NONLINEAR ATTITUDE CONTROL OF PLANAR STRUCTURES IN SPACE USING ONLY INTERNAL CONTROLS

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Abstract

An attitude control strategy for maneuvers of an interconnection of planar bodies in space is developed. It is assumed that there are no exogeneous torques and that torques generated by joint motors are used as means of control so that the total angular momentum of the multibody system is a constant, assumed to be zero in this paper. The control strategy utilizes the nonintegrability of the expression for the angular momentum. Large angle maneuvers can be designed to achieve an arbitrary reorientation of the multibody system with respect to an inertial frame. The theoretical background for carrying out the required maneuvers is summarized.

1. Introduction

In this paper we develop an attitude control strategy for a system of N planar rigid bodies in space which are interconnected by ideal frictionless pin joints in the form of an open kinematic chain. Angular momentum preserving controls, e.g. torques generated by joint motors, are considered. The N-body system is assumed to have zero initial angular momentum. Our earlier work[1,2] demonstrated that reorientation of a planar multibody system with three or more interconnected bodies using only joint torque inputs is an inherently nonlinear control problem which is not amenable to classical methods of nonlinear control. The goal of this study is to indicate how control strategies can be explicitly constructed to achieve the desired absolute reorientation of the N-body system. The key is to excite certain oscillatory motions in the shape of the structure, thereby providing a capability for reorientation of the structure with respect to an inertial frame.

There are many physical advantages in using internal controls, e.g. joint torque controls, to carry out the desired multibody reorientation maneuvers. First of all, this control approach does not modify the total angular momentum of the multibody system. In addition, internal controls have obvious advantages in terms of energy conservation. Moreover, they can be implemented using standard electrical servo motors, a simple and reliable control actuator technology.

The formal development in this paper is concerned with control of a multibody interconnection in space which has zero angular momentum. Although these results are formulated in a general setting, we have been motivated by several classes of specific problems. Several potential applications of our general results are now described.

Manipulators mounted on space vehicles and space robots have been envisioned to carry out construction, maintenance and repair tasks in an external space environment. Previous research on maneuvering of such space multibody systems has mainly focused on maneuvers which achieve desired orientation of some of the

bodies, e.g. an end effector, while the orientation of some of the remaining bodies cannot be specified, at least using the methodologies employed[3-8]. Another related application is the performance by astronauts of reorientation maneuvers in space. Previous research in this area[9] has emphasized dynamics issues. Closely related research has focused on describing the reorientation maneuvers of a falling cat[10]. Finally, we mention another area of potential application of the results of this paper, namely the development of deployment maneuvers for multibody antennas connected to a spacecraft. It is expected that such an approach would have many advantages over the use of existing passive antenna deployment mechanisms[11].

This paper is organized as follows. In Section 2, a mathematical model for a planar multibody system in space is derived. We then formulate an attitude control problem associated with the planar multibody system. In Section 3, we first summarize several relevant theoretical results. We then introduce an attitude control strategy to solve this reorientation problem. Section 4 consists of a summary of the main results and concluding remarks about our continuing research. Although a complete treatment of the topics in the paper requires use of differential geometric tools, our presentation avoids these tools and uses only elementary mathematical methods. However, references to relevant literature are provided throughout.

2. Mathematical Model for Planar Multibody System

We consider a system of N planar rigid bodies interconnected by frictionless one degree of freedom joints in the form of an open kinematic chain. The configuration space, for an observer at the center of mass of the system of rigid bodies, is N dimensional. Since we assume an open kinematic chain there are exactly $N - 1$ joints. We consider controlling the rotational motion of the system using torques at the joints; each joint is assumed to be actuated so as to permit free adjusment of the joint angle. It is assumed that there are no external torques acting on the system. It is clear that the configuration of the N bodies can be described by the absolute angle of any one of the bodies (say body 1) and $N - 1$ joint angles. Denote by θ_1 the absolute angle of body 1, and by the $(N - 1)$-vector $\psi = (\psi_1, \cdots, \psi_{N-1})$

the joint angle vector. Clearly, (θ_1, ψ) is a generalized coordinate vector for the rotational motion. It can be shown that the Lagrangian (which is equal to the rotational kinetic energy under the above assumptions), written in terms of these coordinates and their time derivatives, does not contain θ_1 explicitly, i.e. θ_1 is a cyclic or ignorable coordinate. Consequently, the generalized momentum associated with the cyclic coordinate θ_1 is conserved. This conserved quantity is the first integral of the motion corresponding to conservation of angular momentum of the system. In this paper we assume zero initial angular momentum so that angular momentum remains zero throughout a maneuver.

It is clear that Lagrange's equations describe the motion on the joint angle space, and the evolution of θ_1 can be obtained from the expression for conservation of angular momentum. Thus, the motion of a planar multibody system, under the above assumptions, can be described by the following reduced order equations

$$J_s(\psi)\ddot{\psi} + F_s(\psi, \dot{\psi}) = \tau \ , \tag{1}$$

$$\dot{\theta}_1 + s'(\psi)\dot{\psi} = 0 \tag{2}$$

where $\tau = (\tau_1, \cdots, \tau_{N-1})$ denotes the $(N-1)$-vector of joint torques, $J_s(\psi)$ is a symmetric positive definite $(N-1) \times (N-1)$ matrix function; and $s(\psi)$, $F_s(\psi, \dot{\psi})$ are $(N-1)$-vector functions. Note that in this paper a "prime" denotes tranpose. The explicit specifications of these functions can be found in the literature[1,2,12].

State space equations for (1) and (2) are

$$\dot{\theta}_1 = -s(\psi)'\omega \ , \tag{3}$$

$$\dot{\psi} = \omega \ , \tag{4}$$

$$\dot{\omega} = -J_s^{-1}(\psi)F_s(\psi, \omega) + J_s^{-1}(\psi)\tau \ . \tag{5}$$

Note that equations (4),(5) are expressed in terms of the joint phase variables $(\psi, \dot{\psi})$ only. Hence the joint angle space constitutes a reduced configuration space

for the system. This reduced configuration space is also referred to as the "shape space" of the system[12-16]. It is possible to consider control problems expressed solely in terms of the shape space; such problems can be solved using classical methods. However, in our work we are interested in the more general control problems associated with the complete dynamics of the multibody system defined by equations (1)-(2) (or (3)-(5)).

Note that equations (4)-(5) only, which represent the projection of the motion onto the shape phase space, are feedback linearizable using the feedback transformation

$$u = -J_s^{-1}(\psi)F_s(\psi,\omega) + J_s^{-1}(\psi)\tau \tag{6}$$

where $u \in R^{N-1}$. The above feedback transformation yields the following normal form equations

$$\dot{\theta}_1 = -s(\psi)'\omega \ , \tag{7}$$

$$\dot{\psi} = \omega \ , \tag{8}$$

$$\dot{\omega} = u \ . \tag{9}$$

We remark here that it is impossible to completely linearize the system defined by equations (3)-(5) using static or dynamic feedback combined with any coordinate transformation.

Note that an equilibrium solution of equations (3)-(5) corresponding to $\tau = 0$ (or equivalently an equilibrium solution of equations (7)-(9) for $u = 0$) is given by $(\theta_1^e, \ \psi^e, \ 0)$, where (θ_1^e, ψ^e) is referred to as an equilibrium configuration. Hence an equilibrium solution corresponds to a trivial motion of the system for which all the configuration space variables remain constant.

Note also that equation (3) represents conservation of angular momentum. This equation is not integrable for $N \geq 3$ (i.e. if the multibody system consists of three

or more links). This fact has important implications in terms of controllability properties of the system as will be shown in the subsequent development. As a consequence of the symmetry possessed by the system, θ_1 does not appear explicitly in equation (3). Mechanical systems with such symmetry properties are referred to as Caplygin systems[17-21]. As a consequence of the nonintegrability for $N \geq 3$, the scalar analytic functions

$$H_{ij}(\psi) = \frac{\partial s_i(\psi)}{\partial \psi_j} - \frac{\partial s_j(\psi)}{\partial \psi_i} \ , \ (i,j) \in I^2 \ , \tag{10}$$

where $I = \{1, \cdots, N-1\}$, do not all vanish, except possibly on a set which has measure zero with respect to the shape space.

3. Attitude Control Problem

In this section, we address the following control problem associated with planar multibody systems described by equations (1)-(2) :

Problem : *Given an initial state $(\theta_1^0, \psi^0, \omega^0)$ and a desired equilibrium solution $(\theta_1^e, \psi^e, 0)$, determine a motion $(\theta_1(t), \psi(t), \omega(t))$, $0 \leq t \leq t_f$, such that $(\theta_1(0), \psi(0), \omega(0)) = (\theta_1^0, \psi^0, \omega^0)$, $(\theta_1(t_f), \psi(t_f), \omega(t_f)) = (\theta_1^e, \psi^e, 0)$ and $(\theta_1(t), \psi(t), \omega(t))$ satisfies equations (1)-(2) for some control function $t \longmapsto \tau(t)$.*

Note that, in particular, if $\omega^0 = 0$ then the above problem corresponds to a rest-to-rest maneuver.

The existence of solutions to the above control problem was demonstrated in our earlier work[1,2]. In particular, we studied the nonlinear control system described by equations (7)-(9) and employed certain results from nonlinear control theory to characterize controllability properties of planar multibody systems described by equations (1)-(2). These results not only prove the existence of solutions of the above problem but they also provide a theoretical basis for construction of nonlinear control strategies required to achieve the desired maneuver. We next

summarize those results[1,2].

Under the stated assumptions, a planar multibody system has the following properties if $N \geq 3$, i.e. if it consists of three or more links:

1. *The system is strongly accessible.*
2. *The system is small time locally controllable from any equilibrium.*
3. *The system can be transferred from any initial condition to any desired equilibrium in arbitrarily small time.*

If $N = 1$ or $N = 2$, then the system is not even accessible, not small time locally controllable and there exist initial conditions which cannot be transferred to a desired equilibrium.

The proofs[1,19] of the first two results depend on showing that certain Lie algebraic conditions are satisfied if $N \geq 3$. The third result is proved[1,19] constructively.

It should be emphasized that the subsequent development is assumed to be carried out for multibody systems consisting of three or more links ($N \geq 3$) Note that the reorientation or attitude control problem generally has many solutions. In this paper, we describe one solution approach, outline the theory behind it, and present some data from simulations. The key observation is that there is nonlinear coupling between changes in the shape of the structure and the rotational motion of the structure as a whole; this coupling is used to achieve reorientation of the structure.

Consider equation (3). Assume that joint angles are controlled in such a way that $\psi(t)$, $0 \leq t_1 \leq t \leq t_2$, describes a closed path γ in the shape space. Integrating both sides of equation (3) from $t = t_1$ to $t = t_2$ and using the fact that $d\psi = \dot{\psi}dt$, we obtain

$$\theta_1(t_2) - \theta_1(t_1) = \oint_\gamma s'(\psi)d\psi \ . \tag{11}$$

Thus by proper selection of a path γ in shape space, any desired geometric phase

(which is a rotation of link 1) can be obtained. By the nonintegrability property mentioned previously, the above integral is in fact path dependent thereby guaranteeing the existence of (many) such paths.

Note that in differential geometry the quantity

$$\alpha(\gamma) = \oint_\gamma s'(\psi)d\psi$$

is referred to as the geometric phase (or holonomy) of the closed path γ. This quantity depends only on the geometry of the closed path and is independent of the speed at which the path is traversed.

Note that Stokes' formula can be applied to obtain an equivalent formula for $\alpha(\gamma)$ as a surface integral. For simplicity, assume that $N = 3$, i.e. the shape space is the (ψ_1, ψ_2) plane. Also, let γ be traversed counterclockwise. Then by Stokes' theorem the above formula can be written as

$$\alpha(\gamma) = \int_S \left(\frac{\partial s_2}{\partial \psi_1} - \frac{\partial s_1}{\partial \psi_2}\right)d\psi_1 d\psi_2$$

where S is the surface within the boundary γ. In the case that the path is traversed clockwise, the surface integral is equal to $-\alpha(\gamma)$.

More information concerning geometric phases can be found in the literature[15]. Geometric phase ideas have proved useful in a variety of inherently nonlinear control problems[19-21]. These ideas have also been used for a class of path planning problems based solely on kinematic relations[13,14,16].

We now describe a control strategy, using the above geometric phase relation (11), which solves the reorientation problem.

Let $(\theta_1^e, \psi^e, 0)$ denote the desired equilibrium solution. We refer to (θ_1^e, ψ^e) and ψ^e as the desired equilibrium configuration and the desired equilibrium shape, respectively. We describe four steps involved in construction of an open loop control function $u_{[0,t_f)} = (u_1, \cdots, u_{N-1})'$ which transfers any initial state $(\theta_1^0, \psi^0, \omega^0)$ to

$(\theta_1^e, \psi^e, 0)$ in time exactly t_f, where $t_f > 0$ is arbitrary.

Let $0 < t_1 < t_2 < t_3 < t_f$ denote an arbitrary partition of the time interval $[0, t_f)$.

Step 1 : *Transfer the system to the desired equilibrium shape, i.e. find a control which transfers the initial state $(\theta_1^0, \psi^0, \omega^0)$ to $(\theta_1^1, \psi^e, 0)$ at time t_1, for some θ_1^1.*

Since the dynamics on the shape phase space are so simple, namely decoupled double integrators, Step 1 has many solutions which are easily obtained using classical methods. One such control function is

$$u_{[0,t_1)} = \begin{cases} -\frac{\pi\omega^0}{t_1}\cos\left(\frac{\pi t}{t_1}\right) & t \in [0, 0.5t_1) \\ \frac{8\pi(\psi^e - \psi^0 - \omega^0 t_1(0.5 - \pi^{-1}))}{t_1^2}\sin\left(\frac{2\pi(2t - t_1)}{t_1}\right) & t \in [0.5t_1, t_1) \, . \end{cases} \tag{12}$$

Next, we select a closed path γ (or a series of closed paths - see Remark 1 below) in the shape space which achieves the desired geometric phase. There are many ways to accomplish such a construction; in our work we have found it convenient to use only two joint motions, keeping the other joints locked, and to use a square path in the restricted two dimensional shape space. It is convenient to select the center of the square path in a region of the shape space which corresponds to a "large" geometric phase change (see Remark 2 below).

To make the above ideas more concrete, we present a specific construction. Let $(i,j) \in I^2$, $i \neq j$, denote a pair of joints. Assume that for $t \geq t_1$ only this pair of joints are actuated while all the other joints are kept fixed. This is equivalent to locking all the joints except the ones labelled i and j and treating the N bodies as three interconnected bodies, for $t \geq t_1$. In this case the desired geometric phase formula can be written as

$$\theta_1(t_f) - \theta_1^1 = \pm\alpha(\gamma)$$

where $+(-)$ corresponds to counterclockwise (clockwise) traversal of the closed path γ. Since we desire to make $\theta_1(t_f) = \theta_1^e$, the closed path γ should be selected to

satisfy

$$\theta_1^e - \theta_1^1 = \pm\alpha(\gamma) \ .$$

The path γ lies in the two dimensional (ψ_i, ψ_j) plane, so that

$$\alpha(\gamma) = \oint_\gamma \tilde{s}_i(\psi_i, \psi_j)d\psi_i + \tilde{s}_j(\psi_i, \psi_j)d\psi_j$$

where the scalar functions $\tilde{s}_i(\psi_i, \psi_j)$ and $\tilde{s}_j(\psi_i, \psi_j)$ are obtained by evaluating $s_i(\psi)$ and $s_j(\psi)$ at $\psi_k = \psi_k^e, \quad \forall k \in I$ where $k \neq i, j$.

As mentioned above we choose γ to be a square path in the (ψ_i, ψ_j) plane which is centered at the shape defined by ψ^* and which has side of length z^*, where z^* satisfies

$$\pm\alpha(\gamma_{z^*}) + \theta_1^1 - \theta_1^e = 0 \ .$$

Here γ_z indicates the dependence of the square path on the size parameter z. In most cases, this equation is easily solved using standard numerical procedures.

Remark 1 : Note that here, for notational simplicity in presenting the main idea, we assume that the desired geometric phase can be obtained by a single closed path. In general, more than one closed path may be required to produce the desired geometric phase; for such cases γ can be viewed as a concatenation of a series of closed paths. In any event, the motion along such a closed path defines a periodic motion corresponding to a change in the shape of the structure.

Remark 2 : Selection of the center point ψ^* of the path is rather arbitrary, e.g. one selection is $\psi^* = \psi^e$. However, other choices may provide a greater change in the geometric phase for a given size path. In this regard, the use of Stoke's theorem, as indicated previously, suggests that ψ^* should be chosen where

$$|\frac{\partial s_j(\psi)}{\partial \psi_i} - \frac{\partial s_i(\psi)}{\partial \psi_j}|$$

is a maximum.

We now describe the remaining three steps as follows.

Step 2 : *Transfer the system from state* $(\theta_1^1, \psi^e, 0)$ *to a state corresponding to the corner of γ closest to ψ^e, along an arbitrary path in the shape space, in $t_2 - t_1$ units of time.*

As an example, if p_1^* is the corner of γ closest to ψ^e we propose the following control function for Step 2.

$$u_{[t_1,t_2)} = \frac{2\pi(p_1^* - \psi^e)}{(t_2 - t_1)^2} \sin(\frac{2\pi(t - t_1)}{(t_2 - t_1)}) \ . \tag{13}$$

Step 3 : *Traverse the selected square path (counterclockwise or clockwise, depending on the sign of the desired geometric phase value), in $t_3 - t_2$ units of time; the resulting change in the angle θ_1 is necessarily $\theta_1^e - \theta_1^1$.*

Without loss of generality, we assume that the desired geometric phase value is obtained by counterclockwise traversal of the closed path starting and ending at p_1^*. Then, the following control functions guarantee traversal of the closed path, thereby accomplishing Step 3.

$$u_{[t_2,t_2+h)} = \frac{2\pi(p_2^* - p_1^*)}{h^2} \sin(\frac{2\pi(t - t_2)}{h}) \ , \tag{14}$$

$$u_{[t_2+h,t_2+2h)} = \frac{2\pi(p_3^* - p_2^*)}{h^2} \sin(\frac{2\pi(t - t_2 - h)}{h}) \ , \tag{15}$$

$$u_{[t_2+2h,t_2+3h)} = \frac{2\pi(p_4^* - p_3^*)}{h^2} \sin(\frac{2\pi(t - t_2 - 2h)}{h}) \ , \tag{16}$$

$$u_{[t_2+3h,t_3)} = \frac{2\pi(p_1^* - p_4^*)}{h^2} \sin(\frac{2\pi(t - t_2 - 3h)}{h}) \tag{17}$$

where $h = (t_3 - t_2)/4$.

Step 4 : *Transfer the system back to the desired equilibrium shape ψ^e following the path used in Step 2, in $t_f - t_3$ units of time; thereby guaranteeing that the desired final state $(\theta_1^e, \psi^e, 0)$ is reached at time t_f.*

The following control function

$$u_{[t_3,t_f)} = \frac{2\pi(\psi^e - p_1^*)}{(t_f - t_3)^2} \sin\left(\frac{2\pi(t - t_3)}{(t_f - t_3)}\right)$$ (18)

accomplishes Step 4.

The corresponding control torque τ can be computed using equation (6). It is clear that the constructed control torque transfers the initial condition of the system (1)-(2) to the desired equilibrium configuration at time t_f. It is important to emphasize that the above construction is based on a priori selection of a square as the closed path in the shape space. Selection of square paths simplifies computation of the controls; however other path selections, e.g. corresponding to sinusoidal changes in the shape of the structure, could be made. There are infinitely many choices for control functions which accomplish the above four steps, and the total time required is arbitrary.

4. Conclusions

In this paper we have developed an attitude control strategy for planar rigid bodies interconnected by ideal pin joints in the form of an open kinematic chain. The control strategy utilizes the nonintegrability of the expression for angular momentum. We have demonstrated that large angle maneuvers can be designed to achieve an arbitrary reorientation of the multibody system with respect to an inertial frame; the maneuvers are performed using internal controls, e.g. servo torque motors located at the joints of the body segments. The theoretical background for carrying out the required maneuvers has been briefly summarized. We mention two nontrivial extensions of the approach in this paper which are currently being developed. The first extension is to non-planar reorientation maneuvers of multibody systems consisting of rigid and flexible links; in this case the dynamics issues are much more complicated but in principle the approach is viable[22]. Another extension is the development of feedback implementations of the controls presented in this paper; some results have been obtained[19] using a (necessarily)

discontinuous feedback strategy. These important extensions generally require the use of differential geometric methods for a complete treatment.

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