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A Stochastic Approach to Robust Broadband Structural Control

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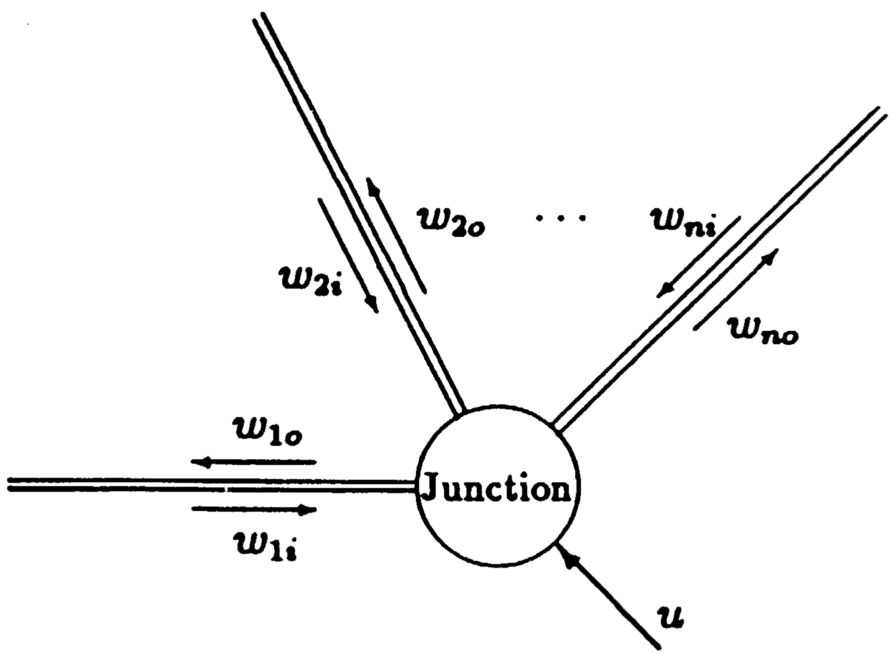
Goal

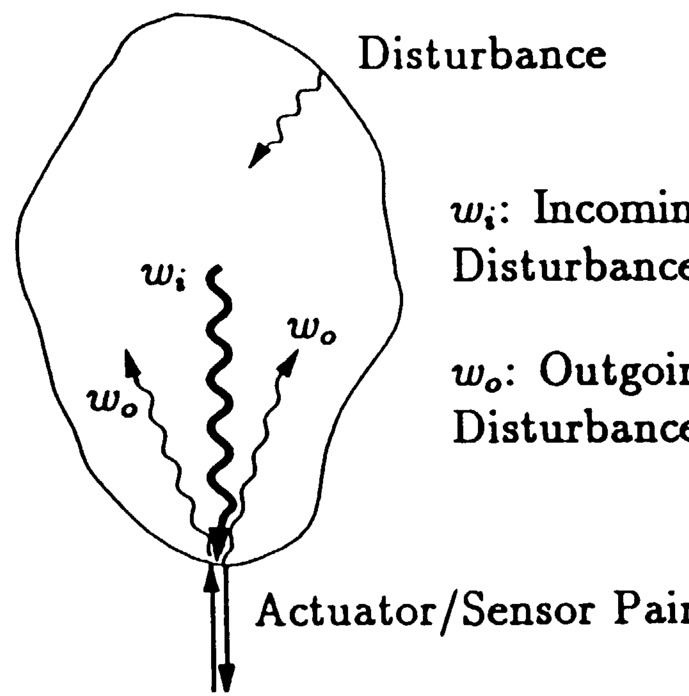
- Active broadband control of uncertain modally dense structures.
- Use collocated feedback.
  - Positive real controller guarantees stability.
  - Low authority or local control ("active damping.")
- Use local acoustic or statistical model of structure.
- Maximize power dissipation.
  - Equivalent to impedance matching.
  - Cannot match impedance exactly at all frequencies due to causality constraint.
- Experimental demonstration on complex structures.

Local Models

Travelling wave model



Dereverberated mobility model

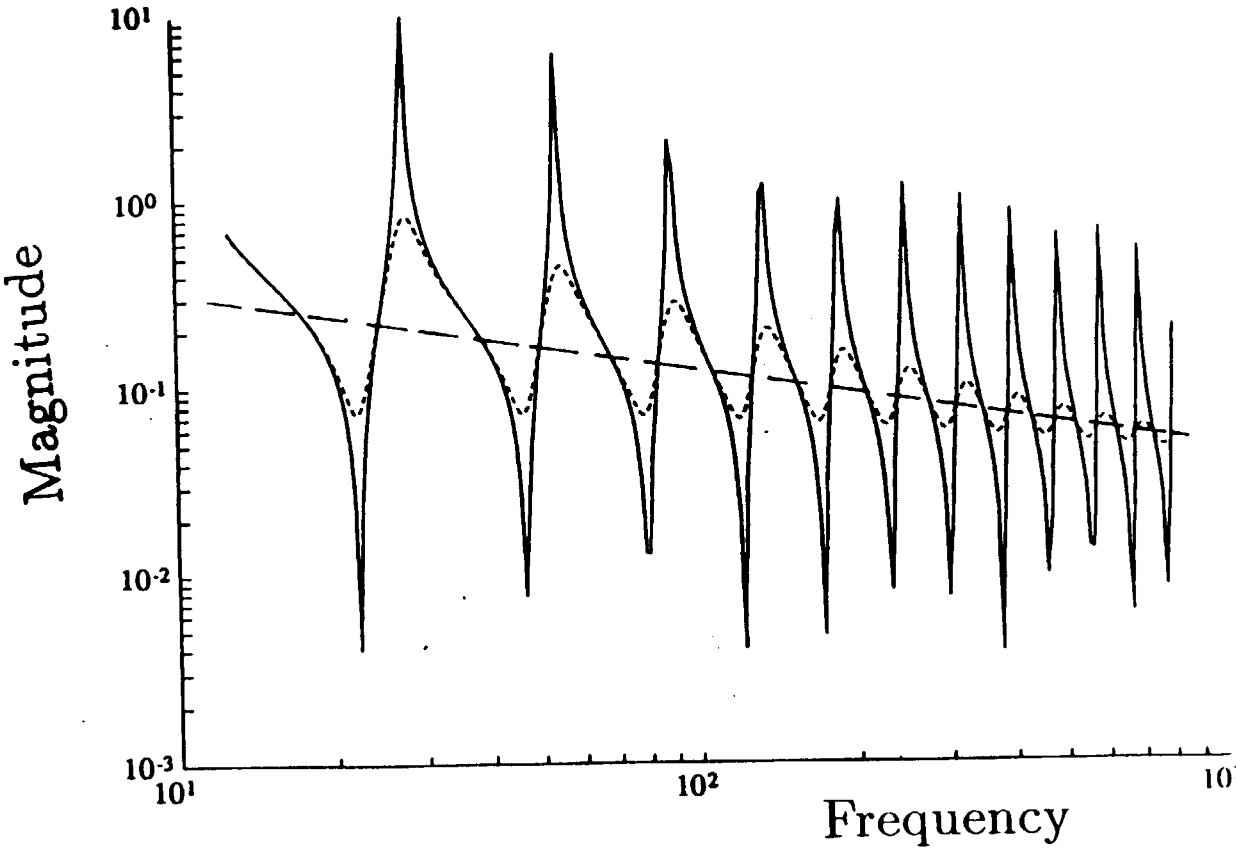


w_i: Incoming Disturbances

w_o: Outgoing Disturbances

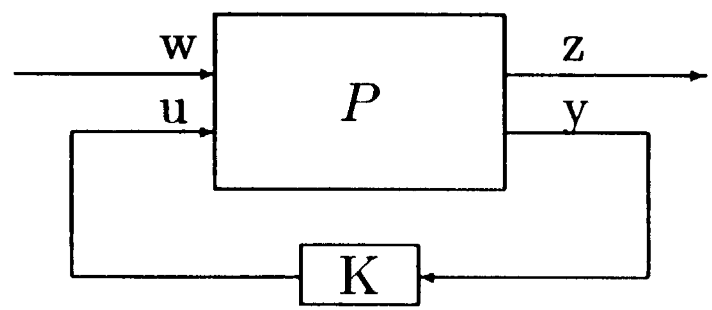
Computation of Dereverberated Mobility

- From averaging transfer function:



Power Flow

- Use local model only.
- Power dissipation related to input mobility G:



$$\begin{Bmatrix} z \\ y \end{Bmatrix} = \begin{bmatrix} G_1 I & G_1 G_0^{\sim} \\ G_0 & G \end{bmatrix} \begin{Bmatrix} w \\ u \end{Bmatrix}$$

- Define $H(s) = \dfrac{z(s)}{w(s)}$.
  - Analagous to reflection coefficient
  - Reflected power is $(H^* H) w^* w$
  - Dissipated power is $(I - H^* H) w^* w$

Impedance Matching

- Maximum power dissipation is obtained if the compensator is the conjugate of the structural impedance.
  - This is noncausal $\Rightarrow$ need best causal approximation.
- $\mathcal{H}_2$ optimization: No guarantee of stability on actual structure.
- $\mathcal{H}_\infty$ optimization: guarantees stability, but doesn't minimize desired performance metric.
- Goal: minimize actual rms cost and guarantee stability using only local information.

Stochastic Systems

- Investigate systems of the form

$$\dot{x} = A(\sigma)x + w$$

  - σ is a random variable with known probability distribution.
  - Uncertainly is only in the imaginary part of the eigenvalues.
- The average covariance $\langle xx^T \rangle_{\sigma,w}$ satisfies
  - Incoherence: the amplitudes of distinct modes are uncorrelated,
  - Equipartition: the average kinetic and potential energy of each mode is the same, and
  - Conservation of energy (for an undamped system.)
- These are the key assumptions made in Statistical Energy Analysis.

Control Problem

- Minimize "global" mean-square performance metric $\langle y^T y \rangle$ using only local knowledge, local control.
- Use characteristics of stochastic systems.
- Use conservation of energy!

Control of Stochastic Systems

- Incoherence $\Rightarrow$

$$\langle y^T y \rangle = \sum_{n=0}^{\infty} C_n E_n \simeq \int_{-\infty}^{\infty} C(\omega) E(\omega)$$

- Incoherence & Equipartition $\Rightarrow$ incoming structural power is proportional to structural energy.
  - Total power dissipated is therefore

$$\Pi_{diss} = \underbrace{(I - H(j\omega)^* H(j\omega))}_{\textit{Relative Dissipation}} E(\omega)$$

- Conservation of energy $\Rightarrow$

$$\Pi_{diss} = \Pi_{in}$$

- The average (over uncertainty) cost is therefore:

$$J = \frac{1}{2\pi} \int_{-\infty}^{\infty} C(\omega) \left[(I - H^* H)^{-1} H^* H \right] \Pi_{in}(\omega) \, \mathrm{d}\omega$$

Properties

- Guarantees $\mathcal{H}_\infty$ norm bound, $\|H\|_\infty < 1$.
  - Guarantees $\left\{\begin{array}{l}\text{positive real}\\ \text{dissipates power}\\ \text{stable closed loop}\end{array}\right.$
- Overbounds weighted $\mathcal{H}_2$ cost.
  - Minimizes desired performance metric.

Using Cost Functional

- Cost is related to other $\mathcal{H}_2/\mathcal{H}_\infty$ approaches, and to differential games.
- Can evaluate cost for a state space system with solution to a Riccati equation and a Lyapunov equation.
- Can optimize cost using Lagrange multiplier approaches
  - Yields coupled Riccati and Lyapunov equations.
  - Cannot be solved explicitly.
  - Can be solved using numerical optimization.

Example: Bernoulli-Euler Beam

- Free-free beam, force actuator and velocity sensor at left end, disturbance force at right end.
- Minimize difference between end-point displacements.

- Dereverberated mobility: (describes local dynamics)

$$G(s) = \frac{\sqrt{2}}{(\rho A)^{3/4}(EI)^{1/4}} \cdot \frac{1}{\sqrt{s}}$$

- Non-causal impedance match: (maximum dissipation)

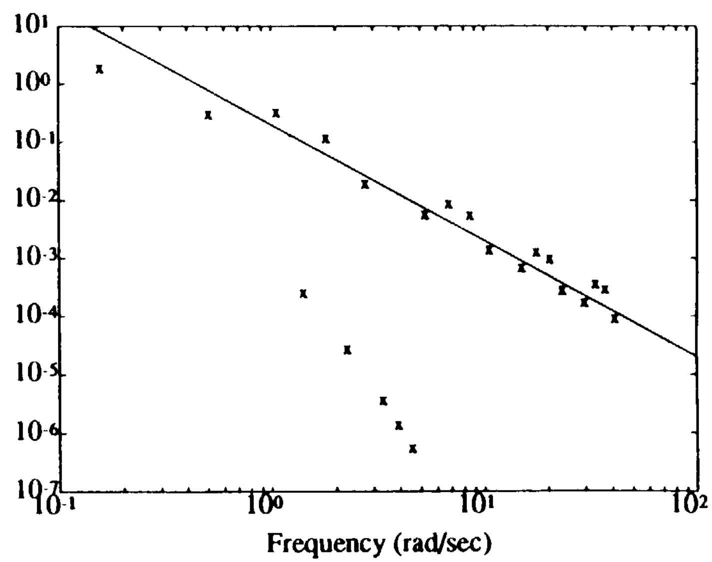
$$K(s) = \frac{(\rho A)^{3/4}(EI)^{1/4}}{\sqrt{2}} \cdot \sqrt{-s}$$

Compensator Design

- Choose $\Pi_{in}(\omega)$ based on disturbance spectrum $V(\omega)$ and dereverberated input mobility at disturbance location $G_d(\omega)$.

$$\Pi_{in} = (G_d + G_d^*)V$$

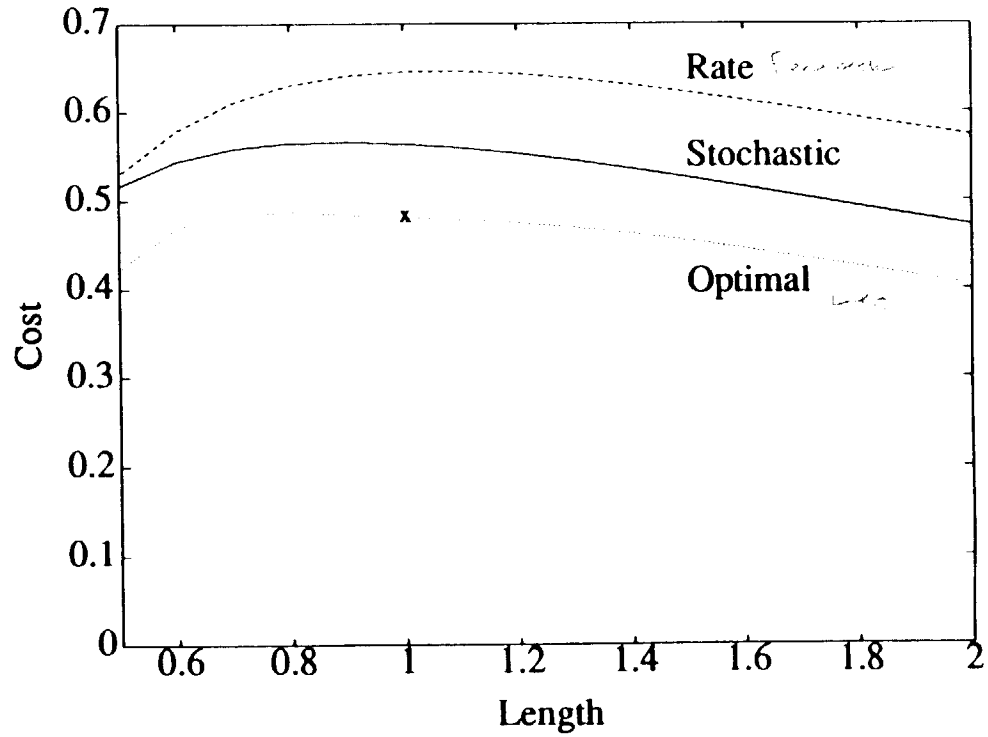
- Choose $C(\omega)$ to approximate modal cost:



- Choose desired compensator order and use numerical optimization.

Performance

- Compare cost versus length of beam for various compensators.
  - LQG compensator unstable for small changes in length.



"Power" Dual Variables

- Force into structure and relative velocity across active strut are dual.
- Piezo stack stiffness is high $\Rightarrow$ commands displacement.

  –Can also command relative velocity.
- Want compensator $K(s)$ such that

$$\begin{aligned} \dot{x} &= K(s)f \\ \Rightarrow x &= K(s)\left(\frac{f}{s}\right) \end{aligned}$$

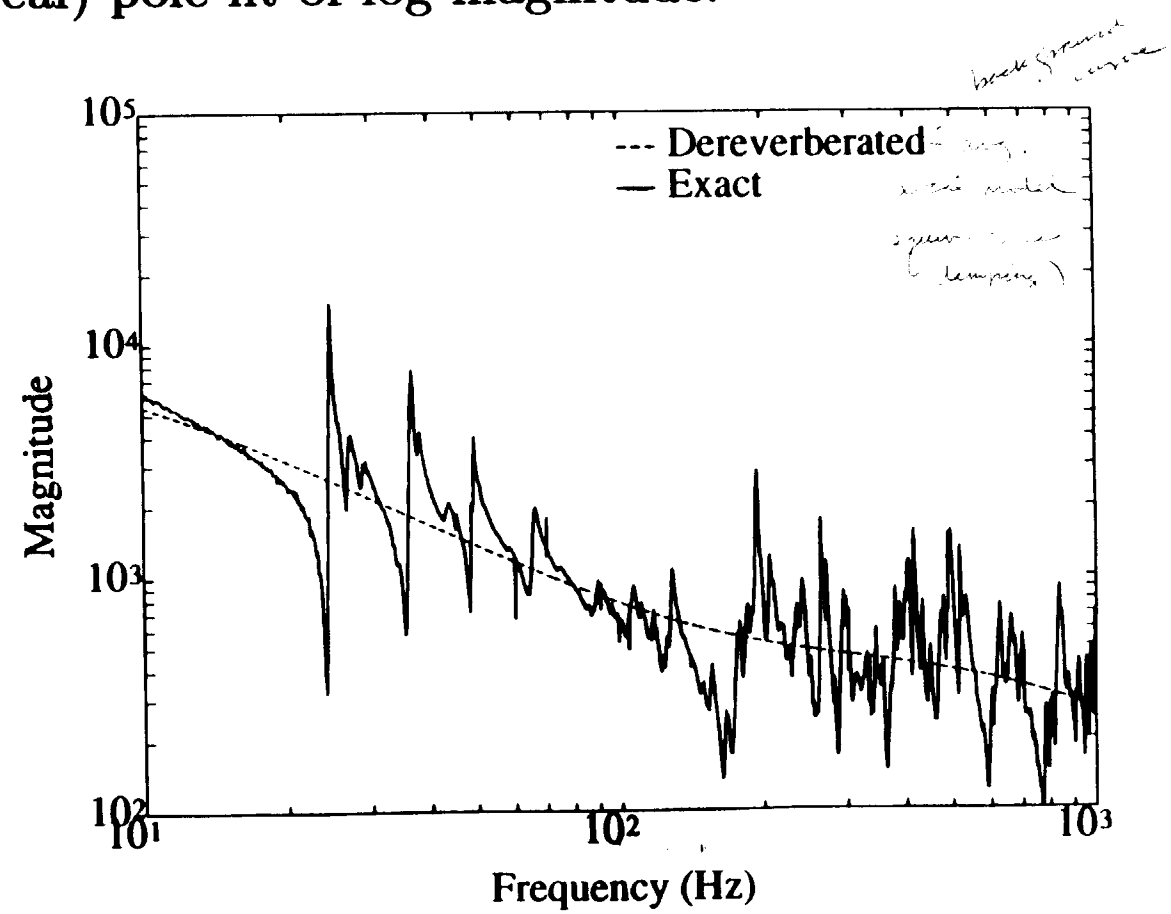
  –Use integral of force feedback.

Dereverberation of Complex Structure

- Wave approach: truss behaves like a beam at low frequencies.
- Compute "best" fit of log magnitude using only real poles and zeroes.
- Fit transfer function using complex poles and zeroes, and add damping to resulting model.

Dereverberated Transfer Function

- Open loop transfer function from displacement to integrated force.
- Three (real) pole fit of log magnitude.



Conclusions

- Use local model of structure for broadband control.
  - Dereverberated mobility or travelling wave model.
  - Ideal compensator is a non-causal impedance match.
- Parameter-robust control for structures must take advantage of conservation of energy.
- Average covariance exhibits equipartition and incoherence.
- A cost functional can be obtained that uses these properties, and guarantees both stability and performance robustness.