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INHIBITING MULTIPLE MODE VIBRATION IN CONTROLLED FLEXIBLE SYSTEMS

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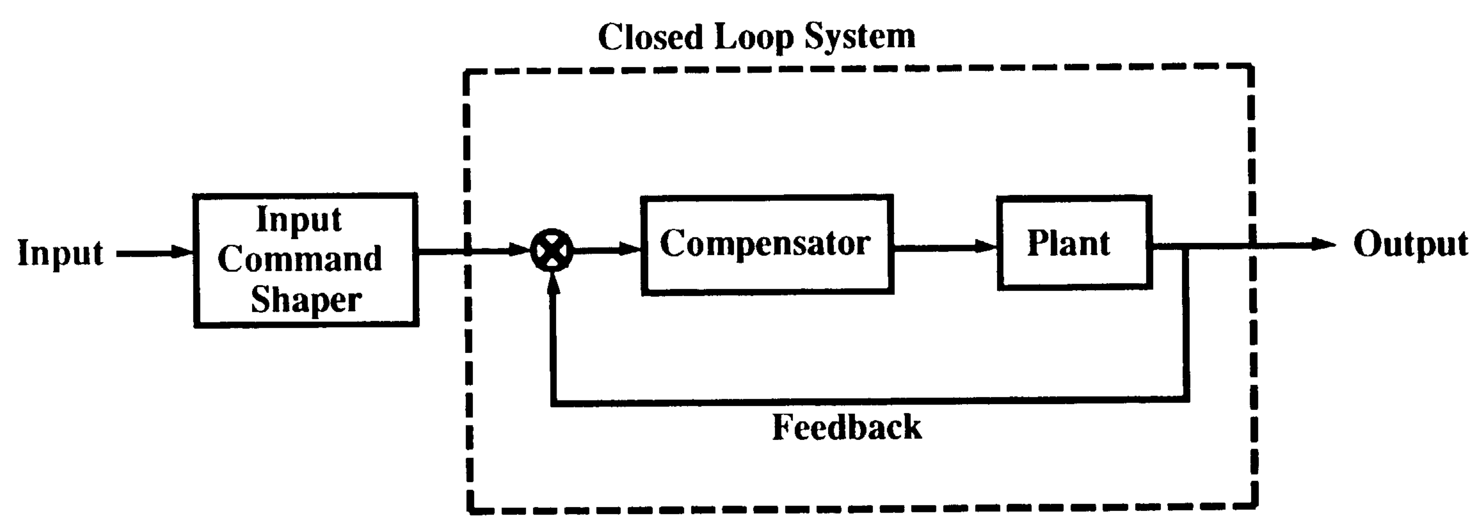
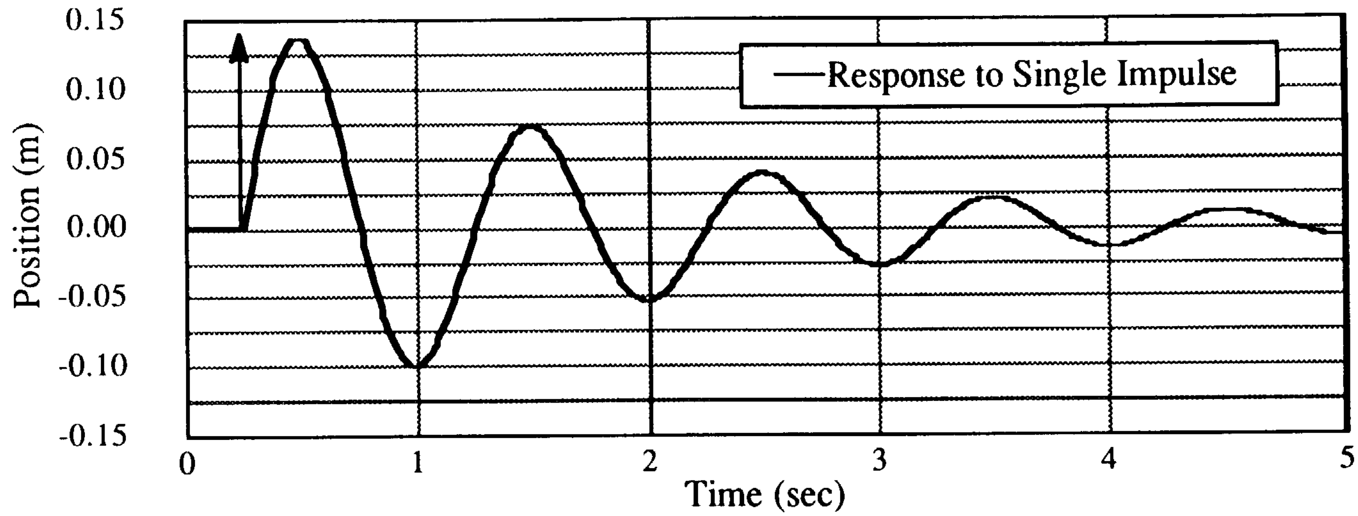
OUTLINE

- Input Pre-Shaping Background
- Developing Multiple-Mode Shapers
- The MACE Test Article
- Tests and Results

SHAPER POSITION IN CONTROL SYSTEM



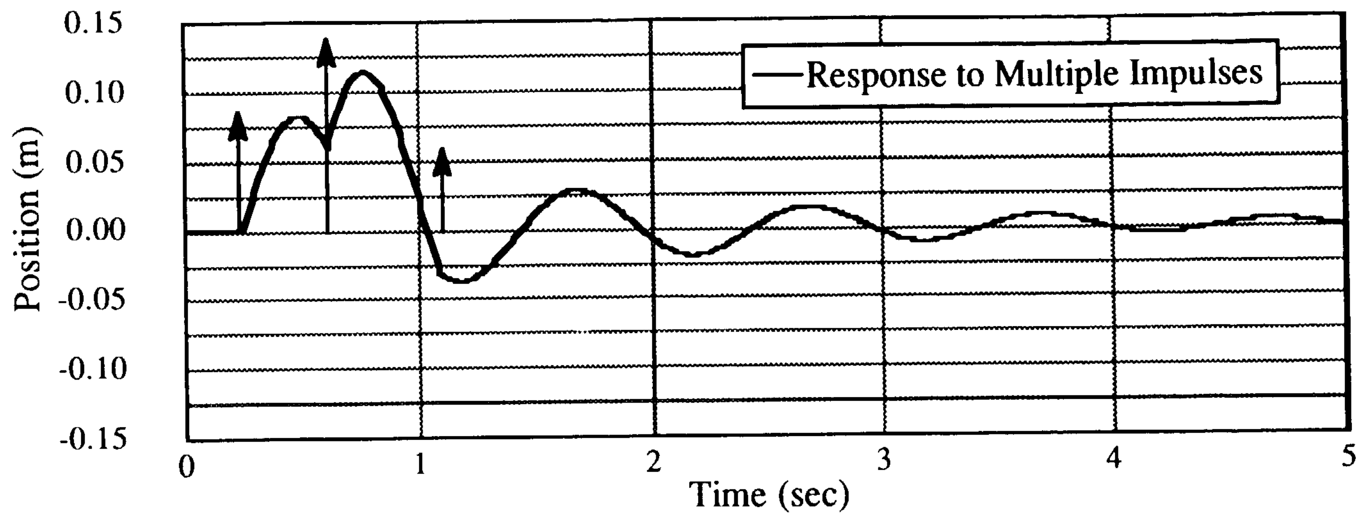
LINEAR SYSTEM IMPULSE RESPONSE



$$y_i(t) = A_i\, e^{-\zeta\,\omega\,(t - t_i)} \sin\left((t - t_i)\,\omega\,\sqrt{1 - \zeta^2}\right)$$

y_i	Response to Impulse *i*
A_i	Magnitude of Impulse *i*
t_i	Time of Impulse *i*
ω	System Natural Frequency
ζ	System Damping Ratio

RESPONSE TO "N" IMPULSES



$$y_i(t) = \sum_{i=1}^{N} A_i \, e^{-\zeta\,\omega\,(t - t_i)} \sin\left((t - t_i)\,\omega\,\sqrt{1 - \zeta^2}\right)$$

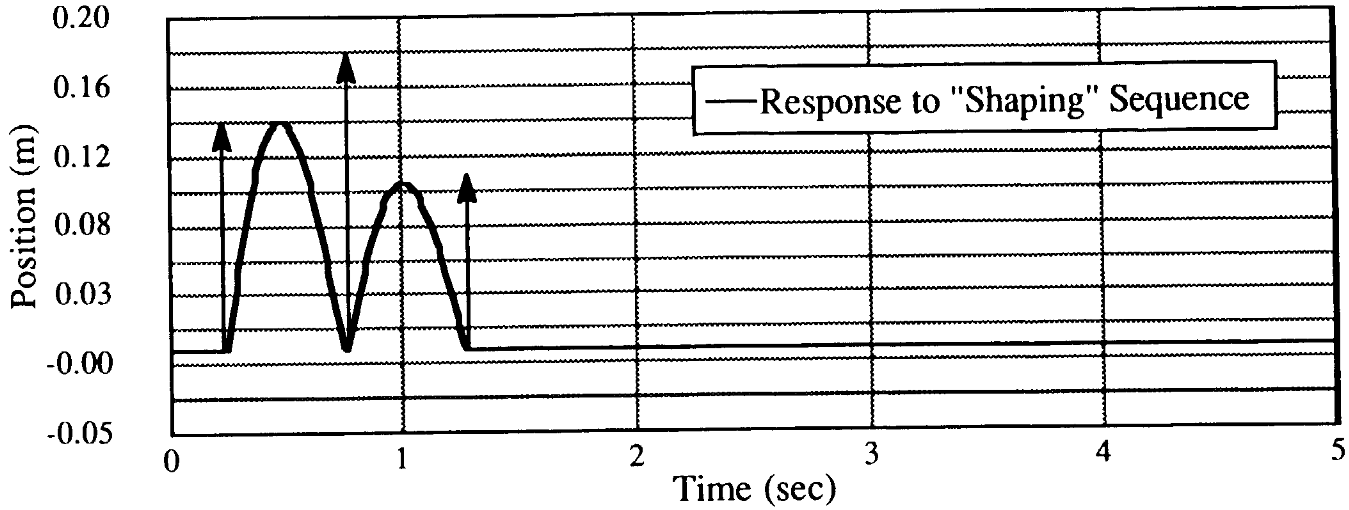
i	Impulse Counter
N	Number of Impulses

AMPLITUDE OF THE MULTIPLE-IMPULSE RESPONSE ENVELOPE

$$Amp = \left[\left(\sum_{i=1}^{N} A_i e^{-\zeta \omega (t_N - t_i)} \sin\left(t_i \omega \sqrt{1-\zeta^2}\right) \right)^2 + \left(\sum_{i=1}^{N} A_i e^{-\zeta \omega (t_N - t_i)} \cos\left(t_i \omega \sqrt{1-\zeta^2}\right) \right)^2 \right]^{1/2}$$

Expression for envelope amplitude at t_N, the time of the final impulse.

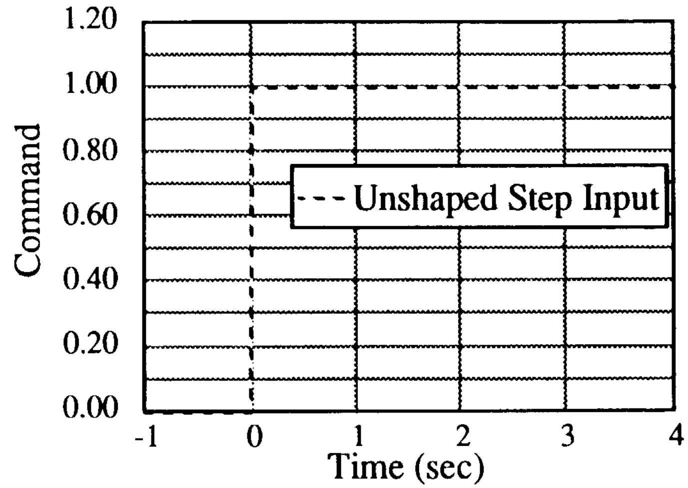
ELIMINATING RESIDUAL VIBRATION



$$\sum_{i=1}^{N} A_i e^{-\zeta \omega t_i} \sin\left(t_i \omega \sqrt{1-\zeta^2}\right) = 0 \qquad \sum_{i=1}^{N} A_i t_i e^{-\zeta \omega t_i} \sin\left(t_i \omega \sqrt{1-\zeta^2}\right) = 0$$

$$\sum_{i=1}^{N} A_i e^{-\zeta \omega t_i} \cos\left(t_i \omega \sqrt{1-\zeta^2}\right) = 0 \qquad \sum_{i=1}^{N} A_i t_i e^{-\zeta \omega t_i} \cos\left(t_i \omega \sqrt{1-\zeta^2}\right) = 0$$

USING THE IMPULSE SEQUENCE



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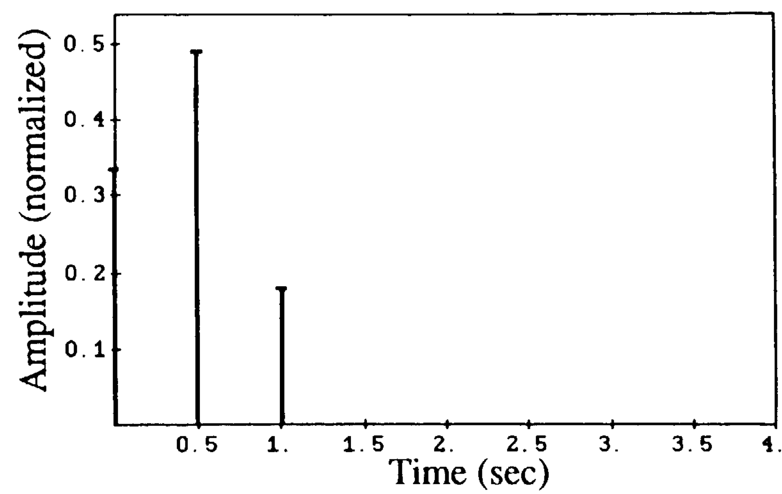
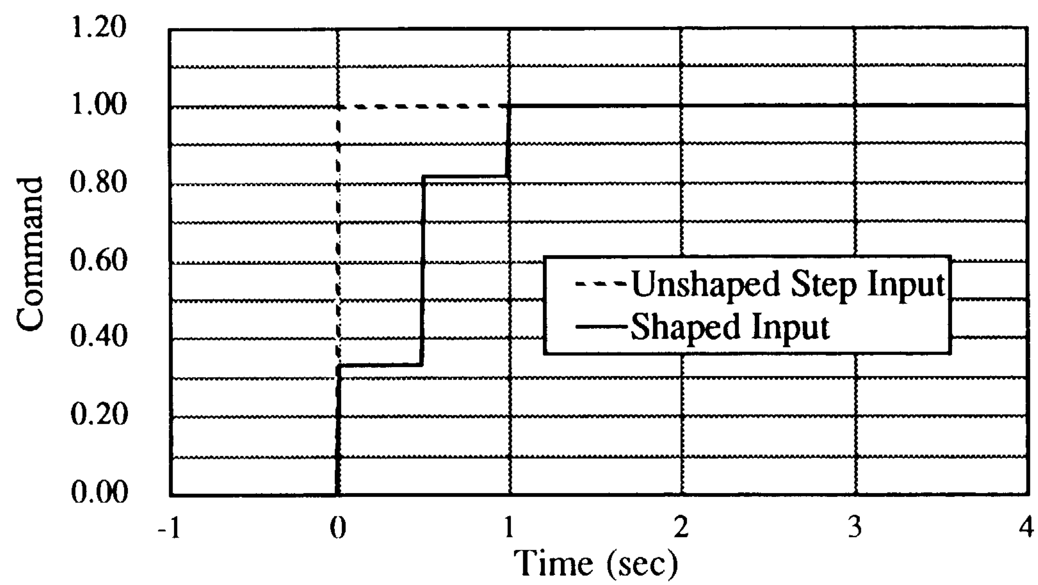
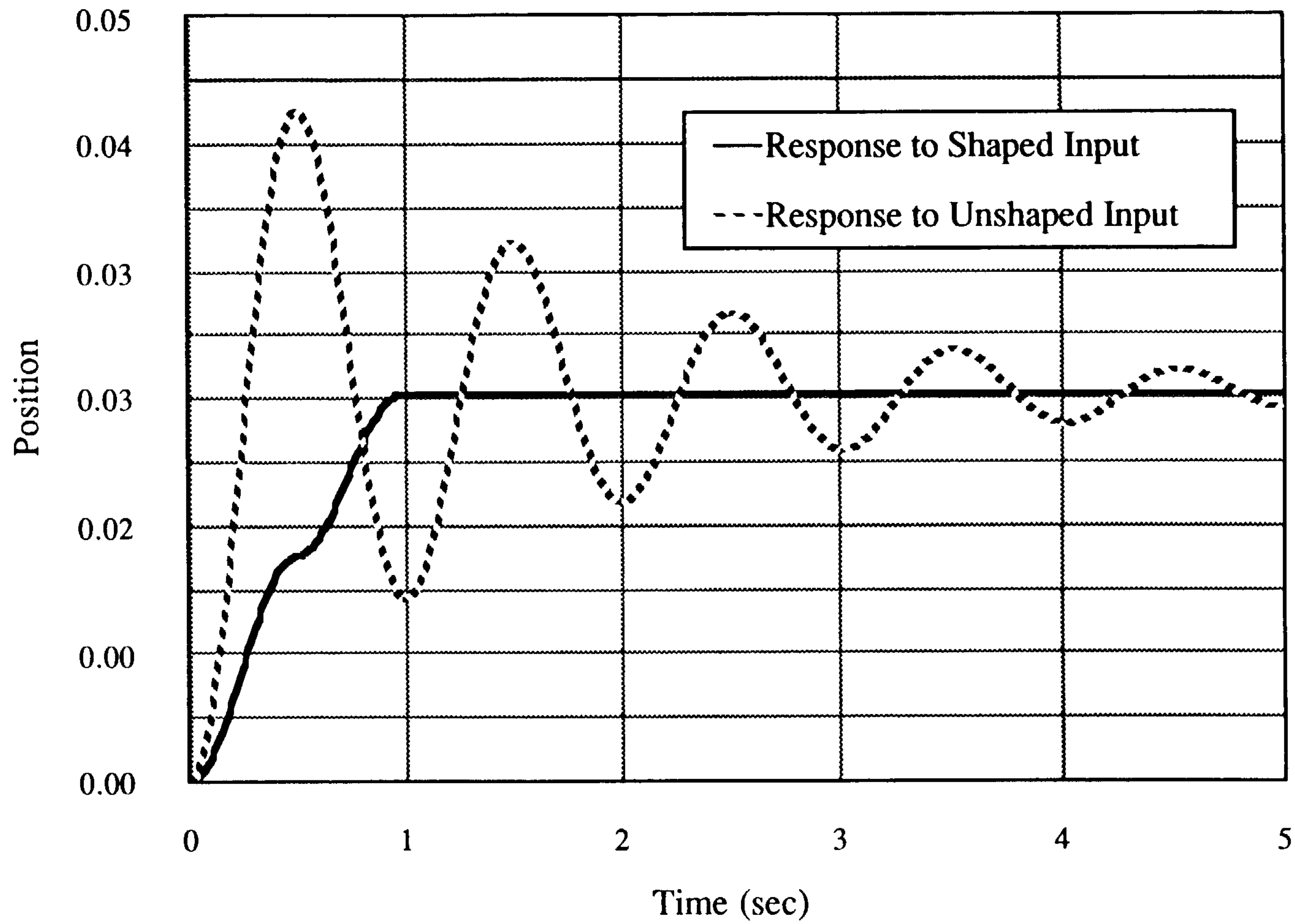
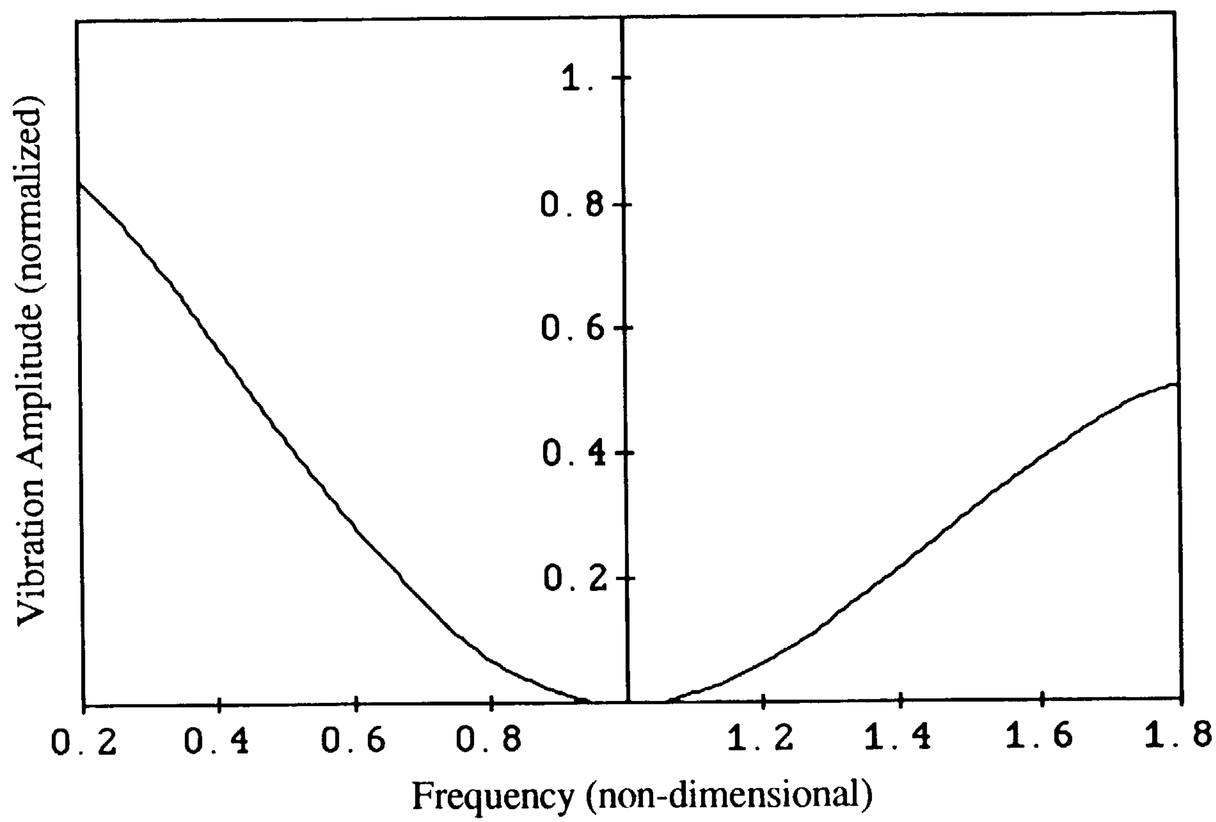




RESPONSE TO INPUTS

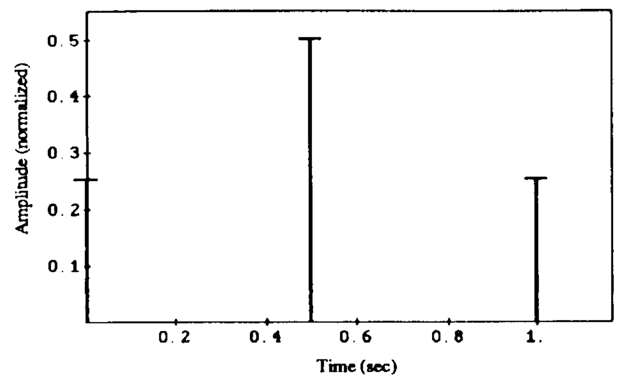


INSENSITIVITY OF THREE-IMPULSE SEQUENCE



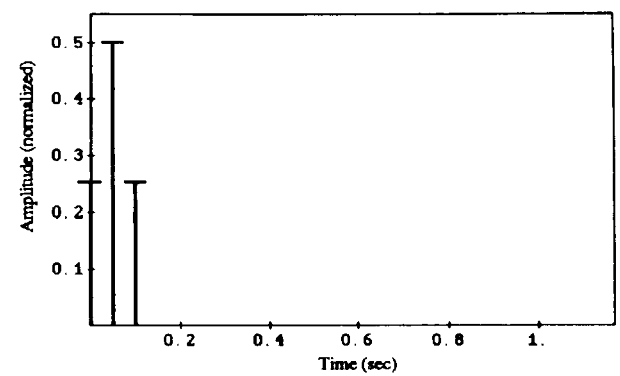
EXTENDING TO MULTIPLE MODE PROBLEMS

CONVOLUTION

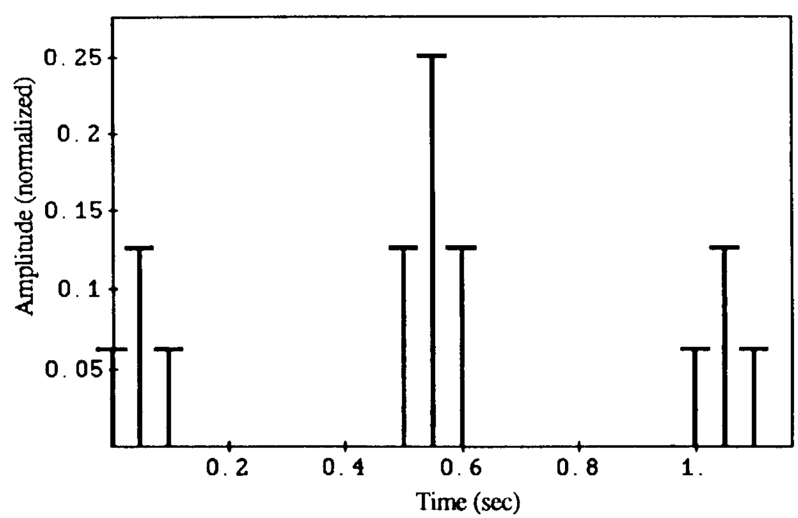


1 Hz Shaper

*

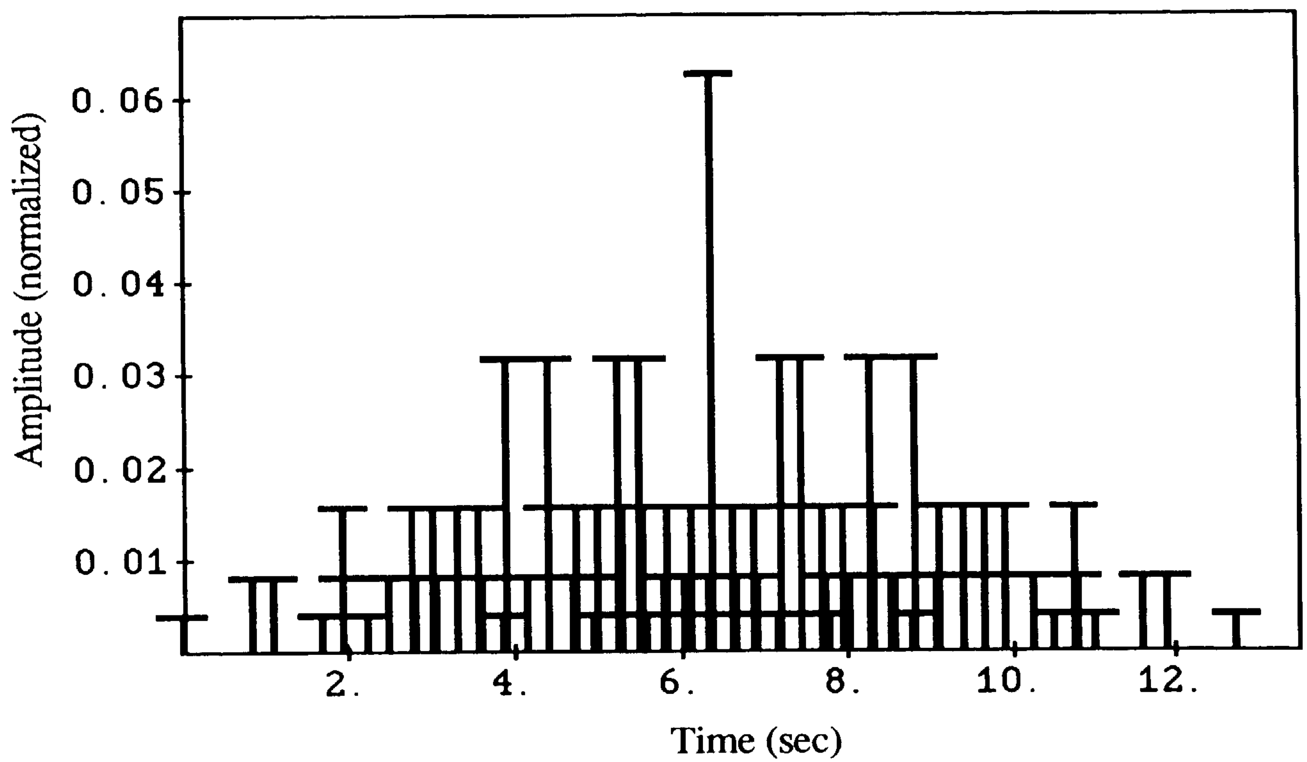


10 Hz Shaper



Convolved Shaper

CONVOLUTION SEQUENCE PROBLEMS



$\omega_1 = 0.20$ Hz $\quad \omega_2 = 0.26$ Hz

$\omega_3 = 0.45$ Hz $\quad \omega_4 = 0.59$ Hz

DIRECT SOLUTION CONSTRAINT EQUATIONS

$$\sum_{i=1}^{N} A_i \, e^{-\zeta_j \omega_j t_i} \sin\left(t_i \, \omega_j \sqrt{1-\zeta_j^2}\right) = 0$$

$$\sum_{i=1}^{N} A_i \, e^{-\zeta_j \omega_j t_i} \cos\left(t_i \, \omega_j \sqrt{1-\zeta_j^2}\right) = 0$$

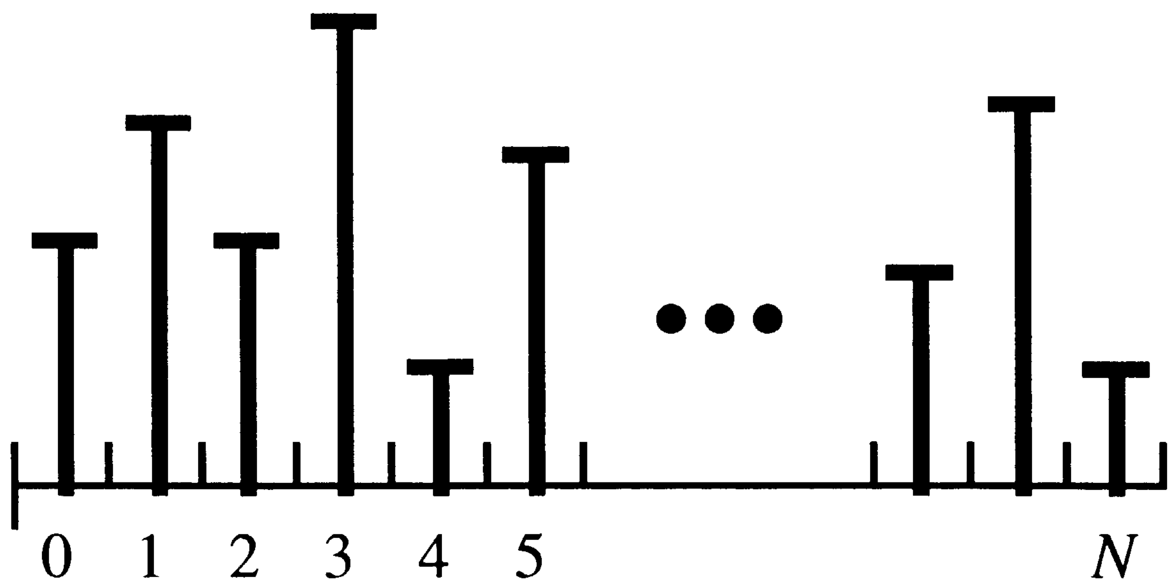
$$\sum_{i=1}^{N} A_i \, t_i \, e^{-\zeta_j \omega_j t_i} \sin\left(t_i \, \omega_j \sqrt{1-\zeta_j^2}\right) = 0$$

$$\sum_{i=1}^{N} A_i \, t_i \, e^{-\zeta_j \omega_j t_i} \cos\left(t_i \, \omega_j \sqrt{1-\zeta_j^2}\right) = 0$$

These four equations are repeated for each mode "j"

LINEARIZING THE EQUATIONS

Define time mesh with N slots



Impulse times (t_i) : Known

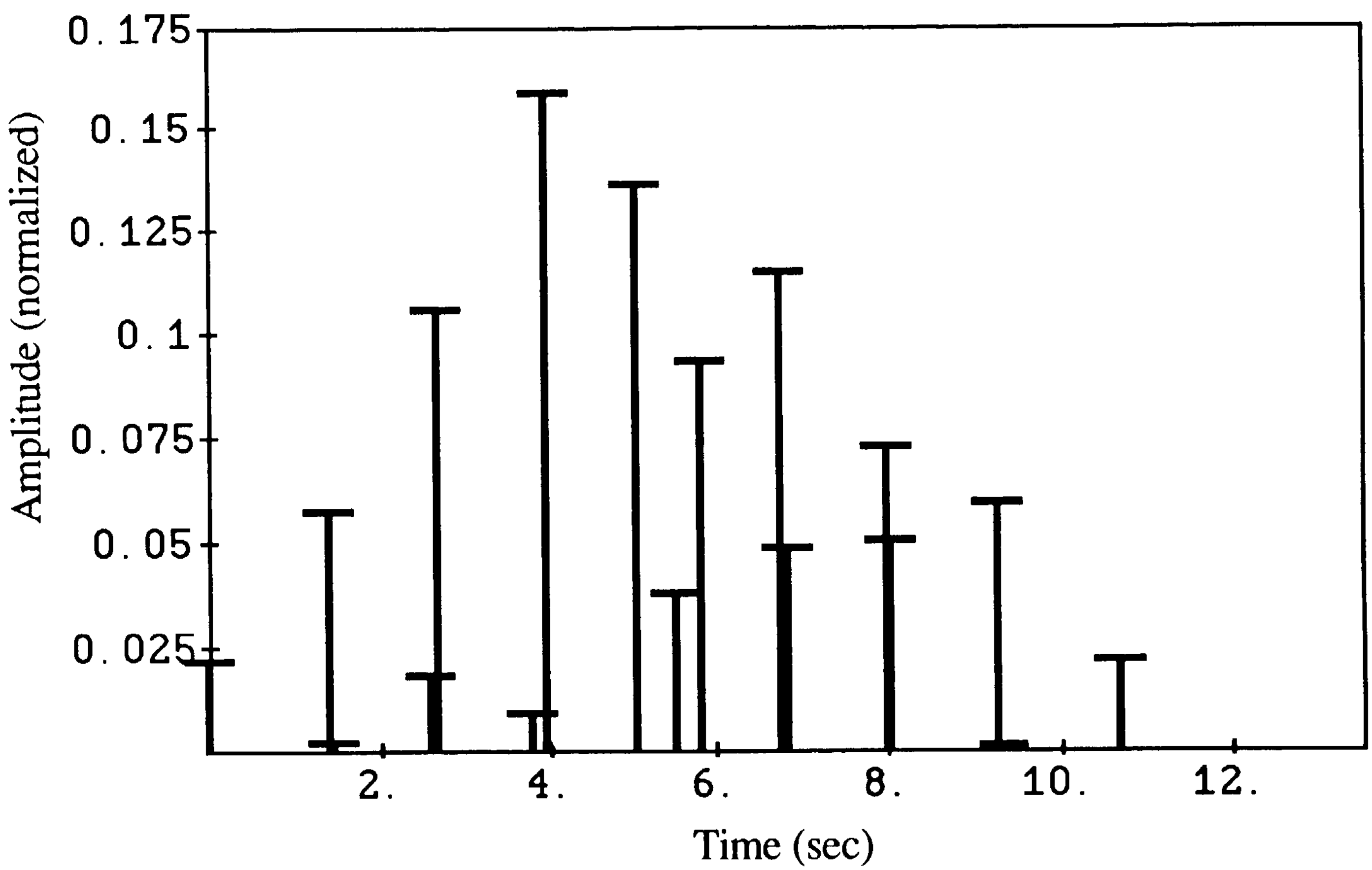
Impulse amplitudes (A_i) : Unknown

COST FUNCTION

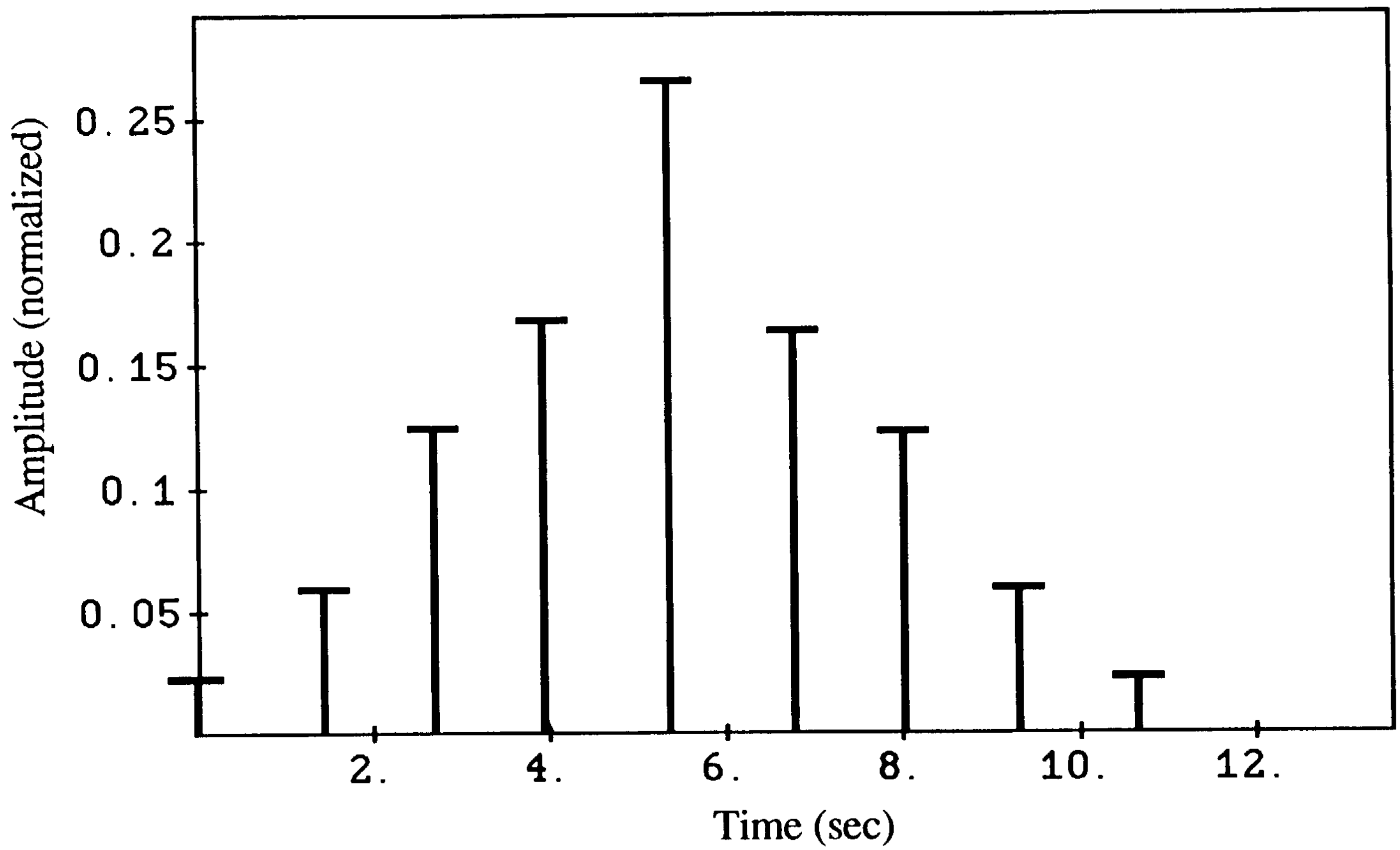
$$Cost = \sum_{j=1}^{M} \left[\left(\sum_{i=1}^{N} A_i\, t_i^2\, e^{-\zeta_j \omega_j t_i} \sin\left(t_i\, \omega_j \sqrt{1-\zeta_j^2}\right) \right)^2 + \left(\sum_{i=1}^{N} A_i\, t_i^2\, e^{-\zeta_j \omega_j t_i} \cos\left(t_i\, \omega_j \sqrt{1-\zeta_j^2}\right) \right)^2 \right]$$

M Number of modes
j Modal index

LINEAR APPROXIMATION SEQUENCE



INTERPRETED LINEAR SEQUENCE



EXACT DIRECT SOLUTION SEQUENCE