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Finite Element Model and Identification Procedure

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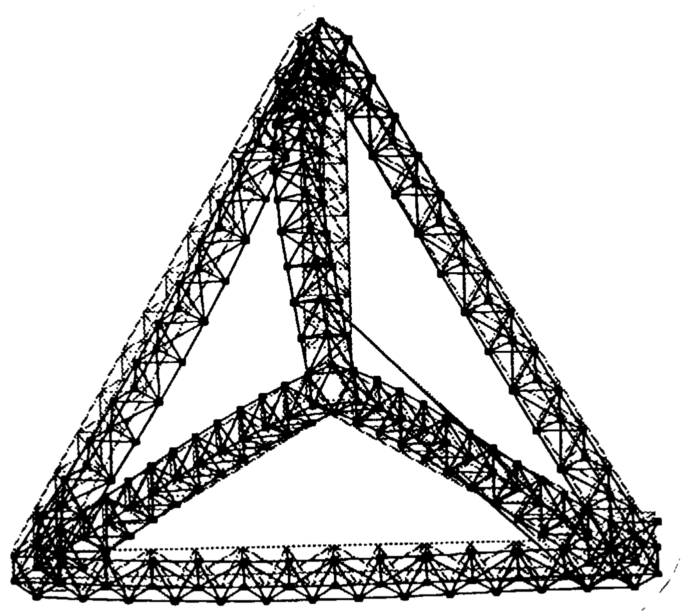
SERC Steering Committee Meeting
22 January, 1992

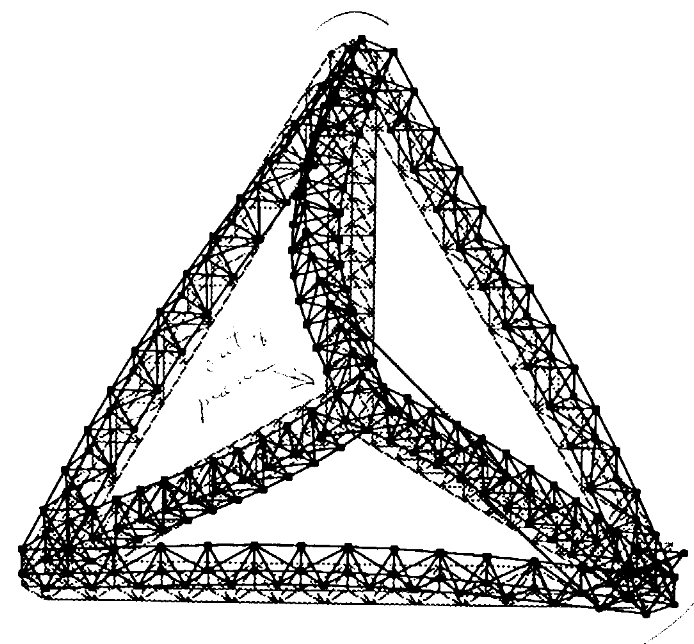
Interferometer Finite Element Model

- ADINA model, with 1500 degrees of freedom.
- Important attributes:
  - 1 beam element per strut
  - consistent mass matrix used
  - node flexibility incorporated through measured strut component test data
  - wires modelled as distributed masses
  - damping not modelled directly, included as modal damping in post processing
  - closely spaced modes due to near symmetries in structure
  - requires approximately 2 mins of Cray CPU time for the first 40 flexible modes.

Testbed Mode Shapes

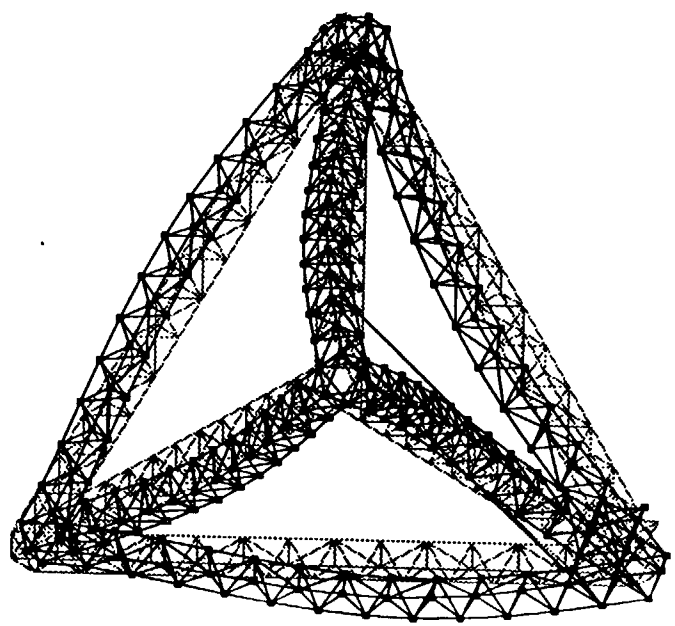


Mode 1 (25.8 Hz)

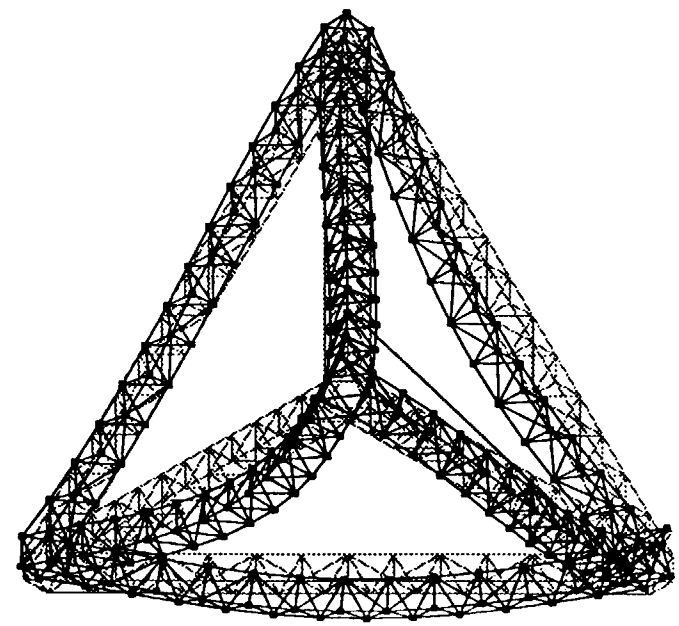


Mode 2 (27.2 Hz)

Testbed Mode Shapes



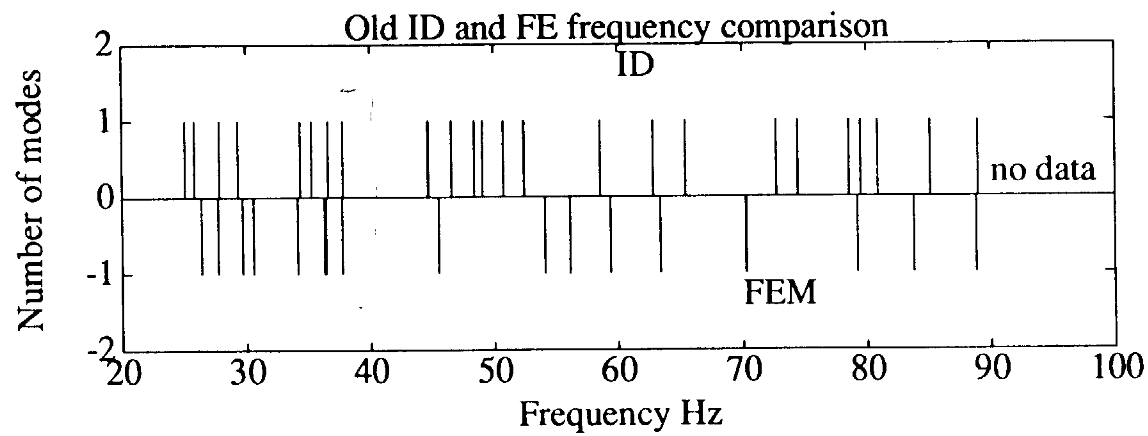
Mode 6 (36.1 Hz)



Mode 7 (36.3 Hz)

Finite Element Model Update

- Large discrepancy between finite element model and identified frequencies indicate that update required.



- Agreement of modal frequency distribution:
  - poor at high frequencies
  - better for lower frequency modes dominated by first leg bending modes.
- Better model needed for sensor, actuator, damper placement, and initial control designs.

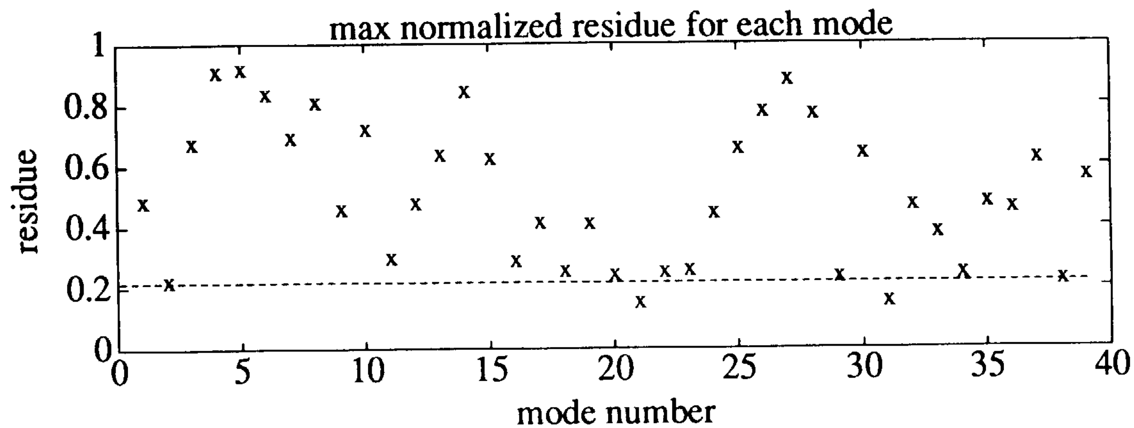
Identification Procedure

- Hardware:
  - 29 Kistler, 9 Sunstrand accelerometers
  - Bruel and Kjaer electromagnetic shaker
  - Tektronix scanner used to simultaneously measure all 38 channels.
- Selection of shaker locations: [illegible] on FE model
  - 2142 possible locations reduced to 24 based on rankings using *mean* and *maximum* modal controllability.
  - goal: maximize controllability of least-controllable mode

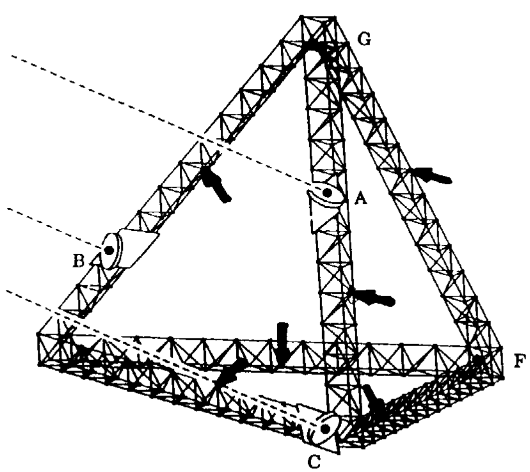
$$\max_i \left(\min_r |A_i^r| \right) \qquad \begin{cases} A &= \text{modal residue at input} \\ i &= \text{input dof (24)} \\ r &= \text{mode number (20)} \end{cases}$$

Shaker Locations

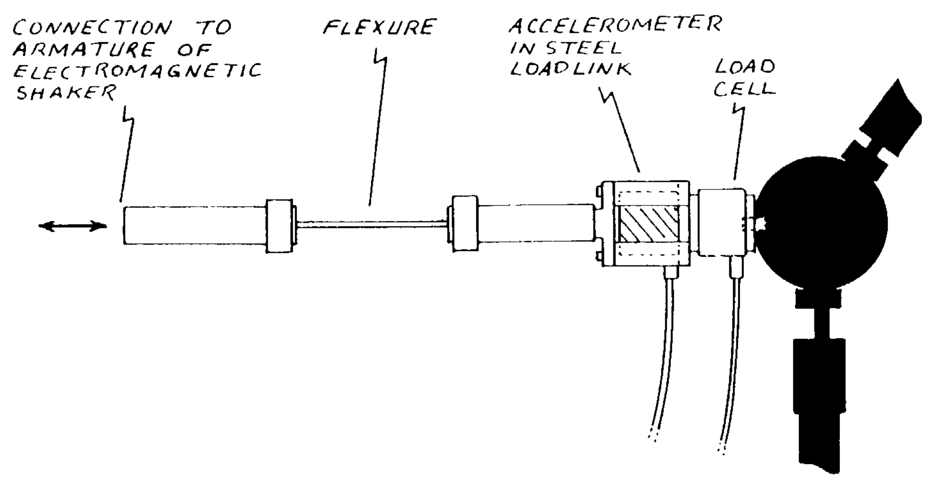
- Analysis resulted in one shaker location in each truss leg.



- Shaker locations:



Shaker Tip:



Data Analysis

- Transfer functions fit with modified least squares approach (R. Smith, UCSB).
- Leads to state space representation of measured data:

$$G_{fit}(s) := \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

– state space representation of each mode:

$$A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix} \qquad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C_i = \begin{bmatrix} c_{1i} & c_{2i} \end{bmatrix} (38 \times 2)$$

– full model:

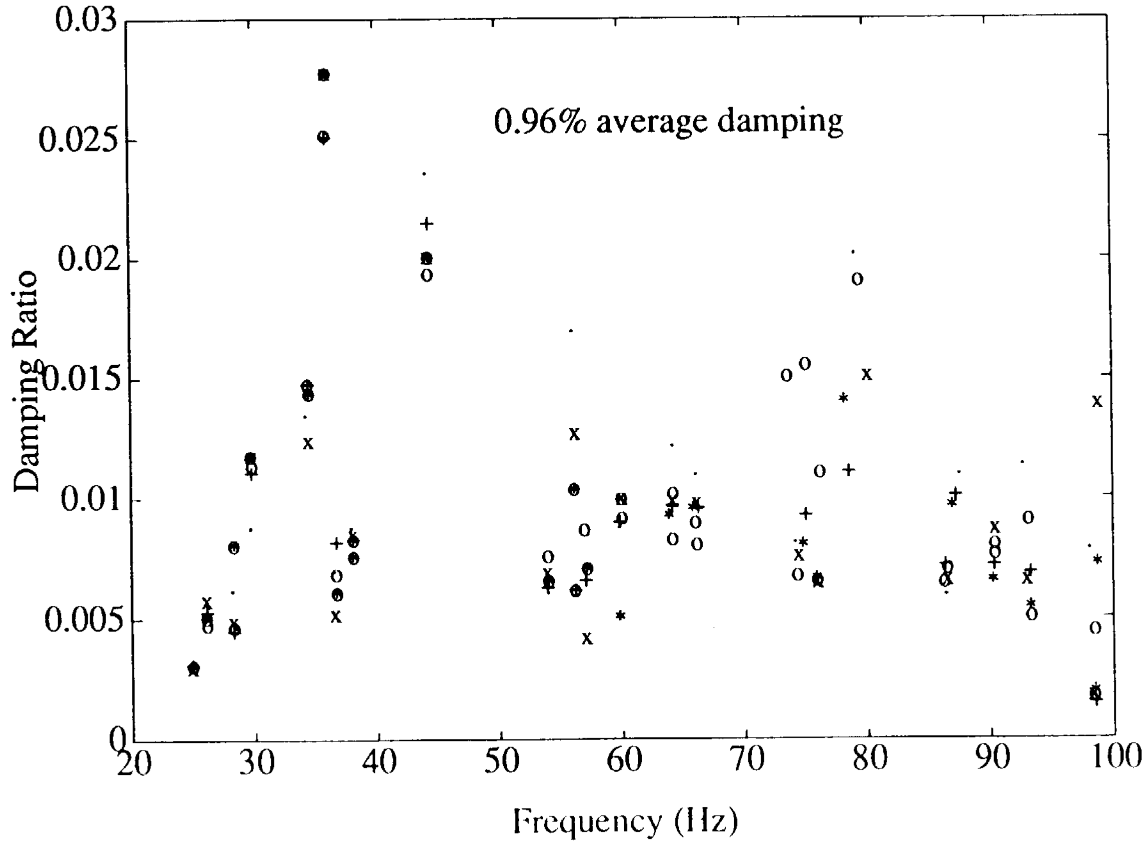
$$A = \mathrm{BlockDiag}\ (A_i)\ ,\ B = \mathrm{Col}\ (B_i)\ ,\ C = \mathrm{Row}\ (C_i)\ ,\ D$$

Data Analysis

- The A and B matrices held fixed for each shaker location.
- Full C matrix adds extra flexibility to approach.
- Modal frequency and damping computed with invfreqs function in MATLAB on several transfer functions.
- Note: good fits require good estimates of the frequency and damping of every mode in the frequency range of interest.
- One of several curve fitting approaches employed at SERC.

Modal Frequency and Damping Comparison

- Six A matrices agree well in frequency, less so in damping.



Computational Procedure

- C and D matrix rows independently selected for each sensor.
- Example: Pick D

$$E_r = \left(\sum_{i=1}^{m} \|Y(j\omega_i) - G(j\omega_i)U(j\omega_i)\|^2 \right)^{\frac{1}{2}}$$

$$\begin{aligned} Z(j\omega_i) &= Y(j\omega_i) - C_b(j\omega_i - A_b)^{-1} B_b U(j\omega_i) \\ E_r(j\omega_i) &= Z(j\omega_i) - D(j\omega_i)U(j\omega_i) \end{aligned}$$

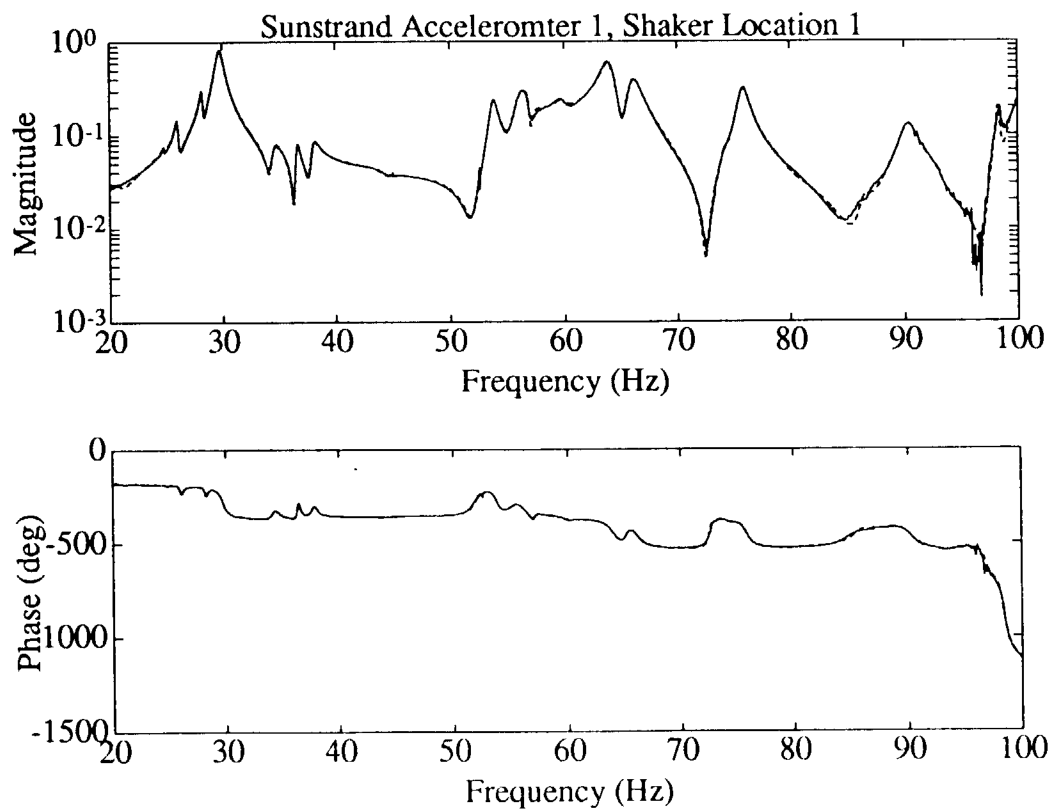
$$\text{Then } D_a = [\mathrm{Re}(\overline{Z})\ \mathrm{Im}(\overline{Z})] \,/\, [\mathrm{Re}(\overline{U})\ \mathrm{Im}(\overline{U})]$$

$$\text{where } \overline{Z} = [Z(1)\, Z(2)\, \ldots Z(m)]$$

- Similar for the C matrix. Several iterations required.
- Software exists to perform an overall A matrix update.

Fit Comparison

- Final fit comparison:



- Procedure effectively fits hundreds of transfer functions. Results good enough for control designs.

Residue Analysis

- Need to compute displacement residues from approximate accelerometer transfer function.

$$G_{fit}(s) = \sum_{i=1}^{m} \frac{c_{1i} + c_{2i}s}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} + d \approx \frac{\ddot{y}}{f}$$

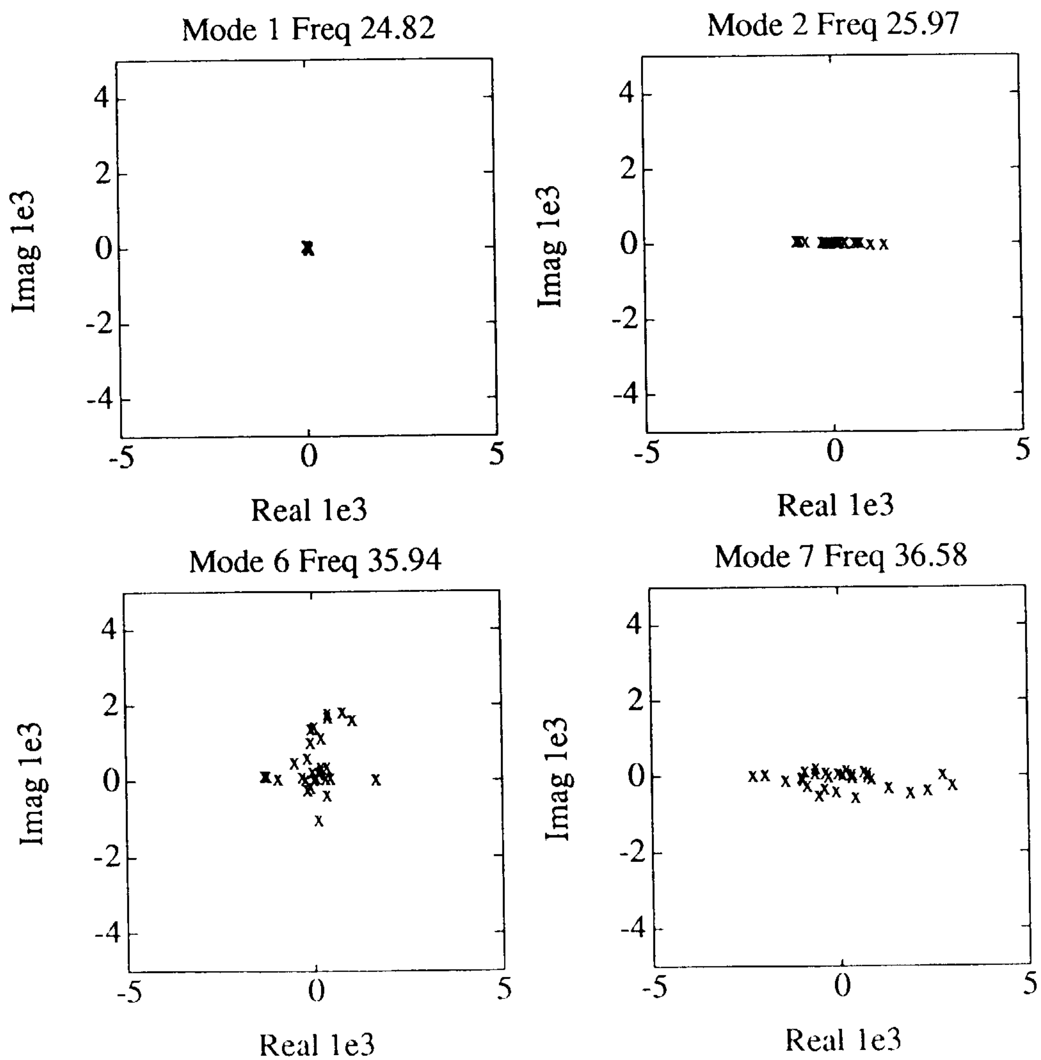
$$\overline{G}_{fit}(s) = \sum_{i=1}^{m} \frac{b_{1i} + b_{2i}s}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} + \frac{h(s)}{s^2} \approx \frac{y}{f}$$

where
$$b_{1i} = -\frac{(1-4\zeta^2)}{\omega_i^2}\, c_{1i} - \frac{2\zeta}{w_i}\, c_{2i}$$
$$b_{2i} = \frac{2\zeta}{\omega_i^3}\, c_{1i} - \frac{1}{w_i^2}\, c_{2i}$$

$$\text{Residue}: \ \phi_i(x_{act})\phi_i(x_{sens})^H = (b_{1i} + b_{2i}s)\,\big|_{s=j\omega_i}$$

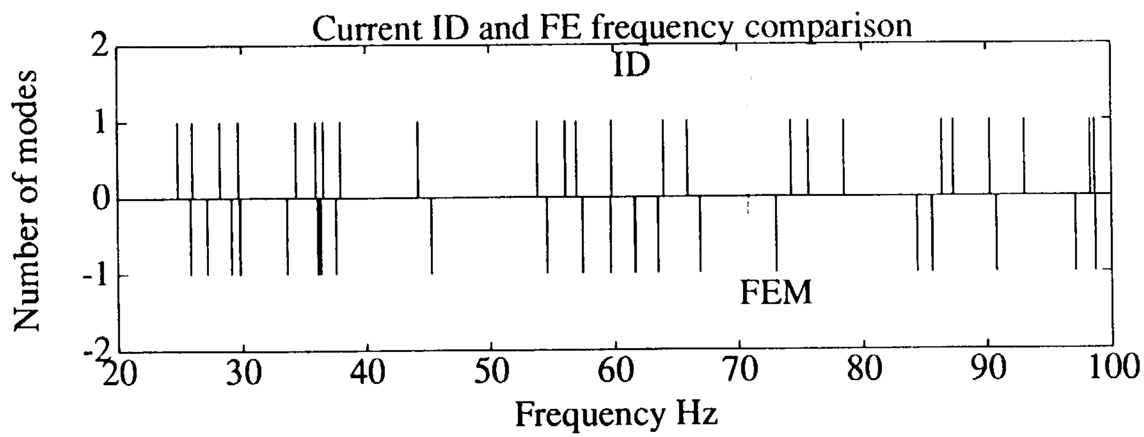
Typical Residues

- Residues rotated by phase at sensor collocated with shaker.



Current Status

- Frequency comparison after structural and model updates:

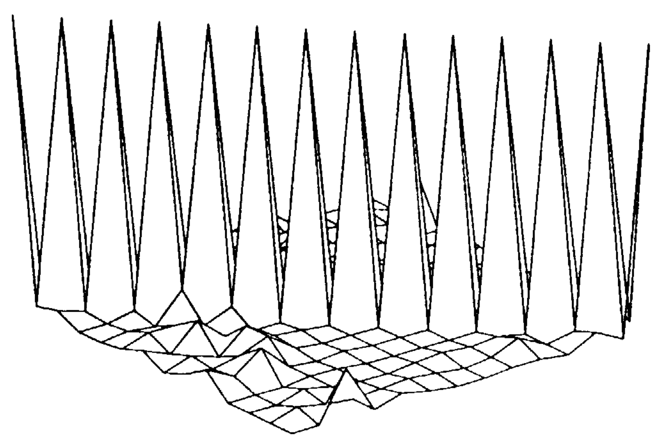


- Modifications:
  - eigenvector studies illustrated importance of plate flexibility, inclusion in the FEM lead to improved frequency agreement (¡4 % error in first 9)
  - fourth vertex stiffened to improve optical alignment, and better agreement indicates prior presence of local modes.

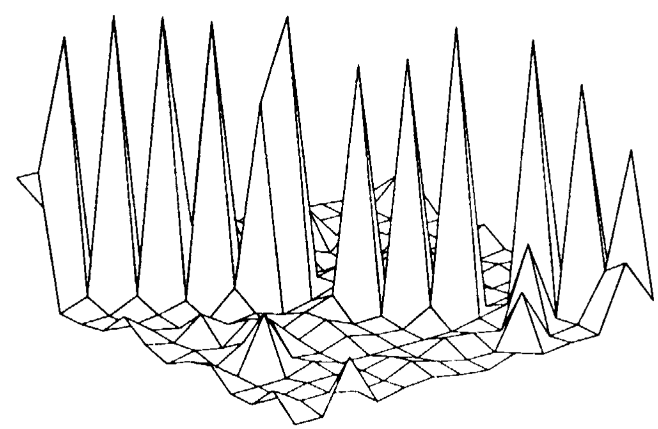
Identification/FEM Residual Comparison

- Correlate identified and FEM residues for first 14 modes.
- Modal Assurance Criterion:

$$\mathrm{mac}(x_1, x_2) = \frac{\| \Sigma_{i=1}^{m} \phi(x_1)_i \phi(x_2)_i^H \|^2}{\left(\Sigma_{j=1}^{m} \phi(x_1)_j \phi(x_1)_j^H\right)\left(\Sigma_{j=1}^{m} \phi(x_2)_j \phi(x_2)_j^H\right)}$$



Model/Model



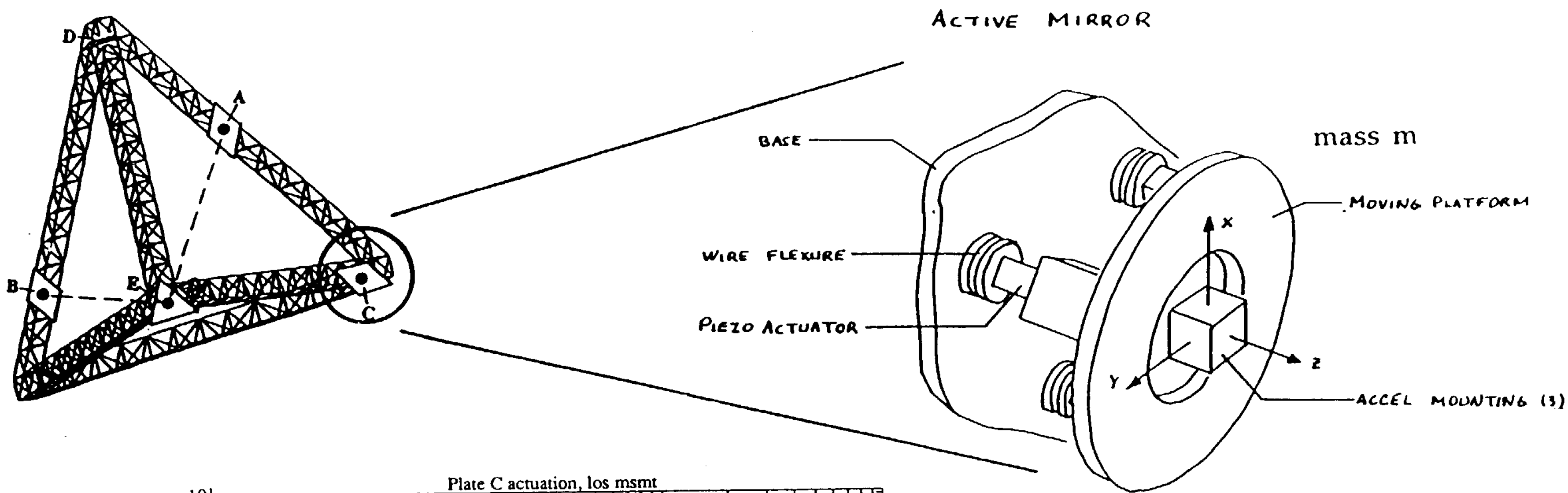
Model/Measured

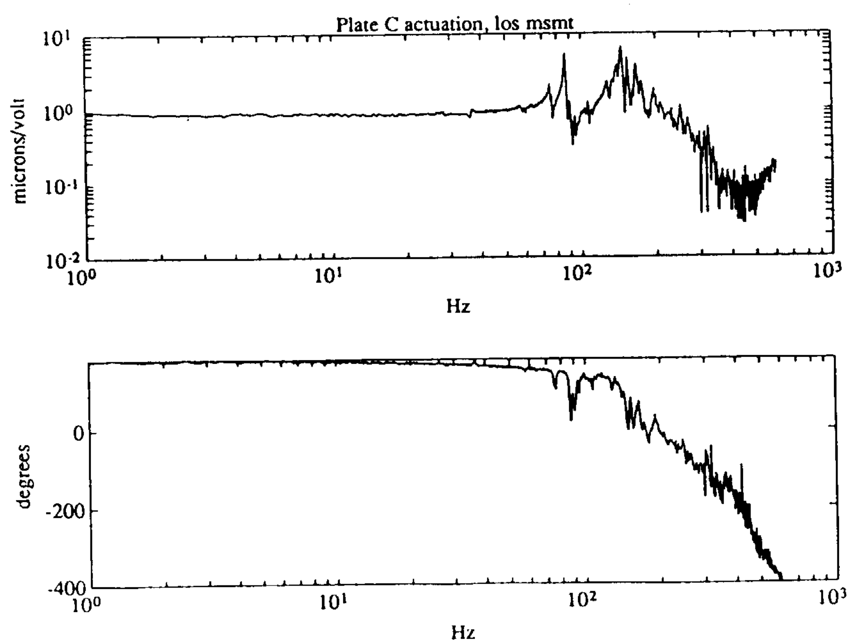
- Issue: FEM modes real, measured residues complex.

Future Work

- Continue coarse FEM changes to correct plate flexibility and mass distribution assumptions.
- Apply gradient type updates on the stiffness and mass matrices to match residues of higher frequency modes.
- Improve FEM suspension model with ID data.
- Develop state space model that can be used for sensor, actuator, damper placement, and initial control designs.

Pathlength Control Using Isolation Mounts





Input: piezo voltage

Ouput: pathlength C-E (microns)