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Reference Equations of Motion for Automatic Rendezvous and Capture

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TRW/Houston

1.0 INTRODUCTION

The analysis presented in this paper defines the reference
coordinate frames and control parameters necessary to model the
relative motion and attitude of spacecraft in close proximity with
another space system during the Automatic Rendezvous and Capture
phase of an on-orbit operation. Translation motion equations
relative to the geodesic LVLH frame are developed yielding the
Clohessy-Wiltshire differential equations of motion.
Perturbations of the gravity partial derivative matrix due to a
4x4 gravity model show the expected deviations from the Clohessy-
Wiltshire approximation. For the docking case, however, as the
relative separation distances become small, both relative
gravitational and inertial accelerations can be omitted from the
translation equations of motion.

Euler's equation for the rotational rate accelerations of a rigid
body are developed with expected torque effects indicated. The
actual torque computations are problematic and these specific
models are not developed as part of this work.

The relative docking port target position vector and the attitude
control matrix are defined based upon an arbitrary spacecraft
design. These translation and rotation control parameters could
be used to drive the error signal input to the vehicle flight
control system. Measurements for these control parameters would
become the bases for an autopilot or FCS design for a specific
spacecraft.

Cartesian analysis is used throughout the paper for the clarity
and compactness in representing component equations. The use of
Cartesian tensor notation makes the task of repeated
differentiation of transformation equations considerably easier
and more understandable. For instance, the position vector in
inertial space is given by

$$x = a \bar{x} + X \tag{1.1}$$

where $a\bar{x}$ is a transformation matrix times the vector $\bar{x}$. Equation
1.1 defines both rotation and translation in the inertial frame.
Differentiating Equation 1.1 for velocity is simply,

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$$v = a\bar{v} + V \tag{1.2}$$

where

$$\bar{v} = \overline{W}\,\mathfrak{X} + \bar{v} \tag{1.3}$$

and $\overline{W}$ is the skew-symmetry angular velocity matrix. The vector $\bar{v}$ is recognized from vector analysis [see Reference 4.0, Page 208] as the operator,

$$\left(\bar{\omega}\pmb{\times} + \frac{d}{dt} \right)\bar{x} \tag{1.4}$$

However, in Equation 1.2 the inertial coordinates are preserved and no ambiguities arise as to which coordinate system the vectors components are represented.

2.0 TRANSLATIONAL DYNAMIC MOTION OF CENTER OF MASS

Let (o) be the origin of an imaginary coordinate frame attached to a mass in orbital motion about the planet's center. Further assume that there is no contact force acting upon this mass so that it moves along the geodesic arc in the inertial frame whose origin coincides with the center of mass of the planet. Also attached to the origin (o) is the Local-Vertical-Local-Horizontal (LVLH) coordinate reference frame. For circular orbital motion the LVLH system rotates downward about its y-axis at the orbital rate.

Figure 2-1
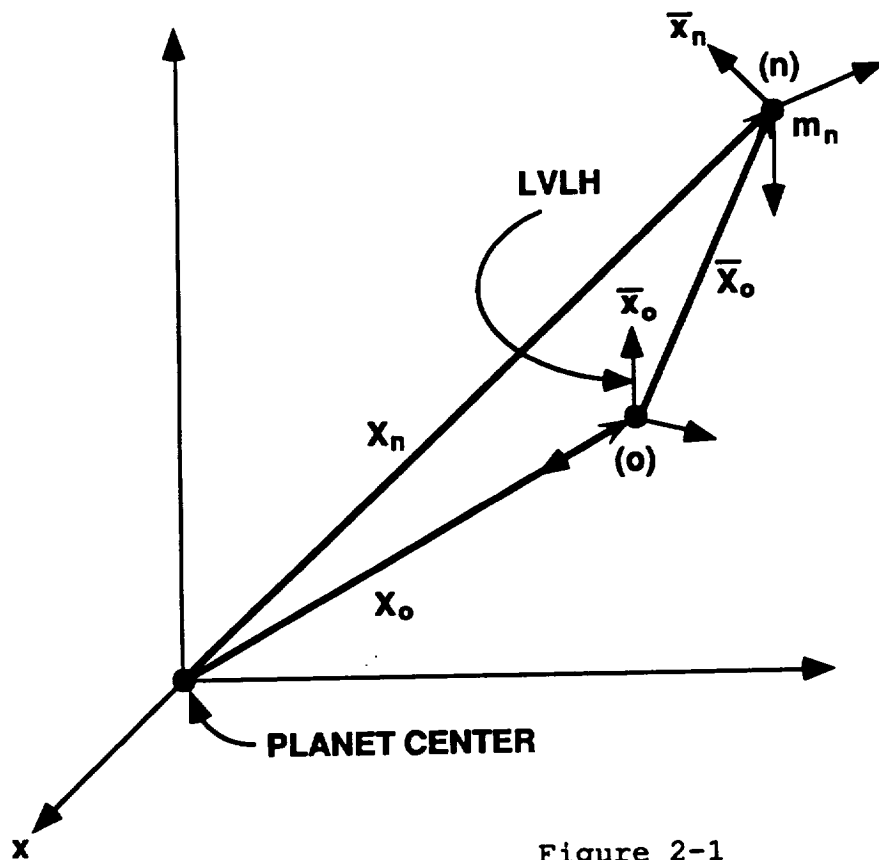
The Cartesian equation,

$$x_{on} = a_o \bar{x}_{on} + X_o \tag{2.1}$$

Represents the components of the inertial vector to any coordinate center located at (n) as seen from (o), where a_o is the transformation matrix LVLH to inertial and

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$$X_n = a_o \tilde{X}_{on} + X_o \qquad (2.2)$$

represents vectors to the center of mass m_n at (n) or

$$a_o \tilde{X}_{on} = X_n - X_o. \qquad (2.3)$$

This is their relative position relationship at any time t. Differentiating Equation 2.3 for velocity

$$a_o \tilde{v}_{on} = V_n - V_o \qquad (2.4)$$

where

$$\tilde{v}_{on} = \overline{W} \tilde{X}_{on} + \tilde{v}_{on}$$

and

$$\overline{W} = \begin{pmatrix} 0 & -\bar{\omega}_3 & \bar{\omega}_2 \\ \bar{\omega}_3 & 0 & -\bar{\omega}_1 \\ -\bar{\omega}_2 & \bar{\omega}_1 & 0 \end{pmatrix}$$

is the skew-symmetric angular velocity matrix. Solving for relative velocity in the LVLH frame,

$$\tilde{v}_{on} = (a_o)^T (V_n - V_o) - \overline{W} \tilde{X}_{on}.$$

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Differentiating Equation 2.4 for acceleration

$$a_o \dot{a}_{on} = \dot{V}_n - \dot{V}_o \qquad (2.5)$$

where

$$\bar{a}_{on} = (\overline{WW} + \dot{\overline{W}})\; \bar{x}_{on} + 2\overline{W}\; \bar{v}_{on} + \dot{\bar{v}}_{on}$$

and the relative LVLH acceleration vector as seen from (o) is

$$\dot{\bar{v}}_{on} = (a_o)^T\; (\dot{V}_n - \dot{V}_o) - (\overline{WW} + \dot{\overline{W}})\; \bar{x}_{on} - 2\,\overline{W}\; \bar{v}_{on}. \qquad (2.6)$$

The acceleration vectors are

$$\dot{V}_n = G(X_n) + A_n \quad \text{and} \quad \dot{V}_o = G(X_o) + A_o \qquad (2.7)$$

But $A_o = 0$, since we have defined (o) to be an inertial frame moving along the geodesic arc about the planet center. Therefore,

$$\dot{V}_n - \dot{V}_o = G(X_n) - G(X_o) + A_n \qquad (2.8)$$

and Equation 2.5 becomes

$$a_o \bar{a}_{on} = G(X_n) - G(X_o) + A_n. \qquad (2.9)$$

If X_n and X_o are very close to one another. The difference in gravitation accelerations is simply a linear variation in acceleration about the origin (o),

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$$\delta G(X_o) \cong G(X_n) - G(X_o)$$

and

$$\delta G(X_o) = \frac{\delta G}{\delta X_o} \delta X_o = \frac{\delta G}{\delta X_o} (X_n - X_o) = P_X (X_n - X_o), \qquad (2.10)$$

therefore, Equation 2.9 becomes

$$a_o \bar{a}_{on} = P_X (X_n - X_o) + A_n. \qquad (2.11)$$

From Equation 2.3 we have

$$a_o \bar{a}_{on} = P_X (a_o \bar{x}_{on}) + A_n \qquad (2.12)$$

and solving for the acceleration in the LVLH frame,

$$\bar{a}_{on} = [(a_o)^T P_X a_o] \bar{x}_{on} + (a_o)^T A_n \qquad (2.13)$$

$$\bar{a}_{on} = \bar{P}_X \bar{x}_{on} + \bar{A}_n \qquad (2.14)$$

where

$$\bar{A}_n = (a_o)^T A_n \quad \text{and} \quad \bar{P}_X = [(a_o)^T P_X a_o].$$

The $\overline{P}_x$ matrix is defined from the similarity transformation for matrices as described in Reference 5.0, Pages, 317 and 318, then Equation 2.14 becomes,

$$\dot{\overline{v}}_{on} = (\overline{P}_x - \overline{WW} - \dot{\overline{W}}) \; \overline{x}_{on} - 2\overline{W} \; \overline{v}_{on} + \overline{A}_n . \tag{2.15}$$

Equation 2.15 is the equation for motion for mass m in the LVLH coordinates system attached to the geodesic point (o) in Figure 1.0. The $\overline{P}_x$ matrices are used here for simplicity and also contribute to understanding in the analysis, however, it should be pointed out that the $\overline{P}_x$ matrices are non-linear and their region of numeric stability about the geodesic origin is small. Hence, for very accurate relative motion computations the difference equations of Equation 2.9 become a more reliable relationship on which to base computational algorithms. Equation 2.15 can be represented by the sum of two vectors,

$$\dot{\overline{v}}_{on} = \overline{A}_{In} + \overline{A}_n , \tag{2.16}$$

where $\overline{A}_{In}$ are the combined gravitational and inertial accelerations effecting the relative motion of the center of mass.

The $\overline{P}_x$ matrix reduces to (see Appendix C)

$$\overline{P}_x = -\mu R^{-3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} = -\omega_o^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} . \tag{2.17}$$

In a spherical gravitational field, i.e., where all gravity harmonics are assumed zero,

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$$\overline{WW} = \begin{pmatrix} 0 & 0 & -\omega_o \\ 0 & 0 & 0 \\ \omega_o & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -\omega_o \\ 0 & 0 & 0 \\ \omega_o & 0 & 0 \end{pmatrix} = \begin{pmatrix} -\omega_o^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\omega_o^2 \end{pmatrix} \qquad (2.18)$$

and for uniform circular orbit motion

$$\dot{\overline{W}} = 0. \qquad (2.19)$$

However, in the "real world" small oscillatory motions do occur about the LVLH axes due to the gravity harmonics. Then Equation 2.15 reduces to

$$\begin{cases} \dot{v}_x = 2\omega_o\, v_z + \overline{A}_x & = \overline{A}_{Ix} + \overline{A}_x \\[4pt] \dot{v}_y = -\omega_o^2\, \overline{y} + \overline{A}_y & = \overline{A}_{Iy} + \overline{A}_y \\[4pt] \dot{v}_z = 3\omega_o^2\, \overline{z} - 2\omega_o\, v_x + \overline{A}_z & = \overline{A}_{Iz} + \overline{A}_z \end{cases} \qquad (2.20)$$

which are recognized as the Clohessy-Wiltshire approximation for the differential equations of relative motion [as described in Reference 1.0]. The $\overline{A}_I$ in Equation 2.20 are the combined relative gravitational and inertial acceleration effects in the LVLH system.

3.0 ROTATIONAL DYNAMICS

The vehicle angular momentum in body axis coordinates transforms into the inertial frame via

$$L = a \bar{L} \tag{3.1}$$

and $\bar{L}$ for any rigid body is given by $\bar{I}\bar{\omega}$ where $\bar{I}$ is the body axis moment of inertia matrix and $\bar{\omega}$ is the body axis rotation rate vector. Equation 3.1 becomes

$$L = a\bar{I}\bar{\omega}. \tag{3.2}$$

Using Newton's Law;

$$\frac{dL}{dt} = N. \tag{3.3}$$

Equation (3.2) becomes

$$\frac{dL}{dt} = a\overline{WI}\bar{\omega} + a\bar{I}\dot{\bar{\omega}} = N = a\bar{N} \tag{3.4}$$

where N and $\bar{N}$ are the applied torques in the inertial and body axis coordinate frames, respectively. Equation 3.4 yields Euler's Equation of motion for a rigid body,

$$\bar{N} = \overline{WI}\bar{\omega} + \bar{I}\dot{\bar{\omega}}, \tag{3.5}$$

and solving for $\dot{\bar{\omega}}$, we have,

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$$\dot{\overline{\omega}} = \overline{J} \ (\overline{N} - \overline{WI}\overline{\omega}) \tag{3.6}$$

where $\overline{J} = \overline{I}^{-1}$, is the inverse of the body axis moment of inertia matrix. The applied torques are composed of

$$\overline{N} = \overline{N}_G + \overline{N}_T + \overline{N}_A + \overline{N}_P \tag{3.7}$$

gravity, thrusting, aerodynamic and plume impingement torques, respectively, and Equation 3.6 becomes

$$\dot{\overline{\omega}} = \overline{J} \ [\overline{N}_G + \overline{N}_T + \overline{N}_A + \overline{N}_P + \overline{N}_I] \tag{3.8}$$

where the inertial torques are $\overline{N}_I = -\overline{WI}\overline{\omega}$, which results due to rotational motion of the vehicle itself. Integrals of this equation [see Reference 3.0] provide the body axis rotation rate vector as a function of time.

4.0 SIX DEGREES-OF-FREEDOM EQUATIONS OF MOTION

From Equation (2.20),

$$\dot{\bar{v}}_n = \frac{g_o}{W_n} \sum \bar{F}_n + \bar{A}_{In} \qquad (4.1)$$

for translational dynamics and Euler's equation

$$\dot{\bar{\omega}} = \bar{J}_n \, (\bar{N}_n - \bar{W}_n \, \bar{I}_n \, \bar{\omega}_n) \qquad (4.2)$$

for rotation dynamics, where;

$\dfrac{g_o}{W_n} \sum \bar{F}_n$	$\tilde{}$ Summation of contact forces acting on vehicle n (LVLH Ref)
$\bar{A}_{In}$	$\tilde{}$ Inertial and gravitational accelerations of CM of vehicle n (LVLH Ref)
$\bar{I}_n$	$\tilde{}$ Moment of inertia matrix of vehicle n
$\bar{J}_n$	$\tilde{}$ Inverse of moment of inertia matrix of vehicle n
$\bar{N}_n$	$\tilde{}$ Body axis torques about the CM of vehicle n
$\bar{W}_n$	$\tilde{}$ Skew symmetric angular velocity matrix (vehicle n)
$\bar{\omega}$	$\tilde{}$ Body axis rotation rate vector (vehicle n)
$\dot{\bar{\omega}}_n$	$\tilde{}$ Body axis rotation rate acceleration vector for vehicle n.

For example an integrable set of differential equations could be represented as follows,

$$\dot{\bar{v}}_1 = \frac{g_o}{W_1} (-C_{D1} \, \bar{q} \, S_1 \, \bar{u}_{REL1} + T_1) + \bar{A}_{I1} \qquad (4.3)$$

$$\dot{\bar{v}}_2 = \frac{g_o}{W_2} (-C_{D2} \, \bar{q} \, S_2 \, \bar{u}_{REL2}) + \bar{A}_{I2}. \qquad (4.4)$$

Assume that vehicle one is active (with thrust) and that vehicle two is passive (no thrusting) and

$$\dot{\bar{v}}_1 = -B_1 g_o \, \bar{q} \, u_{REL1} + \frac{g_o}{W_1} T_1 + \bar{A}_{I1}$$

$$\dot{\bar{v}}_2 = -B_2 g_o \, \bar{q} \, u_{REL2} + \bar{A}_{I2}, \qquad (4.5)$$

where u_{REL} is a unit vector in the LVLH geodesic system and is in the direction of the motion of the geodesic frame and,

$$B_i \text{ is the ballistic coefficient } \left(\frac{C_{Di} S_i}{W_i} \right)$$

$\bar{q}$ is the fluid dynamic pressure, $\bar{q} = (1/2 \, \rho \, (V_R)^2)$ where,

V_R ~ relative velocity magnitude

ρ ~ fluid density

and the $\bar{u}_{REL}$ vector in the LVLH reference frame is approximately,

$$\bar{u}_{REL} = \begin{pmatrix} 1.0 \\ 0.0 \\ 0.0 \end{pmatrix}. \qquad (4.7)$$

With the above example in mind, Appendix A shows a suggested integrable parameter list for a time-wise step solution to the problem. More precise equations of motion can be defined by computing the driving values for $\bar{F}_n$ and $\bar{N}_n$ in Equations 4.1 and 4.2, respectively, however, the same 6-DOF solution procedures will apply.

5.0 TRANSLATION CONTROL PARAMETERS

As in Figure 2-1, the geodesic center is the motion of an imaginary point falling along the local gravity geodesic arc. No drag or other contact forces are acting at the geodesic center. The rotating LVLH coordinate axes are attached to the geodesic center. The position vector in the inertial coordinate system to a point in space can be computed from each of the coordinate systems shown in the following figure.
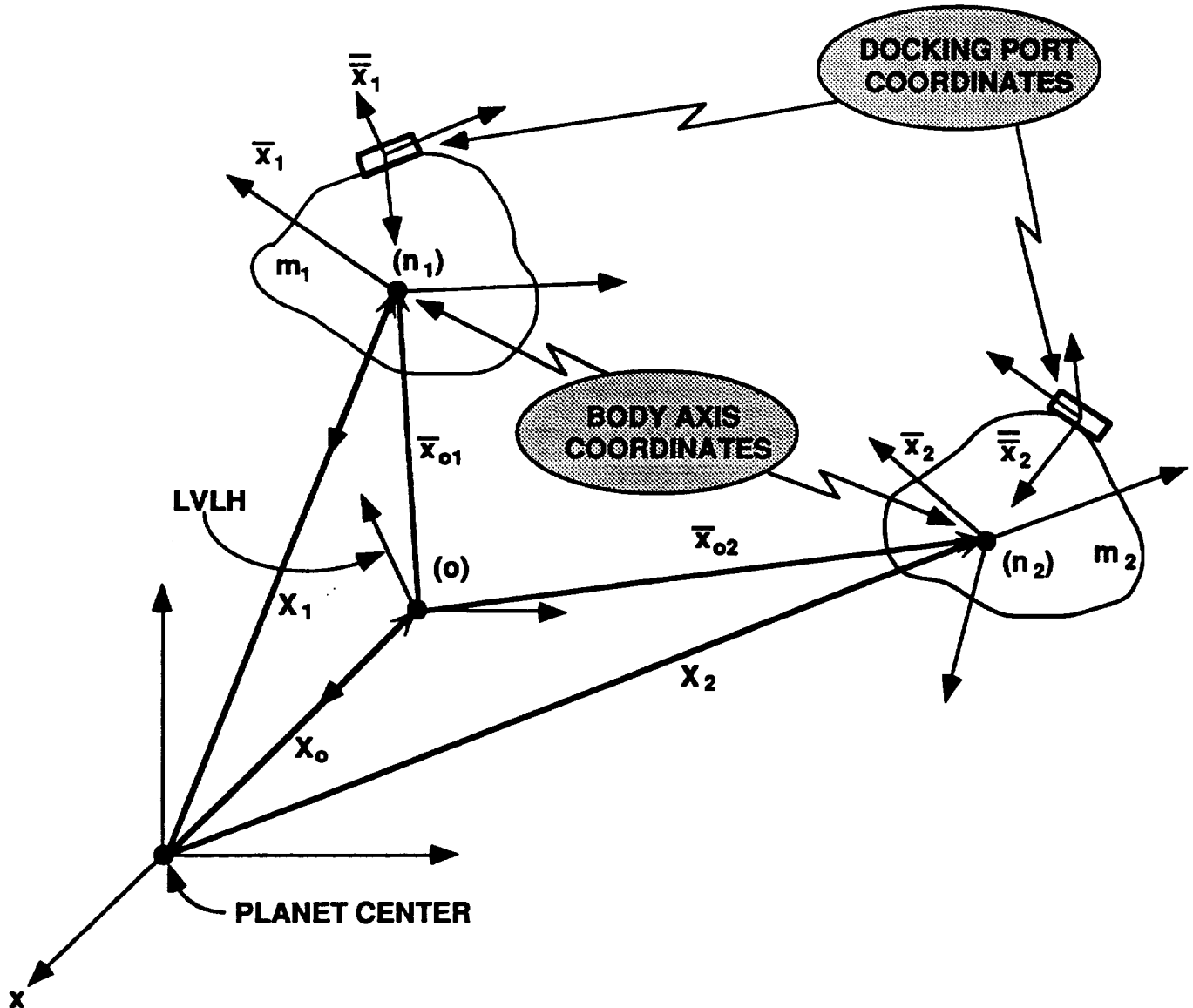
Figure 5-1

The Cartesian equations relating the coordinates as shown in Figure 5.1 can be written as follows;

$$x_0 = a_0\bar{x}_0 + X_0 \qquad (5.1)$$

from the geodesic frame and likewise,

$$x_1 = a_1\bar{x}_1 + X_1 \qquad (5.2)$$

is from the center of mass of vehicle one. From coordinates x_1 on vehicle one,

$$\bar{x}_1 = c_1 \ \bar{\bar{x}}_1 + \overline{X}_1, \qquad (5.3)$$

is from the center of the docking port located on vehicle one. Similarly for vehicle two,

$$x_2 = a_2\bar{x}_2 + X_2 \qquad (5.4)$$

from the center of mass of vehicle two

$$\bar{x}_2 = c_2\bar{\bar{x}}_2 + \overline{X}_2 \qquad (5.5)$$

and from the center of the docking port located on vehicle two.

The following definitions apply to Equations 5.1 through 5.5

a_o ~ The LVLH to inertial transformation matrix attached to the geodesic coordinate center (a time varying matrix)

a_i ~ ith body axis to inertial transformation matrix (a varying matrix)

X_i ~ The inertial coordinates of the ith coordinate center (a dynamic vector in motion about the planet center)

c_i ~ ith docking port to body axis transform matrix (a constant matrix)

$\overline{X}_i$ ~ Body axis coordinates of the ith body docking port coordinate center (a constant vector).

The target docking port inertial position is

$$x_T = a_2 \; \tilde{x}_2 + X_2$$

$$x_T = a_2 \; (c_2\tilde{\overline{x}}_2 + \overline{X}_2) + X_2 \tag{5.6}$$

but

$$\tilde{x}_2 = 0,$$

since the target is the docking port coordinate center and therefore Equation 5.6 reduces to

$$x_T = a_2 \; \overline{X}_2 + X_2, \tag{5.7}$$

the target location in inertial coordinate. Hence, the position relative geometry in the inertial coordinate reference frame becomes

$$a_2 \, \overline{X}_2 \, + \, X_2 \, = \, a_1 \, (c_1 \bar{X}_1 + \overline{X}_1) \, + \, X_1 \tag{5.8}$$

from Equation 5.1

$$X_1 \, = \, a_o \mathfrak{X}_{o1} \, + \, X_o \quad \text{and} \quad X_2 \, = \, a_o \mathfrak{X}_{o2} \, + \, X_o \tag{5.9}$$

and substituting these into Equation 5.8

$$a_2 \overline{X}_2 \, + \, a_o \mathfrak{X}_{o2} \, = \, a_1 \, (c_1 \, \bar{X}_1 + \overline{X}_1) \, + \, a_o \, \mathfrak{X}_{o1} \tag{5.10}$$

where the $\bar{X}_{o2}$ and $\mathfrak{X}_{o2}$ are the position vectors to the vehicle one and two center of mass measured in the LVLH geodesic coordinate frame of reference.

By multiplying Equation 5.10 by the transformation matrix $(a_o)^T$, the equation is transformed to the LVLH frame,

$$(a_o)^T a_2 \, \overline{X}_2 \, + \, \mathfrak{X}_{o2} \, = \, (a_o)^T a_1 \, (c_1 \bar{X}_1 + \overline{X}_1) \, + \, \mathfrak{X}_{o2}. \tag{5.11}$$

Now let $b_1 = (a_o)^T a_1$ and $b_2 = (a_o)^T a_2$, be the transformation matrices for the body axes to the LVLH reference frame. Then Equation 5.11 becomes

$$(\bar{X}_{o2} - \mathfrak{X}_{o1}) \, = \, b_1 \, (c_1 \bar{X}_1 + \overline{X}_1) \, - \, b_2 \overline{X}_2, \tag{5.12}$$

the relative position vector between the two vehicles in the LVLH frame and solving for $\bar{X}_1$,

$$\bar{X}_1 \, = \, (c_1)^T [\, (b_1)^T \{ (\mathfrak{X}_{o2} - \mathfrak{X}_{o1}) + b_2 \overline{X}_2 \} - \overline{X}_1] . \tag{5.13}$$

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We have the position control vector as seen from the vehicle one docking port coordinate center. The GN&C systems should drive this vector to zero for docking contact. Written in body axis coordinates we have,

$$\bar{x}_1 = c_1 \bar{x}_1 = (b_1)^T \ (b_2 \overline{X}_2 + (\bar{x}_{o2} - \bar{x}_{o1})) - \overline{X}_1, \tag{5.14}$$

the control vector as seen from the active vehicle (vehicle one). Hence, a candidate commanded velocity in the vehicle frame becomes,

$$\bar{v}_c = K_R \ \bar{x}_1 \tag{5.15}$$

where the gain,

$$K_R = -\left(\frac{k_R}{\Delta t}\right),$$

can be applied in an Flight Control System (FCS) with appropriate limiters for proper closing rates. For the velocity of the target docking port we differentiate Equation 5.9

$$a_2 \overline{W}_2 \overline{X}_2 + a_o \overline{W}_o \bar{x}_{o2} + a_o \bar{v}_{o2} = a_1 \overline{W}_1 \ (c_1 \bar{x}_1 + \overline{X}_1)$$

$$+ \ a_1 c_1 \bar{v}_1 + a_o \overline{W}_o \bar{x}_{o1} + a_o \bar{v}_{o1}. \tag{5.16}$$

Again by multiplying by $(a_o)^T$ the equation is reduced to LVLH coordinates,

$$(a_o)^T a_2 \overline{W}_2 \; \overline{X}_2 \; + \; \overline{W}_o \hat{X}_{o2} \; + \; \hat{V}_{o2} \; = \; (a_o)^T a_1 \overline{W}_1 \; (c_1 \hat{X}_1 + \overline{X}_1)$$

$$+ \; (a_o)^T a_1 c_1 \hat{\overline{V}}_1 \; + \; \overline{W}_o \hat{X}_{o1} \; + \; \hat{V}_{o1} \tag{5.17}$$

and the relative velocity difference in the LVLH frame is,

$$(\hat{V}_{o2} - \hat{V}_{o1}) \; = \; b_1 \overline{W}_1 (c_1 \hat{X}_1 + \overline{X}_1) \; + \; b_1 c_1 \hat{\overline{V}}_1 \; - \; b_2 \overline{W}_2 \overline{X}_2 \; - \; \overline{W}_o (\hat{X}_{o2} - \hat{X}_{o1}) \tag{5.18}$$

and the docking port relative velocity of the docking port coordinate center on vehicle two (the target) is

$$\hat{\overline{V}}_1 \; = \; (c_1)^T [(b_1)^T \{ (\hat{V}_{o2} - \hat{V}_{o1}) + \overline{W}_o (\hat{X}_{o2} - \hat{X}_{o1}) + b_2 \overline{W}_2 \overline{X}_2 \} - \overline{W}_1 (c_1 \hat{X}_1 + \overline{X}_1)]. \tag{5.19}$$

The control velocity vector in vehicle body axis coordinates becomes,

$$V_1 \; = \; c_1 \hat{\overline{V}}_1 \; = \; (b_1)^T [(\hat{V}_{o2} - \hat{V}_{o1}) + \overline{W}_o (\hat{X}_{o2} - \hat{X}_{o1}) + b_2 \overline{W}_2 \overline{X}_2] \; - \overline{W}_1 (c_1 \hat{X}_1 + \overline{X}_1). \tag{5.20}$$

Using Equations 5.15 and 5.20, the ΔV command for translation control becomes,

$$\Delta \overline{V}_c \; = \; V_c \; - \; V_1, \tag{5.21}$$

and when applied with appropriate limiters or closing gates, the algorithm can be used to the control closing conditions for safe docking. The acceleration of the control vector is found by differentiation of Equation 5.16 and becomes

$$\dot{\bar{\bar{v}}}_1 = (c_1)^T \ [\ (b_1)^T \ b_2 \ (\overline{W}_2\overline{W}_2 + \dot{\overline{W}}_2)\ \overline{X}_2 + (b_1)^T \{\ (\overline{W}_o\overline{W}_o + \dot{\overline{W}}_o)\ (\bar{x}_{o2} - \bar{x}_{o1})$$

$$+\ 2\ (\overline{W}_o)\ (\dot{\bar{v}}_{o2} - \dot{\bar{v}}_{o1}) + (\dot{\bar{v}}_{o2} - \dot{\bar{v}}_{o1})\ \}$$

$$-\ (\overline{W}_1\overline{W}_1 + \dot{\overline{W}}_1)\ (c_1\bar{x}_1 + \overline{X}_1) - 2\overline{W}c_1\bar{v}_1] \tag{5.22}$$

and in body axes coordinates

$$(c_1\dot{\bar{v}}_1)\ =\ (b_1)^T\ b_2\ (\overline{W}_2\overline{W}_2 + \dot{\overline{W}}_2)\ \overline{X}_2\ +\ (b_1)^T\ \{\ (\overline{W}_o\overline{W}_o + \dot{\overline{W}}_o)\ (\bar{x}_{o2} - \bar{x}_{o1})$$

$$+\ 2\ \overline{W}_o\ (\dot{\bar{v}}_{o2} - \dot{\bar{v}}_{o1}) + (\dot{\bar{v}}_{o2} - \dot{\bar{v}}_{o1})\ \}$$

$$-\ (\overline{W}_1\overline{W}_1 + \dot{\overline{W}}_1)\ (c_1\bar{x}_1 + \overline{X}_1) - 2\ \overline{W}_1 c_1 \bar{v}_1. \tag{5.23}$$

6.0 ROTATION CONTROL PARAMETERS

The attitude control matrix may be derived as from rotation equations as follows

$$x_1 = a_1 \bar{x}_1 \qquad\qquad x_2 = a_2 \bar{x}_2$$

$$\bar{x}_1 = c_1 \tilde{x}_1 \qquad\qquad \bar{x}_2 = c_2 \tilde{x}_2$$

$$x_1 = [a_1 c_1]\ \tilde{x}_1 \qquad\qquad x_2 = [a_2 c_2]\tilde{x}_2 \qquad\qquad (6.1)$$

The desired attitude condition at docking is that the two systems have approximately the same orientation in inertial space and, therefore,

$$a_1 c_1 \triangleq a_2 c_2. \qquad\qquad (6.2)$$

If vehicle number one is the active control vehicle,

$$a_{1d} = a_2 c_2 (c_1)^T \qquad\qquad (6.3)$$

is the desired attitude matrix for vehicle number one. The control matrix becomes

$$a_1 \tilde{x}_1 = a_{1d} \tilde{x}_{1d}$$

$$\tilde{x}_1 = [(a_1)^T a_{1d}] \tilde{x}_{1d}$$

$$\tilde{x}_1 = [(a_1)^T a_2 c_2 (c_1)^T] \tilde{x}_{1d}$$

$$C_N = [(a_1)^T a_2 c_2 (c_1)^T] \quad \text{(Attitude Control Matrix)} \qquad (6.4)$$

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That is, the C_N matrix will become the unit matrix when the docking ports are aligned. By extracting the Hamilton Quaternion from the C_N matrix a desired rotation can be found to align the docking ports as follows;

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} = \begin{pmatrix} \cos(\omega/2) \\ \sin(\omega/2)\cos\alpha \\ \sin(\omega/2)\cos\beta \\ \sin(\omega/2)\cos\gamma \end{pmatrix} \tag{6.5}$$

where

$$\omega = 2\cos^{-1}(q_1), \tag{6.6}$$

is the instantaneous angular control rotation required to align the docking ports.

The eigen axis about which the controlled rotation of vehicle one must occur becomes

$$\begin{pmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{pmatrix} = \frac{1}{\sin(\omega/2)} \begin{pmatrix} q_2 \\ q_3 \\ q_4 \end{pmatrix}. \tag{6.7}$$

Hence, as ω approaches zero, the C_N matrix approaches the unit matrix and the desired attitude condition of Equation 6.4 can be reached. The vehicle one attitude GN&C functions must be designed to control the spacecraft to this desired attitude.

The soft singularity indicated in Equation 6.7 will not be reached since ω will fall into the control attitude deadband before $\omega = 0$. A candidate commanded body axis rate vector may be estimated by selecting a scaler quantity Ω for the desired angular closing rate for the control quaternion and the desired body axis rotation rate becomes

$$\bar{\omega}_{1d} = \Omega \begin{pmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{pmatrix} = \frac{\Omega}{\sin(\omega/2)} \begin{pmatrix} q_2 \\ q_3 \\ q_4 \end{pmatrix} \qquad (6.8)$$

The candidate commanded body axis rotation rate change is

$$\Delta\bar{\omega}_{1c} = \bar{\omega}_{1d} - \bar{\omega}_1, \qquad (6.9)$$

where $\bar{\omega}_1$ is the instantaneous body axis rotation rate vector for the active vehicle (i.e., vehicle one).

The desired attitude rate constraint can also be derived by equating the two transformation matrices for docking port coordinates to the inertial coordinates as follows,

$$(a_o)^T a_1 c_1 = (a_o)^T a_2 c_2 \qquad (6.10)$$

and then demanding that the rates of the changed resulting matrices be identical,

$$-\bar{W}_o(a_o)^T a_1 c_1 + (a_o)^T a_1 \bar{W}_1 c_1 = -\bar{W}_o(a_o)^T a_2 c_2 + (a_o)^T a_2 \bar{W}_2 c_2 \qquad (6.11)$$

and

$$-a_o\bar{W}_o(a_o)^T a_1 c_1 + a_1 \bar{W}_1 c_1 = -a_o\bar{W}_o(a_o)^T a_2 c_2 + a_2 \bar{W}_2 c_2. \qquad (6.12)$$

Solving for the body axis rotation rate command matrix for vehicle one

$$\bar{W}_{1c} = (a_o)^T\{(a_o\bar{W}_o(a_o)^T)(a_1 - a_{1d}) + a_2\bar{W}_2 c_2(c_1)^T\}. \qquad (6.13)$$

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The a_{1d} matrix is the desired attitude matrix given by Equation 6.3. Note that when the a_1 matrix approaches the a_{1d} (desired attitude matrix) the steady state rotation rate command matrix for vehicle one becomes

$$\overline{W}_{1c} = (a_1)^T a_2 \overline{W}_2 c_2 (c_1)^T.$$

The candidate FCS could monitor the error signal, $(a_1 - a_{1d})$ from Equation 6.13 above and apply appropriate gains and limiters to control the attitude of vehicle one during docking.

7.0 CONCLUSIONS AND RECOMMENDATIONS

Detailed analysis of the Automatic Rendezvous and Capture problem indicate three different regions of mathematical description for the GN&C algorithms; (1) multi-vehicle orbital mechanics to the rendezvous interface point, i.e., within 100 NM, (2) relative motion solutions (such as Clohessy-Wiltshire type) from the far field to the near field interface, i.e., within 1 NM and (3) close proximity motion - the near field motion where relative differences in the gravitational and orbit inertial effects can be neglected from the equations of motion. Limit boundaries to these regions can be precisely defined by further analysis and will also be a function of the tracking measurement accuracies and the computer resources available to the solution algorithms.

The subjects of this paper analyze the regions (2) and (3) above and presents the derivation and discussion of the general case of gravitational perturbed relative motion. Mathematical deviations from the spherical gravity case of Clohessy-Wiltshire are presented in the analysis. Based upon the preliminary analysis of this work it is recommended that further efforts be used to assess the relative position and velocity differences in region (2) due to non-spherical effects (especially that of J_2) in the gravity perturbations. Future GN&C systems using relative motion measurements from either an onboard laser tracking systems or from the GPS will possibly be able to detect and predict these perturbations in the relative motion.

The docking port relative position and velocity target vectors as outlined in this work couples the effects of the translation and rotation control activity. Based on these preliminary control parameters it is recommended that guidance and control systems functions couple translation and attitude control in region (3) for safe docking maneuvers. In region (2) the relative range between the docking port targets is assumed large enough so that translation and rotation G&C functions can be independent of one another.

8.0 REFERENCES

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SUGGESTED INTEGRABLE PARAMETER LIST

APPENDIX A

The following list contains the fundamental parameters necessary to solve for the motion and attitudes of two vehicles relative to the geodesic LVLH frame and the body axis frame, respectively. A numerical process to perform the first order solution of this set of simultaneous differential equations is required. This process will perform the integral

$$x(t+\Delta t) = x(t) + \int_{t}^{t+\Delta t} \dot{x}(t) \, dt.$$

The process of Runge-Kutta-Gill is recommended, however, in some applications requiring high frequencies (i.e., small Δt) the Cauchy-Euler solution method is adequate for an accurate solution.

Suggested list

		$x(t)$	$\dot{x}(t)$
Vehicle Number One	1. 2. 3.	$\bar{X}_{o1}$ (Position) LVLH Ref.	$\bar{V}_{o1}$
	4. 5. 6. 7.	$\bar{V}_{o1}$ (Velocity) LVLH Ref.	$-B_1 \bar{q} g_o \bar{u}_{VREL} + \dfrac{g_o}{W_1} \bar{T}_1 + \bar{A}_{I1}$
	8. 9. 10.	q_1 (Quaternion) (Body axis to inertial)	$\dot{q}_1$
	11. 12. 13.	$\bar{\omega}_1$ (Body Axis Rotation Rates)	$\bar{J}_1 (\bar{N}_1 - \bar{W}_1 \bar{I}_1 \bar{\omega}_1)$
Vehicle Number Two	14. 15. 16.	$\bar{X}_{o2}$	$\bar{V}_{o2}$
	17. 18. 19. 20.	$\bar{V}_{o2}$	$-B_2 \bar{q} g_o u_{vREL} + \bar{A}_{I2}$
	21. 22. 23.	q_2	$\dot{q}_2$
	24. 25. 26.	$\bar{\omega}_2$	$\bar{J}_2 (\bar{N}_2 - \bar{W}_2 \bar{I}_2 \bar{\omega}_2)$

In the event that the absolute inertial state is required, the following additions can be made to the integrable parameter list

		$x(t)$	$\dot{x}(t)$
Inertial State	27. 28. 29. 30.	X_o	V_o
	31. 32.	V_o	$G_o = \dot{V}_o$

where G_o is the acceleration due to gravity at the point X_o, the geodesic coordinate center (see Figure 2-1).

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Using the Newtonian formulation of gravity the inertial
gravitational acceleration vector in a spherical gravity field is

$$G(x) = -\mu R^{-3} x \tag{B.1}$$

where x is the column vector for position from the gravitating
mass center, that is,

$$x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{B.2}$$

To transform B.1 into the LVLH coordinate frame use Equation 2.1
in B.1,

$$G(\bar{x}) = -\mu R^{-3} a_0 \hat{x} - \mu R^{-3} x_0 \tag{B.3}$$

and transform $G(\hat{x})$ to the LVLH frame,

$$(a_0)^T G(\hat{x}) = \overline{G}(\hat{x}) = -\mu R^{-3}(\hat{x}) - \mu R^{-3}(a^T x_0). \tag{B.4}$$

Since the LVLH coordinate axes are aligned with the Z-axis in the
-R direction, Equation B.4 reduces to

$$\overline{G}(\hat{x}) = -\mu R^{-3} \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} - R_0 \end{pmatrix}. \tag{B.5}$$

The gravitational acceleration vector in a field perturbed by non-spherical gravity harmonics, we write,

$$G(x) = - \mu R^{-3} (I+\mathfrak{R}) x \qquad (B.6)$$

where I is the unit matrix an $\mathfrak{R}$ is a diagonal matrix,

$$\mathfrak{R} = \begin{pmatrix} \mathfrak{R}_x & 0 & 0 \\ 0 & \mathfrak{R}_y & 0 \\ 0 & 0 & \mathfrak{R}_z \end{pmatrix} \qquad (B.7)$$

For example, using the J_2 harmonic only as shown in Reference 2, Pages 50 and 51,

$$\begin{pmatrix} \mathfrak{R}_x \\ \mathfrak{R}_y \\ \mathfrak{R}_z \end{pmatrix} = \begin{pmatrix} 3/2 \ J_2 \left(\dfrac{R_E}{R}\right)^2 \left(1-5\left(\dfrac{z}{R}\right)^2\right) \\ 3/2 \ J_2 \left(\dfrac{R_E}{R}\right)^2 \left(1-5\left(\dfrac{z}{R}\right)^2\right) \\ 3/2 \ J_2 \left(\dfrac{R_E}{R}\right)^2 \left(3-5\left(\dfrac{z}{R}\right)^2\right) \end{pmatrix} \qquad (B.8)$$

Here, R_E is the reference planetary equatorial radius and J_2 is the first zonal harmonic. For instance, the J_2 harmonic for the earth is $-1.0826271 \times 10^{-2}$ (unitless) and has the most pronounced effect on the motion of the low earth orbit spacecraft.

Transforming $G(x)$ in Equation B.6 into $\overline{G}(\overline{x})$ we have

$$\overline{G}(\overline{x}) = -\mu R^{-3} (I+\overline{\mathfrak{R}}) \begin{pmatrix} \overline{x} \\ \overline{y} \\ \overline{z}-R_o \end{pmatrix} \qquad (B.9)$$

where I is the unit matrix and $\overline{\mathfrak{R}}$ is given by the similarity transformation,

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$$\overline{\mathfrak{R}} = a^T \, \mathfrak{R} \, a, \tag{B.10}$$

and becomes a full matrix in the LVLH coordinate frame. Using the J_2 harmonic only from Equation B.8 and reducing to LVLH coordinates Equation B.9 becomes,

$$\overline{G}(\bar{x}) = -\mu R^{-3} \left[I + 3/2 \; J_2 \left(\frac{R_E}{R} \right)^2 \right.$$

$$\left. \left(a^T [K] a - 5 \left(\frac{z}{R} \right)^2 I \right) \right) \left(\begin{matrix} \bar{x} \\ \bar{y} \\ \bar{z} - R_o \end{matrix} \right) \tag{B.11}$$

where the matrix [K] is a constant matrix given by

$$[K] = \begin{pmatrix} 1.0 & 0 & 0 \\ 0 & 1.0 & 0 \\ 0 & 0 & 3.0 \end{pmatrix}. \tag{B.12}$$

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PARTIAL DERIVATIVES OF THE
GRAVITATIONAL ACCELERATION VECTOR

APPENDIX C

The $\overline{P}_x$ Matrix of Equation 2.15 for a spherical gravitational field is found by partial differentiation of Equation B.5 with respect to the $\overline{x}$ coordinates and is

$$\overline{P}_x = \frac{\partial \overline{G}}{\partial \overline{x}} = -\mu R^{-3} \begin{pmatrix} \left(1-3\left(\dfrac{\overline{x}}{R}\right)^2\right) & -3\left(\dfrac{\overline{x}\overline{y}}{R^2}\right) & \dfrac{-3\overline{x}(\overline{z}-R_o)}{R^2} \\[3ex] -3\left(\dfrac{\overline{x}\overline{y}}{R^2}\right) & \left(1-3\left(\dfrac{\overline{y}}{R}\right)^2\right) & \dfrac{-3\overline{y}(\overline{z}-R_o)}{R^2} \\[3ex] \dfrac{-3\overline{x}(\overline{z}-R_o)}{R^2} & \dfrac{-3\overline{y}(\overline{z}-R_o)}{R^2} & \left(1-3\left(\dfrac{\overline{z}-R_o}{R}\right)^2\right) \end{pmatrix} \quad (C.1)$$

The Clohessy-Wiltshire relative motion equations result when $\overline{x}$, $\overline{y}$ and $\overline{z}$ are very small relative to the radius vector magnitude R (thus R approaches R_o) and Equation C.1 is approximated by

$$\overline{P}_x \cong -\mu R^{-3} \begin{pmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & -2.0 \end{pmatrix}. \quad (C.2)$$

When the gravity field is perturbed due to non-spherical harmonics as in a real world situation the P_x matrix is found by partial differentiation of Equation B.9 with respect to the $\overline{x}$ coordinates and can be written out in component form as follows:

$$\frac{\partial \overline{G}_{\mathcal{R}}}{\partial \bar{x}} = \mu R^{-3} \left(1 + \overline{\mathfrak{R}}_{11} + \left(\frac{\partial \overline{\mathfrak{R}}}{\partial \bar{x}} - 3 \frac{\bar{x}}{R^2} (1 + \overline{\mathfrak{R}}_{11}) \right) \bar{x} \right.$$

$$\left. + \left(\frac{\partial \overline{\mathfrak{R}}_{12}}{\partial \bar{x}} - 3 \frac{\bar{x}}{R^2} \right) \bar{y} + \left(\frac{\partial \overline{\mathfrak{R}}_{13}}{\partial \bar{x}} - 3 \frac{\bar{x}}{R^2} \right) (\bar{z} - R_o) \right) \qquad (C.3)$$

$$\frac{\partial \overline{G}_{\mathcal{R}}}{\partial \bar{y}} = \mu R^{-3} \left(\overline{\mathfrak{R}}_{12} + \left(\frac{\partial \overline{\mathfrak{R}}_{11}}{\partial \bar{y}} - 3 \frac{\bar{y}}{R^2} (1 + \mathfrak{R}_{11}) \right) \bar{x} \right.$$

$$+ \left(\frac{\partial \overline{\mathfrak{R}}_{12}}{\partial \bar{y}} - 3 \frac{\bar{y}}{R^2} \mathfrak{R}_{12} \right) \bar{y}$$

$$\left. + \left(\frac{\partial \overline{\mathfrak{R}}_{13}}{\partial \bar{y}} - 3 \frac{\bar{y}}{R^2} \overline{\mathfrak{R}}_{13} \right) (\bar{z} - R_o) \right) \qquad (C.4)$$

$$\frac{\partial \overline{G}_{\mathcal{R}}}{\partial \bar{z}} = \mu R^{-3} \left(\overline{\mathfrak{R}}_{13} + \left(\frac{\partial \overline{\mathfrak{R}}_{11}}{\partial \bar{z}} - 3 \frac{(\bar{z} - R_o)}{R^2} (1 + \overline{\mathfrak{R}}_{11}) \right) \bar{x} \right.$$

$$+ \left(\frac{\partial \overline{\mathfrak{R}}_{12}}{\partial \bar{y}} - 3 \frac{(\bar{z} - R_o)}{R^2} \mathfrak{R}_{12} \right) \bar{y}$$

$$\left. + \left(\frac{\partial \overline{\mathfrak{R}}_{13}}{\partial \bar{z}} - 3 \frac{(\bar{z} - R_o)}{R^2} \mathfrak{R}_{13} \right) (\bar{z} - R_o) \right) \qquad (C.5)$$

$$\frac{\partial \overline{G}_{\varphi}}{\partial \bar{x}} = \mu R^{-3} \left(\overline{\mathfrak{R}}_{21} + \left(\frac{\partial \overline{\mathfrak{R}}_{21}}{\partial \bar{x}} - 3 \frac{\bar{z}}{R^2} \overline{\mathfrak{R}}_{21} \right) \hat{x} \right.$$

$$+ \left(\frac{\partial \overline{\mathfrak{R}}_{22}}{\partial \hat{x}} - 3 \frac{\hat{x}}{R^2} (1+\overline{\mathfrak{R}}_{22}) \right) \hat{y}$$

$$\left. + \left(\frac{\partial \overline{\mathfrak{R}}_{23}}{\partial \bar{x}} - 3 \frac{\bar{z}}{R^2} \mathfrak{R}_{23} \right)(\hat{z}-R_o) \right) \qquad (C.6)$$

$$\frac{\partial \overline{G}_{\varphi}}{\partial \hat{y}} = \mu R^{-3} \left(1 + \overline{\mathfrak{R}}_{22} + \left(\frac{\partial \overline{\mathfrak{R}}_{21}}{\partial \hat{y}} - 3 \frac{\hat{y}}{R^2} \overline{\mathfrak{R}}_{21} \right) \hat{x} \right.$$

$$+ \left(\frac{\partial \overline{\mathfrak{R}}_{22}}{\partial \hat{y}} - 3 \frac{\hat{y}}{R^2} (1+\overline{\mathfrak{R}}_{22}) \right) \hat{y}$$

$$\left. + \left(\frac{\partial \overline{\mathfrak{R}}_{23}}{\partial \hat{z}} - 3 \frac{\hat{y}}{R^2} \overline{\mathfrak{R}}_{23} \right)(\hat{z}-R_o) \right) \qquad (C.7)$$

$$\frac{\partial \overline{G}_{\varphi}}{\partial \hat{z}} = \mu R^{-3} \left(\overline{\mathfrak{R}}_{23} + \left(\frac{\partial \overline{\mathfrak{R}}_{21}}{\partial \hat{z}} - 3 \frac{(\hat{z}-R_o)}{R^2} (1+\overline{\mathfrak{R}}_{21}) \right) \hat{x} \right.$$

$$+ \left(\frac{\partial \overline{\mathfrak{R}}_{22}}{\partial \hat{z}} - 3 \frac{(\hat{z}-R_o)}{R^2} (1+\mathfrak{R}_{22}) \right) \hat{y}$$

$$\left. + \left(\frac{\partial \overline{\mathfrak{R}}_{23}}{\partial \hat{z}} - 3 \frac{(\hat{z}-R_o)}{R^2} \overline{\mathfrak{R}}_{23} \right)(\hat{z}-R_o) \right) \qquad (C.8)$$

$$\frac{\partial \overline{G}_z}{\partial \overline{x}} = \mu R^{-3} \left[\overline{\Re}_{31} + \left(\frac{\partial \overline{\Re}_{31}}{\partial \overline{x}} - 3 \frac{\overline{x}}{R^2} \overline{\Re}_{31} \right) \overline{x} \right.$$

$$+ \left(\frac{\partial \overline{\Re}_{32}}{\partial \overline{x}} - 3 \frac{\overline{x}}{R^2} \overline{\Re}_{32} \right) \overline{y}$$

$$\left. + \left(\frac{\partial \overline{\Re}_{33}}{\partial \overline{x}} - 3 \frac{\overline{x}}{R^2} (1+\Re_{33}) \right) (\overline{z}-R_o) \right] \qquad (C.9)$$

$$\frac{\partial \overline{G}_z}{\partial \overline{y}} = \mu R^{-3} \left[\overline{\Re}_{32} + \left(\frac{\partial \overline{\Re}_{31}}{\partial \overline{y}} - 3 \frac{\overline{y}}{R^2} \overline{\Re}_{31} \right) \overline{x} \right.$$

$$+ \left(\frac{\partial \overline{\Re}_{32}}{\partial \overline{y}} - 3 \frac{\overline{y}}{R^2} \overline{\Re}_{32} \right) \overline{y}$$

$$\left. + \left(\frac{\partial \overline{\Re}_{33}}{\partial \overline{y}} - 3 \frac{\overline{y}}{R^2} (1+\overline{\Re}_{33}) \right) (\overline{z}-R_o) \right] \qquad (C.10)$$

$$\frac{\partial \overline{G}_{\overline{x}}}{\partial \overline{z}} = \mu R^{-3} \left[1 + \overline{\Re}_{33} + \left(\frac{\partial \overline{\Re}_{31}}{\partial \overline{z}} - 3 \frac{(\overline{z}-R_o)}{R^2} \overline{\Re}_{31} \right) \overline{x} \right.$$

$$+ \left(\frac{\partial \overline{\Re}_{32}}{\partial \overline{z}} - 3 \frac{(\overline{z}-R_o)}{R^2} \overline{\Re}_{32} \right) \overline{y}$$

$$\left. + \left(\frac{\partial \overline{\Re}_{33}}{\partial \overline{z}} - 3 \frac{(\overline{z}-R_o)}{R^2} (1+\overline{\Re}_{33}) \right) (\overline{z}-R_o) \right] \qquad (C.11)$$

As $\bar{x}$, $\bar{y}$ and $\bar{z}$ approach zero (R approaches R_o) and Equations C.3 through C.11 form the $\overline{P}_x$ matrix as in Equation C.2 but for non-spherical case,

$$\overline{P}_x \cong -\mu R^{-3} \begin{pmatrix} \left(1+\overline{\mathfrak{R}}_{11}-R_o\dfrac{\partial\overline{\mathfrak{R}}_{13}}{\partial\bar{x}}\right) & \left(\overline{\mathfrak{R}}_{12}-R_o\dfrac{\partial\overline{\mathfrak{R}}_{13}}{\partial\bar{y}}\right) & \left(\overline{\mathfrak{R}}_{13}-R_o\dfrac{\partial\overline{\mathfrak{R}}_{13}}{\partial\bar{z}}\right) \\[12pt] \left(\overline{\mathfrak{R}}_{21}-R_o\dfrac{\partial\overline{\mathfrak{R}}_{23}}{\partial\bar{x}}\right) & \left(1+\overline{\mathfrak{R}}_{22}-R_o\dfrac{\partial\overline{\mathfrak{R}}_{23}}{\partial\bar{y}}\right) & \left(\overline{\mathfrak{R}}_{23}-R_o\dfrac{\partial\overline{\mathfrak{R}}_{23}}{\partial\bar{z}}\right) \\[12pt] \left(\overline{\mathfrak{R}}_{31}-R_o\dfrac{\partial\overline{\mathfrak{R}}_{33}}{\partial\bar{x}}\right) & \left(\overline{\mathfrak{R}}_{32}-R_o\dfrac{\partial\overline{\mathfrak{R}}_{33}}{\partial\bar{y}}\right) & \left(-2+2\overline{\mathfrak{R}}_{33}-R_o\dfrac{\partial\overline{\mathfrak{R}}_{33}}{\partial\bar{z}}\right) \end{pmatrix} \quad (C.12)$$

Typical values for the $\overline{P}_x$ matrix in a 4x4 gravity field at 190 NM altitude (orbit inclination 28.5°) as a function of orbital longitude (L) are;

For $L = 0.0°$,

$$\overline{P}_x = -\mu R^{-3} \begin{pmatrix} 0.100211132E+1 & -0.127455301E-2 & 0.860795280E-4 \\ -0.127455552E-2 & 0.100374459E+1 & 0.141282142E-5 \\ 0.441508313E-4 & -0.992324472E-4 & -0.200586443E+1 \end{pmatrix} \quad (C.13)$$

For $L = 91.914°$

$$\overline{P}_x = -\mu R^{-3} \begin{pmatrix} 0.999755725E+0 & 0.656077200E-4 & -0.128398836E-3 \\ 0.658219319E-4 & 0.100197258E+1 & -0.497582344E-2 \\ -0.780933175E-4 & -0.490363083E-2 & -0.200185338E+1 \end{pmatrix} \quad (C.14)$$

For $L = 184.114°$

$$\overline{P}_x = -\mu R^{-3} \begin{pmatrix} 0.100217612E+1 & 0.119984812E-2 & 0.174780536E-3 \\ 0.119982659E-2 & 0.100371725E+1 & 0.414833236E-3 \\ 0.223653423E-3 & 0.303452941E-3 & -0.200589351E+1 \end{pmatrix} \quad (C.15)$$

For $L = 276.309°$

$$\overline{P}_x = -\mu R^{-3} \begin{pmatrix} 0.999822606E+0 & -0.917771447E-4 & -0.183744890E-3 \\ -0.916192270E-4 & 0.100211408E+1 & 0.474542054E-2 \\ -0.217657097E-3 & 0.482754340E-2 & -0.200180233E+1 \end{pmatrix} \quad (C.16)$$

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The analysis presented in this paper defines the reference coordinate frames, equations of motion and control parameters necessary to model the relative motion and attitude of spacecraft in close proximity with another space system during the Automatic Rendezvous and Capture phase of an on-orbit operation.

The relative docking port target position vector and the attitude control matrix are defined based upon an arbitrary spacecraft design. These translation and rotation control parameters could be used to drive the error signal input to the vehicle flight control system. Measurements for these control parameters would become the bases for an autopilot or FCS design for a specific spacecraft.

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