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FUZZY FORECASTING AND DECISION MAKING IN SHORT DYNAMIC TIME SERIES

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ABSTRACT

This paper focuses on the usage of the fuzzy set theory in forecasting and decision-making in areas with short dynamic time series.

INTRODUCTION

In different areas of human activities, there exists the objective necessity of decision-making and appraising future tendencies of processes under research. When there exists complete and adequate statistical figures or materials on the behavior of the object under research, it is adequate to use well-known orthodox methods or a combination of them to achieve the set objective.

Unfortunately today, there are very many areas of knowledge where, due to many objective reasons, there is the lack of adequate and complete statistical data, i.e. exhaustiveness of basic information.

However, even in cases like this, there is the need for a glance at the future (extrapolation) and a decision made based on that, using the available data. For example: We have a set of factors described by $Y = (y_1, y_2, ..., y_k)$, the activities of which affects a set of other factors described by $F = (f_1, f_2, ...f_\ell)$. It is necessary to access the future trend of F and how it is affected by the factors Y, and arrive at a decision.

There exists about three possible directions of solving this type of problems:

i. Classical and traditional (orthodox) methods,
ii. Expert evaluation methods,
iii. Fuzzy mathematics.

Classical and well-known traditional methods (like correlation-regression analysis, linear programming, etc.) require the length of the time-series to be about 4-6 times longer than the range of forecast, i.e. $4n > m$, where n = length of the time-series, and m = range of forecast.

Forecasting methods based on expert evaluation allows for the "informal" usage or part-usage of the statistical information at hand and the subjective evaluation of the experts involved.

In other words, classical methods of forecasting in cases of short time-series are usually not applicable, since they do not satisfy the methodical assumptions and propositions of mathematical statistics. Other methods based on expert evaluation also cannot be used because of the possibility of giving subjective (un-objective) estimates and the "in-complete" usage of the available statistical data.

In view of these problems, it is necessary to advance that type of technique based on a complete statistical assessment of data and excluding all sorts of subjective estimates.

CALCULATION OF COMPOSED MEMBERSHIP FUNCTION

Let the linguistic meanings of forecasting the economic indices of the economic problem be expressed with the help of the membership function $m_k(u) \varepsilon [0,1]$, where $u = [0,1]$, $k = \overline{1,N}$

The matrix $M = \| m_k(u) \|$ consists of a set of the membership function (MF) that in general characterizes the fuzzy model of the economic indices under review.

The composed membership function of the model we suggest should be calculated as the linear functions as below:

$$F = A^{-1} M,$$

where $A = HG^{1/2}$ - matrix of the weighting coefficients of the principal components;

G - eigenvalue vector of the matrix R;
H - matrix of eigenvalue vectors of matrix R
$R = 1/N \, MM^T$ - correlation matrix of the MF for an index.

To calculate the elements of the vector G and matrix H, the following matrix equations are solved:

$$\| R - gE \| = 0,$$
$$g \varepsilon G$$
$$(R\text{-}gE)\, H = 0,$$

where $E = \| e_{ij} \|$ is the matrix, in which $eii = 1$; $e_{ij} = e_{ji} = 0$, $i \neq j$.

More precise description of this method calculating the composed MF can be found in [1].

Let's examine closely an example of decision-making on an economic problem concerning, for instance, the production (or estimated level of output) of a new commodity. At our disposal are three known factors like

(i) cost of production (cost price),
(ii) per capital output (capital intensity),
(iii) taxation policy,

affecting production level.

On these three factors we have only a limited dynamic time-series of 3-4 years. However, it is necessary to make a decision on the production of the new commodity.

Let the following fuzzy linguistic variables express the membership function of the above factors under review:

< cost of production > ———> < satisfactory good >
< per capital output > ———> < average >
< taxation policy > ———> < good >

Employing known formulae of the membership functions' setting we can determine the meanings:

	0	.2	.4	.6	.8	1.0
<cost price>	0	.0	.6	.8	.9	1.0
<capital intensity>	0	.0	.6	1.0	.6	.0
<taxation policy>	0	.0	.4	.6	.8	1.0

Using the method of principal components, the values of the first principal component weighting coefficient for the three factors are as follows:

$$a_1 = 0.3894;\ a_2 = 0.253;\ a_3 = 0.363.$$

Then the generalized estimation of our decision-making process has the following composed membership function:

$$MF = 0 \mid 0 + 0 \mid .2 + .55 \mid .4 + .78 \mid .6 + .75 \mid 1.0$$

with the average linguistic meaning $P = 0.72$, which characterizes the generalized estimation of the decision-making process of the production of the new commodity.

FUZZY ANALOGY OF BROWN SMOOTHING METHOD

Calculation of the prognostic models of the estimated level of output of a new commodity put forward in the previous paragraph based on the 3-4 years time-series cannot be done with the help of know formulae. To this end, we advance a technique, the basic conceptions of which are stated below:

I. The smoothing parameter (@) can be calculated as in [2].

$@ = 2/n + 1$, where n = length of time-series.

II. Generalized fuzzy model for calculating the fuzzy numbers of a dynamic series (i = 1, 2, 3, ..., n) >

$i = 1$:

$Y_1 = Y_1(f) \pm @\, Y_0$

$Y_1\ (\min) = Y_1 - @\, Y_1(f)$

$Y_1\ (MAX) = Y_1 + @\, Y_1(f)$

| where,
| Y_1 - mean (average) value,
| $Y_1(f)$ - actual value of factor for first member of time-series,
| Y_1 (min) - minimal value,
| Y_1 (max) - maximum value.

Hence,

$$\tilde{Y}_1 = Y_1\ (f) \pm @\, Y_1(f)$$

$i = 2$:

$Y_2 = @\, Y_1 + (1 - @)\, Y_2\ (f)$

$Y_2\ (p) = -@\, Y_2\ (1 - @) - (1\text{-}@)\, Y_1\ (\min)\ @$

$Y_2\ (pp) = @\, Y_2\ (1\text{-}@) + (1\text{-}@)\, Y_1\ (\max)\ @$

| where,
| Y_2 (min) - minimal value of second member of time-series.

$$Y_2\,(\min) = Y_2 - Y_2\,(p)$$

$$Y_2\,(\max) = Y_2 + Y_2\,(pp)$$

Hence,

$$\tilde{Y}_2 = [Y_2\,(\min)\,;\ Y_2\,;\ Y_2\,(\max)\,]$$

. . . etc

$i = n$:

$$Y_n = @\,Y_n\text{-}1 + (1\text{-}@)\,Y_n(f)$$

$$Y_n(p) = \text{-}@\,Y_n\,(1\text{-}@) - (1\text{-}@)\,Y_{n\text{-}1}\,(\min)\,@$$

$$Y_n(pp) = @\,Y_n\,(1\text{-}@) + (1\text{-}@)\,Y_{n\text{-}1})\,(\max)\,@$$

$$Y_n\,(\min) = Y_n - Y_n\,(p)$$

$$Y_n\,(\max) = Y_n + Y_n\,(pp)$$

Hence,

$$\tilde{Y}_n = [Y_n\,(\min)\,;\ Y_n\,;\ Y_n\,(\max)\,]$$

| Y_2 - mean (average) value of second member time-series | etc.

| where,

| $Y_{n\text{-}1}$ -fuzzy number of n-1 time-series

The generalized forecast model of the fuzzy analogy of Brown Smoothing Method can be expressed thus:

$i = n + 1$:

$$Y_{n+1}\,(\text{prog.}) = @\,\tilde{Y}_n + (1 = @)\,Y_n\,(f)$$

where,

Y_{n+1} (prog.) - forecast level for the year $n + 1$,

$\tilde{Y}_n$ - fuzzy number of factor for the year n,

$Y_n(f)$ - actual value of factor for the year n.

Based on the calculated fuzzy numbers of the factors for the period (3-4 years) and applying the generalized forecast fuzzy analogy of Brown Smoothing Method, the estimated output level of the new commodity can be calculated.

FUZZY DECISION MAKING

Based on the basic directive (requirement) measuring the efficiency and profitability of the decision-making process to engage in the production of a new commodity for instance, and the estimates described above, a procedure is developed for decision-making in these conditions.

The composed membership function (MF) of the indices, i.e. the average linguistic meaning P, serves as a means of making a decision on the production of a new commodity.

The procedure assumes the comparison of two fuzzy numbers based on a set of index values. The result is an interval (span) between the set of fuzzy numbers describing the basic directive of profitability of the economic index on one hand, and the set of fuzzy numbers describing the economic estimates, i.e. the average linguistic meaning P, on the other hand.

This interval is measured from 0.5 on the relative estimate scale, i.e. decision scale. If any element of the set describing the basic directive of profitability is less (or smaller) than any element of the set of fuzzy numbers describing the economic index estimates, i.e. average linguistic meaning *P*, then the interval is positive, if not, then it is negative.

In other words, if the average linguistic meaning *P* describing the economic index estimates is to the left of the fuzzy numbers of the basic directive of profitability, then the estimate is negative, i.e. not good enough, and if it is to the right, then it is positive.

The intersection of the basic directive of profitability and the economic index estimates is measured on the decision scale from 0.5 to both sides of the scale. So, if the average linguistic meaning *P* falls to the right of this intersection, then the decision-making process based on this is positive. If however, it falls to the left then it is negative.

The composed membership function (MF) on the decision scale describes the fuzzy number corresponding to the profitability of the economic index estimates (estimated level of output of a new product) and the fuzzy decision based on that estimate.

The correctness, i.e. trustworthiness, of the estimate is calculated as a measure of the fuzziness of the resulting fuzzy number.

The validity of the decision is estimated by the square of the fuzzy decision at the interval [0.5, 1] if the mode of the function is between 0 and 0.5, and in the interval [0, 0.5] if the mode is between 0.5 and 1.

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