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Can Bose Condensation of Alpha Particles Be Observed in Heavy Ion Collisions?

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Abstract

Using a fully self-consistent quantum statistical model, we demonstrate the possibility of Bose condensation of alpha particles with a concomitant phase transition in heavy ion collisions. Suggestions for the experimental observation of the signature of the onset of this phenomenon are made.

Theory and Results

For some time, there has been an intrinsic interest in Bose condensation of particles with a concomitant phase transition in nuclear physics. Studies of multifragment final states over a broad range of incident energies may provide information about liquid-vapor and other types of nuclear phase transitions, and ultimately information about the equation of state of nuclear matter (refs. 1 through 12), provided that multifragment disintegrations are related to bulk instabilities of nuclear matter at low densities. Most experimental observations (ref. 8) provide end-point information and do not necessarily provide information about transient phenomena. Here we demonstrate that Bose condensation of alpha particles is such a transient phenomena, which has thus far eluded experimental observation. The aim of this letter is two-fold: first, to establish the possibility of Bose condensation of alpha particles and the concomitant phase transition; second, to suggest specific experimental observations for its verification. The smallness of the finite volumes being considered could mask a clear signal of the phase transition. However, by exercising sufficient care, experimental signatures for the onset of alpha condensation may be observable.

Bose condensation of pions was predicted earlier (ref. 6) to occur in the expansion phases of nuclear collisions. There is, however, a subtle difference between Bose condensation of alphas and that of pions. In the formation of alpha

particles, one requires four nucleons to be present in close physical and momentum proximity. Therefore, matter present must have some minimum density for condensation to occur. On the other hand, pion production is basically a one-nucleon phenomenon and therefore does not depend upon the presence of a minimum density of nuclear matter. Hence, pion condensation is observable in end-point measurements, such as total yields or cross sections, but alpha condensation is not. Its observation must be related to pre-ultimate stages of rarefaction (e.g., compression).

In this letter, we show that for a given excitation energy (represented by a temperature T), one must reach a critical density (ρ_c) to observe the phase transition. Although the condensed alphas are present at densities below ρ_c, their presence is masked by other (noncondensed) alphas produced in the collision. Our results indicate that the number of condensed alpha particles increases as the density increases toward ρ_c. Therefore, our approach to investigating this phenomenon is to start at minimum (zero) density and compress the nuclear matter in the collision until the critical density (where the system becomes unstable) is reached.

To focus on the physics of the phenomenon, a simple model is assumed. More detailed and reliable predictions would certainly require a more sophisticated

and realistic model; nevertheless, a simple model will suffice to demonstrate the phenomenon.

We begin with an initial assembly of nucleons consisting of N_{in} neutrons and Z_{in} protons which have been compressed and are evolving in thermodynamic equilibrium at an excitation energy (temperature) T to form clusters of light ions (^{4}He - α, ^{3}He - h, tritons - t, deuterons - d) together with neutrons (n) and protons (p). (Although the existence of a fully equilibrated system in heavy ion collisions is still debated, it is generally believed (refs. 4, 5, 9, 10, 11, 12) that equilibrium thermodynamics is a viable theory which gives reliable predictions). In the model, charge and baryon number are conserved using

$$N_p + N_d + 2N_h + N_t + 2N_\alpha = Z_{in}$$

$$N_n + N_d + N_h + 2N_t + 2N_\alpha = N_{in},$$ (1)

where the N_i refer to the number of particles of species i. These particles have finite dimensions and initially move within a volume V_{in}. For convenience, we treat the particles as point particles (no physical dimensions) which move in a reduced volume given by

$$V_{pt} = V_{in} - \sum_i N_i V_i,$$ (2)

where V_i is the eigen volume of the ith particle. The number (matter) density is

$$\rho = (N_{in} + Z_{in})/V_{in}.$$ (3)

The density of point particles is

$$\rho_{pt} = (N_{in} + Z_{in})/V_{pt} , \qquad (4)$$

where $\rho_{pt} > \rho$ since $V_{pt} < V_{in}$. Our results will be presented in terms of ρ_{pt}.

For a distribution of fermions, quantum statistics (ref. 13) yields

$$N_i = 2g_i \, V_{pt} \, \pi^{-1/2} \, \lambda_i^{-3} \, F_{FD} (\mu_i/k_B T), \qquad (5)$$

where $g_i = 2 S_i + 1$ is the spin degeneracy, μ_i is the chemical potential, λ_i is the thermal wavelength for the ith particle of mass m_i

$$\lambda_i = 2 \, \pi \, h \, (2\pi \, m_i \, k_B T)^{-1/2}, \qquad (6)$$

where k_B is Boltzmann's constant, and the species index is i (= p. n. h, or t) as appropriate. Values for the Fermi functions (spin $\frac{1}{2}$)

$$(7)$$

$$F_{FD} (v) = \int_0^\infty \frac{x^{1/2}}{1 + e^{x - v}} \, dx,$$

are tabulated in the literature (ref. 14) and are increasing functions of v. For bosons (refs. 13, 15, 16), we have

$$N_i = \left[\exp\left(-\mu_i/k_B T\right) - 1\right]^{-1}$$

$$+ \, g_i \, V_{pt} \, \lambda_i^{-3} \, F_{BE} \left(-\mu_i/k_B T\right), \qquad (8)$$

where the first term on the right side of equation (8) gives the number of condensed bosons and the species index is i (= d, α).

In equation (8), the Bose-Einstein functions are given by

(9)

$$F_{BE}(\nu) = \int_0^\infty \frac{x^{1/2}}{e^{x+\nu} - 1} \, dx.$$

In equilibrium, the chemical potential for the ith species is

$$\mu_i = z_i \, \mu_p + n_i \, \mu_n + \epsilon_i,$$
(10)

where z_i and n_i are the number of protons and neutrons in the ith species and ϵ_i is

its binding energy. This completes the description of the model used in this work.

To illustrate the application of these methods to a model system, consider an

assembly of $N_{in} = 40$ neutrons and $Z_{in} = 40$ protons, such as might exist in a

^{40}Ca - ^{40}Ca central collision. Figure 1 displays the distributions of light clusters for T

= 1, 2.5, 5, and 7.5 MeV as a function of the density ratio ρ_{pt}/ρ_o where $\rho_o = 0.17$ fm^{-3}.

Note that the distribution of clusters is completely dominated by alpha particles at

these excitations. Other particles/clusters begin to appear at higher temperatures

and lower densities. From the figure, we observe that the total number of alphas

(condensed and other) remains constant as the system is compressed, i.e., as density

is increased. However, the percentage of condensed alphas increases with increasing

density (i.e., compression) so that at higher densities (for a given temperature) the

dominant contribution comes from condensed alphas. It is these condensed alphas

which ultimately control the highest stable density (critical density ρ_c) beyond which

instability sets in. This instability is evident in our results by a lack of stable solutions for densities greater than the critical values. The critical density ratios (ρ_c/ρ_o) for the onset of instability at T = 1, 2.5, 5, and 7.5 MeV are 0.5, 2.1, 6.1, and 11.3, respectively. For comparison purposes, we note that the corresponding freeze-out density ratios (taking into account the finite cluster volumes) are 0.5, 2.05, 4.61, and 4.29, respectively.

Mathematically, the instability is related to the singularity resulting from $\mu \rightarrow 0$ in the first term on the right-hand side of equation (8). Physically, the instability is related to the phase transition, which would be observable if the nuclear systems were large enough (much greater number of particles in the ensemble), and if the system could be observed for a long time. In ideal quantum gases, this phase transition occurs with Bose condensation. At this point, alpha particles could be created with little or no energy investment. In our work, it is important to recognize that the alpha condensation occurring during compression is a hot one. Its existence is not limited to near zero temperatures. A remarkable feature is that this condensation, if it occurs, should be experimentally observable.

The question is, how do we observe it? The most common observable for any distribution of particles is to measure total yields/cross sections. A measurement of the total yield of alphas, however, would not confirm or deny the existence of

condensed alphas. From Figure 1, we note that the total yield of alphas (condensed and noncondensed) is constant and essentially independent of the density when below the critical density. Only the relative proportions of condensed to noncondensed alphas vary. Consequently, experimental observation/detection must be based upon methods which distinguish between these two categories of alphas. One such method of detection might involve measuring the momentum distributions of the alphas. In the center-of-mass system, the condensed alpha particles will be produced with near-zero momenta, whereas the noncondensed alphas will have much larger momentum values. Therefore, a possible signature for alpha condensation might be found in experimental measurements of total transverse momentum $(P_\perp)$ or total in-plane transverse momentum (P_x) of alphas which should **decrease** with **increasing** density. Ideally, for an infinitely-large system, the presence of a phase transition would be evident by a sudden (discontinuous) decline in measured momenta as the critical density is reached. For real, finite systems, any discontinuous reductions in measured momenta may be obscured by the small sizes of the systems considered; nevertheless, significant decreases in measured momentum components of alpha particles as the matter is compressed will provide indisputable evidence for Bose condensation of alpha particles with the concomitant phase transition.

Another experimental observable which may be linked to the Bose condensation of alpha particles is the possible occurrence of a critical maximum density ρ_c for a given excitation (T), a density above which stable nuclear matter is non-existent. Recall from our model that the critical density arises mathematically from the singularity occurring in the term [equation (8)] representing Bose condensation. This is less definitive, however, than directly observing alphas with near-zero momentum as discussed earlier.

Figure 2 displays the number of condensed alpha particles as a function of ρ_{pt}/ρ_0 for different temperatures. From the figure, it is apparent that the critical density where the instability occurs **does** depend on the temperature. Clearly, however, the ratio of condensed to noncondensed alphas at the critical density is independent of temperature. It is also interesting to note that for temperatures below the binding energy per nucleon of the alpha particle the number of condensed alphas reaches the same fixed value ($\approx$ 17 here) before the critical density is reached, irrespective of the system temperature.

Finally, we emphasize that the findings and conclusions of the present work, although definitive and quantitative in nature, need further study and refinement using more sophisticated models than the one used here. Nevertheless, experimental verification of our proposal should be possible even though difficult

because of the finite sizes of nuclear ensembles. In view of the importance of the

phenomena, verification of its existence merits concerted efforts by experimental

investigators.

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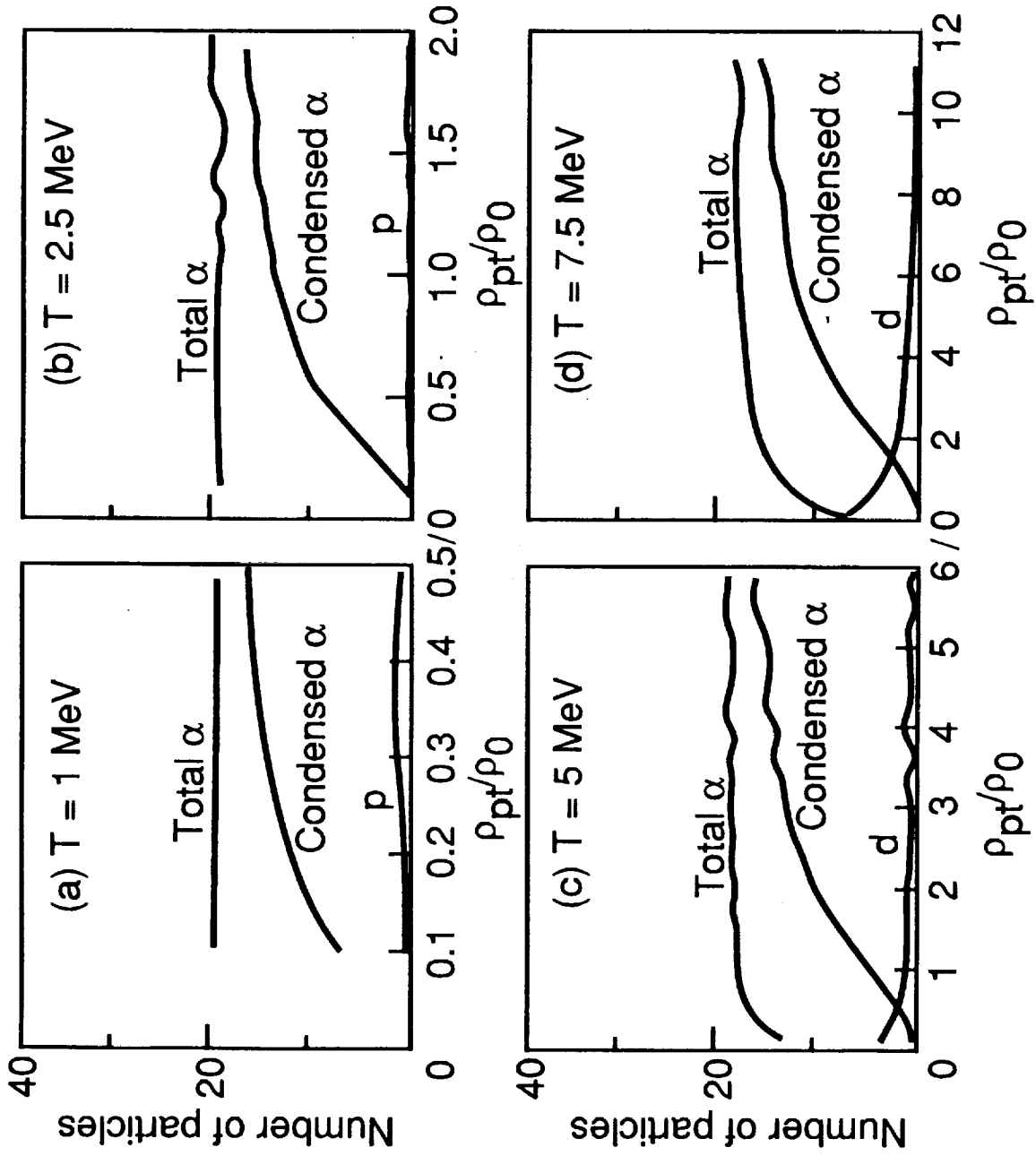
Figure 1. Distribution of particles up to critical density ratio for various temperatures. Curves for total alphas (condensed and noncondensed), condensed alphas and the next-most abundant other species are displayed. The initial ensemble contained 40 protons and 40 neutrons.
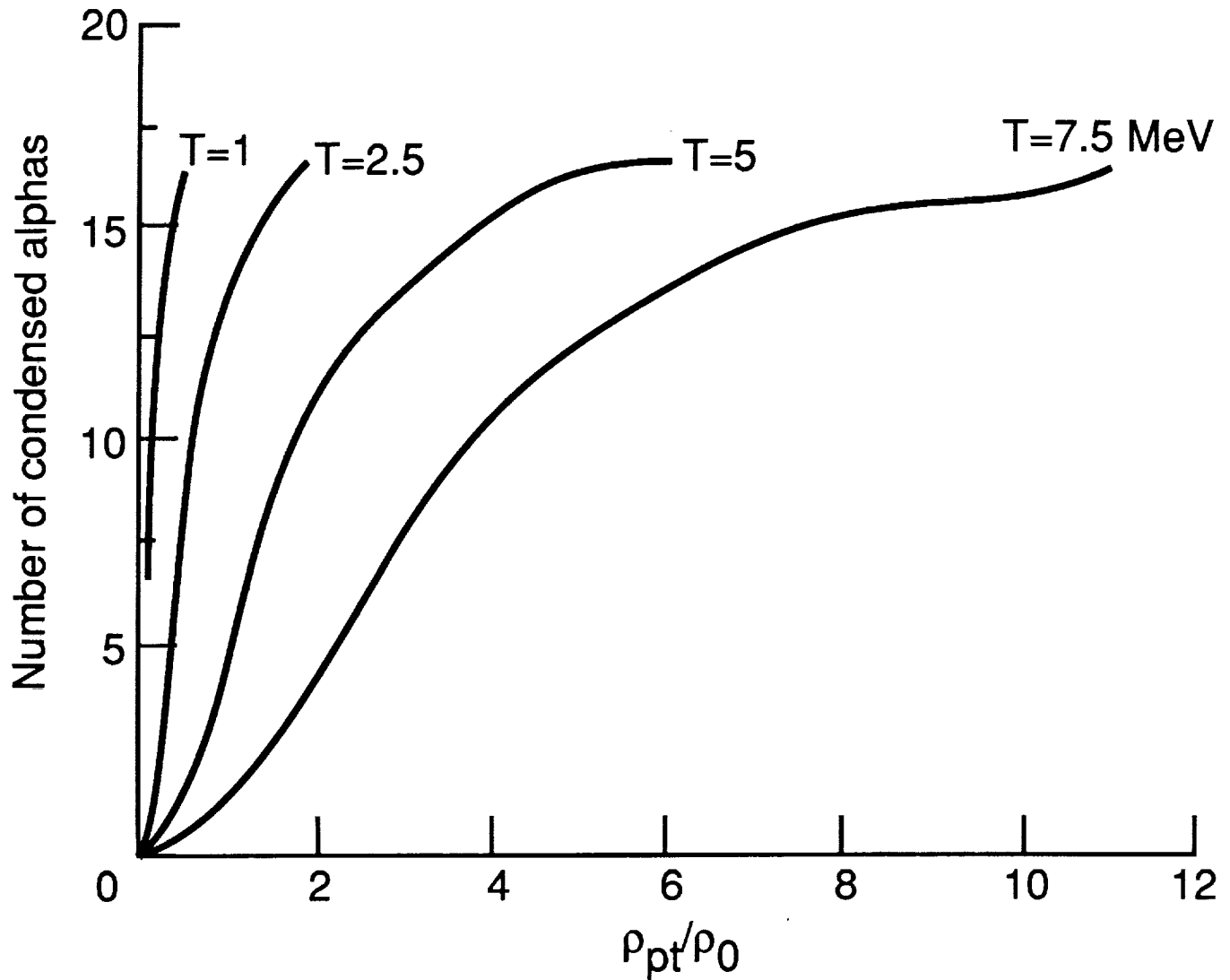
Figure 2. Number of condensed alphas as a function of density ratio at temperatures of 1, 2.5, 5 and 7.5 MeV. Each curve terminates at the critical density ratio for that temperature. Note that the number of condensed alphas reaches the limiting value of 16.7 at the critical density, irrespective of temperature.

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