MIT
Space
Engineering
Research
Center

Active Impedance Matching of Complex Structural Systems

Douglas G. MacMartin
David W. Miller
Steven R. Hall

July 1, 1991

Goal

- Active broadband control of uncertain modally dense structures.
- Use collocated feedback.
  - –Positive real controller guarantees stability.
  - –Low authority or local control ("active damping.")
- Use local acoustic or statistical model of structure.
- Maximize power dissipation.
  - –Equivalent to impedance matching.
  - –Cannot match impedance exactly at all frequencies due to causality constraint.
- Experimental demonstration on complex structures.

Travelling Wave Model

- Describe junction dynamics using waves.
- Relate physical variables to wave modes:

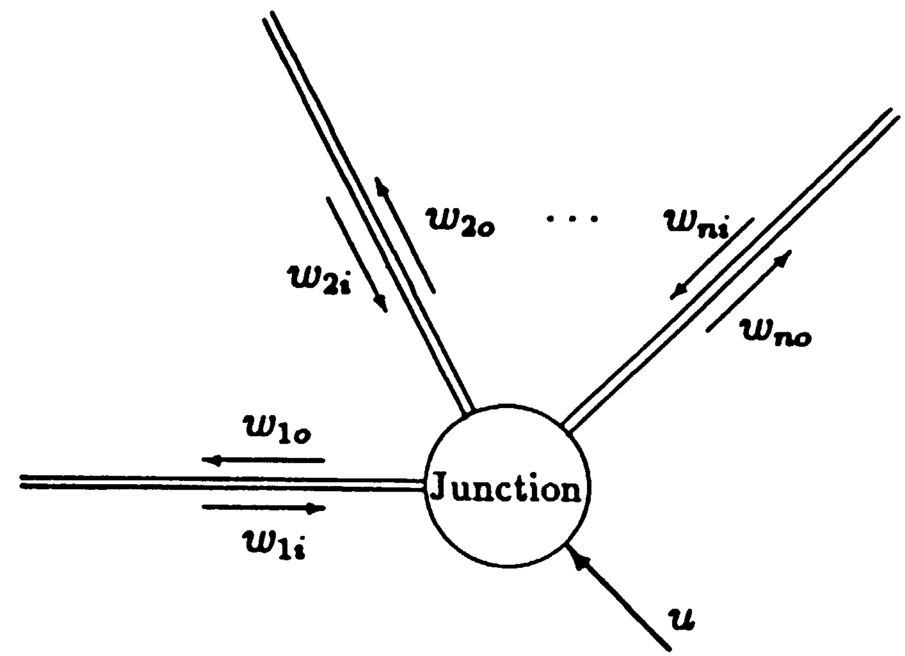
$$\begin{bmatrix} q \\ f \end{bmatrix} = \begin{bmatrix} Y_{qi} & Y_{qo} \\ Y_{fi} & Y_{fo} \end{bmatrix} \begin{bmatrix} w_i \\ w_o \end{bmatrix}$$

- Scattering and generation of waves:

$$w_o(s) = S(s)w_i(s) + \Psi(s)u(s)$$

- Relate physical variables to control:

$$q = (Y_{qo}\Psi)u + (Y_{qi} + Y_{qo}S)w_i$$

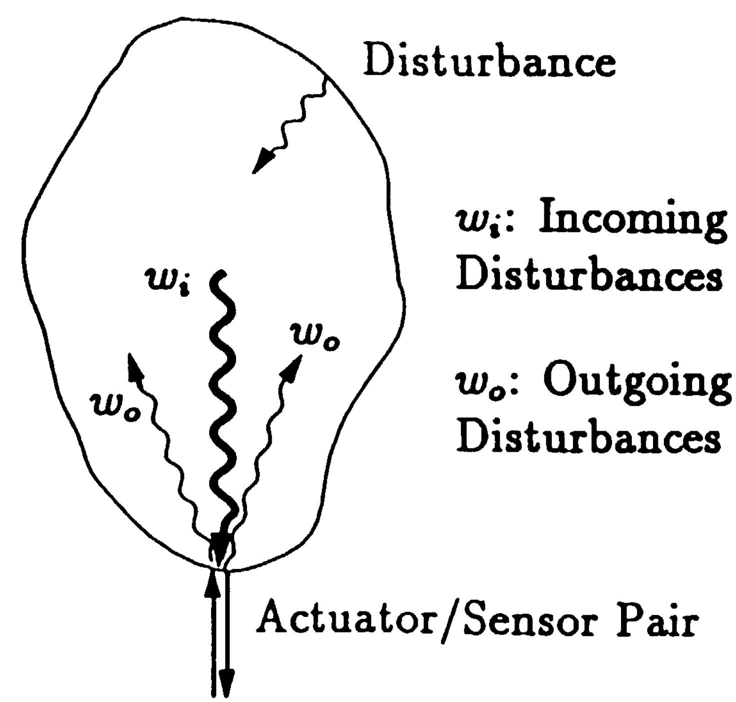


Dereverberated Mobility Model

- Total Response = Direct Field + Reverberant Field
- Response of form:

$$y(s) = G(s)u(s) + d(s)$$

- Gives accurate model of local dynamics.
- Approach applicable to arbitrary structure.

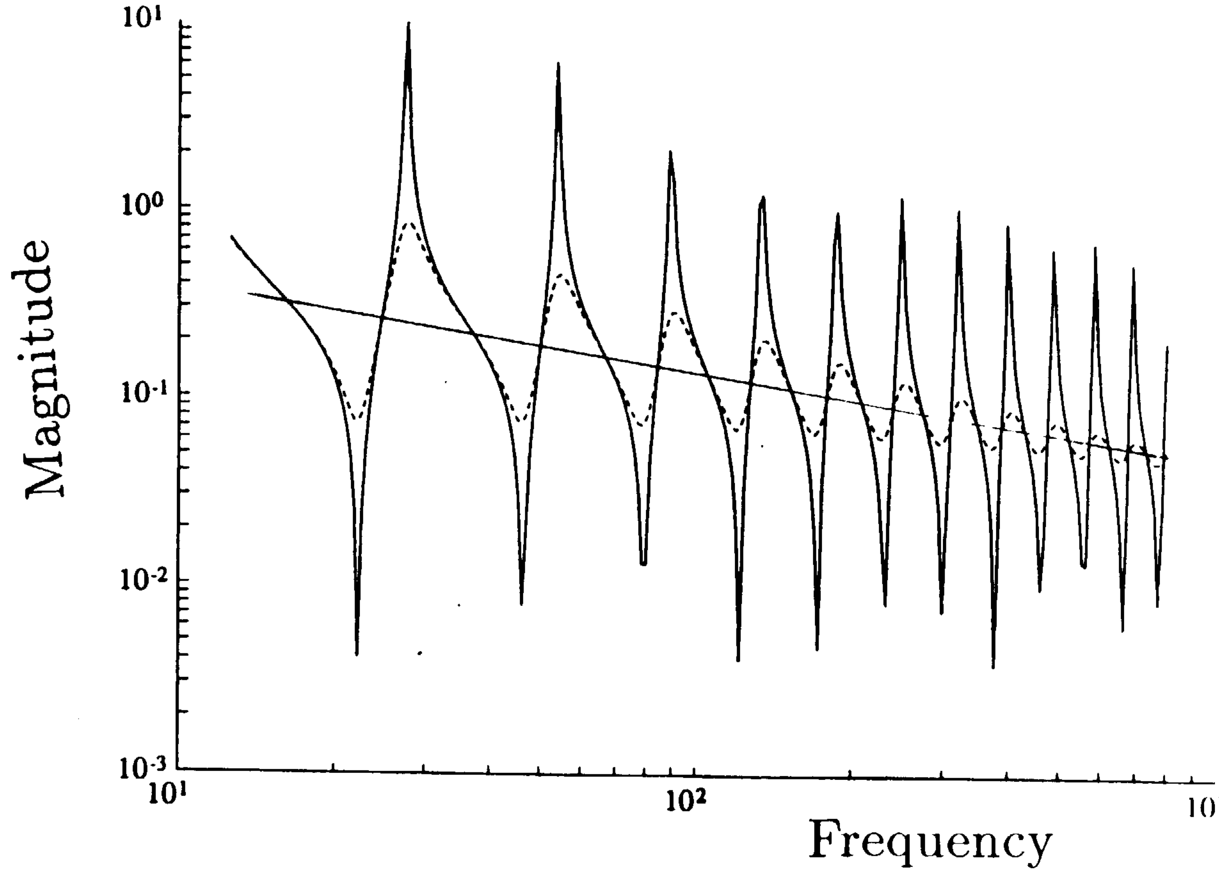


Computation of Dereverberated Mobility

- From wave model:

$$G = sT(Y_{qo}\Psi)$$

- From averaging transfer function:



Control Problem: Optimal Impedance Matching

- Design compensator based on local model and apply to real structure.
- Minimize power flow into structure.

$$\mathbb{P}(\omega) = \mathrm{tr}\left\{\Phi_{uy}(\omega) + \Phi_{yu}(\omega)\right\}$$

- Maximum dissipation is obtained if the compensator is the conjugate of the structural impedance:

$$K(s) = \left(G(-s)^T\right)^{-1}$$

- This is noncausal!
- Problem is to match impedance as well as possible, subject to causality.

$\mathcal{H}_2$-Optimal Solution

- Minimize rms power flow using Wiener-Hopf or LQG to guarantee causality.

$$\begin{aligned} J &= \int_{-\infty}^{\infty} \mathbb{P}(\omega)\mathrm{d}\omega \\ &= \int_{-\infty}^{\infty} \mathrm{tr}\left\{\Phi_{uy}(\omega) + \Phi_{yu}(\omega)\right\} \mathrm{d}\omega \end{aligned}$$

- Requires knowledge of disturbance spectrum Φ_{dd}.
- Local model is not conservative: departing energy does not return.
- No guarantee of stability on actual structure.
  - May add power at certain frequencies to achieve greater dissipation at others.

$\mathcal{H}_\infty$-Optimal Solution

- Guarantee stability by guaranteeing power dissipation at all frequencies:

$$\mathbb{P}(\omega) = \mathrm{tr}\left\{\Phi_{uy}(\omega) + \Phi_{yu}(\omega)\right\} < 0 \quad \forall\, \omega$$

- Instead, guarantee that the reflected power into the structure is less than the incoming power.

  –This is a standard $\mathcal{H}_\infty$ control problem.

- Guaranteeing $\|T_{zw}\|_\infty < 1$ guarantees power dissipation at all frequencies.

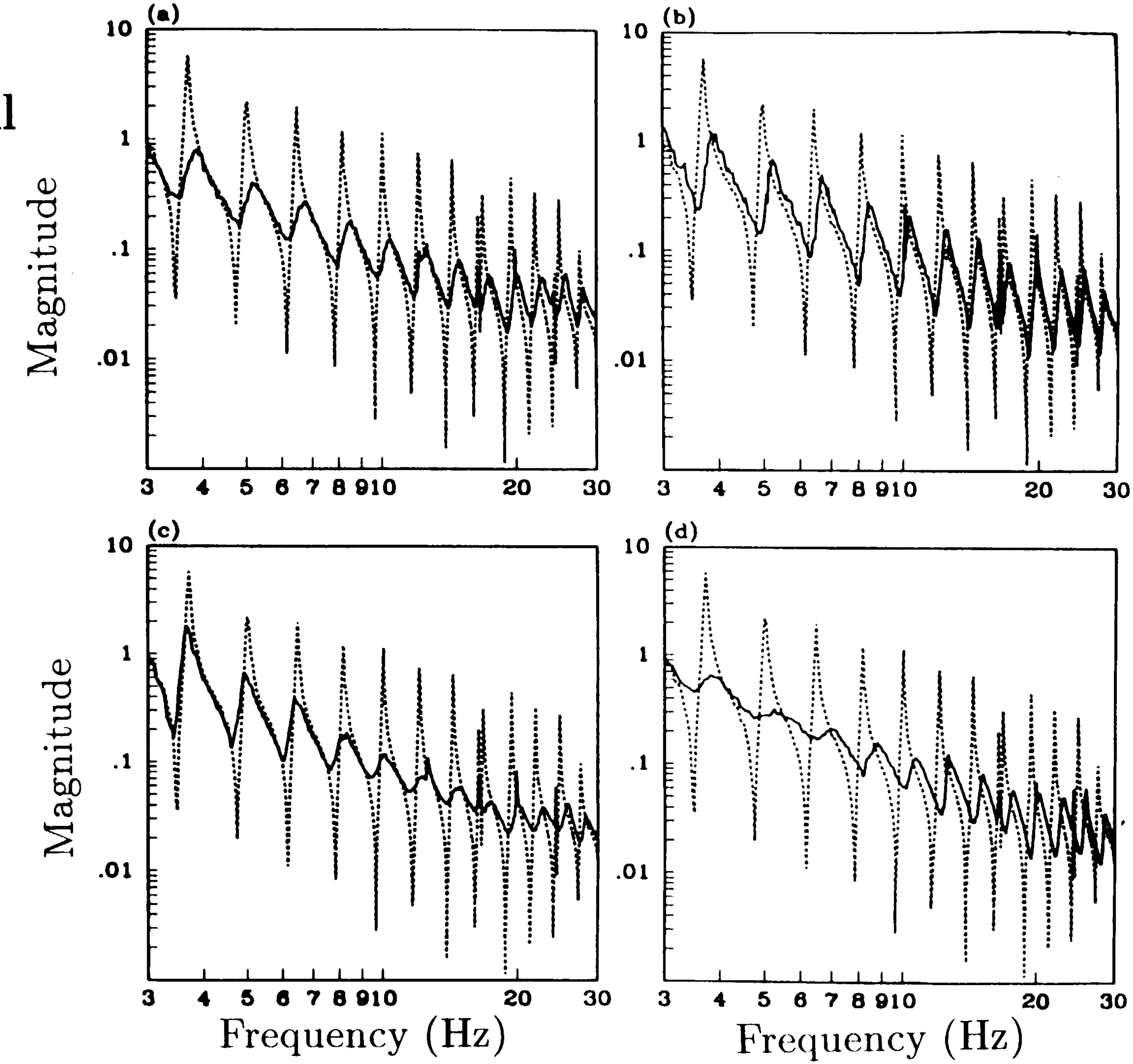
- This also guarantees a positive real compensator.

Statistical Energy Analysis (SEA) Solution

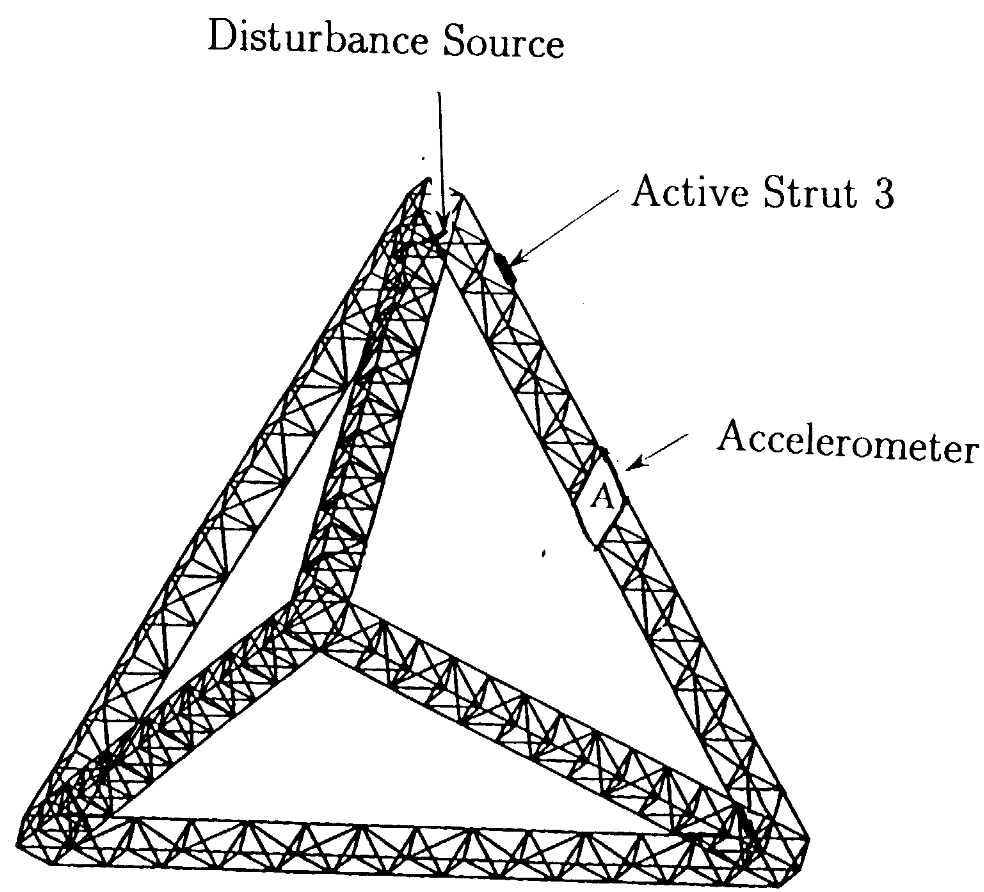
- Model structure with average driving point mobility (a generalization of the dereverberated mobility.)
- Include knowledge that structure conserves energy.
- Guaranteed finite energy $\Rightarrow$ Guaranteed stability.
- Minimize desired rms cost, expressed in terms of power flow.

Experimental Transfer Functions

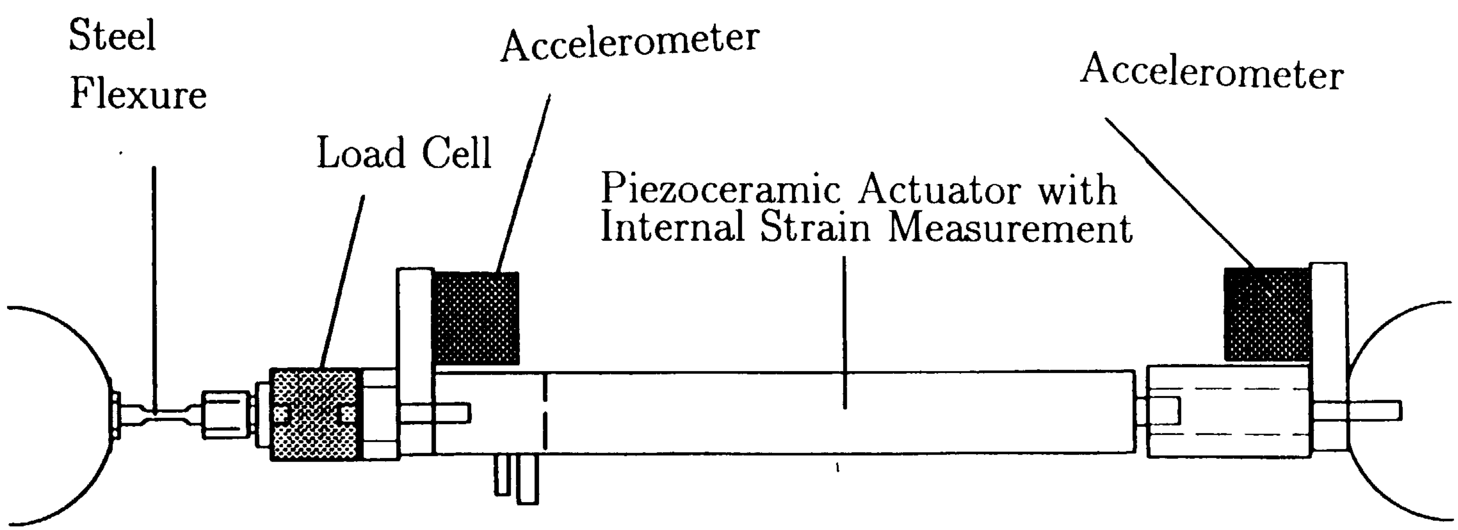
a) Rate
b) $\mathcal{H}_\infty$
c) $\mathcal{H}_2$
d) Weighted $\mathcal{H}_\infty$



Interferometer Actuator and Sensor Locations



Active Strut Configuration



“Power” Dual Variables

- Force into structure and relative velocity across active strut are dual.
- Piezo stack stiffness is high $\Rightarrow$ commands displacement.

  –Can also command relative velocity.
- Want compensator $K(s)$ such that

$$\begin{aligned} \dot{x} &= K(s)f \\ \Rightarrow x &= K(s)\left(\frac{f}{s}\right) \end{aligned}$$

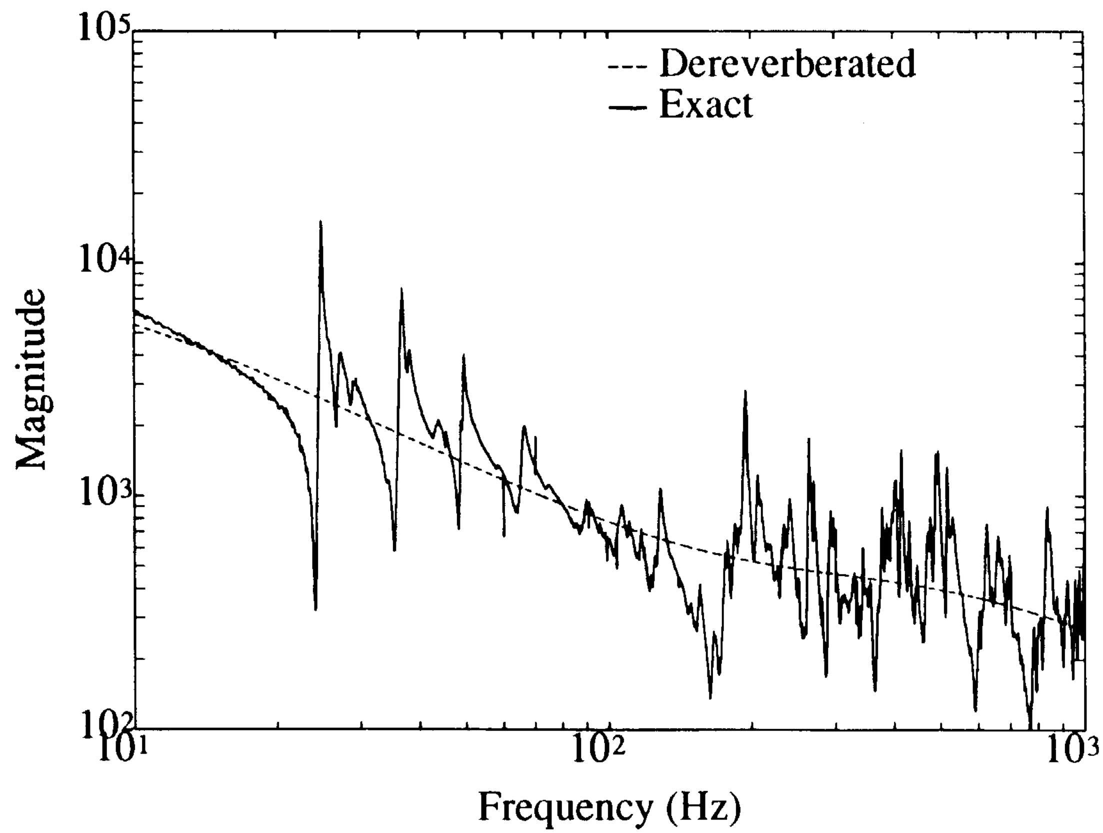
  –Use integral of force feedback.

Dereverberation of Complex Structure

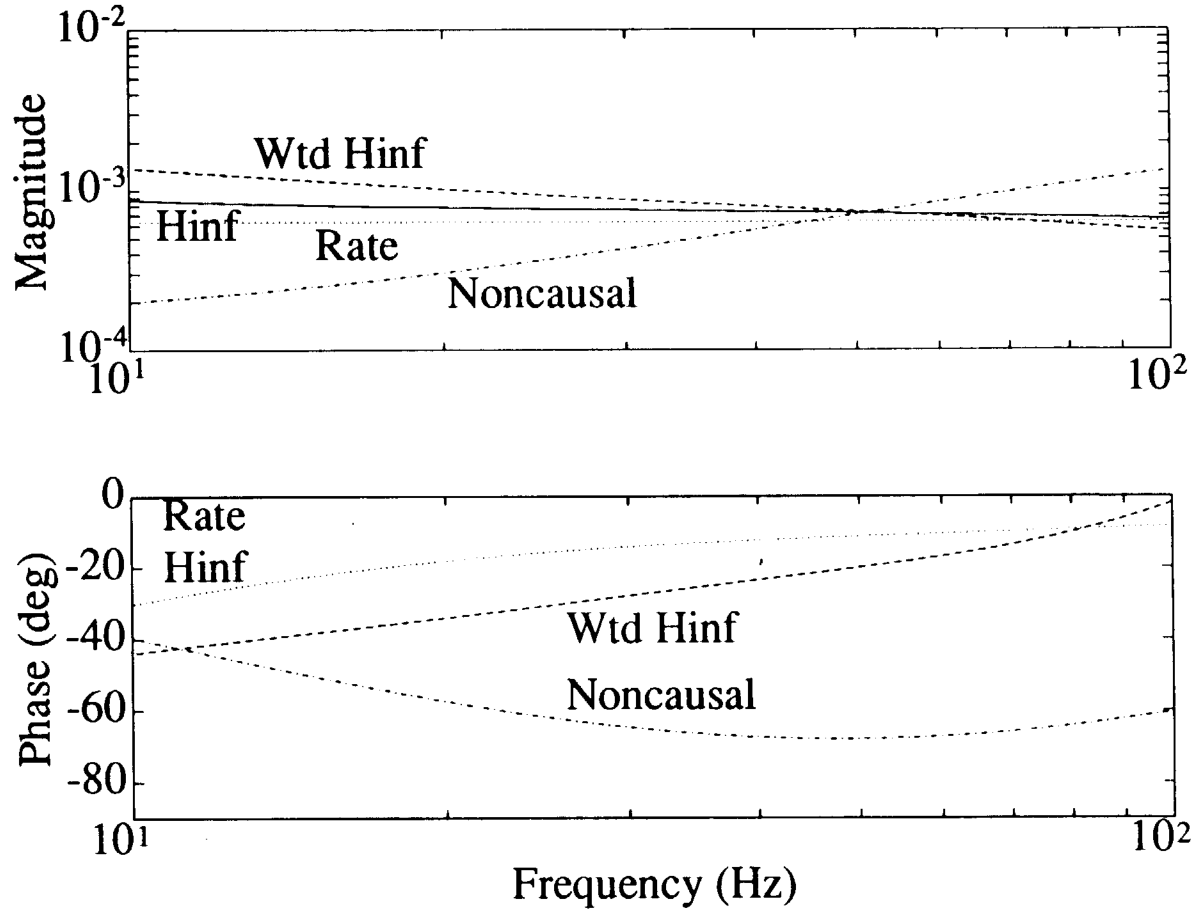
- Wave approach: truss behaves like a beam at low frequencies.
- Compute "best" fit of log magnitude using only real poles and zeroes.
- Fit transfer function using complex poles and zeroes, and add damping to resulting model.

Dereverberated Transfer Function

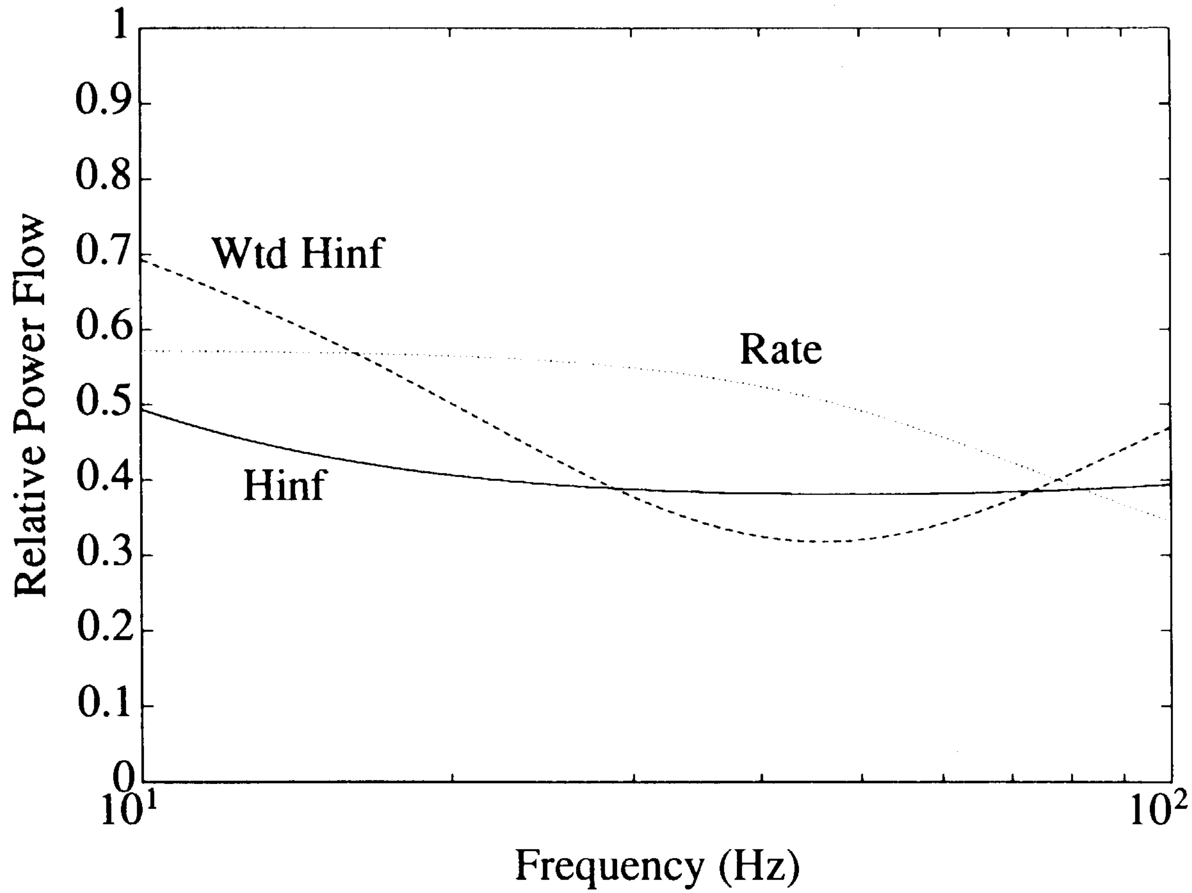
- Open loop transfer function from displacement to integrated force.
- Three (real) pole fit of log magnitude.



Compensators

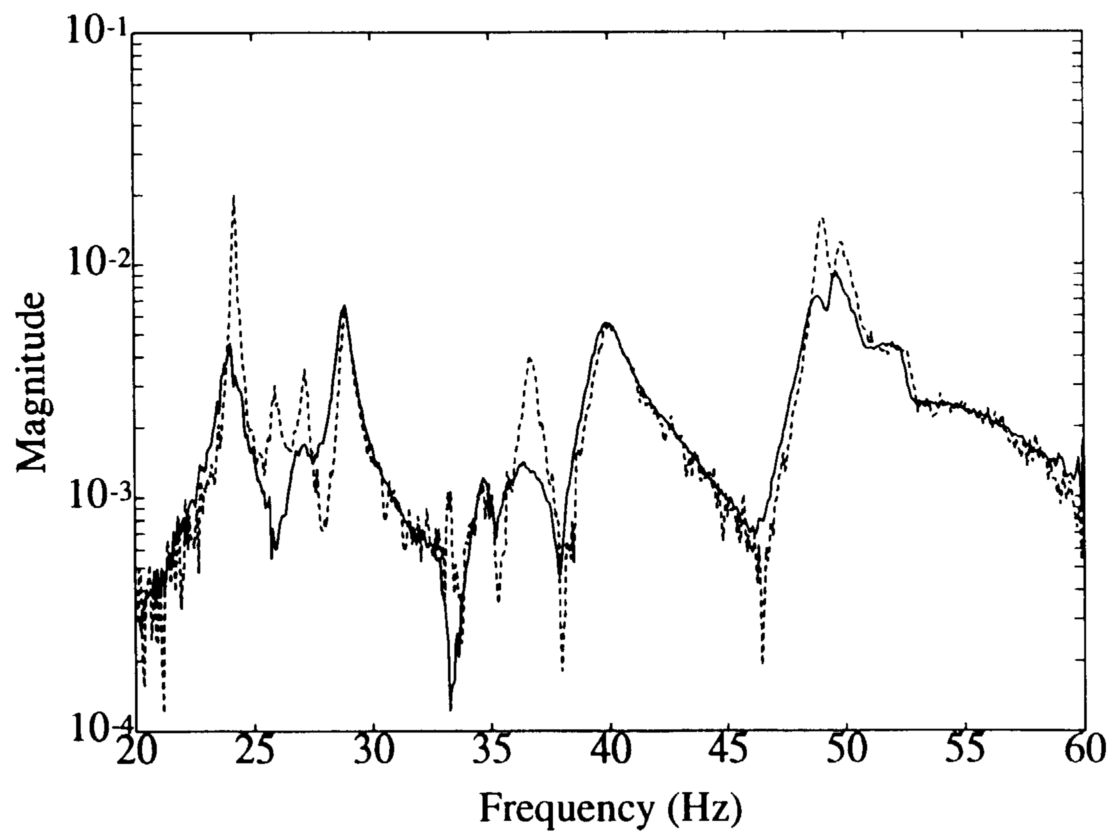


Relative Power Flow



Preliminary Experimental Results

- Open and closed loop transfer functions from disturbance source to siderostat acceleration.
- Single constant gain loop closed around active strut # 3.



Conclusions

- Local model can be used for control design for uncertain modally dense systems.
  - Travelling wave or dereverberated mobility model.
- Ideal compensator for power dissipation is usually noncausal.
  - This is an impedance matching problem.
- $\mathcal{H}_2$ or $\mathcal{H}_\infty$ optimal matches give good performance.
  - $\mathcal{H}_\infty$ approach guarantees stability.
  - Weighting functions introduce flexibility.
- Concepts can be applied to arbitrarily complex structural systems.
  - Some damping added with simple compensator.
  - Expect more damping possible with better impedance match.