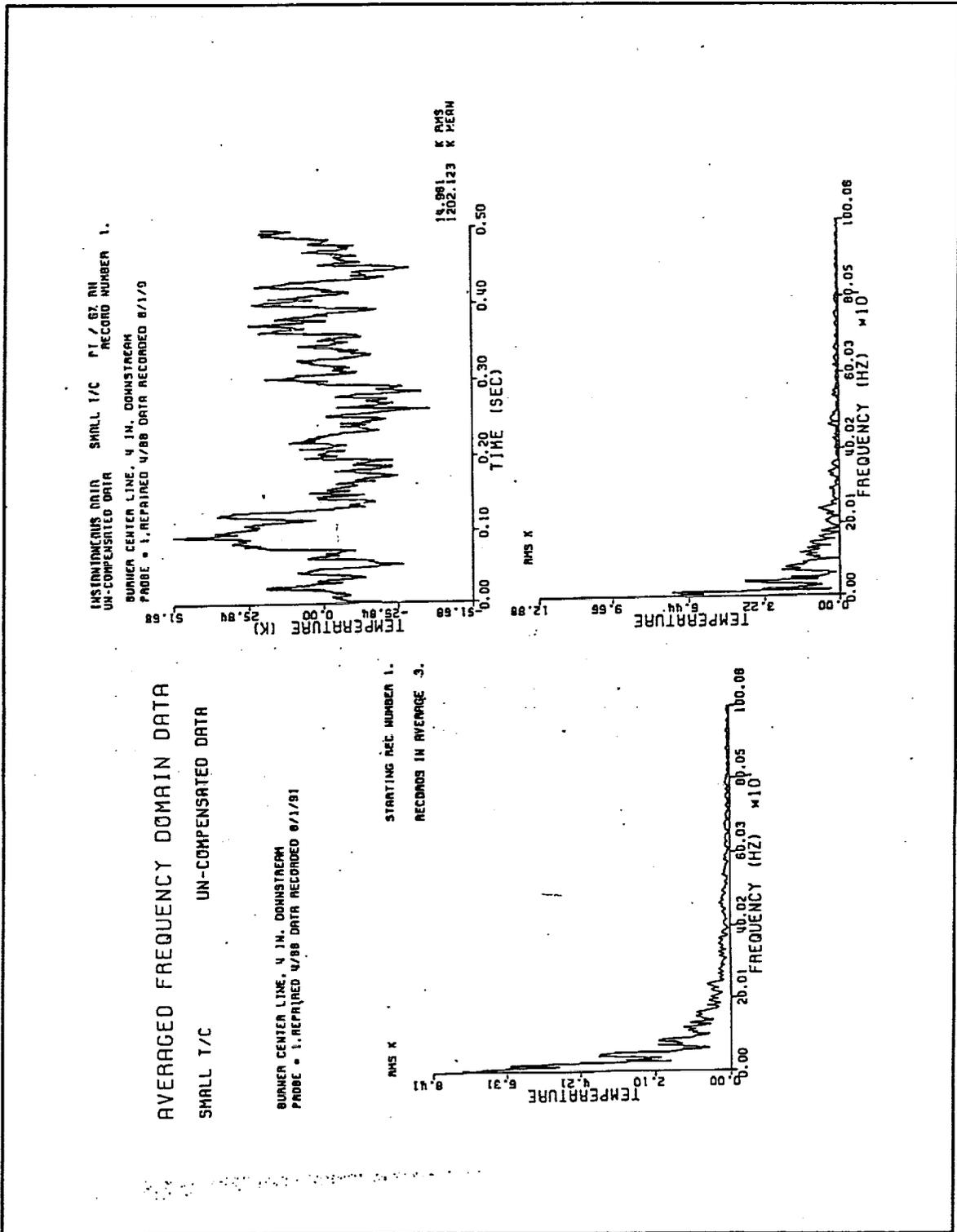


Figure D.6: Uncompensated Data: Averaged Narrowband Power Spectral Density (K rms), Instantaneous Temperature (K), Instantaneous Narrowband Power Spectral Density (K rms)

COMPOSIT INSTANTANEOUS TIME WAVEFORM

SMALL T/C COMPENSATED DATA
INSTANTANEOUS DATA, RECORD NUMBER 1.

BURNER CENTER LINE, 4 IN. DOWNSTREAM
PROBE = 1, REPAIRED 4/88 DATA REC

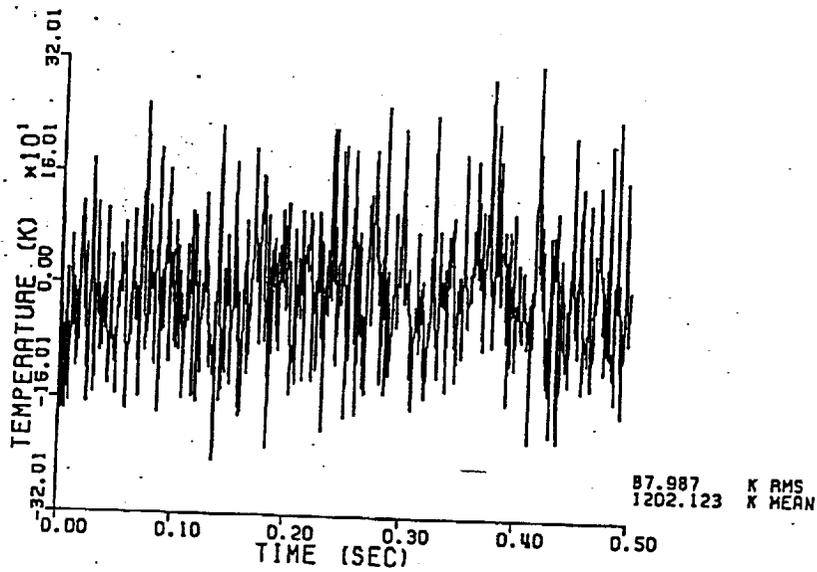


Figure D.7: Compensated Data: Instantaneous Temperature (K)

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