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# Bounds on Internal State Variables in Viscoplasticity

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# BOUNDS ON INTERNAL STATE VARIABLES IN VISCOPLASTICITY

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**Abstract-** A typical viscoplastic model will introduce up to three types of internal state variables in order to properly describe transient material behavior; they are: the back stress, the yield stress, and the drag strength. Different models employ different combinations of these internal variables—their selection and description of evolution being largely dependent on application and material selection. Under steady-state conditions, the internal variables cease to evolve and therefore become related to the external variables (stress and temperature) through simple functional relationships. A physically motivated hypothesis is presented that links the kinetic equation of viscoplasticity with that of creep under steady-state conditions. From this hypothesis one determines how the internal variables relate to one another at steady state, but most importantly, one obtains bounds on the magnitudes of stress and back stress, and on the yield stress and drag strength.

## EVOLUTION OF INELASTICITY

The evolution of plastic strain, $\dot{\varepsilon}^p_{ij}$, in the classical theory of creep (ODQVIST [1974]) is given by

$$\dot{\varepsilon}^p_{ij}\Big|_{ss} = \frac{1}{2}\,\|\dot{\varepsilon}^P\|_{ss}\,\frac{S_{ij}}{\|\mathbf{S}\|}\,, \tag{1}$$

with the subscript '$ss$' implying steady state, and where $\|\dot{\varepsilon}^P\|$ denotes the magnitude of the plastic strain-rate, and $\|\mathbf{S}\|$ denotes the magnitude of the deviatoric stress, $S_{ij}$. The subscript signifying steady-state is not attached to $S_{ij}$ because stress is a controllable variable, whereas, creep-rate is a response variable and hence its attachment. This equation states that an increment in creep strain accumulates in the current direction of the deviatoric stress. A dot is placed over a variable to signify its time rate-of-change.

The evolution of plastic strain in viscoplasticity (PRAGER [1949]) is defined as

$$\dot{\varepsilon}^p_{ij} = \frac{1}{2}\,\|\dot{\varepsilon}^P\|\,\frac{S_{ij} - B_{ij}}{\|\mathbf{S} - \mathbf{B}\|}\,, \tag{2}$$

where $B_{ij}$ is the back stress which represents the anisotropic aspect of internal stress. This equation states that an increment in plastic strain accumulates in the direction of an effective stress, $S_{ij} - B_{ij}$.

The norms, or magnitudes, pertaining to the deviatoric tensors of this paper are defined by

$$\|\mathbf{I}\| = \sqrt{2\,I_{ij}\,I_{ij}} \quad \text{and} \quad \|\mathbf{J}\| = \sqrt{\tfrac{1}{2}\,J_{ij}\,J_{ij}}\,, \tag{3}$$

where $I_{ij}$ is any deviatoric 'strain-like' tensor, and $J_{ij}$ is any deviatoric 'stress-like' tensor. These are the norms of VON MISES [1913], where the coefficients under the radical signs scale the theory for shear.

## KINETICS OF EVOLUTION

In the theory of creep, $\|\dot{\varepsilon}^p\|_{ss}$ is described by a kinetic equation, *i.e.* an equation of state. ZENER & HOLLOMON [1944] determined that such a kinetic equation can, to a good approximation, be decomposed into a product of two functions; in particular, at steady state

$$\|\dot{\varepsilon}^p\|_{ss} = \vartheta[T]\,Z_{ss}\left[\frac{\|\mathbf{S}\|}{C}\right] \geq 0\,, \tag{4}$$

where $\vartheta > 0$ is a thermal function, $Z \geq 0$ is the Zener parameter, and $C > 0$ is a strength parameter that normalizes the stress. The Zener parameter is a temperature normalized measure of the plastic strain-rate. Square brackets, $[\cdot]$, are used throughout this paper to denote 'function of', and are therefore kept logically separate from parentheses, $(\cdot)$, which are used for mathematical groupings.

The kinetics of viscoplasticity are also taken to be described by a ZENER & HOLLOMON [1944] decomposition of state, *viz.* (FREED <u>et al.</u> [1991])

$$\|\dot{\varepsilon}^p\| = \vartheta[T]\,Z\left[\left\langle \frac{\|\mathbf{S}-\mathbf{B}\| - Y}{D} \right\rangle\right] \geq 0\,, \tag{5}$$

where the Macauley bracket, $\langle(\|\mathbf{S}-\mathbf{B}\| - Y)/D\rangle$, has either a value of 0 whenever $\|\mathbf{S}-\mathbf{B}\| < Y$ (defining the elastic domain), or a value of $(\|\mathbf{S}-\mathbf{B}\| - Y)/D$ whenever $\|\mathbf{S}-\mathbf{B}\| > Y$ (defining the viscoplastic domain), with $\|\mathbf{S}-\mathbf{B}\| = Y$ establishing the yield surface. The internal state variables—$B_{ij}$, $D$ and $Y$—are described by evolution equations that are functions of state. Remarkably, details of their evolution are not required in order for us to bound their values, with one exception, the back stress at steady state must be coaxial with the deviatoric stress so that the directions of plastic strain-rate defined by Eqns (1 & 2) are equivalent at steady state.

## BOUNDS ON INTERNAL STATE

In order for a viscoplastic theory to reduce analytically to creep theory when at steady state (*i.e.* when $\dot{\mathbf{B}} = \mathbf{0}$, $\dot{D} = 0$ and $\dot{Y} = 0$ for $\|\dot{\varepsilon}^p\| > 0$) two conditions must be satisfied. First, the back stress at steady state must be coaxial with the deviatoric stress, as stated above. And second, it is necessary that the kinetics of viscoplasticity reduce analytically to the kinetics of creep under steady-state conditions. To satisfy this second constraint, I hypothesize relationships between the steady-state and transient Zener parameters, and between the internal and external variables, when at steady state (FREED & WALKER [1990]).

In support of the second constraint, given to secure equivalence between creep and viscoplasticity when at steady state, I shall suppose that (FREED & WALKER [1993])

$$Z \equiv Z_{ss}\left[\left\langle \frac{\|\mathbf{S}-\mathbf{B}\| - Y}{D} \right\rangle\right]. \tag{6}$$

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This relationship implies that the transient Zener parameter, $Z$, has the same functional form as the steady-state Zener parameter, $Z_{ss}$, but with a different argument.

Furthermore, to also support this second constraint, I shall suppose that

$$\|\mathbf{B}\|_{\rightarrow} = f\, \iota_{ss}[\|\mathbf{S}\|]\,\|\mathbf{S}\| \quad , \quad D_{ss} = D_0 + \delta\,\|\mathbf{S}\| \quad \text{and} \quad Y_{ss} = (1-f)\iota_{ss}[\|\mathbf{S}\|]\,\|\mathbf{S}\| , \qquad (7)$$

where $\iota_{ss} > 0$ and $\delta > 0$ are the steady-state fractions of applied stress that are associated with the internal stress (i.e. the back and yield stresses) and the drag strength, respectively, such that $0 < \iota < 1$. The parameter $f$ partitions the internal stress between isotropic and kinematic contributions, such that $0 < f < 1$. The fact that the drag strength is taken to be proportional to the applied stress at saturation is consistent with TAYLOR'S [1934] concept of a material's innate strength to resist plastic flow, i.e. $D$ is a strength parameter—not a stress parameter. We take the internal stress to be a nonlinear function of the applied stress at saturation because that is what the experimental data of ARGON & TAKEUCHI [1981] and ČADEK [1987] suggest.

Because the applied stress and the back stress must be coaxial at steady state, as discussed above, it follows that

$$\|\mathbf{S}-\mathbf{B}\|_{ss} = \|\mathbf{S}\| - \|\mathbf{B}\|_{ss} . \qquad (8)$$

Therefore, upon equating arguments of the Zener parameters given in Eqns (4 & 5) in accordance with the hypothesis of Eqn. (6), while utilizing Eqns (7 & 8), one obtains the result

$$\iota_{ss} = \frac{C - D_0 - \delta\,\|\mathbf{S}\|}{C} = \frac{C - D_0 + \sqrt{(C - D_0)^2 - 4\,\delta\,C(\|\mathbf{B}\|_{ss} + Y_{ss})}}{2\,C} . \qquad (9)$$

If one uses Eqn. (7) and writes $\|\mathbf{B}\|_{ss} + Y_{ss} = \iota_{ss}[\|\mathbf{S}\|]\,\|\mathbf{S}\|$, then from Eqn. (9) one determines that $\|\mathbf{B}\|_{ss} + Y_{ss} = (C - D_0 - \delta\|\mathbf{S}\|)\|\mathbf{S}\|/C$. Because $\partial(\|\mathbf{B}\|_{ss}+Y_{ss})/\partial\|\mathbf{S}\| = 0$ establishes the maximum state of internal stress, one is lead to the result

$$\|\mathbf{S}\|_{max} = \frac{C - D_0}{2\,\delta} . \qquad (10)$$

Substituting this relationship back into Eqns (7 & 9) gives the additional upper bounds for: the back stress,

$$\|\mathbf{B}\|_{max} = f\,\frac{(C - D_0)^2}{4\,\delta\,C} , \qquad (11)$$

the drag strength,

$$D_{max} = \tfrac{1}{2}\left(C + D_0\right) , \qquad (12)$$

and the yield stress,

$$Y_{max} = (1 - f)\frac{(C - D_0)^2}{4\,\delta\,C} . \qquad (13)$$

*It is a remarkable fact that one can bound the stress and internal state variables without specifying the details of how these internal variables evolve.* Similar bounds are given in FREED & WALKER [1993] for the case where the internal stress is composed of short- and long-range back stresses with no yield stress.

Restricting $\iota_{ss}$ to be real valued, and considering $\iota_{min}$ to be associated with the maximum attainable magnitude of internal stress, one finds that on approaching the limit of zero stress the ratio of internal stress to applied stress at steady state is at its maximum, i.e.

$$\lim_{\|\mathbf{S}\|\to 0} \iota_{ss} \equiv \iota_{max} = \frac{C - D_0}{C} \simeq 1 , \qquad (14)$$

which is in reasonable agreement with the experimental observations of ARGON & TAKEUCHI [1981] and ČADEK [1987]. Approaching the limit of maximum stress, this ratio attains its minimum, *i.e.*

$$\lim_{\|S\|\to\|S\|_{max}} \iota_{ss} \equiv \iota_{min} = \frac{C - D_0}{2C} \simeq 1/2 \,, \tag{15}$$

which is in reasonable agreement with the experimental observations of LOWE & MILLER [1983] and ARGON & BHATTACHARYA [1987].

To be physically meaningful, $\|B\| \geq 0$, $D > 0$ and $Y \geq 0$. Furthermore, their steady-state values ought to increase monotonically with increasing stress (FREED and WALKER, 1990). This is verified easily for my hypothesis, Eqns (6, 7 & 8), as long as $0 \leq \|B\| \leq \|B\|_{max}$, $D_0 \leq D \leq D_{max}$ and $0 \leq Y \leq Y_{max}$.

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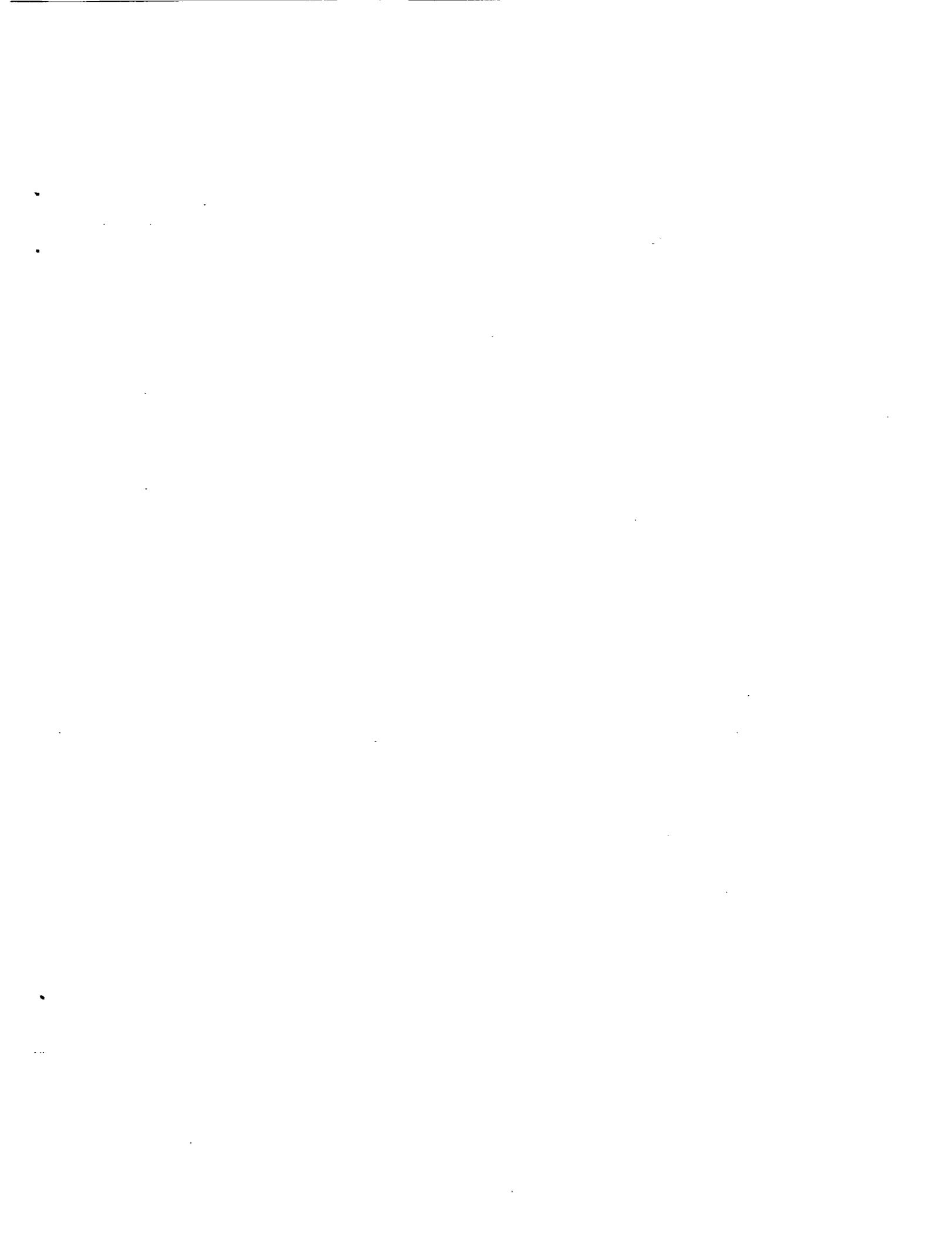
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