N93-29990
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ATTITUDE STABILIZATION OF A RIGID SPACECRAFT USING TWO MOMENTUM WHEEL ACTUATORS

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Abstract

It is well known that three momentum wheel actuators can be used to control the attitude of a rigid spacecraft and that arbitrary reorientation maneuvers of the spacecraft can be accomplished using smooth feedback. If failure of one of the momentum wheel actuators occurs, we demonstrate that two momentum wheel actuators can be used to control the attitude of a rigid spacecraft and that arbitrary reorientation maneuvers of the spacecraft can be accomplished. Although the complete spacecraft equations are not controllable, the spacecraft equations are small time locally controllable in a reduced nonlinear sense. The reduced spacecraft dynamics cannot be asymptotically stabilized to any equilibrium attitude using a time-invariant continuous feedback control law, but discontinuous feedback control strategies are constructed which stabilize any equilibrium attitude of the spacecraft in finite time. Consequently, reorientation of the spacecraft can be accomplished using discontinuous feedback control.

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1. Introduction

We consider the attitude control of a spacecraft modeled as a rigid body. It is well known that three actuators, either gas jets or momentum wheels, can be used to control the attitude of a rigid spacecraft and that arbitrary reorientation maneuvers of the spacecraft can be accomplished using smooth feedback[1-7]. If failure of one of the actuators occurs, then one is left with only two actuators. In this paper, the attitude stabilization problem of a rigid spacecraft using only two control torques supplied by momentum wheel actuators is considered. Since we are considering a space-based system, the problem considered here, namely, the attitude stabilization of a spacecraft operating in an actuator failure mode, is an important control problem. It is assumed that the center of mass of the system consisting of the spacecraft and the momentum wheel actuators is fixed in space.

Attitude stabilization of a rigid spacecraft using two momentum wheel actuators is not a mature subject in the literature. Controllability results for a rigid spacecraft controlled by momentum wheel actuators are presented in Ref. 8. We mention that most of the previous researchers have considered the problem of controlling a rigid spacecraft using two gas jet actuators[8-22]. Attitude stabilization of a rigid spacecraft using two gas jet actuators is considered in Refs. 8-13. Refs. 14-22 consider only the stabilization of the angular velocity equations of a rigid spacecraft using two gas jet actuators.

We consider the attitude stabilization of a spacecraft using control torques supplied by two momentum wheel actuators about axes spanning a two dimensional plane orthogonal to a principal axis of the spacecraft. The linearization of the complete spacecraft dynamic equations at any equilibrium attitude has an uncontrollable eigenvalue at the origin. Consequently, controllability and stabilizability properties of the spacecraft cannot be inferred using classical linearization ideas. The complete spacecraft dynamics is, in fact, not controllable. Under a rather weak assumption, the spacecraft dynamics is small time locally controllable at any equilibrium attitude in a reduced nonlinear sense. The reduced spacecraft dynamics cannot be asymptotically stabilized to any equilibrium attitude using time-invariant continuous feedback. Nevertheless, two different discontinuous feedback control strategies are constructed which achieves reorientation of the spacecraft in finite time. Using the concept of geometric phase[23], a discontinuous feedback control strategy is presented based on the nonholonomic control theory in Ref. 24. An alternate discontinuous feedback control strategy, based on the fact that rigid body rotations do not commute, is also presented.

This paper is based on our ealier work presented in Ref. 10 and is a companion to Ref. 11 and Ref. 12, which treat the attitude stabilization of a rigid spacecraft using two gas jet actuators.

2. Kinematic and Dynamic Equations

The orientation of a rigid spacecraft can be specified using various parametrizations of the special orthogonal group $SO(3)$. Here we use the Z-Y-X Euler angle convention for parametrizing the orientation of the rigid spacecraft[25]. The corresponding rotation matrix is denoted as $R(\psi, \theta, \phi)$, where ψ, θ, ϕ are the Euler angles. We assume that the Euler angles are limited to the ranges $-\pi < \psi < \pi$, $-\pi/2 < \theta < \pi/2$, $-\pi < \phi < \pi$. Suppose ω_1, ω_2, ω_3 are the principal axis components of the absolute angular velocity vector ω of the spacecraft. Then we have[25]

$$\dot{\phi} = \omega_1 + \omega_2 \sin\phi \tan\theta + \omega_3 \cos\phi \tan\theta, \tag{2.1}$$

$$\dot{\theta} = \omega_2 \cos\phi - \omega_3 \sin\phi, \tag{2.2}$$

$$\dot{\psi} = \omega_2 \sin\phi \sec\theta + \omega_3 \cos\phi \sec\theta. \tag{2.3}$$

Next we consider the dynamic equations which describe the evolution of the angular velocity components of the spacecraft. Consider two momentum wheel actuators spinning about axes defined by unit vectors b_1, b_2 fixed in the spacecraft such that the center of mass of the i-th wheel lies on the axis defined by b_i, and a control torque $-\bar{u}_i$ is supplied to the i-th wheel about the axis defined by b_i by a motor fixed in the spacecraft. Consequently, an equal and opposite torque $\bar{u}_i$ is exerted by the wheel on the spacecraft. We assume that b_i defines a principal axis for the i-th wheel which is symmetric about b_i. Further b_1 and b_2 span a two dimensional plane which is orthogonal to a principal axis of the spacecraft and, without loss of generality, b_i are assumed to be of the form

$$b_i = (b_{i1}, b_{i2}, 0)^T, \ i = 1, 2. \tag{2.4}$$

The mass of spacecraft, wheel 1 and wheel 2 are denoted as m_1, m_2 and m_3 respectively, and ρ_1, ρ_2, ρ_3 denote the position vectors of the center of mass of the spacecraft, wheel 1 and wheel 2 respectively with respect to the center of mass of the whole system. Thus from the location of the wheels

$$\rho_2 = \rho_1 + d_1 b_1, \tag{2.5}$$

$$\rho_3 = \rho_1^1 + d_2 b_2, \tag{2.6}$$

where d_1, d_2 are constants. Since, by the definition of center of mass,

$$\sum_{i=1}^{3} m_i \rho_i = 0, \tag{2.7}$$

further manipulation of equations (2.5)-(2.7) gives expressions for ρ_1, ρ_2 and ρ_3 which we denote as $\rho_i = (c_{i1}, c_{i2}, 0)^T$, $i = 1, 2, 3$. The total angular momentum vector of the system is given, in the spacecraft body frame, by

$$R(\psi, \theta, \phi)H = J\omega + v, \tag{2.8}$$

where

$$J = [I_1 + \sum_{i=1}^{3} \overline{I_i} + \sum_{i=2}^{3} (I_i - \underline{I_i})], \tag{2.9}$$

$$\overline{I_i} = m_i \begin{bmatrix} c_{i2}^2 & -c_{i1}c_{i2} & 0 \\ -c_{i1}c_{i2} & c_{i1}^2 & 0 \\ 0 & 0 & c_{i1}^2 + c_{i2}^2 \end{bmatrix}, \; i = 1, 2, 3, \tag{2.10}$$

$$\underline{I_2} = b_1 b_1^T j_1, \tag{2.11}$$

$$\underline{I_3} = b_2 b_2^T j_2, \tag{2.12}$$

$$v = \underline{I_2}(\omega + b_1 \dot{\theta}_1) + \underline{I_3}(\omega + b_2 \dot{\theta}_2), \tag{2.13}$$

where I_1, I_2, and I_3 denote the inertia tensors of the spacecraft, wheel 1 and wheel 2 respectively, j_1 is the moment of inertia of wheel 1 about the axis defined by b_1, j_2 is the moment of inertia of wheel 2 about the axis defined by b_2, and θ_1, θ_2 are the angles of rotation of wheel 1 and wheel 2 about the axes defined by b_1 and b_2 respectively. Here H denotes the angular momentum vector of the system expressed in the inertial coordinate frame. The angular momentum vector H is a constant since there is no external moment about the center of mass of the system. Suppose $\bar{u}_1$ and $\bar{u}_2$ are the control torques; then

$$\dot{v} = -(b_1 \bar{u}_1 + b_2 \bar{u}_2). \tag{2.14}$$

Differentiating (2.8) with respect to time we obtain

$$J\dot{\omega} = S(\omega) R(\psi, \theta, \phi) H + b_1 \bar{u}_1 + b_2 \bar{u}_2, \tag{2.15}$$

where

$$S(\omega) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix} .$$

Note that

$$I_1 = \mathrm{diag}(I_{11}, I_{12}, I_{13}),$$

$$I_2 = \text{block diag}(I_{21}, I_{22}),$$

$$I_3 = \text{block diag}(I_{31}, I_{32}),$$

where I_{21}, I_{31} are invertible 2×2 matrices, $I_{11}, I_{12}, I_{13}, I_{22}, I_{32}$ are nonzero real numbers and therefore J is a positive definite matrix of the form

$$J = \text{block diag } (J_1, J_2), \tag{2.16}$$

where J_1 is an invertible 2×2 matrix and J_2 is a nonzero real number.

3. Controllability and Stabilizability Properties

In this section we consider the controllability and stabilizablity properties of the spacecraft dynamics controlled by two momentum wheel actuators. Define

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = J_1^{-1} \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \end{bmatrix} .$$

From Section 2 the complete spacecraft dynamics can be rewritten as

$$\dot{\omega} = \begin{bmatrix} J_1^{-1} & 0_{(2\times 1)} \\ 0_{(1\times 2)} & J_2^{-1} \end{bmatrix} S(\omega) R(\psi, \theta, \phi) H + \begin{bmatrix} u_1 \\ u_2 \\ 0 \end{bmatrix}, \tag{3.1}$$

$$\dot{\phi} = \omega_1 + \omega_2 \sin\phi \tan\theta + \omega_3 \cos\phi \tan\theta, \tag{3.2}$$

$$\dot{\theta} = \omega_2 \cos\phi - \omega_3 \sin\phi, \tag{3.3}$$

$$\dot{\psi} = \omega_2 \sin\phi \sec\theta + \omega_3 \cos\phi \sec\theta, \tag{3.4}$$

where H is a constant vector.

The linearization of the complete spacecraft dynamic equations (3.1)-(3.4) at any equilibrium attitude has an uncontrollable eigenvalue at the origin. Consequently, the controllability and stabilizability properties of the complete spacecraft dynamics cannot be inferred using classical linearization ideas. However, from equations (2.4), (2.11)-(2.13) and the definition

$$c = (0, 0, 1)^T, \tag{3.5}$$

we have $c^T v = 0$. Therefore from equation (2.8) we have

$$c^T R(\psi, \theta, \phi) H = c^T J \omega. \tag{3.6}$$

Since H is a constant vector, this equation represents a constraint on the motion of the spacecraft irrespective of the controls applied. Thus the complete spacecraft dynamics is not completely controllable. Therefore we ask the following question: what restricted control and stabilization properties of the spacecraft can be demonstrated in this case? Our analysis begins by demonstrating that, under an appropriate restriction of interest, the spacecraft equations have restricted controllability and stabilizability properties.

Consider equations (3.1)-(3.4) and suppose the angular momentum vector H of the system is zero. From equations (2.16), (3.5) and (3.6) it follows that the angular velocity component of the spacecraft about the uncontrolled principal axis is identically zero, i.e., $\omega_3 \equiv 0$. Under such a restriction, the reduced spacecraft dynamics are described by

$$\dot{\omega}_1 = u_1, \tag{3.7}$$

$$\dot{\omega}_2 = u_2, \tag{3.8}$$

$$\dot{\phi} = \omega_1 + \omega_2 \sin\phi \tan\theta, \tag{3.9}$$

$$\dot{\theta} = \omega_2 \cos\phi, \tag{3.10}$$

$$\dot{\psi} = \omega_2 \sin\phi \sec\theta. \tag{3.11}$$

Notice that the linearization of the equations (3.7)-(3.11) at any equilibrium has an uncontrollable eigenvalue at the origin. Therefore analysis of the controllability and stabilizability properties of the reduced spacecraft dynamics requires inherently nonlinear techniques. The following results follow directly based on an analysis similar to that in Ref. 24.

Theorem 3.1: The reduced dynamics of a spacecraft controlled by two momentum wheel actuators as described by equations (3.7)-(3.11) are small time locally controllable at any equilibrium.

Theorem 3.2: The reduced dynamics of a spacecraft controlled by two momentum wheel actuators as described by equations (3.7)-(3.11) cannot be asymptotically stabilized to any equilibrium using a time-invariant continuous feedback control law, but the reduced dynamics can be asymptotically stabilized to any equilibrium using a piecewise continuous feedback control law.

Theorem 3.1 follows from the fact that a sufficient condition for small time local controllability given in Ref. 26 is satisfied by the equations (3.7)-(3.11). The first part of Theorem 3.2 follows from the fact that a necessary condition for the existence of a time-invariant continuous feedback control law given in Ref. 17 is not satisfied by equations (3.7)-(3.11); the second part is a consequence of small time local controllability[26]. The implications of the properties stated above are as follows. Suppose the angular momentum vector H is zero. Then the spacecraft controlled by two momentum wheel actuators can be controlled to any equilibrium attitude but the feedback control law must necessarily be discontinuous. Thus arbitrary reorientation of the spacecraft can be achieved under the restriction $H = 0$; If $H \neq 0$, equation (3.6) implies that reorientation of the spacecraft to an equilibrium attitude cannot be achieved.

4. Feedback Stabilization Algorithms

We restrict our study to the class of discontinuous feedback controllers in order to asymptotically stabilize the reduced spacecraft dynamics described by state equations (3.7)-(3.11). Clearly, traditional nonlinear control design methods are of no use since there is no general procedure for the design of a discontinuous feedback control. However, an algorithm generating a discontinuous feedback control which asymptotically stabilizes an equilibrium can be constructed, as suggested by the controllability properties of the system. Without loss of generality, we assume that the equilibrium to be stabilized is the origin. We present two different discontinuous control strategies which stabilize the origin of equations (3.7)-(3.11) in finite time.

4.1. Feedback stabilization based on nonholonomic control theory

Consider a diffeomorphism defined by

$$y_1 = \cos\phi \, ln(\sec\theta + \tan\theta) + \psi\sin\phi, \tag{4.1}$$

$$y_2 = \omega_2\sec\theta - y_4 y_5, \tag{4.2}$$

$$y_3 = \phi, \tag{4.3}$$

$$y_4 = \omega_1 + \omega_2 \sin\phi \tan\theta, \tag{4.4}$$

$$y_5 = \sin\phi \ln(\sec\theta + \tan\theta) - \psi\cos\phi, \tag{4.5}$$

If we now define the feedback relations

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -\sin\phi\sin\theta & (1 - y_5\sin\phi\sin\theta) \\ \cos\theta & y_5\cos\theta \end{bmatrix} \left\{ \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \cos\phi y_5 \sin\phi \sec^2\theta\omega_2^2 \\ 0 \end{bmatrix} \right.$$

$$\left. - \begin{bmatrix} -y_4^2 y_1 + \cos\phi(\sec\theta\tan\theta\omega_2^2 - y_5 y_4 \tan\theta\omega_2) \\ \cos\phi(y_4\tan\theta\omega_2 + \sin\phi\sec^2\theta\omega_2^2) \end{bmatrix} \right\}, \tag{4.6}$$

then the reduced spacecraft dynamics (3.7)-(3.11) are described in the new variables by the normal form equations

$$\dot{y}_1 = y_2, \tag{4.7}$$

$$\dot{y}_2 = v_1, \tag{4.8}$$

$$\dot{y}_3 = y_4, \tag{4.9}$$

$$\dot{y}_4 = v_2, \tag{4.10}$$

$$\dot{y}_5 = y_4 y_1. \tag{4.11}$$

From equations (4.1)-(4.5), notice that $\omega_1 = \omega_2 = \phi = \theta = \psi = 0$ implies that $y_1 = y_2 = y_3 = y_4 = y_5 = 0$. Hence asymptotic stabilization of equations (3.7)-(3.11) to the origin is equivalent to asymptotic stabilization of the normal form equations (4.7)-(4.11) to the origin; hence we consider asymptotic stabilization of the normal form equations. The normal form equations (4.7)-(4.11) are in a familiar form which has been studied in Ref. 24 and therefore can be stabilized by the following discontinuous control strategy.

- First, transfer the initial state of the normal form equations (4.7)-(4.11) to the equilibrium state $(0, 0, 0, 0, y_5^1)$, for some y_5^1, in finite time.
- Next, traverse a closed path γ in the (y_1, y_3) space in finite time, where the path γ is selected to satisfy

$$-y_5^1 = \int_\gamma y_1 dy_3; \tag{4.12}$$

this transfers the state $(0, 0, 0, 0, y_5^1)$ to the origin in finite time.

Here we consider a rectangular path γ in the (y_1, y_3) space formed by line segments from $(0, 0)$ to $(y_1^*, 0)$, from $(y_1^*, 0)$ to (y_1^*, y_3^*), from (y_1^*, y_3^*) to $(0, y_3^*)$, and from $(0, y_3^*)$ to $(0, 0)$. For such a path, the line integral in equation (4.12) can be explicitly evaluated as $y_1^* y_3^*$ so that equation (4.12) becomes

$$-y_5^1 = y_1^* y_3^*, \tag{4.13}$$

and the parameters y_1^* and y_3^* specifying the particular rectangular path are chosen to satisfy the above equation.

Throughout, assume $k > 0$, and define

$$G(x_1, x_2) = \begin{cases} k & \text{if } \{x_1 + \frac{x_2|x_2|}{2k} > 0\} \text{ or} \\ & \{x_1 + \frac{x_2|x_2|}{2k} = 0 \text{ and } x_2 > 0\} \\ -k & \text{if } \{x_1 + \frac{x_2|x_2|}{2k} < 0\} \text{ or} \\ & \{x_1 + \frac{x_2|x_2|}{2k} = 0 \text{ and } x_2 < 0\} \\ 0 & \text{if } \{x_1 = 0 \text{ and } x_2 = 0\} \end{cases}.$$

We use the well-known property that any initial state of the system

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = -G(x_1 - \bar{x}_1, x_2),$$

is transferred to the final state $(\bar{x}_1, 0)$ in a finite time.

We now present a specific feedback control algorithm which stabilizes the spacecraft to the origin in finite time; this feedback control algorithm implements the approach just described.

Maneuver 1: Apply

$$v_1 = -G(y_1, y_2),$$

$$v_2 = -G(y_3, y_4),$$

until $(y_1, y_2, y_3, y_4, y_5) = (0, 0, 0, 0, y_5^1)$ where y_5^1 is arbitrary; then go to Maneuver 2.

Maneuver 2: If $y_5^1 \geq 0$, choose $y_1^* = -y_3^* = \sqrt{(y_5^1)}$; else choose $y_1^* = y_3^* = \sqrt{(-y_5^1)}$; Apply

$$v_1 = -G(y_1 - y_1^*, y_2) ,$$

$$v_2 = -G(y_3, y_4) ,$$

until $(y_1, y_2, y_3, y_4, y_5) = (y_1^*, 0, 0, 0, y_5^1)$; then go to Maneuver 3.

Maneuver 3: Apply

$$v_1 = -G(y_1 - y_1^*, y_2) ,$$

$$v_2 = -G(y_3 - y_3^*, y_4) ,$$

until $(y_1, y_2, y_3, y_4, y_5) = (y_1^*, 0, y_3^*, 0, 0)$; then go to Maneuver 4.

Maneuver 4: Apply

$$v_1 = -G(y_1, y_2) ,$$

$$v_2 = -G(y_3 - y_3^*, y_4) ,$$

until $(y_1, y_2, y_3, y_4, y_5) = (0, 0, y_3^*, 0, 0)$; then go to Maneuver 5.

Maneuver 5: Apply

$$v_1 = -G(y_1, y_2) ,$$

$$v_2 = -G(y_3, y_4) ,$$

until $(y_1, y_2, y_3, y_4, y_5) = (0, 0, 0, 0, 0)$; then go to Maneuver 2.

It can be verified that the execution of Maneuver 1 transfers the initial state of the normal form equations to the equilibrium state $(0, 0, 0, 0, y_5^1)$, for some y_5^1, in finite time. Subsequent execution of Maneuvers 2 through 5 then transfers the state $(0, 0, 0, 0, y_5^1)$ to the origin in finite time. This control algorithm is nonclassical and involves switching between various feedback functions. Justification that it stabilizes the origin of the normal form equations (4.7)-(4.11) in finite time follows as a consequence of the construction procedure. Since stabilization of the normal form equations to the origin is equivalent to stabilization of the state equations (3.7)-(3.11) to the origin, we conclude that the control inputs u_1 and u_2 given by equation (4.6) with v_1 and v_2 defined by the above control algorithm stabilizes the reduced spacecraft dynamics described by equations (3.7)-(3.11) to the equilibrium $(\omega_1, \omega_2, \phi, \theta, \psi) = (0, 0, 0, 0, 0)$ in finite time. A computer implementation of the feedback control strategy can be easily carried out.

4.2. Feedback stabilization based on rigid body rotational characteristics

We now present an alternate discontinuous feedback control strategy for stabilizing the origin of equations (3.7)-(3.11) in finite time. This strategy requires that the spacecraft undergo a sequence of specified maneuvers and is based on the fact that rigid body rotations do not commute. The physical interpretation of the sequence of maneuvers that transfers any initial state of equation (3.7)-(3.11) to the origin is as follows.

- Transfer the initial state of equations (3.7)-(3.11) to any equilibrium state in finite time; i.e. bring the spacecraft to rest.
- Transfer the resulting state to an equilibrium state where $\phi = 0$ in finite time; i.e. so that the spacecraft is at rest with $\phi = 0$.
- Transfer the resulting state to an equilibrium state where $\phi = 0$, $\theta = 0$ in finite time; i.e. so that the spacecraft is at rest with $\phi = 0$, $\theta = 0$.
- Transfer the resulting state to an equilibrium state where $\phi = \frac{\pi}{2}$, $\theta = 0$ in finite time; i.e. so that the spacecraft is at rest with $\phi = \frac{\pi}{2}$, $\theta = 0$.
- Transfer the resulting state to the equilibrium state $(0, 0, \frac{\pi}{2}, 0, 0)$ in finite time.
- Transfer the equilibrium state $(0, 0, \frac{\pi}{2}, 0, 0)$ to the equilibrium state $(0, 0, 0, 0, 0)$ in finite time.

We now present a feedback control algorithm which stabilizes the spacecraft to the origin in finite time; this feedback control algorithm implements the approach just described.

Maneuver 1. Apply

$$u_1 = -k \,\mathrm{sign}\omega_1 ,$$

$$u_2 = -k \,\mathrm{sign}\omega_2 ,$$

until $(\omega_1, \omega_2) = (0, 0)$; then go to Maneuver 2.

Maneuver 2: Apply

$$u_1 = -G(\phi, \omega_1) ,$$

$$u_2 = 0 ,$$

until $(\omega_1, \omega_2, \phi) = (0, 0, 0)$; then go to Maneuver 3.

Manuever 3: Apply

$$u_1 = 0 ,$$

$$u_2 = -G(\theta, \omega_2) ,$$

until $(\omega_1, \omega_2, \phi, \theta) = (0, 0, 0, 0)$; then go to Maneuver 4.

Maneuver 4: Apply

$$u_1 = -G(\phi - \frac{\pi}{2}, \omega_1) ,$$

$$u_2 = 0 ,$$

until $(\omega_1, \omega_2, \phi, \theta) = (0, 0, \frac{\pi}{2}, 0)$, then go to Maneuver 5.

Maneuver 5: Apply

$$u_1 = 0 ,$$

$$u_2 = -G(\psi, \omega_2) ,$$

until $(\omega_1, \omega_2, \phi, \theta, \psi) = (0, 0, \frac{\pi}{2}, 0, 0)$; then go to Maneuver 6.

Maneuver 6: Apply

$$u_1 = -G(\phi, \omega_1) ,$$

$$u_2 = 0 ,$$

until $(\omega_1, \omega_2, \phi, \theta, \psi) = (0, 0, 0, 0, 0, 0)$; then go to Maneuver 1.

It can be verified that the execution of Maneuver 1 transfers the initial state of equations (3.7)-(3.11) to the equilibrium state $(0, 0, \phi^1, \theta^1, \psi^1)$, for some ϕ^1, θ^1, ψ^1, in finite time. Execution of Manuever 2 then transfers the state $(0, 0, \phi^1, \theta^1, \psi^1)$ to the state $(0, 0, 0, \theta^1, \psi^1)$; execution of Manuever 3 then transfers the state $(0, 0, 0, \theta^1, \psi^1)$ to the state $(0, 0, 0, 0, \psi^1)$; execution of Manuever 4 then transfers the state $(0, 0, 0, 0, \psi^1)$ to the state $(0, 0, \frac{\pi}{2}, 0, \psi^1)$; execution of Manuever 5 then transfers the state $(0, 0, \frac{\pi}{2}, 0, \psi^1)$ to the state $(0, 0, \frac{\pi}{2}, 0, 0)$; finally, execution of Manuever 6 transfers the state $(0, 0, \frac{\pi}{2}, 0, 0)$ to the state $(0, 0, 0, 0, 0)$. This strategy is discontinuous and nonclassical in nature. A computer implementation of the feedback control strategy can be easily carried out.

4.3 Comments

We have introduced two different control laws which transfer any initial state of equations (3.7)-(3.11) to the origin in finite time. Each of these control laws is in feedback form, since the control values depend on the current state; and each control law is discontinuous. The first construction procedure makes use of the nonholonomic features of the reduced spacecraft dynamics, while the second construction procedure uses physical insight about rigid body rotations. The first control law constructed makes use of both control actuators simultaneously, while the second control law (after Maneuver 1) uses only a single actuator at a time. The two discontinuous feedback control laws exhibited are illustrations of the class of control laws which asymptotically stabilize equations (3.7)-(3.11) to the origin. There are other maneuver sequences, and corresponding feedback control laws, which will also achieve the desired attitude stabilization of the spacecraft. But each such strategy is necessarily discontinuous.

One of the advantages of the development in Sections 4.1 and 4.2 is that feedback control strategies are constructed which guarantee attitude stabilization in a finite time. The total time required to complete the spacecraft reorientation is the sum of the times required to complete the sequence of maneuvers described. It should be clear that the time required to complete each maneuver depends on the single positive parameter k in the corresponding control law. There is a trade off between the required control levels, determined by the selection of k, and the resulting times to complete each of the maneuvers and hence the total time required to reorient the spacecraft. In particular, the time to reorient the spacecraft from a given initial state to the origin can be expressed as a function of the value of the parameter k and of the initial state.

We have demonstrated, by construction, the closed loop properties for the special feedback control strategies presented. Our analysis was based on an ideal model assumption. Further robustness analysis is required to determine effects of model uncertainities and external disturbances. Unfortunately, such robustness analysis is quite difficult since the closed loop vector fields are necessarily discontinuous. Perhaps, feedback control strategies which stabilize the spacecraft attitude, different from ones presented in this paper, would provide improved closed loop robustness. These issues are to be studied in future research.

5. Simulation

We illustrate the results of the paper using an example. Consider a rigid spacecraft with no control torque about the third principal axis and two control torques, generated by momentum wheel actuators, are applied about the other two principal axes. Therefore the vectors b_1 and b_2 are given by $b_1 = (1, 0, 0)^T$, $b_2 = (0, 1, 0)^T$. For our simulation, we use the spacecraft parameters used in Ref. 2. The mass of the spacecraft, m_1, is 500 Kg, and the masses of the momentum wheels, m_2 and m_3, are each 5 Kg. The center of mass of the momentum wheels are located at a distance 0.2 m from the center of mass of the spacecraft, i.e., $d_1 = d_2 = 0.2$ m. The moment of inertia of the wheels about its axis of rotation is 0.5 Kg.m^2, i.e., $j_1 = j_2 = 0.5$. The inertia tensor of the spacecraft and the two momentum wheels are

$$I_1 = \text{diag}\ (86.215,\ 85.07,\ 113.565)\ \text{Kg.m}^2\ ,$$

$$I_2 = \text{diag}\ (0.5,\ 0.25,\ 0.25)\ \text{Kg.m}^2\ ,$$

$$I_3 = \text{diag}\ (0.25,\ 0.5,\ 0.25)\ \text{Kg.m}^2\ .$$

Using these parameters, the inertia matrix J can be calculated which equals

$$J = \text{diag}(86.7,\ 85.5,\ 114.5)\ \text{Kg.m}^2\ ,$$

approximately. The complete dynamics of the spacecraft system defined by equations (3.1)-(3.4) is not controllable, but we consider the restriction that the angular momentum vector $H = 0$. Consequently, we are interested in stabilizing the reduced spacecraft dynamics described by equations (3.7)-(3.11) to the equilibrium $(\omega_1, \omega_2, \phi, \theta, \psi) = (0, 0, 0, 0, 0)$. The spacecraft is initially at rest (i.e., $\omega_1^0 = \omega_2^0 = 0$) with an initial orientation given by the Euler angles $\phi^0 = \pi$, $\theta^0 = 0.25\pi$ and $\psi^0 = -0.5\pi$.

First, a computer implementation of the feedback control algorithm specified in Section 4.1 was used to stabilize the spacecraft to the origin. The value of the gain k was chosen as 1. The time responses of the Euler angles, angular velocities and the control torques are shown in Fig. 1, Fig. 2 and Fig. 3 respectively. After a total maneuver time of 11.77 seconds, $\omega_1 = \omega_2 = \phi = \theta = \psi = 0$. Next, a computer implementation of the feedback control algorithm specified in Section 4.2 was used to stabilize the spacecraft to the origin. The value of the gain k was chosen as 1. The time responses of the Euler angles, angular velocities and the control torques are shown in Fig. 4, Fig. 5 and Fig. 6 respectively. After a total maneuver time of 13 seconds, $\omega_1 = \omega_2 = \phi = \theta = \psi = 0$.

6. Conclusion

The attitude stabilization problem of a spacecraft using control torques supplied by two momentum wheel actuators about axes spanning a two dimensional plane orthogonal to a principal axis has been considered. The complete spacecraft dynamics are not controllable. However, the spacecraft dynamics are small time locally controllable in a reduced sense. The reduced spacecraft dynamics cannot be asymptotically stabilized using time-invariant continuous feedback, but discontinuous feedback control strategies have been constructed which stabilizes the spacecraft (in the reduced sense) to an equilibrium attitude in finite time. The results of the paper show that although classical nonlinear control techniques do not apply, it is possible to construct control laws based on the particular spacecraft dynamics.

Acknowledgements

This work was partially supported by NSF Grant No. MSS - 9114630 and by NASA Grant No. NAG - 1 - 1419.

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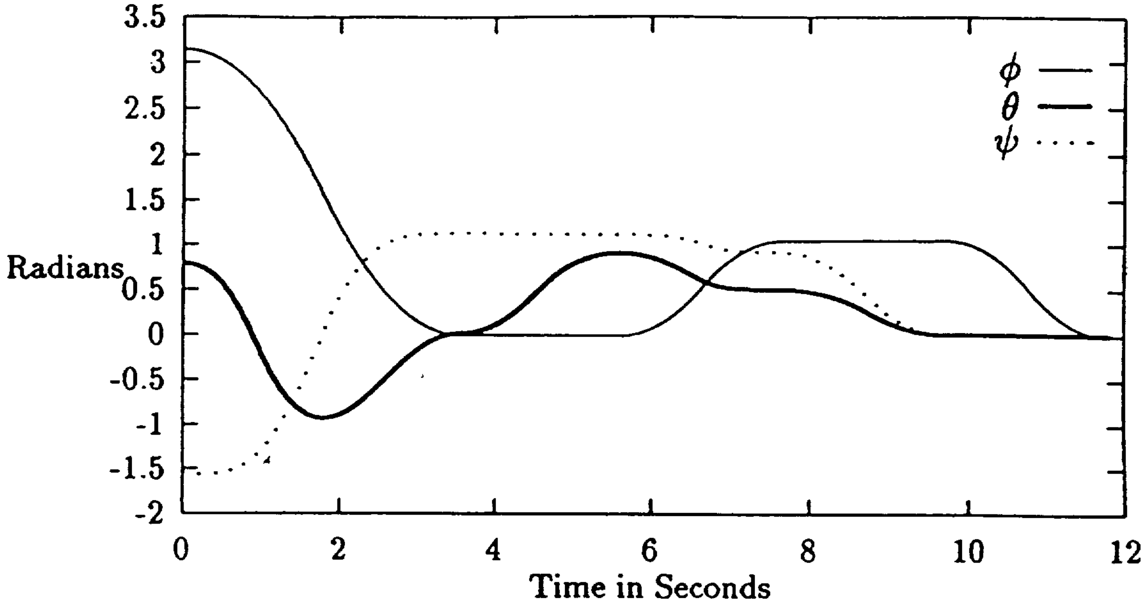

Fig. 1: Euler Angles

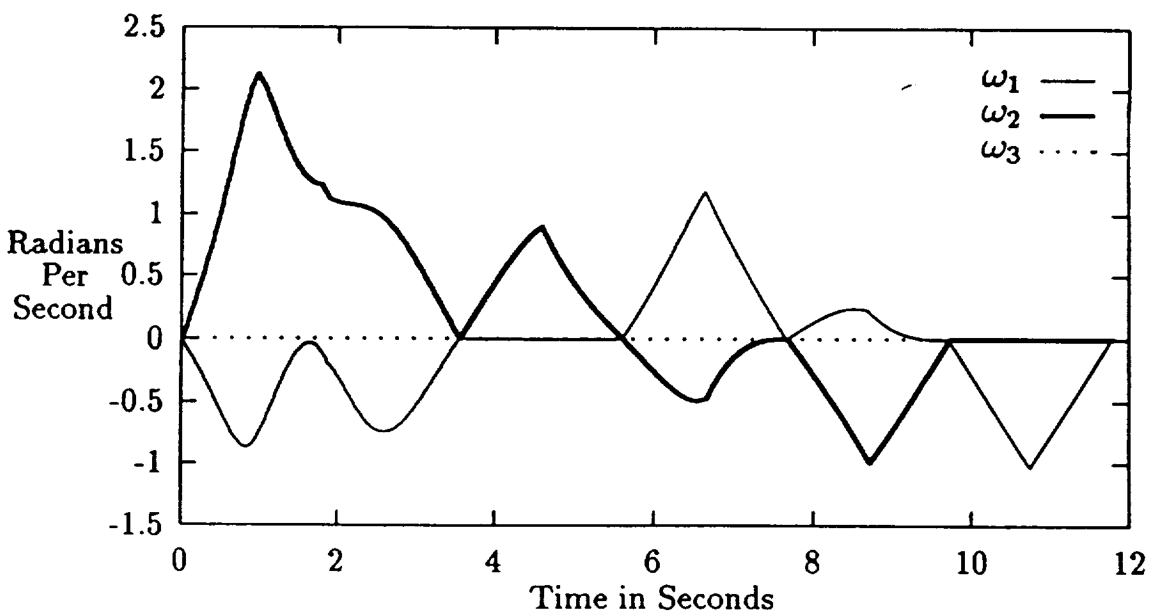

Fig. 2: Anglular Velocities

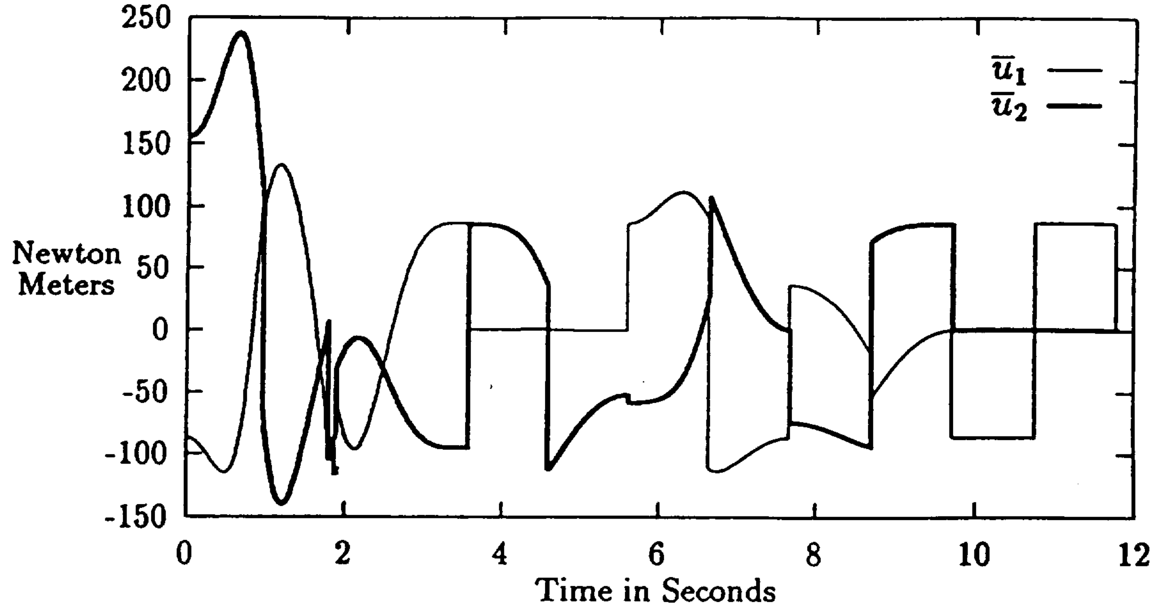

Fig. 3: Control Torques

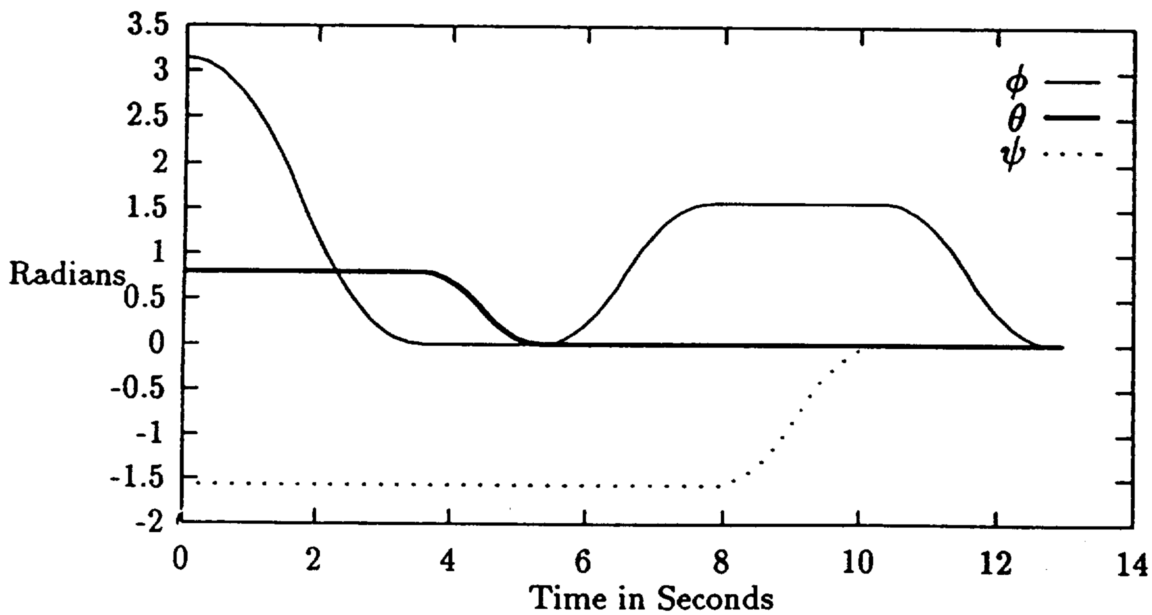

Fig. 4: Euler Angles

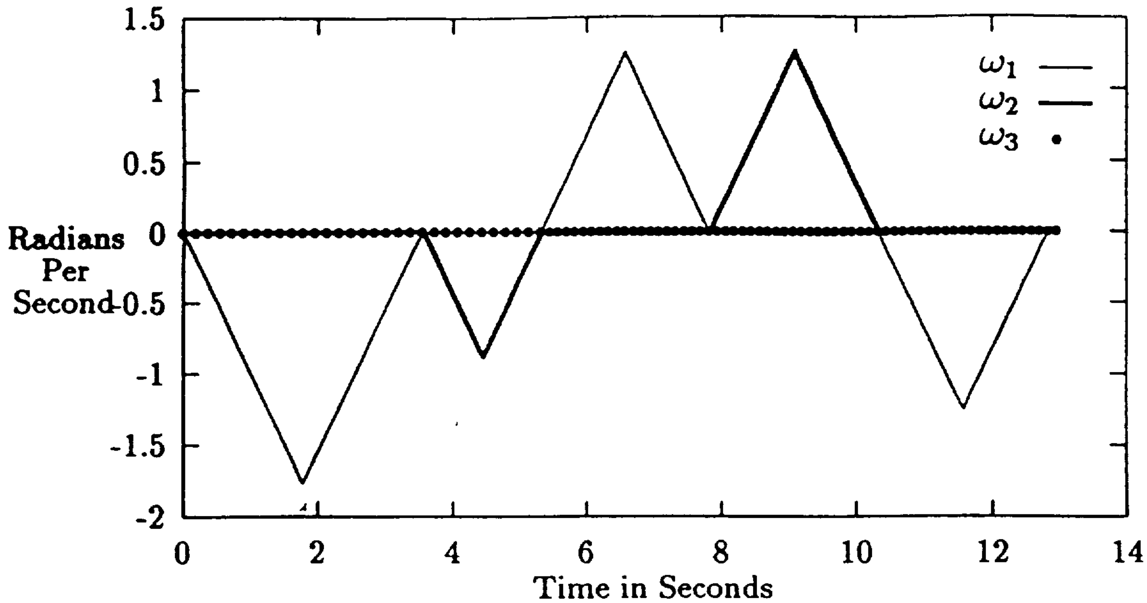

Fig. 5: Anglular Velocities

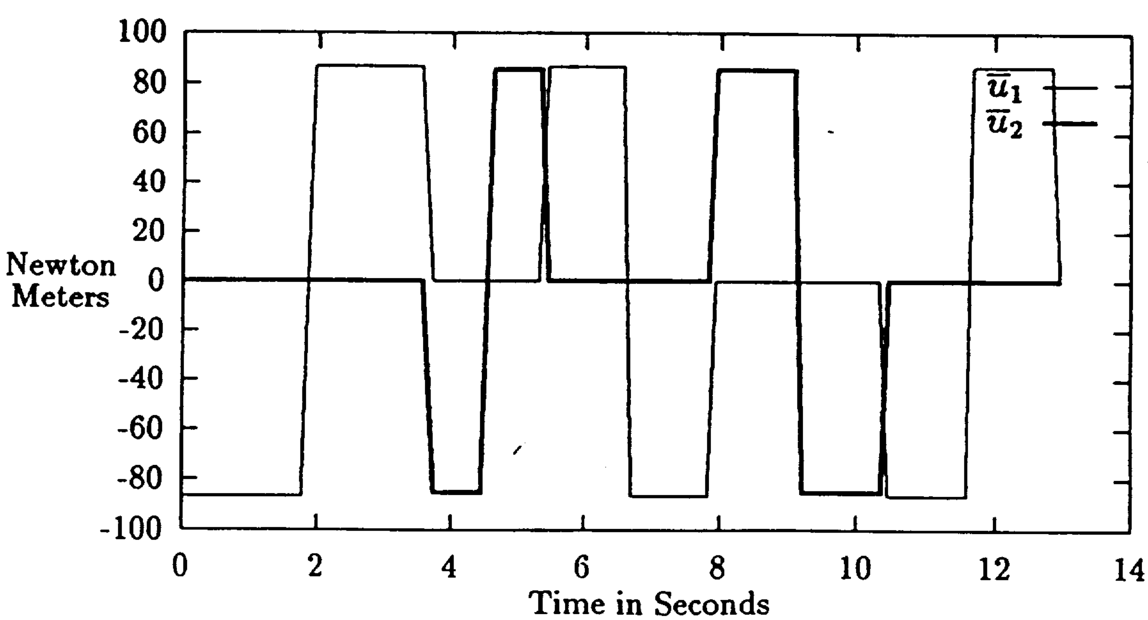

Fig. 6: Control Torques